Fee versus royalty licensing in a Cournot duopoly with increasing marginal costs.*

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July 15, 2021

Abstract

We consider a symmetric homogeneous Cournot duopoly operating under (linear) increasing marginal costs. One of the firms owns a patented superior technology that reduces the intercept of the marginal cost function. We compare the incentives of the insider patentee to license the thechnology to the rival firm either through a fixed fee or through a royalty. We obtain that royalty licensing is preferred if the innovation is small. Moreover, we show that our model is able to replicate the results in Wang (2002), which analyzes the same question in a differentiated duopoly with constant marginal costs.

JEL Classification: L13;L41;L42

keywords: patent licensing, royalty, fixed fee, increasing marginal costs

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*Financial support from the Spanish Ministry of Science and Innovation (Grant PID2019-107081GB-I00/AEI/10.13039/501100011033) and Generalitat Valenciana (Grant PROMETEO 2019/037) is gratefully acknowledged.

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1 Introduction

The literature on patent licensing is huge and most of the papers share the assumption that firms operate at constant marginal costs. Lately, some papers have relaxed this assumption by introducing non-constant returns in licensing models (Sen and Stamatopoulos (2009, 2016), Mukherjee (2014) and Karakitsiou and Mavrommati (2013). Mukherjee (2014) considers that firms produce with increasing linear marginal costs and deals with the case of a drastic process innovation. Considering "auction plus royalty" licensing, he obtains that the technology is licensed to all firms both for the cases of an outsider and an insider innovator. Licensing to more firms has the advantage of reducing costs and possibly the disadvantage of increasing (profit reducing) competition. However, this problem can be avoided by raising the royalty rate, which explains the result.

The most closely related paper to ours is Sen and Stamatopoulos (2009). They consider two firms that operate with linear marginal costs, but they allow for both the case of increasing and decreasing marginal costs. An external patentee has an innovation that reduces the intercept of the marginal cost and uses two-part tariff licensing contracts. The distinction between increasing and decreasing marginal costs is important because it leads to very different results in terms of diffusion of the technology and the form of the licensing contracts. With increasing marginal cost, the tecnology is licensed to both firms and the licensing contract includes a royalty. With decreasing marginal costs, however, the technology is licensed to only one firm with no royalty in the contract.

In this paper, we focus on the case of increasing linear marginal costs and the innovation also reduces its intercept. The difference is that we consider an internal patentee and restrict attention to two simple licensing policies, either a fixed fee or a royalty. We show that a royalty contract is chosen if the marginal cost is not very steep. This generalizes the result obtained by Wang (1998) for the case of constant marginal costs. However, if the marginal cost is steep enough, the patentee uses a royalty contract in equilibrium only for small enough innovations. Compared to fixed fee licensing, royalty licensing has the advantage of protecting the patentee's market profits by limiting competition. However, it has the disadvantage of introducing productive inefficiencies. Cost minimization would require that total output is split equally between the two firms but, under a royalty contract, firm 1 produces more than firm 2 in equilibrium. Under a fixed fee contract, costs are minimized, as both firms produce the same amount. It turns out that the positive effect on profits dominates the negative effect on costs only when the innovation is small.

An interesting aspect in our framework that has not been realized so far and that we pose as the main message of the present paper, is that many of the results of these new brand models can be traced back to the abundant literature of licensing models where firms sell differentiated goods. Both increasing marginal costs and product differentiation increase the incentives of a patent holder to license the new technology to a competitor. As stated in Katz and Shapiro (1985): "In practice, there are many motivations for patent licensing.

If the patent holder faces capacity constraints, or in general produces subject to increasing marginal costs, he may license his innovation to rivals to expand the scale of use of the new technology. Similarly, the innovator may have limited expertise in some markets where his discovery can reduce costs; licensing to the producer of a differentiated product is an example". What we claim is that, in the linear case, we can import some of the results obtained in the licensing literature dealing with constant marginal costs and product differentiation, to the licensing literature with increasing marginals costs and homogeneous goods.

To understand this relationship, we have to introduce some notation. Following Vives (2002), we can transform a duopoly model with linear demand and quadratic cost (linear marginal cost) into a model of product differentiation with linear demand and constant marginal cost. The former model leads to a firm's profit function:

$$(a - q_i - q_j)q_i - (cq_i + fq_i^2).$$

Manipulating the expression we have:

$$\begin{aligned}&(a - (1+f)q_i - q_j)q_i - cq_i = \\ &(1+f)\left(\frac{a}{1+f} - q_i - \frac{q_j}{1+f}\right)q_i - cq_i,\end{aligned}$$

which represents a firm's profit function in a model of product differentiation with constant marginal cost and a substitution coefficient

$$d = \frac{1}{1+f}. \tag{1}$$

Notice that parameter c stands for the intercept of the (increasing) marginal cost in the former model and for a constant marginal cost in the latter.

Wang (2002) also compares the performance of fixed fee licensing and royalty licensing in a Cournot duopoly where one of the firms owns an innovation that reduces the intercept of the marginal cost. The distinctive assumptions between our model and Wang (2002) is that he considers that: i) firms sell differentiated goods with inverse demands $P_i = a - q_i - dq_j$ and ii) firms produce with constant marginal cost. Interestingly, we show that we can replicate the results in Wang (2002) with our model, just by replacing f with $\frac{1}{d} - 1$.[1] However, in doing the comparison, we realized that there is a flaw in the proof of Proposition 3 in Wang (2002) and that, once corrected, the region where fixed fee licensing is superior to royalty licensing is enlarged.

The rest of the paper is organized as follows. In the next Section, we present the model. Section 3 studies the situation without licensing. The optimal licensing contracts with fixed fee licensing and royalty licensing are studied in Sections 4 and 5 respectively. In Section 6 we compare, from the licensor's

[1] We obtain that expression by solving for f in condition (1).

point of view, both licensing methods and derive the main results of the paper. In Section 7, we compare the performance of fixed fee licensing and royalty licensing for the case where the innovation reduces the slope of the marginal cost. Concluding remarks put the paper to an end. All the formal proofs have been relegated to an Appendix.

2 The Model

We have two Cournot duopolists (firm 1 and firm 2) that compete in a homogenous good market with inverse demand $P = a - Q$. Firm 1 owns a patented superior technology that allows to produce the good with quadratic costs $C_1(q_1) = (c-\varepsilon)q_1 + fq_1^2$ whereas firm 2 produces with costs $C_2(q_2) = cq_2 + fq_2^2$ with $a > c \geq \varepsilon > 0$ and $f \geq 0$. Firm 1 has to decide whether to license its superior technology to Firm 2. The licensing mechanisms available are fixed fee licensing and royalty licensing.

We analyze the following three stage game. In the first stage, firm 1 sets a fixed-fee or a royalty rate. In stage 2, firm 2 decides whether to accept the licensing contract. In the third stage, both firms compete à la Cournot with the cost functions inherited from the licensing stage. We will obtain the subgame perfect Nash equilibrium, solving the proposed game by backward induction.

3 The benchmark case: no licensing

Let us separate the cases of drastic and non-drastic innovations. If the innovation is non-drastic (which occurs when $\varepsilon < \varepsilon_D = (a-c)(1+2f))^2$, simple computations show that the firms' equilibrium outputs and profits are given by:

$$q_1^{NL} = \frac{(a-c+\varepsilon)(1+2f)+\varepsilon}{3+4f(2+f)}, \quad q_2^{NL} = \frac{(a-c)(1+2f)-\varepsilon}{3+4f(2+f)};$$

$$\Pi_1^{NL} = (1+f)\left(\frac{(a-c+\varepsilon)(1+2f)+\varepsilon}{3+4f(2+f)}\right)^2, \quad \Pi_2^{NL} = (1+f)\left(\frac{(a-c)(1+2f)-\varepsilon}{3+4f(2+f)}\right)^2.$$

If the innovation is drastic (which occurs when $\varepsilon \geq \varepsilon_D = (a-c)(1+2f))^3$, firms' equilibrium outputs and profits are given by:

$$q_1^{NL} = \frac{a-c+\varepsilon}{2(1+f)}, \quad q_2^{NL} = 0;$$

$$\Pi_1^{NL} = \frac{(a-c+\varepsilon)^2}{4(1+f)}, \quad \Pi_2^{NL} = 0.$$

[2] Following the patent licensing literature, we define an innovation as drastic in our setting if the monopoly price with the new technology is below (or equal to) the lowest possible marginal cost under the old technology (which would be c) so that the firm using the old technology would be driven out of the market.

[3] To allow for the possibility of drastic innovations, given that $0 < \varepsilon \leq c$, we assume that $a - c$ is small enough.

Notice that if $f = \frac{1}{d} - 1$, the above condition for an innovation to be drastic turns out to be $\varepsilon \geq \frac{(a-c)(2-d)}{d}$, which is exactly the condition for a drastic innovation in Wang (2002). From now on, we will remark the correspondence between our results and the ones in Wang (2002).

4 Fixed fee licensing

Under fixed fee licensing, the patent holder, firm 1, offers to the potential licensee, firm 2, a fixed fee contract F. If firm 2 does not accept the licensing contract, it obtains equilibrium profits Π_2^{NL} in the third stage. If firm 2 accepts the licensing contract, both firms face the same cost function $C_i(q_i) = (c-\varepsilon)q_i + fq_i^2$. In this case, equilibrium outputs and profits are given respectively by:

$$\begin{aligned} q_1^F &= q_2^F = \frac{a-c+\varepsilon}{3+2f}; \\ \Pi_1^F &= \Pi_2^F = (1+f)\left(\frac{a-c+\varepsilon}{3+2f}\right)^2. \end{aligned}$$

In the second stage, firm 2 will accept any contract F such that:

$$\Pi_2^F - F \geq \Pi_2^{NL} \tag{2}$$

In the the first stage, if firm 1 decides to license the technology, it will set the highest F that satisfies (2) i.e. $\overline{F} = \Pi_2^F - \Pi_2^{NL}$. Therefore, fixed fee licensing is profitable for firm 1 if:

$$\Pi_1^F + \overline{F} = \Pi_1^F + \Pi_2^F - \Pi_2^{NL} \geq \Pi_1^{NL}. \tag{3}$$

Notice that (3) holds as long as technology licensing increases industry profits, because the previous condition can be rewritten as:

$$\Pi_1^F + \Pi_2^F - \left(\Pi_1^{NL} + \Pi_2^{NL}\right) \geq 0. \tag{4}$$

Looking for the conditions such that 4 holds, we can write the following proposition:

Proposition 1 *Under fixed fee licensing, firm 1 will license both non-drastic and drastic innovations if* $f \geq 0.2071$*. If* $f < 0.2071$*, firm 1 will only license non-drastic innovations satisfying* $\varepsilon \leq \frac{2(a-c)(1+2f)^2}{3-4f^2}$.

The result that drastic innovations are licensed when decreasing returns are large enough may look surprising at first sight because, when the innovation is drastic, no licensing implies that firm 1 becomes a monopolist and one could think that industry profits are maximized under monopoly. This holds true under constant marginal costs[4] but it is not necesarily true under decreasing returns. It can be seen that for convex enough cost functions, industry profits in a duopolistic industry may be higher than industry profits under monopoly.[5] The intuition for the result in Proposition 1 is the following: when decreasing returns are important enough ($f \geq 0.2071$), the cost saving incentive associated to fixed fee licensing is so important that it suffices to explain its profitability, regardless of the size of the innovation. When decreasing returns are not so important however ($f < 0.2071$), the cost savings produced by fixed fee licensing have to be compared with its negative competition effect, which increases with the size of the innovation. As a result of the comparison, we get that only for small enough innovations the efficiency effect dominates, making fixed fee licensing profitable.

Notice that if $f = \frac{1}{d} - 1$, condition $f \geq 0.2071$ turns out to be $d \leq 0.8284$, which is the cut-off value for the substitution parameter in Proposition 1 in Wang (2002). And $\varepsilon < \frac{2(a-c)(1+2f)^2}{3-4f^2}$ becomes $\varepsilon < \frac{2(2-d)^2(a-c)}{8d-4-d^2}$, which coincides with condition (12) in Proposition 1 in Wang (2002). Therefore, Proposition 1 in our setting (with homogenous product and increasing marginal costs) replicates the result in Proposition 1 in Wang (2002) (in a setting with product differentiation and constant marginal costs).

5 Royalty licensing

If firm 2 accepts a royalty contract r, its cost function will be $C_2(q_2) = (c - \varepsilon + r)q_2 + fq_2^2$. Notice that firm 2 will accept any royalty contract such that $r \leq \varepsilon$. In this case, the equilibrium outputs and profits will be given respectively by:

$$q_1^R = \frac{(a-c+\varepsilon)(1+2f)+r}{3+4f(2+f)}, \quad q_2^R = \frac{(a-c+\varepsilon)(1+2f)-2(1+f)r}{3+4f(2+f)} \quad \text{and}$$

$$\Pi_1^R = (1+f)\left(\frac{(a-c+\varepsilon)(1+2f)+r}{3+4f(2+f)}\right)^2, \quad \Pi_2^R = (1+f)\left(\frac{(a-c+\varepsilon)(1+2f)-2(1+f)r}{3+4f(2+f)}\right)^2.$$

In the first stage, firm 1 will maximize its total income $\Pi_1^R + rq_2^R$ subject to $r \leq \varepsilon$. The objective function $\Pi_1^R + rq_2^R$ is strictly concave in r, with a global maximum in:

[4] Wang (1998) shows that under constant marginal costs, an insider innovator would never license a drastic innovation. It would prefer, instead, to become a monopolist in the market.

[5] For example, the equilibrium industry profits in a market where n symmetric firms compete à la Cournot with inverse demand $P = a - Q$ and costs $C(q) = (c-\varepsilon)q + fq^2$, are given by $\Pi(n) = \frac{n\,(a-c+\varepsilon)^2(1+f)}{(1+2f+n)^2}$. In this setting, $\Pi(2) - \Pi(1) \geq 0$ if $f \geq 0.2071$. (see Amir (2003).

$$r^* = \frac{(a-c+\varepsilon)(1+2f)(5+2f(5+2f))}{2(1+f)(5+8f(2+f))}. \qquad (5)$$

Therefore, the optimal royalty will be r^*, if $r^* \leq \varepsilon$. This condition is satisfied whenever:

$$\varepsilon \geq \widehat{\varepsilon}, \text{ where } \widehat{\varepsilon} = (a-c)\left(1 - \frac{2f}{5+2f(11+4f(3+f))}\right). \qquad (6)$$

When (6) is not satified, the optimal royalty is ε. It is direct to check that (6) is satisfied for drastic innovations, which implies that the optimal royalty in this case is r^*. Again, if $f = \frac{1}{d} - 1$, (5) turns out to be the optimal royalty in equation (20) in Wang (2002) and (6) coincides with condition (19) in Wang (2002).

It is very intuitive that royalty licensing is always strictly profitable for firm 1 in our setting (it always prefers royalty licensing to no licensing). If it does not license its superior technology, firm 1 obtains Π_1^{NL}. By setting $r = \varepsilon$, it obtains Π_1^{NL} plus the royalty revenues, which are positive if the innovation is non-drastic. If the innovation is drastic, one can check that $\Pi_1^R + rq_2^R$ evaluated at $r = r^*$ is higher than Π_1^{NL}, except for the case $f = 0$, in which case both coincide. Notice that if $f = 0$ (constant marginal costs) we are back to Wang (1998), where drastic innovations are not licensed.

6 Comparison: fixed fee versus royalty licensing

We have to compare firm 1's total income under fixed fee licensing and under royalty licensing. This comparison is very easy when firm 1 does not license the technology under a fixed fee policy. Proposition 1 shows that this is the case when $f < 0.2071$ and $\varepsilon > \frac{2(a-c)(1+2f)^2}{3-4f^2}$. In this case, firm 1's total income is higher under royalty licensing than under fixed fee licensing for the same reason that, in the previous section, we showed that royalty licensing is always better than no licensing.[6]

For the rest of the cases, we have to distinguish three different regions in the domain of the difference between firm 1's total income under fee licensing and under royalty licensing, because it has different functional forms in each region:

- If condition (6) does not hold, the optimal royalty is $r = \varepsilon$.

- If condition (6) holds and the innovation is non-drastic, the optimal royalty is r^*.

- If the innovation is drastic, the optimal royalty is r^*.

We devote Proposition 2 to study the case of drastic innovations and Proposition 3 to the case of non-drastric innovations both when (6) holds and when (6) does not hold.

[6] The only exception being when $f = 0$ and the innovation is drastic.

Proposition 2 *With a drastic innovation ($\varepsilon \geq \varepsilon_D$), fixed fee licensing is superior to royalty licensing for firm 1 if* $f > 0.2693$*, whereas royalty licensing dominates if* $0 \leq f \leq 0.2693$.

Again, if $f = \frac{1}{d} - 1$, condition $f > 0.2693$ turns out to be $d \leq 0.7878$ in Proposition 4 in Wang (2002). Nevertheless, discrepancies between our results and those in Wang (2002) appear in the comparison between fixed fee licensing and royalty licensing for the case of non-drastic innovations. The reason is that there is a mistake in Wang (2002) for the case in which the optimal royalty is r^* (condition (19) in Wang (2002) and condition (6) in our case). Wang (2002) claims that, in this case, royalty licensing is always superior to fixed fee licensing and we find that this is the case only for small innovations. Therefore, comparing our results with those in Wang (2002), we enlarge the region of parameters where fixed fee licensing dominates.

Proposition 3 *With a non-drastic innovation, if licensing does not occur under fixed fee licensing then royalty licensing is superior to fixed fee licensing. If licensing occurs under fixed fee licensing, two threshold values for the size of the innovation* ε_1 *and* ε_2 *exist such that:*

i) if (6) does not hold, then

i1) if $f \leq 0.3735$*, royalty licensing is superior to fixed fee licensing for firm 1.*

i2) if $f > 0.3735$*, royalty licensing is superior to fixed fee licensing for firm 1 if* $\varepsilon < \varepsilon_1$ *and fixed fee licensing dominates otherwise.*

ii) if (6) holds, then

ii1) if $f > 03735$*, fixed fee licensing is superior to royalty licensing for firm 1.*

ii2) if $0.2693 < f \leq 0.3735$*, royalty licensing is superior to fixed fee licensing for firm 1 if* $\varepsilon < \varepsilon_2$ *and fixed fee licensing dominates otherwise.*

ii3) if $f \leq 0.2693$*, royalty licensing is superior to fixed fee licensing for firm 1.*

It is interesting to remark that the results in the two previous propositions are consistent, in the sense that we could summarize both results stating that there exists a cut-off value for the size of the innovation such that below this cut-off, royalty licensing is superior to fixed fee licensing and above the cut-off, fixed fee licensing is superior to royalty licensing, whenever decreasing returns are important enough. Notice that, stated in this way, the previous sentence is true for the particular cases of drastic and non-drastic innovations. The following proposition formalizes the previous intuition, summarizing Propositions 2 and 3.

Proposition 4 *Two threshold values for the size of the innovation exist such that:*

if $f > 0.3735$*, royalty licensing is superior to fixed fee licensing for firm 1 if* $\varepsilon < \varepsilon_1$ *and fixed fee licensing dominates otherwise.*

if $0.2693 < f \leq 0.3735$*, royalty licensing is superior to fixed fee licensing for firm 1 if* $\varepsilon < \varepsilon_2$ *and fixed fee licensing dominates otherwise.*

if $f \leq 0.2693$ *royalty licensing is superior to fixed fee licensing for firm 1.*

Notice that when f is low, we are back to the result in Wang (1998) showing that under constant marginal costs, royalty licensing is superior to fixed fee licensing. However, if decreasing returns are important enough (when f is high enough), we obtain the clear-cut result that royalty licensing is superior to fixed fee licensing only for small enough innovations. Compared to fixed fee licensing, royalty licensing has the advantage of protecting firm 1's profits by limiting competition. However, it has the disadvantage of introducing productive inefficiencies. Cost minimization requires that total output is split equally between the two firms but, with royalty licensing, firm 1 ends up producing more than firm 2. With fixed fee licensing, however, costs are minimized because both firms produce the same amount. It turns out that the positive effect on profits dominates the negative effect on costs only when the innovation is small.

7 The innovation reduces the slope of the marginal cost

So far, we have studied the case in which the innovation reduces the intercept of the marginal cost. In this section, we extend the model to consider the case in which the innovation reduces not the intercept but the slope of the marginal cost function. In this new scenario, we assume that the patentee (firm 1) has total costs given by $C_1(q_1) = fq_1^2$ and firm 2 by $C_2(q_2) = dq_1^2$ where $d > f \geq 0$. Firm 1 has to decide whether to license its superior technology to firm 2. The higher the value of d, the larger the size of the innovation. Notice that if firm 1 sets the monopoly price, firm 2 would always respond with a positive output because its marginal cost is zero when its output is zero, regardless of the value of d. In other words, no innovation is drastic in this scenario, regardless of d. However, we can approximate the drastic case in the limit, when parameter d tends to infinity.

We obtain similar results as the ones obtained in the previous sections. Fixed fee licensing is preferred to royalty licensing when the marginal cost of the patentee is steep enough (high f) and /or when the size of the innovation is large enough (high d). We discuss the results in more detail below and relegate the technical details to the Appendix.

The profitability of fixed fee licensing follows the same pattern as in Proposition 1. It is always profitable if $f \geq 0.2071$ (observe that it is the same cut-off value) and it is profitable only for small innovations otherwise.

The main complication in the analysis appears when computing the optimal royalty. The licensee accepts the royalty contract if the profits it gets under royalty licensing are higher than its profits under no licensing. This condition collapses to $r \leq \varepsilon$ when the innovation reduces the intercept (it is fully determined by the technology). When the innovation affects the slope, however, the condition becomes more complicated:

$$r \leq \frac{a(1+2f)[(1+f)^2 - \frac{(1+2f)(3+2f)\sqrt{(1+d)(1+f)^3}}{3+4f+4d(1+f)}]}{2(1+f)^3}.$$

In order to be able to derive explicit results, we restrict the analysis to three particular cases: (i) the case where $f = 0$ and, hence, the patentee produces with constant marginal costs as in Wang (1998); (ii) the case where $f = 1$ and (iii) the drastic case, which is approximated when d tends to infinity.

In all three cases, royalty licensing is profitable. In Case (i) royalty licensing is always superior to fixed fee licensing, as in Wang (1998). In Case (ii), fixed fee licensing is superior to royalty licensing if $d > 5.31$. Finally, in Case (iii), fixed fee licensing is superior to royalty licensing if $f \geq 0.2693$. Observe that this is the same threshold as the one obtained in Proposition 2.

8 Conclusion

In this paper, we have taken up the classical question in the patent licensing literature about the relative performance of royalties and fixed fees, in a new setting where firms' marginal costs are increasing. We have disclosed the close relationship that exists between our model and the one in Wang (2002), where firms sell differentiated products with constant marginal costs. This might open up the possibility of importing other results from the patent licensing literature with differentiated goods and constant marginal costs to our setting with homogeneous goods and increasing marginal costs.

9 Appendix

Proof of Proposition 1

Let us find conditions such that (4) holds. Let us start with the case of a non-drastic innovation. In this case, (4) results in:

$$\frac{\varepsilon(1+f)(2(a-c)(1+2f)^2 - \varepsilon(3-4f^2))}{(3+4f(2+f))^2} \geq 0. \tag{7}$$

Simple inspection shows that (7) holds if $3 - 4f^2 \leq 0$ i.e. if $f \geq 0.8660$. Now, if $f < 0.8660$, it holds if $\varepsilon < \frac{2(a-c)(1+2f)^2}{3-4f^2}$. It is direct to see that if $f \geq 0.2071$, then $\frac{2(a-c)(1+2f)^2}{3-4f^2} \geq (a-c)(1+2f) > \varepsilon$ holds (notice that the second inequality is just the definition of the non-drastic case), which implies that all non-drastic innovations are licensed. Finally, if $f < 0.2071$, only small non-drastic innovations satisfying $\varepsilon < \frac{2(a-c)(1+2f)^2}{3-4f^2}$ are licensed.

For the case of a drastic innovation, (4) results in:

$$(a-c+\varepsilon)^2\left(\frac{2(1+f)}{(3+2f)^2}-\frac{1}{4(1+f)}\right)\geq 0.$$

It can be checked numerically that the previous condition holds if $f \geq 0.2071$.

Proof of Proposition 2

Let $T(\varepsilon)$ be the difference between firm 1's total income under fixed fee licensing and under royalty licensing when the innovation is drastic, namely, when $\varepsilon \geq (a-c)(1+2f)$. It is direct to check that $sign\{T(\varepsilon)\} = sign\{-5+4f+40f^2+48f^3+16f^4\}$. Numerical computations lead to the result.

Proof of Proposition 3

First, we prove point (i), where $\varepsilon < \widehat{\varepsilon}$. We define $P(\varepsilon) = \dfrac{\varepsilon\left(-(a-c)(1+2f)^2+\varepsilon f(5+4f(2+f)\right)}{(3+4f(2+f))^2}$ as the difference between firm 1's total income under fee licensing and under royalty licensing for $\varepsilon < \widehat{\varepsilon}$ (so that the optimal royalty rate is $r=\varepsilon$). Expression $P(\varepsilon)$ is positive iff $\varepsilon > \varepsilon_1$, with $\varepsilon_1 = \dfrac{(a-c)(1+2f)^2}{f(5+4f(2+f))}$. It can be verified numerically that if $f \leq 0.3735$, then $\varepsilon_1 \geq \widehat{\varepsilon}$ and, therefore, given that we are in the region $\varepsilon < \widehat{\varepsilon}$, we have that $\varepsilon < \varepsilon_1$ holds, so $P(\varepsilon)$ is negative and royalty licensing is superior to fixed fee licensing. This proves point (i1). Now, if $f > 0.3735$, $P(\varepsilon)$ is negative if $\varepsilon < \varepsilon_1$ and positive if $\varepsilon_1 \leq \varepsilon < \widehat{\varepsilon}$. This proves point (i2).

Next, we prove point (ii), where $\varepsilon \geq \widehat{\varepsilon}$. We define $B(\varepsilon) = \frac{2(a-c+\varepsilon)^2(1+f)}{3+2f^2} - \frac{(a-c+\varepsilon)^2(1+2f)^2(5+4f)^2}{4(1+f)(5+8f(2+f))^2} - \frac{(1+f)(c+\varepsilon+2cf-a(1+2cf))^2}{(3+4f(2+f))^2} - \frac{(a-c+\varepsilon)^2 f(1+2f)(5+2f(5+2f))}{(1+f)(5+8f(2+f))^2}$ as the difference between firm 1's total income under fixed fee licensing and under royalty licensing if $\varepsilon \geq \widehat{\varepsilon}$ (so that the optimal royalty rate is $r=r^*$). In a Lemma in the Appendix, we prove that $B'(\varepsilon) > 0$ if $f > 0.2693$. Point (ii1) follows from the Lemma (see below) and from the fact that (i2) implies $P(\widehat{\varepsilon}) > 0$ and, given that by definition $P(\widehat{\varepsilon}) = B(\widehat{\varepsilon})$, it follows that $B(\widehat{\varepsilon}) > 0$. Point (ii2) follows from the Lemma and the fact that (i1) implies $P(\widehat{\varepsilon}) < 0$, and then, $B(\widehat{\varepsilon}) < 0$ holds, and that Proposition 2 implies $T(\varepsilon_D) > 0$, and then, $B(\varepsilon_D) > 0$, because by definition $T(\varepsilon_D) = B(\varepsilon_D)$. Finally, proving point (ii3) requires a different approach, because the Lemma does not apply. For $f \leq 0.2693$, we check numerically that $B"(\varepsilon) < 0$ (so function $B(\varepsilon)$ is concave in that interval) and that $B(\varepsilon_3) < 0$, where ε_3 is such that $B'(\varepsilon_3) = 0$ (that is, ε_3 is the value where function $B(\varepsilon)$ reaches its maximum).

Lemma 1

$B'(\varepsilon) > 0$ if $f > 0.2693$ (and the innovation is non-drastic).

Proof of Lemma 1

$B'(\varepsilon) = (a-c)g(f) + e\ h(f)$, where

$g(f) = \frac{4(1+f)}{(3+2f)^2} + \frac{2(1+f)(1+2f)}{(3+4f(2+f))^2} - \frac{(1+2f)^2(5+4f)^2}{2(1+f)(5+8f(2+f))^2} - \frac{2f(1+2f)(5+2f(5+2f))}{(1+f)(5+8f(2+f))^2}$

$h(f) = \frac{4(1+f)}{(3+2f)^2} - \frac{2(1+f)}{(3+4f(2+f))^2} - \frac{(1+2f)^2(5+4f)^2}{2(1+f)(5+8f(2+f))^2} - \frac{2f(1+2f)(5+2f(5+2f))}{(1+f)(5+8f(2+f))^2}$

One can check numerically that $g(f) > 0$ for all non-negative f and that $h(f) > 0$ if $f > 0.7219$. Then $B'(\varepsilon) > 0$ if $f > 0.7219$. For $f < 0.7219$, $B'(\varepsilon) > B'((a-c)(1+2f))$ because we are considering non-drastic innovations. One can check numerically that $B'((a-c)(1+2f)) > 0$ if $f > 0.2693$.

Cut-off values

$$\varepsilon_1 = \frac{(a-c)(1+2f)^2}{f(5+4f(2+f))},$$

$$\varepsilon_2 = \frac{(a-c)(1+2f)\left(4(1+f)^2(5+8f(2+f)\right)\sqrt{\frac{(-5+4f(1+2f(5+2f(3+f))))}{5+8f(2+f)}}-(15+2f(49+2f(57+4f(16+f(9+2f))))))}{-25+8f(-15+2f(-9+f(6+f(21+4f(4+f)))))},$$

$$\varepsilon_3 = \frac{(a-c)(1+2f)(15+2f(49+2f(57+4f(16+f(9+2f)))))}{25-8f(-15+2f(-9+f(6+f(21+4f(4+f)))))}$$

Technical details of Section 7

Without licensing, we have an asymmetric Cournot model. Straighforward computations lead to the following equilibrium profits for firm 1 and firm 2 respectively:

$$\Pi_1(d,f) = \frac{(1+f)(a+2ad)^2}{(3+4f+4d+4df)^2} \text{ and } \Pi_2(d,f) = \frac{(1+d)(a+2af)^2}{(3+4f+4d+4df)^2}.$$

LIcensing by means of a fixed fee is profitable if it increases industry profits. Next Proposition derives the result:

Proposition 5 *Fixed fee licensing is always profitable if* $f \geq 0.2071$. *Otherwise, it is only profitable if* $d < -1 - \frac{2}{4(1+f)} - \frac{2}{-1+4f(1+f)}$.

Proof. Fixed fee licensing is profitable if the following expression is positive:

$$2\Pi_1(f,f) - \Pi_1(f,d) - \Pi_2(f,d) =$$

$$= \frac{a^2(d-f)(3+4d(1+f)(-1+4f(1+f))+4f(6+f(9+4f)))}{(3+2f)^2(3+4f+4d+4df)^2}.$$

If $-1+4f(1+f) \geq 0$, the previous expression is positive. It holds if $f \geq 0.2071$. If $-1+4f(1+f) < 0$, it is positive if $f < d < -1 - \frac{1}{4(1+f)} - \frac{2}{-1+4f(1+f)}$.
■

With royalty licensing, the situation is asymmetric again. Firm 1 faces cost fq_1^2 and firm 2 cost $rq_2 + fq_2^2$, where r is the royalty rate. Simple computations show that equilibrium outputs are given by:

$$q_1(r) = \frac{a+2af+r}{3+4f(2+f)} \text{ and } q_2(r) = \frac{a+2af-2(1+f)r}{3+4f(2+f)}.$$

They lead to the following equilibrium profits:

$$\Pi_1^R(r) = \frac{(1+f)(a+2af+r)^2}{[3+4f(2+f)]^2} \text{ and } \Pi_2^R(r) = \frac{(1+f)(a+2af-2(1+f)r)^2}{[3+4f(2+f)]^2}.$$

Firm 2 will accept the royalty licensing scheme whenever:

$$\Pi_2^R(r) \geq \Pi_2(f,d).$$

This condition holds if

$$r \leq \frac{a(1+2f)[(1+f)^2 - \frac{(1+2f)(3+2f)\sqrt{(1+d)(1+f)^3}}{3+4f+4d(1+f)}]}{2(1+f)^3} = \overline{r}(f,d).$$

Firm 1 will set r to maximize:

$$\begin{aligned} & \Pi_1^R(r) + rq_2(r) \\ s.t.\ r & \leq \overline{r}(f,d). \end{aligned}$$

Then, the optimal royalty is given by:

$$\begin{aligned} r^* &= \min\{\overline{r}(f,d), \widehat{r}(f)\} \\ \text{where } \widehat{r}(f) &= \frac{a(1+2f)(5+2f(5+2f))}{2(1+f)(5+8f(2+f))}. \end{aligned}$$

Lemma 6 *If $f > 0$, there exists $\overline{d}$ such that $r^* = \overline{r}(f,d)$ if $d < \overline{d}$ and $r^* = \widehat{r}(f)$ otherwise.*

Proof. $\overline{r}(f,d)$ is increasing in d. $\overline{r}(f,f) = 0 < \widehat{r}(f) < \underset{d->\infty}{Lim}\overline{r}(f,d) = \dfrac{a+2af}{2+2f}$ imply the result. ■

We have to check whether firm 1 obtains more profits under royalty licensing than under no licensing.

If $d \geq \overline{d}$, it is possible to check that royalty licensing is profitable i.e.

$$\Pi_1^R(\widehat{r}(f)) + \widehat{r}(f)q_2(\widehat{r}(f)) > \Pi_1(d,f) \tag{8}$$

However, if $d < \overline{d}$ and the optimal royalty is $\overline{r}(f,d)$, the difference in profits between royalty licensing and no licensing can not be signed in general. Therefore, we restrict the analysis to three particular cases: i) the case where $f = 0$ and the patentee produces with constant marginal cost as in Wang (1998), ii) the case where $f = 1$ and iii) the drastic innovation case, which we approximate when d tends to infinity.

Case i: When $f = 0$, $\overline{r}(0,d) < \widehat{r}(0)$. Then, we have that:

$$\Pi_1^R(\overline{r}(0,d)) + \overline{r}(0,d)q_2(\overline{r}(0,d)) - \Pi_1(d,0) = \frac{3a^2d}{4(3+4d)^2} > 0.$$

and royalty licensing is profitable.

Case ii: If $f = 1$, $\overline{d} = 14.03$. For $d \geq 14.03$, we know from the general case that royalty licensing is profitable. For $d < 14.03$, we can check numerically that :

$$\Pi_1^R(\overline{r}(1,d)) + \overline{r}(1,d)q_2(\overline{r}(1,d)) - \Pi_1(d,1) > 0,$$

implying that royalty licensing is profitable.

Case iii: When d tends to infinity and $f > 0$, we have $\overline{r}(f,d) > \widehat{r}(f)$. In this case, we know from (8) that royalty licensing is profitable.

Finally, we compare the performance of royalty licensing and fixed fee licensing. This comparison is straightforward when licensing does not take place under a fixed fee, because we have checked that royalty licensing is always profitable. For the other cases, we have to compare the revenues obtained under fixed fee licensing $2\Pi_1(f,f) - \Pi_2(d,f)$ with the ones obtained under royalty licensing $\Pi_1^R(r^*) + r^* q_2(r^*)$.

Case i: When $f = 0$, $\overline{r}(0,d) < \widehat{r}(0)$. Then to get the result we have to compute:

$$\begin{aligned} & 2\Pi_1(0,0) - \Pi_2(0,d) - \Pi_1^R(\overline{r}(0,d)) - \overline{r}(0,d)q_2(\overline{r}(0,d)) \\ = & -\frac{a^2 d(15+16d)}{36(3+4d)^2} < 0. \end{aligned}$$

Therefore, royalty licensing is always superior to fixed fee licensing as in Wang (1998).

Case ii: If $f = 1$, we know from Proposition 1 that the patentee would always find profitable to license the technology though a fixed fee. As far as royalty licensing is concerned, in this case $\overline{d} = 14.03$. Then, we have to compare the fixed fee licensing and royalty licensing in two different regions: i) when $d < 14.03$ and the royalty is $\overline{r}(1,d)$ and ii) when $d \geq 14.03$ and the royalty is $\widehat{r}(1)$. Numerical computations lead to the following results:

$$\begin{aligned} 2\Pi_1(1,1) - \Pi_2(1,d) - \Pi_1^R(\overline{r}(1,d)) - \overline{r}(1,d)q_2(\overline{r}(1,d)) &> 0 \\ if\ d &> 5.31. \end{aligned}$$

and that

$$\begin{aligned} 2\Pi_1(1,1) - \Pi_2(1,d) - \Pi_1^R(\widehat{r}(1)) - \widehat{r}(1)q_2(\widehat{r}(1)) &> 0 \\ if\ d &> 14.03. \end{aligned}$$

This implies that fixed fee licensing is more profitable than royalty licensing for the patentee if $d > 5.31$. This result corresponds to the one obtained in Proposition 4, where royalties are used when the innovation is small.

Case iii: When d tends to infinity and $f > 0$, we have $\overline{r}(f,d) > \widehat{r}(f)$. Then, we have to compute (given that $\Pi_2(f,d)$ tends to zero):

$$\begin{aligned} & 2\Pi_1(f,f) - \Pi_1^R(\widehat{r}(f)) - \widehat{r}(f)q_2(\widehat{r}(f)) \\ = & \frac{a^2(-5+4f+40f^2+48f^3+16f^4)}{4(1+f)(3+2f)^2(5+16f+8f^2)}. \end{aligned}$$

The previous expression is positive if $f \geq 0.269308$ and, therefore, we conclude that fixed fee licensing is more profitable than royalty licensing in this case.

10 References

Amir, R. (2003), "Market Structure, Scale Economies and Industry Performance", CORE. Discussion Paper No 2003/65.

Karakitsiou, A. and A. Mavrommati (2013), "Ad-valorem and Royalty Licensing under Decreasing Returns to Scale", Asian Journal of Business and Management 1 (1), 14-20.

Katz, M. L. and C. Shapiro (1985), "On the licensing of innovations", RAND Journal of Economics 16 (4), 504-520.

Mukherjee, A. (2014), "Licensing under convex costs", Journal of Economics 111, 289-299.

Sen, D. and G. Stamatopoulos (2009), "Technology transfer under returns to scale", The Manchester School 77 (3), 337-365.

Sen, D. and G. Stamatopoulos (2016), "Licensing under general demand and cost functions", European Journal of Operational Research, 253, 673-680.

Vives, X. (2002), "Private information, strategic behaviour and efficiency in Cournot markets", RAND Journal of Economics 33 (3), 361-376.

Wang, X.H. (1998), "Fee versus royalty licensing in a Cournot duopoly model", Economics Letters 60, 55-62.

Wang, X.H. (2002), "Fee versus royalty licensing in a differentiated Cournot duopoly", Journal of Economics and Business 54, 253-266.