Bargaining with Evolving Private Information

Juan Ortner

Introduction

I study a bilateral bargaining game in which:

(i) buyer is privately informed about her value

(ii) seller privately observes her *stochastically changing* cost $c_t \in \{c_L, c_H\}$

(iii) seller makes all the offers

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Main novelty: arrival of new private information

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Main novelty: arrival of new private information

How does the arrival of new private information affect bargaining outcomes?

Results I

Focus on PBE under which price offers reveal seller's cost

Provide characterization of set of revealing PBE

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Provide characterization of set of revealing PBE

Under revealing PBE:

(i) trade is inefficiently delayed while costs are high

(ii) trade is fast when costs fall

Inefficiencies driven by seller's info revelation constraints

Equilibria rationalize observed pricing patterns in markets for new durable goods.

Results II

Frequent-offers limit of most efficient revealing PBE

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Limit characterized by system of ODEs
describing evolution of prices and trade probabilities

Comparative statics:

- An increase in seller's high cost increases prices and leads to slower trade
- An increase in value dist increases prices and leads to slower trade
- Inefficiencies increase as lowest buyer value goes to zero

Results III

Compare results with setting in which seller's costs are public (Ortner, 2017)

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Model with public costs retains key Coasian elements:

(i) equilibrium outcome is efficient
(ii) seller can't extract rents

High value buyer better-off under public costs
low value buyer indifferent

Related literature

- One sided priv information Coasian bargaining:
  FLT (1985), GSW (1985), etc.
- Inefficiencies with one-sided priv info:
  Ausubel and Deneckere (1989), Deneckere and Liang (2006), McAffee and Wiseman (2008), etc.
- Inefficiencies with two-sided priv info:
  Cramton (1984, 1992), Cho (1990), Ausubel and Deneckere (1992), etc.
- Bargaining with arrival of public info:
  Fuchs and Skrzypacz (2010), Daley and Green (2020)

Players, actions, information

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discrete time $t = 0, \Delta, 2\Delta, ...$

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- seller privately learns cost $c_0 \in \{c_L = 0, c_H\}$
- seller makes offer p_0; buyer accepts or rejects

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If agreement reached at t:

- buyer gets $e^{-rt}(v - p_t)$
- seller gets $e^{-rt}(p_t - c_t)$

Values, costs

- Buyer's value v drawn from dist F
  $\text{supp}\, F = [\underline{v}, \overline{v}]$, $F'(v) = f(v) > 0 \forall v \in [\underline{v}, \overline{v}]$, $\underline{v} > c_L = 0$

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- For talk, seller's cost c_t evolves as:

$$
\begin{aligned}
& \text{Prob}(c_0 = c_H) = q \in (0,1) \\
\forall t > 0, \quad & \text{Prob}(c_t = c_H | c_{t-\Delta} = c_H) = e^{-\lambda\Delta} \\
& \text{Prob}(c_t = c_H | c_{t-\Delta} = c_L) = 0
\end{aligned}
$$

  Results generalize if c_L is not absorbing

Histories, strategies

- seller history at t: $h_t^S = \{c_\tau, p_\tau\}_{\tau=0}^{t-1}$
- buyer history at t: $h_t^B = \{v, p_\tau\}_{\tau=0}^{t-1}$
- (pure) strategy profile (σ^S, σ^B):

$$\sigma^S(h_t^S) : \{c_L, c_H\} \to \mathbb{R}_+$$
$$\sigma^B(h_t^B) : \mathbb{R}_+ \to \{accept, reject\}$$

- beliefs $\mu = (\mu^S, \mu^B)$

Solution concept

Focus on PBE (σ, μ) such that:

1. for all h_t^S, $\operatorname{supp} \sigma^S(h_t^S)(c_H) \cap \operatorname{supp} \sigma^S(h_t^S)(c_L) = \emptyset$

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2. if $\mu^B(h_t^B) = \mathrm{Prob}(c_t = c_L | h_t^B) = 1$, then $\mu^B(h_\tau^B) = 1$ for all h_τ^B that follow h_t^B

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Let Σ^R denote set of PBE satisfying (1)-(2) such that, for all on-path h_t^S, $\sigma^S(h_t^S)(c_H)$ is a pure action

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Paper also considers mixed strategies and other equilibria.

First-best

Define

$$\rho \equiv \frac{e^{-r\Delta}(1 - e^{-\lambda\Delta})}{1 - e^{-(r+\lambda)\Delta}}$$

Let v^* be the solution to

$$v^* - c_H = \rho v^*.$$

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Proposition 1 (first-best).

Under the first-best outcome, the buyer buys at time $t = 0$ if $v \geq v^*$, and buys at $\tau_L \equiv \min\{t : c_t = c_L\}$ if $v < v^*$.

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Assumption 1.

$v^* \in (\underline{v}, \overline{v})$.

Equilibria: preliminaries

Any PBE in Σ^R satisfies:

1. skimming property: at any t, there exists $\kappa_{t+\Delta}$ s.t. buyer accepts offer iff $v \geq \kappa_{t+\Delta}$;
   $\kappa_{t+\Delta}$ is seller's belief cutoff: $\text{Prob}(v \leq \kappa | h^S_{t+\Delta}) = \frac{F(\kappa)}{F(\kappa_{t+\Delta})}$.

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For $\kappa \in [\underline{v}, \overline{v}]$, let:

- $p^L(\kappa) =$ price offered by seller in one-sided priv info game
- $\pi^L(\kappa) =$ seller's profits in one-sided priv info game

Equilibria: preliminaries

A PBE $(\sigma, \mu) \in \Sigma^R$ induces sequences $\{p^H_\tau, \kappa^H_\tau\}$, with $\{\kappa^H_\tau\}$ decreasing, s.t.:

- if $c_t = c_H$, seller posts price p^H_t
  buyer trades at time t iff $v \in [\kappa^H_{t+\Delta}, \kappa^H_t)$,
- if $c_t = c_L$ and $c_{t-\Delta} = c_H$, seller posts price $p^L(\kappa^H_t)$
  earns cont profits $\pi^L(\kappa^H_t)$.

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Seller's profits under $\{p^H_\tau, \kappa^H_\tau\}$ at time t with $c_t = c_H$:

$$\pi^H_t(\{p^H_\tau, \kappa^H_\tau\}) = (p^H_t - c_H)\left(\frac{F(\kappa^H_t) - F(\kappa^H_{t+\Delta})}{F(\kappa^H_t)}\right) + e^{-(r+\lambda)\Delta}\frac{F(\kappa^H_{t+\Delta})}{F(\kappa^H_t)}\pi^H_{t+\Delta}(\{p^H_\tau, \kappa^H_\tau\}) + e^{-r\Delta}(1 - e^{-\lambda\Delta})\frac{F(\kappa^H_{t+\Delta})}{F(\kappa^H_t)}\pi^L(\kappa^H_{t+\Delta})$$

Equilibria: characterization

Theorem 1.

(i) *Suppose* $(\sigma, \mu) \in \Sigma^R$ *induces* $\{p^H_\tau, \kappa^H_\tau\}$*. Then,* $\{\kappa^H_\tau\}$ *is decreasing, and for all* t*:*

$$\kappa^H_{t+\Delta} - p^H_t = e^{-(r+\lambda)\Delta}(\kappa^H_{t+\Delta} - p^H_{t+\Delta}) + e^{-r\Delta}(1 - e^{-\lambda\Delta})(\kappa^H_{t+\Delta} - p^L(\kappa^H_{t+\Delta})) \tag{1}$$

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(ii) $\exists\overline{\Delta} > 0$ *s.t., if* $\Delta \leq \overline{\Delta}$*, for any* $\{p^H_\tau, \kappa^H_\tau\}$ *with* $\{\kappa^H_\tau\}$ *decreasing satisfying* (1)-(3), $\exists(\sigma, \mu) \in \Sigma^R$ *inducing* $\{p^H_\tau, \kappa^H_\tau\}$.

Proof sketch: part (i)

- Why does (2) hold?
  Consider hist. h_t^S with $c_{t-\Delta} = c_H$ and suppose $c_t = c_L$.
- Seller has incentives to reveal her cost if

$$\frac{F(\kappa_t^H) - F(\kappa_{t+\Delta}^H)}{F(\kappa_t^H)} p_t^H + e^{-r\Delta} \frac{F(\kappa_{t+\Delta}^H)}{F(\kappa_t^H)} \pi^L(\kappa_{t+\Delta}^H) \leq \pi^L(\kappa_t^H)$$

Proof sketch: part (ii)

If $\{p_\tau^H, \kappa_\tau^H\}$ satisfies (1)-(3), construct PBE (σ, μ) s.t.:

- on eq'm path, if $c_t = c_H$:
  seller charges p_t^H
  buyer accepts if $v \in [\kappa_{t+\Delta}^H, \kappa_t^H)$
- if $c_t = c_L$, play cont eq'm with one-sided priv info
- if seller deviates while $c_t = c_H$, buyer assigns prob. 1 to $c = c_L$; only accept low prices ($\approx \underline{v}$ if $\Delta < \overline{\Delta}$).
- PBE (σ, μ) is weakly stationary (as in FLT, GSW)

Equilibria: inefficiencies

Proposition 2.

Suppose $\{p_\tau^H, \kappa_\tau^H\}$ *is induced by an equilibrium* $(\sigma, \mu) \in \Sigma^R$*. Then, for all* t*,* $\kappa_t^H \geq v^*$*.*

Inefficiencies due to too much delay.

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Proof.

Suppose $\kappa_{t+\Delta}^H < v^* \iff (1-\rho)\kappa_{t+\Delta}^H < c_H = (1-\rho)v^*$.

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Proof.

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Price p^H_t satisfies

$$\begin{aligned} \kappa^H_{t+\Delta} - p^H_t &\geq \rho(\kappa^H_{t+\Delta} - p^L(\kappa^H_{t+\Delta})) \\ \iff p^H_t &\leq (1-\rho)\kappa^H_{t+\Delta} + \rho p^L(\kappa^H_{t+\Delta}) \\ &< c_H + \rho p^L(\kappa^H_{t+\Delta}) \end{aligned}$$

Seller would rather sell to type $\kappa^H_{t+\Delta}$ at τ_L (i.e. (3) fails). □

Frequent-offers limit: preliminaries

Under most efficient eq'm in Σ^R, for all t with $\kappa^H_{t+\Delta} > v^*$,

$$\frac{F(\kappa^H_t) - F(\kappa^H_{t+\Delta})}{F(\kappa^H_t)} p^H_t = \pi^L(\kappa^H_t) - e^{-r\Delta} \frac{F(\kappa^H_{t+\Delta})}{F(\kappa^H_t)} \pi^L(\kappa^H_{t+\Delta})$$

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For each Δ, let $\{p^H_\tau(\Delta), \kappa^H_\tau(\Delta)\}$ be induced sequences under most efficient eq'm in Σ^R.

Define $\hat{v} \equiv \lim_{\Delta \to 0} v^* = \frac{r+\lambda}{r} c_H$.

Frequent-offers limit: characterization

Theorem 2.
There exists $p^H : \mathbb{R}_+ \to \mathbb{R}_+$ *and* $\kappa^H : \mathbb{R}_+ \to [\underline{v}, \overline{v}]$ *s.t.* $\lim_{\Delta \to 0} p_t^H(\Delta) = p^H(t)$ *and* $\lim_{\Delta \to 0} \kappa_t^H(\Delta) = \kappa^H(t)$.

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Functions $p^H(t)$ *and* $\kappa^H(t)$ *solve*

$$-\frac{dp^H(t)}{dt} = r(\kappa^H(t) - p^H(t)) + \lambda(\underline{v} - p^H(t)) \tag{4}$$

$$-\frac{d\kappa^H(t)}{dt} = \frac{F(\kappa^H(t))}{f(\kappa^H(t))} \frac{r\underline{v}}{(p^H(t) - \underline{v})} \tag{5}$$

Boundary conditions: $\kappa^H(0) = \overline{v}$, *and* $p^H(\hat{t}) = \hat{v} - \frac{\lambda}{r+\lambda}(\hat{v} - \underline{v})$, *where* $\hat{t} \equiv \inf\{t \geq 0 : \kappa^H(t) = \hat{v}\}$.

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For all $t > \hat{t}$, $\frac{dp^H(t)}{dt} = \frac{d\kappa^H(t)}{dt} = 0$.

Frequent-offers limit: intuition

Equation (4) follows from buyer's indifference equates benefit and cost of delay:

$$-\frac{dp^H(t)}{dt} - \lambda(\underline{v} - p^H(t)) = r(\kappa^H(t) - p^H(t)).$$

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Equation (5) follows from seller IC

$$-\frac{d\kappa^H(t)}{dt}\frac{f(\kappa^H(t))}{F(\kappa^H(t))}(p^H(t) - \underline{v}) = r\underline{v}.$$

De-coupling price ODE

- Let $P^H(\kappa)$ be price at which buyer with value $\kappa \geq \hat{v}$ buys
  For all $t \leq \hat{t}$, $P^H(\kappa^H(t)) = p^H(t)$.

De-coupling price ODE

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  For all $t \leq \hat{t}$, $P^H(\kappa^H(t)) = p^H(t)$.

- Using (4)-(5), for all $\kappa \geq \hat{v}$,

$$\frac{dP^H(\kappa)}{d\kappa} = \Big(r(\kappa - P^H(\kappa)) + \lambda(\underline{v} - P^H(\kappa))\Big)\, \frac{f(\kappa)}{F(\kappa)}\, \frac{(P^H(\kappa) - \underline{v})}{r\underline{v}},$$

with $P^H(\hat{v}) = \hat{v} - \frac{\lambda}{r+\lambda}(\hat{v} - \underline{v})$.

Comparative statics

Proposition 3.

The following comparative statics hold:

(i) *As F increases in terms of its reverse hazard rate, price $P^H(\kappa)$ increases for all $\kappa > \hat{v}$, and the rate of trade $-\frac{d\kappa^H(t)}{dt}\frac{f(\kappa^H(t))}{F(\kappa^H(t))}$ falls.*

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(iii) *As λ increases, price $P^H(\kappa)$ increases for all $\kappa \in [\hat{v}, \tilde{v})$, and decreases for all $\kappa \in (\tilde{v}, \overline{v}]$. The rate of trade $-\frac{d\kappa^H(t)}{dt}\frac{f(\kappa^H(t))}{F(\kappa^H(t))}$ falls for all $t > \tilde{t}$, and increases for all $t < \tilde{t}$.*

No gap limit

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In the limit as $\underline{v} \to 0$,

(i) *the rate of trade* $-\frac{d\kappa^H(t)}{dt}\frac{f(\kappa^H(t))}{F(\kappa^H(t))}$ *goes to zero;*

(ii) *the seller's profits go to zero.*

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As $\underline{v} \to 0$:

- seller profits go to zero (as in FLT, GSW)
- inefficiencies grow

Publicly observable costs

Suppose costs $\{c_t\}$ is publicly observable, as in Ortner (2017).

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For each $\Delta > 0$, let

- $(\sigma^\Delta, \mu^\Delta)$ be weakly stat eq'm of game with public costs
- $\pi^{\text{pub}}(\Delta)$ be seller's profits at $t = 0$ under $(\sigma^\Delta, \mu^\Delta)$ conditional on $c_0 = c_H$.

Publicly observable costs

Suppose costs $\{c_t\}$ is publicly observable, as in Ortner (2017).

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- $(\sigma^\Delta, \mu^\Delta)$ be weakly stat eq'm of game with public costs
- $\pi^{\text{pub}}(\Delta)$ be seller's profits at $t = 0$ under $(\sigma^\Delta, \mu^\Delta)$ conditional on $c_0 = c_H$.

Note: when $c_t = c_L$, cont play coincides with eq'm play of one-sided priv info game in FLT, GSW.

Publicly observable costs

Theorem 3.

Suppose the seller's costs are publicly observable. As $\Delta \to 0$*,*

(i) *the limiting outcome under* $(\sigma^\Delta, \mu^\Delta)$ *is efficient:*
buyer with $v \geq \hat{v}$ *buys at* $t = 0$*; buyer with* $v < \hat{v}$ *buys at* τ_L

(ii) *if* $c_0 = c_H$*, the seller's initial price under* $(\sigma^\Delta, \mu^\Delta)$ *converges to* $c_H + \frac{\lambda}{r+\lambda}\underline{v}$

(iii) *if* $c_0 = c_H$*, seller's profits* $\pi^{pub}(\Delta)$ *converge to* $\frac{\lambda}{r+\lambda}\underline{v}$

Publicly observable costs, cont'd

Eq'm with public costs retains key Coasian features.

As $\Delta \to 0$:

- efficient outcome
- seller doesn't extract rents from high-value buyers

Publicly observable costs, cont'd

Eq'm with public costs retains key Coasian features.

As $\Delta \to 0$:

- efficient outcome
- seller doesn't extract rents from high-value buyers

High value buyers are better-off under public costs
low value buyers are indifferent.

Discussion I: other equilibria

Game admits other equilibria

Semi-separating equilibria:

- Low cost seller mixes between $p^L(\kappa_t^H)$ and p_t^H
- Seller IC constraints binds
- Evolution of prices adjusted

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Pooling equilibria

- High and low cost seller pool for $\tau \geq 1$ periods separate afterwards
- Can construct more efficient pooling eq'm (if $\text{Prob}(c_0 = c_H)$ is large); don't need to deter c_L-seller from mimicking.

Discussion II: increasing costs

Suppose $\{c_t\}$ evolves as:

$$\begin{aligned} & \text{Prob}(c_0 = c_H) = q \in (0,1) \\ \forall t > 0, \quad & \text{Prob}(c_t = c_H | c_{t-\Delta} = c_H) = e^{-\lambda\Delta} \\ & \text{Prob}(c_t = c_L | c_{t-\Delta} = c_L) = e^{-\gamma\Delta}. \end{aligned}$$

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In any weakly stationary revealing PBE
as $\Delta \to 0$, seller's profits converge to $\underline{v}$ when $c_t = c_L$

Discussion III: efficient mechanism

In many bargaining models with inefficient delay:
$\nexists$ IC, IR and BB mechanism achieving efficiency

- e.g., two-sided priv info (Cho (1990)), correlated values (Deneckere-Liang (2006), Fuchs-Skrzypacz (2010)).

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Not necessarily true in current model:

Proposition 5.

If $(1-\rho)\underline{v} \geq (1-F(v^*))c_H$, there exists a mechanism satisfying IC, IR and BB, that implements the first-best.

Efficient mechanism

Under mechanism: buyer reports $\tilde{v} \in [\underline{v}, \overline{v}]$ at $t = 0$;
seller reports $\tilde{c}_t \in \{c_L, c_H\}$ at each $t = 0, \Delta, ...$

Efficient mechanism

Under mechanism: buyer reports $\tilde{v} \in [\underline{v}, \overline{v}]$ at $t = 0$;
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If $\tilde{c}_0 = c_L$: buyer buys $t = 0$, pays $\underline{v}$ (regardless of $\tilde{v}$).

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If $\tilde{c}_0 = c_H$:

- if $\tilde{v} \geq v^*$, buyer buys at $t = 0$, pays $c_H + \rho\underline{v}$
- if $\tilde{v} < v^*$, buyer does not buy at $t = 0$, pays $\rho\underline{v}$
  buyer buys at $\hat{\tau}_L = \min\{t : \tilde{c}_t = c_L\}$, and pays $c_L = 0$

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Seller has incentives to report c_L truthfully at $t = 0$ if

$$\underline{v} \geq (1 - F(v^*))(c_H + \rho\underline{v}) + F(v^*)\rho\underline{v}$$
$$\iff (1 - \rho)\underline{v} \geq (1 - F(v^*))c_H$$