Organized Information Transmission

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Introduction

Information is often transmitted in horizontal or vertical ways.

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- Transmitting information to many people at once, instead of to each of them individually, minimizes the number of communication channels.

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- Examples: hierarchical communication in organizations, viral marketing, etc.
- Delegating information transmission to the receivers themselves can be cost-saving.

Contribution

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Focus on two classes of (indirect) information structures as natural "proofs of concept";

Characterize the outcomes they implement in general finite games;

$\Rightarrow$ Organizational perspective on incomplete information / constrained information design.

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Introduce a general taxonomy of information structures which speaks to the horizontal and vertical dimensions of real-world transmission;

Focus on two classes of (indirect) information structures as natural "proofs of concept";

Characterize the outcomes they implement in general finite games;

$\Rightarrow$ Organizational perspective on incomplete information / constrained information design.

Show their optimality in binary-action environments with complementarities.

Contribution

In comparison, direct information structures, which invoke the Revelation Principle (Myerson (1991)) and make incentive-compatible action recommendations, do not constrain information horizontally or vertically.

This can make them very difficult to implement in reality.

Contribution

"The Revelation Principle in mechanism design is both a blessing and a curse [...] It is a curse because direct mechanisms provide such an unrealistic picture of decision-making in organizations."

Van Zandt (2007)

Related Literature

- **Information Design**
  - **BAS**: Arieli and Babichenko (2019), Candogan and Drakopoulos (2020), Candogan (2020)
  - **Adversarial selection**: Morris, Oyama and Takahashi (2020), Li, Song and Zhao (2019), Inostroza and Pavan (2020), Mathevet, Perego and Taneva (2020)
- **Solution Concepts**

  Forges (1993, 2006), Bergemann and Morris (2016)
- **Communication in Organizations**

  Radner (1993), Van Zandt (1999), Rantakari (2008), Alonso, Dessein, and Matouschek (2008), Hori (2006), Dessein (2002), Crémer, Garicano, and Prat (2007)

Preliminaries

Model & Basic Definitions

- Set of players: $\mathcal{I} = \{1, \ldots, n\}$.
- Uncertain state: $\omega \in \Omega$ (finite).
- Prior: $\mu \in \Delta(\Omega)$.
- Payoffs: $u_i : A \times \Omega \to \mathbb{R}$.

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- Payoffs: $u_i : A \times \Omega \to \mathbb{R}$.

  Optimality results : assume
  $A_i = \{0, 1\}$ and $u_i = 0$ if $a_i = 0$.

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Sms Dh

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- An outcome distribution $p \in BCE(\mu)$ if [Bergemann and Morris 2016]

  1. $p(A \times \{\omega\}) = \mu(\omega)$ for all ω
  2. $\sum_{\omega} \sum_{a_{-i}} p(a, \omega) \left(u_i(a; \omega) - u_i(a_i', a_{-i}; \omega)\right) \geqslant 0$ for all i, a_i, a_i'.

Sms Dh

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- Information structure (S, P): $S = \prod_i S_i$ (finite) and $P = \{P(\cdot|\omega)\}_{\omega \in \Omega}$.
- p is implemented by (S, P) if there is a (pure strategy) BNE a^* s.t.

$$p(a, \omega) = \sum_{s \in S} \mu(\omega) P(\{s : \ a^*(s) = a\}|\omega) \quad \forall a, \omega.$$

Informedness

Let $\mu_i(\cdot|s_i)$ denote i's belief about (ω, s_{-i}).

- i is weakly more informed than j at s, denoted $i \succeq^s_{\text{Inf}} j$, if

$$\mu_i(\omega, s'_{-i}|s_i, s_j) = \mu_i(\omega, s'_{-i}|s_i)$$

for all $\omega \in \Omega$ and $s'_{-i} \in S_{-i}$.

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for all $\omega \in \Omega$ and $s'_{-i} \in S_{-i}$.

- i and j are equally informed at s, denoted $i =^s_{\text{Inf}} j$, if $i \succeq^s_{\text{Inf}} j$ and $j \succeq^s_{\text{Inf}} i$.

Horizontal vs. Vertical Transmission

Horizontal transmission to i and j at s if $i =^s_{\text{Inf}} j$.

Vertical transmission from i to j at s if $i \succeq^s_{\text{Inf}} j$ and i satisfies communication incentives.

General taxonomy (Organized information structures): All information structures can be classified according to the extent to which they allow horizontal and vertical transmission.

We will focus on two special classes of organized information structures:

- single-meeting schemes – illustrating horizontal transmission
- delegated hierarchies – illustrating vertical transmission

Characterization:

Single-Meeting Schemes

Definition

In principle **(1)** the content of a meeting is common knowledge among the participants, **(2)** who also know the non-participants are less informed.

Definition

An information structure (S, P) is a single-meeting scheme if there exist a collection $\{M(s) \subseteq \mathcal{I} : s \in S \text{ s.t. } P(s) > 0\}$ and at most one $\tilde{s}_i \in S_i$ for each i such that:

(1) $i \in M(s)$ implies $i \succsim^s j$ for all $j \in \mathcal{I}$ and

(2) $i \notin M(s)$ implies $s_i = \tilde{s}_i$.

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- Non-participation may carry different information for different players: $\mu_i(\cdot|\tilde{s}_i) \neq \mu_j(\cdot|\tilde{s}_j)$ possible.
- Many possible meetings ex-ante: $\{M(s) \subseteq \mathcal{I} : s \text{ s.t. } P(s) > 0\}$ but only one is ever realized.

Characterization

What strategic outcomes can emerge in a game where incomplete information is described by single-meeting schemes?

Information design: use as constraints in linear programming.

Characterization

Theorem 1

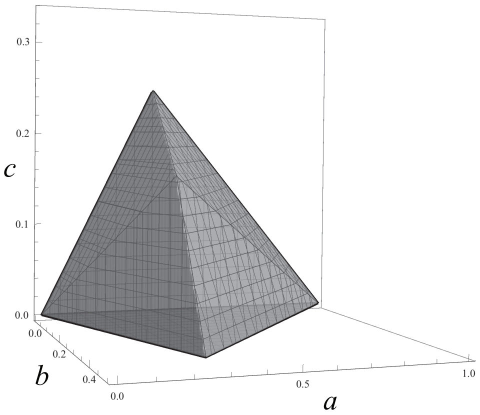
A distribution $p \in BCE(\mu)$ can be implemented by a single-meeting scheme, if and only if, for all $i \in \mathcal{I}$, there is $\tilde{a}_i \in A_i$ such that for all $a_i \in A_i \backslash \{\tilde{a}_i\}$

$$\sum_{\omega \in \Omega} p(a_i, a_{-i}, \omega)\left(u_i(a_i, a_{-i}; \omega) - u_i(a_i', a_{-i}; \omega)\right) \geqslant 0$$

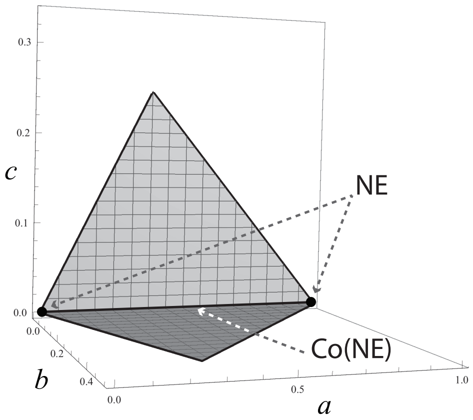
for all $a_i' \in A_i$ and $a_{-i} \in A_{-i}$.

bce

Characterization BoS

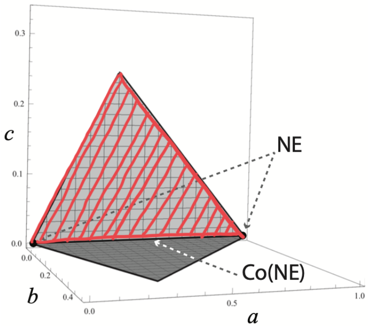


(a) Correlated Equilibria



(b) SMS

Characterization BoS

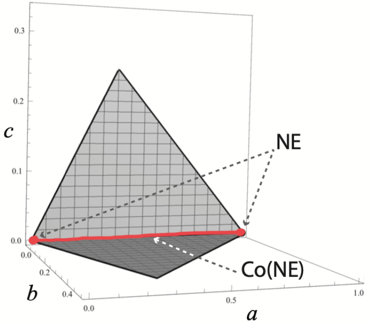


(b) SMS

	0	1
0	a	0
1	c	d

$\tilde{a}_1 = 1, \tilde{a}_2 = 0$

Characterization BoS



(b) SMS

	0	1
0	a	0
1	0	d

$\tilde{a}_1 = 0, \tilde{a}_2 = 0$ (or $\tilde{a}_1 = 1, \tilde{a}_2 = 1$)

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Characterization:

Delegated Hierarchies

Definition

Two ingredients:

1. To *be able* to vertically transfer information to one another, players must be ordered w.r.t. how informed they are.

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Two ingredients:

1. To *be able* to vertically transfer information to one another, players must be ordered w.r.t. how informed they are.
   - Information hierarchy: there exists a total order $\geq$ on $\mathcal{I}$ such that for all s with $P(s) > 0$, $i \succsim^s j$ is equivalent to $i \geq j$.

2. To be *willing* to transfer information to one another, truthful communication incentives must be satisfied.

Definition

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- Sequential cheap talk game beginning with i^*, in which each player receives information, becomes an informed sender to his immediate successor and chooses an action strategically.
- In equilibrium: no profitable deviation from truthful information transmission and from equilibrium action.

Characterization

What strategic outcomes can emerge in a game where incomplete information is described by delegated hierarchies?

Information design: use as constraints in linear programming.

Characterization

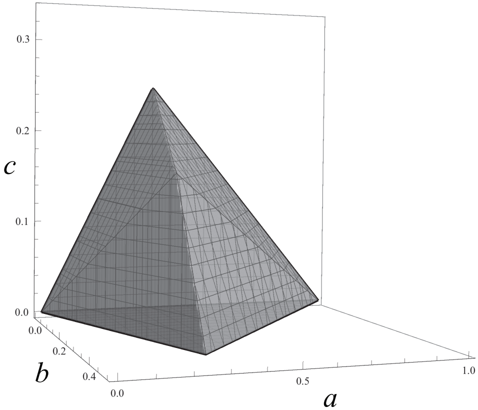
Theorem 2

A distribution $p \in \Delta(A \times \Omega)$ can be implemented by a delegated hierarchy, if and only if, $p(A \times \{\omega\}) = \mu(\omega)$ for all ω and there exists a total order $\succeq$ on $\mathcal{I}$ such that for all a_i, $a_{i\succeq}$ and $i \in \mathcal{I}$,

$$\sum_{\omega \in \Omega} \sum_{a_{i\preceq}} p(a_i, a_{i\preceq}, a_{i\succeq}, \omega)\Big(u_i(a_i, a_{i\preceq}, a_{i\succeq}; \omega) - u_i(a_i', a_{i\preceq}, a_{i\succeq}'; \omega)\Big) \geqslant 0$$

for all a_i' and $a_{i\succeq}'$ such that $p(a_{i\succeq}') > 0$.

bce



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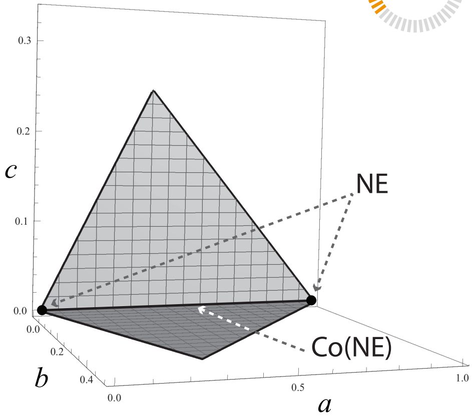

(b) SMS & DH

Optimality

Optimality

In which optimization problems are single-meeting schemes and delegated hierarchies optimal?

Optimality of Single-Meeting Schemes

$v : A \times \Omega \to \mathbb{R}$ is weakly increasing (in A) if

$$a' \geqslant a \Rightarrow v(a'; \omega) \geqslant v(a; \omega) \ \forall \omega \in \Omega.$$

Let $\mathcal{V}^{\mathrm{M}} = \{v : A \times \Omega \to \mathbb{R} : \ v \text{ is weakly increasing}\}.$

Optimality of Single-Meeting Schemes

Assumption 1 (Weak complementarities).

For all $i \in \mathcal{I}$, $u_i(1, a_{-i}; \omega)$ is weakly increasing in a_{-i} for all $\omega \in \Omega$.

Proposition 1

If $\{u_i\}$ satisfy Assumption 1 and $v \in \text{cone}\left(\mathcal{V}^{\mathrm{M}} \cup \{u_i\}\right)$, then there is $p^* \in \operatorname*{argmax}_{p \in BCE(\mu)} \mathbb{E}_p[v]$ that can be implemented by a single-meeting scheme.

Optimality of Delegated Hierarchies

$v : A \times \Omega \to \mathbb{R}$ is supermodular on $(A \times \Omega)$ if for all $a, a' \in A$ and $\omega' \geqslant \omega$,

$$v(a \vee a'; \omega') + v(a \wedge a'; \omega) \geqslant v(a; \omega) + v(a'; \omega').$$

Let $\mathcal{V}^{\mathrm{SM}} =$

$\{v : A \times \Omega \to \mathbb{R} : \ v \text{ is supermodular and weakly increasing}\}.$

Optimality of Delegated Hierarchies

Assumption 2 (Supermodularity).

For all $i \in \mathcal{I}$, u_i is supermodular on $(A \times \Omega)$.

Proposition 2

If $\{u_i\}$ satisfy Assumption 2 and $v \in \text{cone}\left(\mathcal{V}^{\text{SM}} \cup \{u_i\}\right)$, then there is $p^* \in \underset{p \in BCE(\mu)}{\text{argmax}}\, \mathbb{E}_p[v]$ that can be implemented by a delegated hierarchy.

Conclusion and Extensions

- We model the organizational structure of information (with respect to transmission).
- Focus on two classes of (indirect) information structures:
  - Characterize the associated strategic outcomes, which are subsets of (Bayes) correlated equilibria.
  - Show optimality in binary-action environments with complementarities.
- Extensions: many-(simultaneous) meetings and random delegated hierarchies.

Thank you!