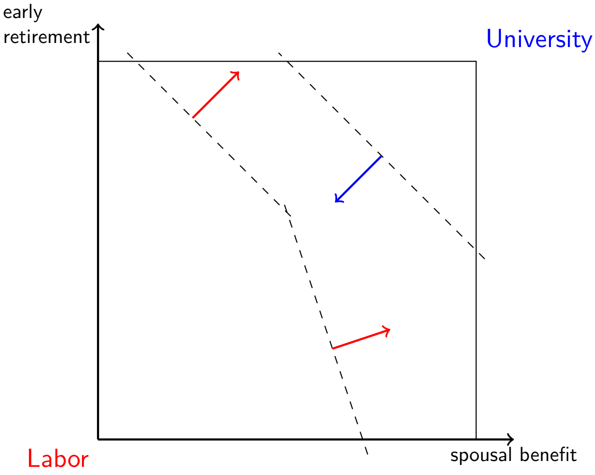
Bargaining with Incomplete Information

Marcin Pęski

University of Toronto

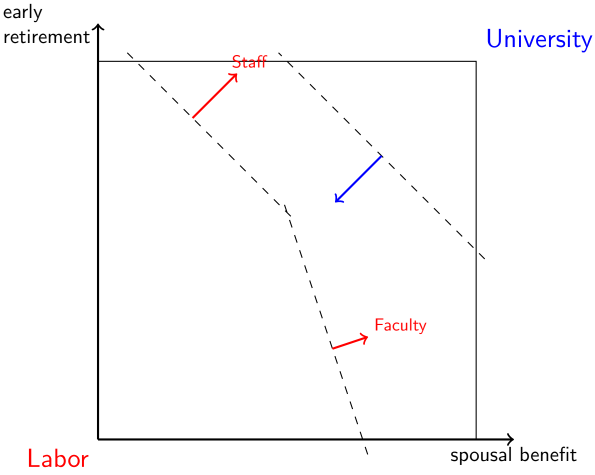
July 22, 2020

Introduction
UofT pension bargaining



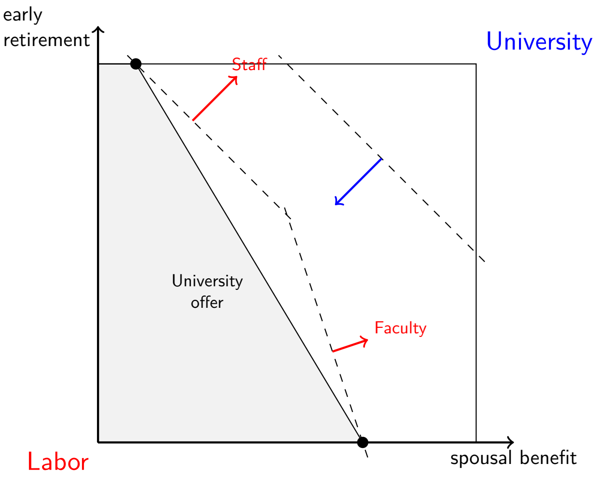
Introduction

UofT pension bargaining



Introduction

UofT pension bargaining



Introduction

Alternating-offer bargaining over heterogeneous pie,

- one-sided incomplete information about preferences,
- mechanisms as offers.

Introduction

- Mechanisms as offers:
  - menus,
  - menus of menus,
  - "I divide and you choose" vs "you divide and I choose",
  - arbitration and general mechanisms,
  - negotiations to create or alter the bargaining protocol,

Literature

- Complete information about preferences:
  - axiomatic: Nash (50, 53)
  - alternating-offer Rubinstein (82)
  - reputational: Myerson (91), Kambe (99), Abreu and Gul (00), Compte and Jehiel (02), Fanning (16)
  - *all solutions the same -> Nash program success!*
- Incomplete information:
  - axiomatic (mechanisms): Harsanyi and Selten (72), Myerson (84)
  - Coasian-bargaining with menus (2 types only): Wang (98), Strulovici (17)
  - alternating-offer with menus (2 types only + refinements): Sen (00), Inderst (03)
  - common knowledge of surplus: Jackson et al (18).
- Dynamic mechanism design without commitment: Skreta (06), Liu et al (19), Doval, Skreta (18).
- Informed principal (...)
  - dynamic informed principal?

Introduction

- **Main result**: When $N = 2$, there is a unique PBE: Bob chooses optimal screening menu st. each Alice type receives complete info. payoff
  - no refinements needed,
  - ex ante, but not ex post efficient
  - constrained commitment solution, non-Coasian result,
  - equilibrium bounds when $N \geq 3$.
- Role of mechanisms:
  - menus help with screening and signaling (inscrutability),
  - menus of menus help with belief punishment,
  - no other mechanisms needed.

Model

Environment

- Alice (informed) and Bob (uninformed).
- Pie $X = \left\{ x \in \left([0,1]^N \right)^2 : \sum_i x_{i,n} \leq 1 \text{ for each } n \right\}$.
  - mostly, $N = 2$.
- Linear preferences $\mathcal{U} := \left\{ u \in \mathbb{R}_+^N : \sum u_n = 1 \right\}$
  - linear utilities $u \in \mathcal{U}$ from $x \in X$: $u(x) = \sum_n u_i x_{i,n}$,
  - Bob's preferences v,
  - Bob's beliefs $\mu \in \Delta\mathcal{U}$ about Alice's preferences u.
- Discounting $\delta < 1$.
- Alternating-offer bargaining with mechanisms as offers

Model

Mechanisms as offers

- Each offer is a *mechanism*: a finite-horizon extensive-form game.
  - $m = \left(\left(S_A^t, S_B^t \right)_{t \leq T}, \chi \right)$
  - allocation: $\chi : \prod_{i,t} S_{i,t} \to X$,
  - $T < \infty$ and S_i^t compact.
- Examples: single-offers, menu, menu of menus:
- $\mathcal{M}$ - "compact" space of all available mechanisms
  - main result hold as long as $\mathcal{M}$ contains menus and menus of menus.

- Perfect Bayesian Equilibrium,
  - existence is an issue.
- (Payoff) outcomes:

$$e_B \in [0,1], e_A : \mathcal{U} \to [0,1].$$

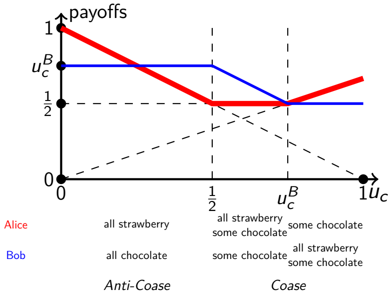
- Limit set of equilibrium outcomes $E^j(\delta, \mu)$:

$$E^j(\mu) = \lim_{\delta \to 1} E^j(\delta, \mu)$$

- Coasian bargaining and dynamic mechanism design without commitment: Doval, Skreta (18), Liu et al (19)
- As in that literature,
  - players cannot *unilaterally* commit to future offers,
  - players are committed to an offer for the period in which the offer is made.
- But, players have also access to a large(-r) space of mechanisms,
  - including mechanism, which offered and accepted *bilaterally*, may commit players to an ex post inefficient allocation.

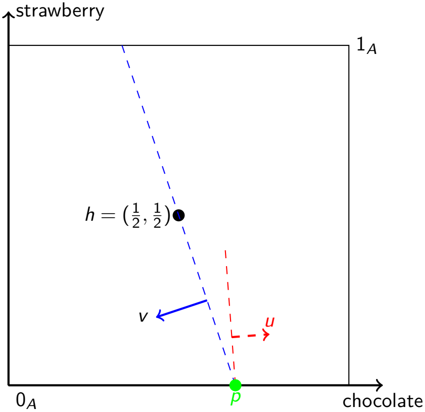
Complete information

- Complete information bargaining: Alice u, and Bob v (fixed).
- Assume
  - $N = 2$ (chocolate, strawberry)
  - assume $v_c > v_s$ (Bob likes chocolate more).
- As $\delta \to 1$, Alice's payoffs converge to the Nash solution: $(\mathcal{N}_A(u), \mathcal{N}_B(u))$.



Complete information

Nash allocations I

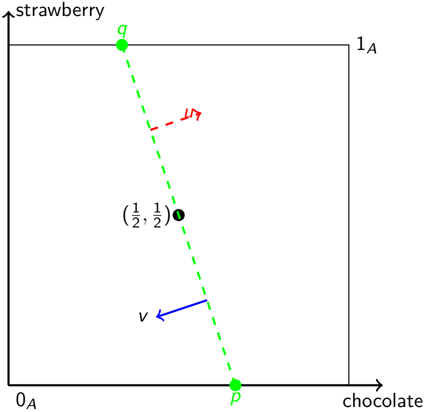


Nash allocations:

- p if $u_c > v_c$, i.e., if Alice likes chocolate more than Bob.

Complete information

Nash allocations II

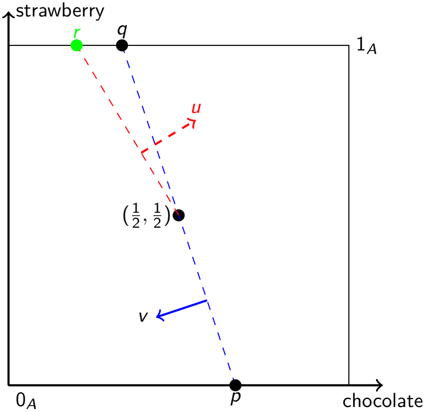


Nash allocations:

- p if $u_c > v_c$,
- $\overline{pq}$ if $u_c = v_c$

Complete information

Nash allocations III

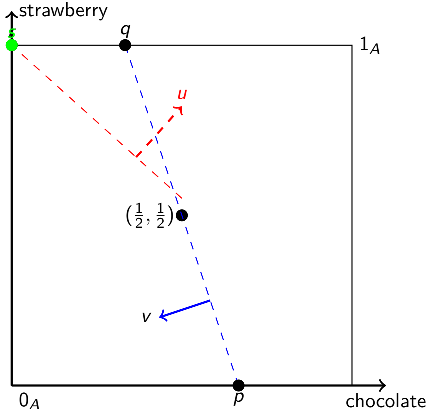


Nash allocations:

- p if $u_c > v_c$,
- $\overline{pq}$ if $u_c = v_c$,
- r if $\frac{1}{2} < u_c < v_c$,

Complete information

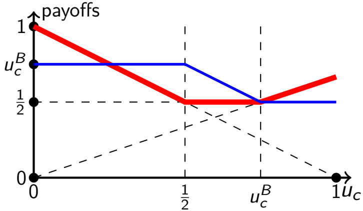
Nash allocations IV



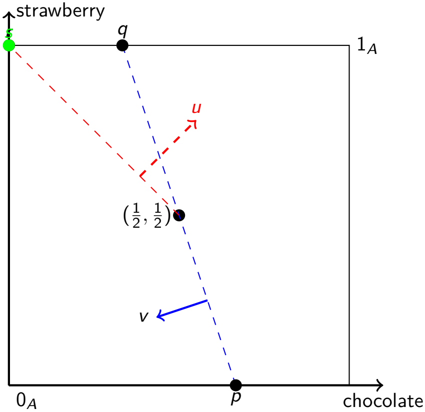
Nash allocations:

- p if $u_c > v_c$,
- $\overline{pq}$ if $u_c = v_c$,
- r if $\frac{1}{2} < u_c < v_c$,
- s if $u_c < \frac{1}{2}$ (i.e., Alice likes strawberry more)

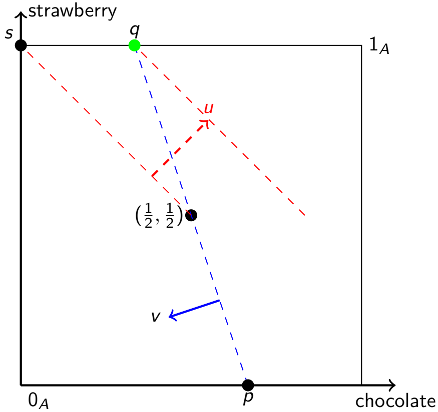
- Nash payoffs:



Alice	all strawberry	all strawberry some chocolate	some chocolate
Bob	all chocolate	some chocolate	all strawberry some chocolate
	Anti-Coase		*Coase*



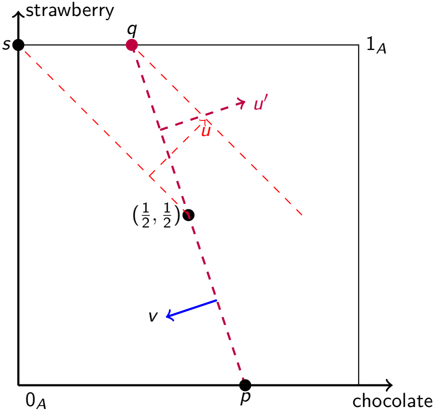
Incentive problem.



Incentive problem.

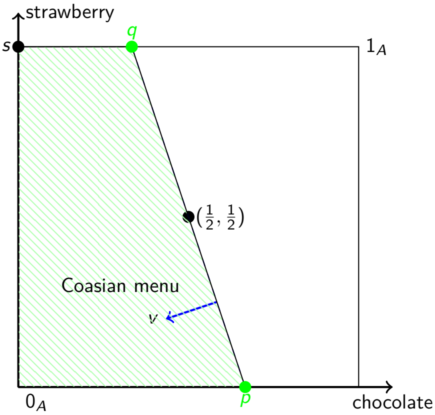
Complete information

Incentive problem III



Incentive problem.

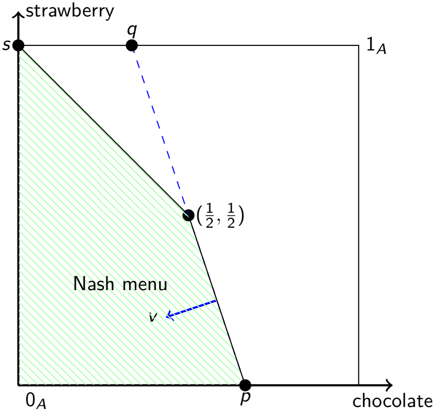
- types $u_c < v_c$ prefer to report $u'_c \approx v_c$



- If we ignore incentive problem, Alice chooses either p or q
- Coasian menu $\{p, q\}$.
- A companion paper studies the same environment,
  - bargaining with reputational types like in Abreu-Gul (00) and Kambe (98)
  - Coasian menu is the unique equilibium outcome.

Complete information

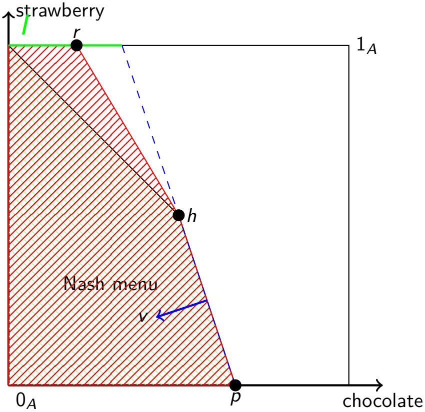
Nash menu



- If we want to ensure that each type of Alice receives her complete information payoff, we can offer Nash menu $\{s, h, p\}$.

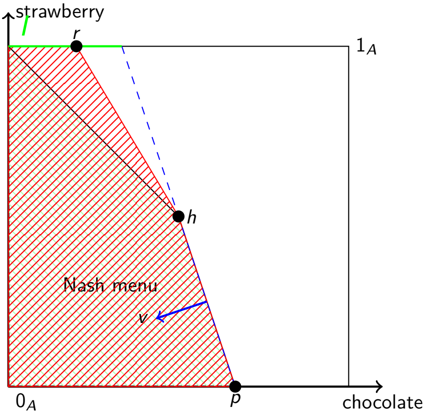
Main result

Main result



- Class of menus
  $m_r = \{p, h, r\}$ for $r \in I$

Main result

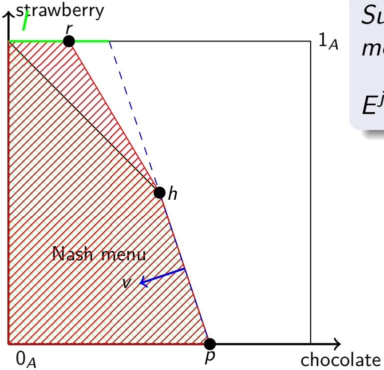


- Alice's payoff:
  $e_A(u; r) = \max_{x \in m_r} u(x)$
- Bob's payoff:

$$e_B(\mu; r)$$
$$= (v_c(1 - r_c))\, \mu\left(u : u_c r_c + u_s \geq \frac{1}{2}\right)$$
$$+ \frac{1}{2}\left(1 - \mu\left(u : u_c r_c + u_s \geq \frac{1}{2}\right)\right).$$

- optimal menus
  $R^*(\mu) = \arg\max_{r \in I} e_B(\mu; r)$.

Main result



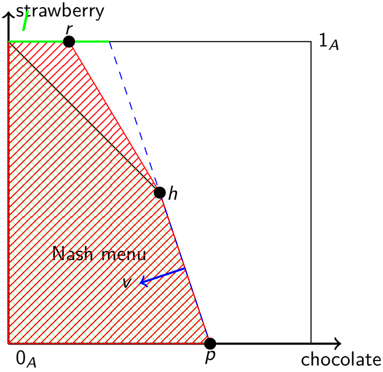
Theorem

Suppose $N = 2$ and $\mathcal{M}$ contains all menus and menus of menus. Then,

$$E^j(\mu) = \{e_A(u; r), e_B(\mu; r) : r \in R^*(\mu)\}.$$

- Bob offers an optimal screening menu.

Main result

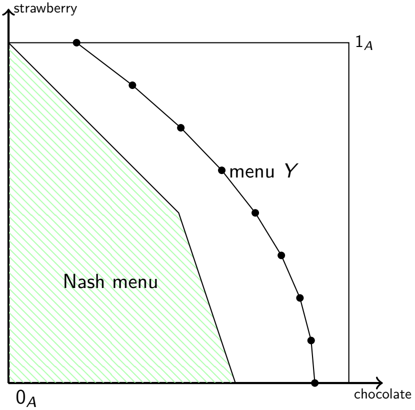


- Bob's payoff is unique and continuous in μ.
  Alice's payoff is "generically" unique.
- Constrained "commitment".
  - not a Coasian menu,
  - not a reputational result.
- Not ex post efficient.

- Suppose that Alice's type u is known.
- Let $\Pi(y) = \max_{x:u(x)\geq y} v(x)$ be Bob's payoff.
- Payoff y is too high for an equilibrium if Alice is not resistant to Bob's deviation:
  - Bob rejects, waits for one period and makes a counter-offer,
  - there exists $y' \geq \delta y$ such that $\delta\Pi(y') > \Pi(y)$.
- Let h be the highest

- Payoffs in menu $Y \subseteq X$:
  - Alice: $y(u;Y) = \max_{x \in Y} u(x)$,
  - Bob (ex post): $\pi(u;Y) = \max_{x \in x(u;Y)} v(x)$, where $x(u;Y) = \arg\max_{x \in Y} u(x)$
  - Bob's expected: $\Pi(\mu;Y) = \int \pi(u;Y)\, d\mu(u)$.
- Observation: if $(e_A, e_B) \in E^j(\delta, \mu)$ are equilibrium payoffs (or payoffs in any IC mechanism), then there is a menu Y such that $e_A = y(.;Y)$ and $e_B \leq \Pi(\mu;Y)$.

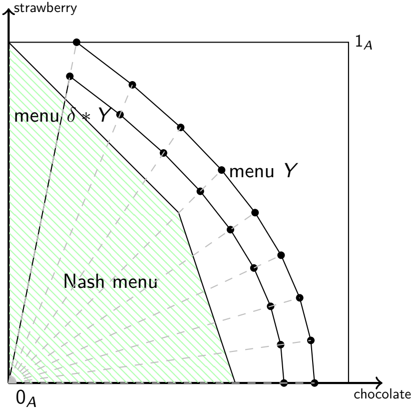
Proof
Upper bound



- Menu Y is too high for Alice if

Proof

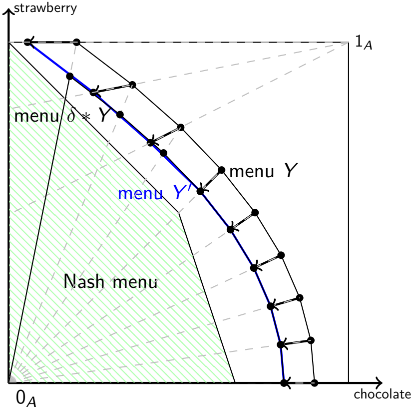
Upper bound



- Menu Y is too high for Alice
  if
  there exists $Y' \supset \delta Y$ s.t.
  $\delta \Pi(Y', \mu) > \Pi(Y, \mu)$.

Proof

Upper bound



- Menu Y is too high for Alice
  if
  there exists $Y' \supset \delta Y$ s.t.
  $\delta\Pi(Y', \mu) > \Pi(Y, \mu)$.
- Any Y that contains a neighborhood of Nash menu is too high.

Proof

Upper bound

- Show that no equilibrium payoff can be uniformly higher than Nash payoffs $\mathcal{N}_A$ on the support of beliefs.
- If so, any menu with payoffs strictly above Nash must be accepted.
- But then, Bob's payoff cannot be lower than

$$\max_{Y \supseteq \text{Nash menu}} \Pi(\mu; Y).$$

- Because things are nice and linear, an optimal solution is

$$m_r \text{ for } r \in R^*(\mu).$$

- If Alice's payoffs are too low, then Alice should have a profitable deviation:
  - a signaling problem: find a deviation that is attractive for Bob *with arbitrary beliefs*,
  - solution: menu of menus

$$W(u, y_u) = \{y \in \mathcal{Y} : y(u) \geq y_u\}.$$

Proof

Lower bound

- Payoff y_u is too low for type u if for any menu Y such that $y_u \geq y(u; Y)$, any beliefs ψ, there exists menu Y' such that

$$\delta y(u; Y') > y \text{ and } \Pi(\psi, Y') > \Pi(\psi; Y).$$

- We show that
  - $y < \frac{1}{2}$ is too low for any type u,
  - $y < 1$ is too low for type who only likes strawberries
  - $y < \frac{1}{2v_c}$ is too low fortype who only likes chocolate.
- Any equilibrium menu must contain Nash menu.

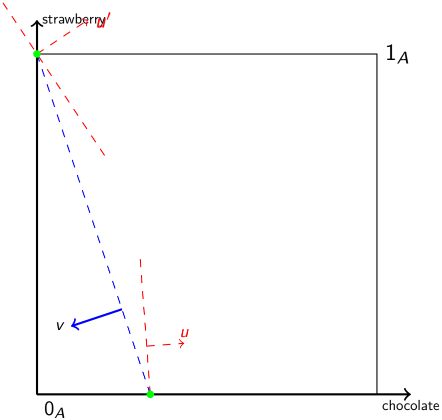
Comments

Single offer

- The ability to offer mechanisms is important for the uniqueness.
- Assume that only single offers are allowed.
- Continuum of equilibria due to signaling issues and punishment with beliefs.

Comments

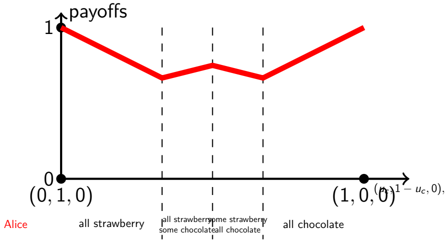
Single offer



- Anti-Coasian equilibrium.
  - punishment of deviations with “bad” beliefs.
- This equilibrium does not survive if Alice can make menus of menus.

Comments

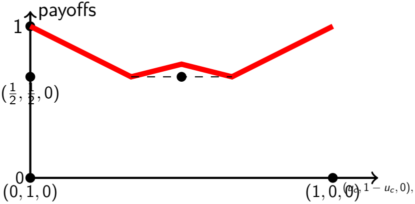
$N > 2$



- Suppose $N = 3$ (chocolate, strawberry, vanilla).
- $v = \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$,
- $\mathcal{N}_A$ is not a menu (it is not convex).

Comments

$N > 2$



- There is an equilibrium st.

$$e_A\left(\frac{1}{2}, \frac{1}{2}, 0\right) \leq \left(\text{Vex}\mathcal{N}^A\right)\left(\frac{1}{2}, \frac{1}{2}, 0\right)$$

.

  - punishment with beliefs

Conclusion

- A model of bargaining with incomplete information about preferences and mechanisms as offers
- Main result: unique outcome (nice!)
  - role of mechanisms in bargaining
  - but not clear what to do about about Nash program,
  - also, a companion paper: reputational types lead to a different result.
- Proof of a concept that bargaining with mechanisms is possible and useful,
  - other environments, two-sided incomplete information