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ABSTRACT

This paper studies the optimal issuance strategy in the US Treasury auctions and clarifies the contributions of the primary dealer system to the debt management objective and the financial stability. The new idea of the paper is that the primary dealer system reduces the volatility of bids and prices in the process of routing indirect bidders’ bids, thus contributes to these policy goals when the integrity of the auction system is maintained with the verifications of indirect bidder bids. We develop a novel framework of uniform price auctions of discrete units with the primary dealer system to partially identify the effect of policy counterfactuals based on bidder private values consistent with the observed market outcomes. Counterfactual simulations find that the primary dealer system provides the lowest price volatilities while maintaining the
equal level of auction prices in comparison with the direct bidding system and the joint bidding system. These properties of the primary dealer system could have been valuable during the period of financial crisis.

Keywords: Treasury auctions, auction theory, debt management.

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1. Introduction.

The United States Treasury markets are the crown jewels of the US and the world economy. First, from the fiscal policy perspective, the Federal government issues debt securities in the primary market to finance its operations and refinance its maturity debt. Second, from the monetary policy perspective, the Federal Reserve conducts the open market operations in the Treasury markets to affect the federal funds rate. Third, from the financial market perspective, the US Treasuries yields are the benchmark for pricing fixed income domestic and foreign securities. Short term Treasury securities are a most widely accepted form of collateral for borrowing and repo financing.

Indeed, the importance of the US Treasury securities has become more pronounced during the recent financial crisis and the great recession. First, from the fiscal policy perspective, the Treasury department increased the issuance from $4.4 trillion in 2006 to $7.9 trillion in 2013. Second, from the monetary policy perspective, the Federal Reserve conducted rounds of the Quantitative Easing to purchase the Treasury securities to lower the long-term rate in the zero interest rate environment. Third, from the financial market perspective, a contraction in short-term repo markets played an important role in the collapse of funding of the shadow banking sector. Then, the analysis and the design of the Treasury auctions to ensure efficient and stable pricing of Treasury securities is a task that can contribute to the public objectives of the smooth funding of the Federal government operation, the effective implementation of the monetary policy, and the financial stability.

The US Treasury department is authorized under 31 U.S.C. Subtitle III of the United States Code to issue Treasury securities and to prescribe terms and conditions for their issuance and sale to finance the excess of outlays over receipts and to service existing and maturing debt. The US Treasury started employing auctions for distributions of bills since December 1929, for notes since November 1970, and for bonds since January 1973. The Treasury department currently uses the mechanism “single price auctions with the primary dealer system” for the distribution of security. The
primary dealer system is a bidder management system where bidders are classified into primary dealers who bid for their own house accounts, indirect bidders who bid through a primary dealer, and direct bidders who bid for their own house accounts. In addition to bidding for their own account, primary dealers route bids by indirect bidders (their customers) and they can observe indirect bidders’ bids before submitting their own bids. Single-price (uniform price) auctions determine the distribution of securities among bidders: the Treasury awards all noncompetitive bids and then accepts competitive bids in ascending order of their yields (lowest from highest) until the quantity of awarded bids reaches the offering amount, and all bidders will receive the same yield at the highest accepted bid.

But recently there have been significant changes and uncertainty in the economic environment concerning Treasury auctions. First, from the fiscal policy perspective, the Treasury has auctioned a large amount of securities since the latter half of 2008 to deal with the Financial Crisis. But the deficit for the fiscal year 2014 was the lowest since 2008 and represents a decrease of 29% from the previous year. The Treasury expects to gradually decrease auction sizes for shorter-dated securities. Second, from the monetary policy perspective, the Federal Reserve recently announced the end of the Quantitative Easing and there have been expectations that the Federal Reserve will start raising short-term interest rates. Third, from the financial market perspective, Basel III requires banks to hold more cash in reserve for assets such as bonds they keep on their balance sheets. In addition, the foreign official holding of US Treasury securities has increased from $400 billion in January 1994 to $3 trillion in June 2010 (US Treasury (2013)).

Then, the US Treasury has asked a following question to the Treasury Borrowing Advisory Committee (Treasury Borrowing Advisory Committee (2014)):

Treasury has used the Primary Dealer model for auctioning and distributing debt for several decades. This highly efficient system has been a key feature for the effective functioning of Treasury auctions. Given the evolution of the

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1Indeed, there are 4 recent incidents that seem to suggest that these changes in economic environments affect the Treasury markets. First, Fleming and Myers (2013) claim that the shares of primary dealer purchases have declined since 2008. Second, Fleming, Keane, Martin, and McMorrow (2014) report that settlement fails in the U.S. Treasury securities spiked to their highest level since May 2009. Third, on October 15, 2014, the yield on the benchmark 10-year US government bond plunged 33 basis points to 1.86 per cent before rising to settle at 2.13 per cent (“Treasury flash crash”). Fourth, a decision of the US Treasury at November 5, 2014 to gradually cut back the size of two-and three-year note auctions over the next three months creates concerns about the risk of auctions (Eddings (2014)).
financial services industry, market structure, regulation and technology over
recent years, does the current structure for distributing Treasury securities
remain optimal? Are there any modifications that could result in a lower
cost of funding for Treasury and/or enhance secondary market liquidity?

The goal of this paper is to contribute to this policy question of the optimality of
the primary dealer system for the distribution of securities by using the economic
theory to interpret observed market outcomes and to predict the outcomes when the
counterfactual policies were to be adopted.

Previous academic studies on Treasury auctions focused on theoretical and em-
pirical comparison between the single price auction method and the multiple price
auction method\(^2\). Regarding the primary dealer system, Smith (1966) studies the
bidding problem of a dealer who faces uncertain resale prices in the secondary mar-
ket. Chatterjee and Jarrow (1998) find that dealers can observe the order flow in the
when-issued-market and, when the short selling is large, the dealer has an incentive
to corner and short squeeze in discriminatory auctions\(^3\). Bikchandani, Edsparr, and
Huang (2000) find that dealers with large demands may forgo buying at low prices on
the when-issued market in order not to reveal the information. Hortacsu and Sareen
(2006) empirically show that a primary dealer observation of the order flow of bids by
customers affects market participants’ bidding behavior in the Canadian Treasury auc-
tions. Hortacsu and Kastl (2012) quantify the economic benefits that dealers derive
from having an access to the customer order information in the Canadian auctions.
Hortacsu, Kastl, and Zhang (2014) examine the US Treasury bills and notes auctions
with the primary dealer system. Boyarchenko, Lucca, and Veldkamp (2014) study the
role of the primary dealers as information aggregators\(^4\).

\(^2\)Theoretical contributions include Vickrey (1961), Friedman (1962), Smith (1966), Reichert
(1968), Wilson (1979), Milgrom (1981), Back and Zender (1993), Simon (1994), Milgrom and
Weber (2000), Ausubel and Cranton (2002), Kremer and Nyborg (2004), and Ausubel, Cranton,
Pycia, Rostek, and Weretka (2014). US studies include Rieber (1965), Bolton (1973), Umlauf
(1991), Simon (1994), and Malvey and Archibald (1998). International evidences include Umlauf
(1993), Nyborg and Sundaresan (1996), Fevrier, Preget, and Visser (2004), Keloharju, Nyborg,
and Rydqvist (2005), Preget and Waelbroeck (2005), Armantier and Sabai (2007), Brenner, Galai,
and Sade (2009), Hortacsu and McAdams (2010), and Kastl (2012). Athey and Haile (2005) and
Hendricks and Porter (2007) are reviews of empirical studies of auctions.

\(^3\)A short squeeze developed in the two-year note auctioned on April 1991. When the squeeze
first manifested itself in mid-May, the yield on the April two-year note moved considerably out of
line with surrounding market rates, and the notes were “on special” in the repurchase agreement
(“repo”) market. Salmon Brothers admitted large bids falsifying the customer bids that lead to the
concentration of auction awards to Salomon and some of its customers.

\(^4\)More generally, a study of the primary dealer system is related to studies of bidder heterogeneities
Then the objective of this paper is to go one step further beyond the previous research by conducting counterfactual analysis of issuance costs and stability of the Treasury auction systems. Specifically we consider 2 questions:

- What was the change in the economic environments in the US Treasury auctions during and after the period of the financial crisis in comparison with the period before the crisis?
- What are the significances of the primary dealer system to the debt management objective and the financial stability in comparison with other policy alternatives such as the direct bidding system and the joint bidding system (syndication system) especially in the economic environments such as the period of financial crisis?

In other words, the concern of this paper is about comparison of market outcomes of the primary dealer system with those from other systems (from the risk management perspective), and is different from previous research of estimating bidder behavior and primary dealer information advantage.

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5 Indeed, Matthew Rutherford, former Under Secretary for Domestic Finance at the US Treasury, discusses the importance of counterfactual analysis in the Quarterly Refunding Webcast on November 11, 2013: “What was the cost to us in issuing Treasury bills? This is a very difficult question for us to answer in part because you need to know what the counterfactual is and it would be difficult to know what this rate is absent this debate.”


7 In terms of risk management perspective, for example, Hortacsu, Kastl, and Zhang (2014) conclude that “…under the assumption that the mechanism is achieving an approximately efficient allo-
The new idea of the paper is that the primary dealer system contributes to the debt management objective and the financial stability by realizing a balance of the price stability and competition when the integrity of the Treasury auction system is maintained by the objective verification of indirect bidder bids. First, by allowing primary dealers to observe the bids by indirect bidders, the primary dealer can have more precise estimates of the order flow. Second, by ensuring that the primary dealers route the bids by indirect bidders, the system will preserve competition and limit the discretion of primary dealers. As a result, the primary dealer system could reduce yield volatility further from single price auctions. We now explain this point using a simple example.

Example. Suppose that a seller sells a unit of security using the single price auction (can be considered as the second price auction with a single unit of sale). Suppose there are 2 bidders. Each bidder has a private value for the security. The distribution of private values among bidders is iid uniform [0,1]. Bidder preferences are quasi-linear and risk-neutral.

Let us consider 3 auction mechanisms that can be used for the distribution of the security. In the direct bidding system, each bidder bids independently as in standard second price auction. In the joint bidding system, 2 bidders bid as a single entity. In the primary dealer system, we assume that bidder 1 is the primary dealer, observes a bid by bidder 2, routes the bid to the auctionner, then submits its own bid.

We now compare the equilibria of these 3 auction mechanisms. The direct bidding system is the standard second price auction. Thus the truthful bidding is a dominant strategy. The seller revenue is the second highest private value. The expected revenue cation, the total bidder surplus in these auctions amounts to about 3 basis point and thus the 3 basis point is the upper bound that one can hope from a redesign of the auction mechanism.” But in reality, even if the primary dealer system has low expected bidder surplus, if it entails significant risks to the Treasury markets, then one would need to reconsider the effectiveness of the primary dealer system. That would imply the significance for further studies beyond Hortacsu, Kastl, and Zhang (2014).

DeBrock and Smith (1983), Krishna and Morgan (1997), and Levin (2002) consider the effect of joint bidding through merging of signals in common value auctions.

Hortacsu and Kastl (2012) already recognize the information advantage of the primary dealer from observing the order flow of the indirect bidders. The new effect considered in this paper is the systemic implication that the primary dealer system can reduce the volatility of the bids and prices and contribute to the financial stability. I thank Lawrence Ausubel and Marek Pycia for helpful discussions on this issue.

After the paper was presented in the Winter Meeting of the Econometric Society in January 2014, an author has become aware of the example in Hortacsu and Sareen (2006) that examines the discriminatory auctions. In contrast, this example studies the primary dealer system in uniform price auctions.

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is 1/3 and the variance is 1/12. In the joint bidding system, there is only one bidder and the seller revenue is 0.

To understand the equilibrium bidding behavior in the primary dealer system, consider a best response of the primary dealer 1 with signal $v_1$ when the primary dealer observes indirect bidder 2’s bid $b_2$. If $v_1 > b_2$, then, primary dealer 1’s best response is to bid any amount above $b_2$. Suppose $v_1 < b_2$. What will be the primary dealer 1’s best response? Suppose the primary dealer 1 bids 0. Then, given this primary dealer 1’s response, indirect bidder 2 will always bid 1 and the primary dealer 1 will always bid 0. Thus bidding 0 does not seem to be a best response for the primary dealer. Indeed, suppose the primary dealer 1 chooses a following strategy:

$$ b_1(v_1, b_2) = \begin{cases} 
    b_2 + \varepsilon & \text{if } v_1 > b_2 \\
    b_2 - \varepsilon & \text{if } v_1 < b_2.
\end{cases} $$

In words, if the indirect bidder bids below the primary dealers’s value, the primary dealer bids just above the bid by the indirect bidder. If an indirect bidder bids above the value, the primary dealer bids just below the bid to deter jump bids\textsuperscript{11}. Given this response, what will be indirect bidder 2’s best response? Indirect bidder 2 wins when $v_1 < b_2$ and the payment is (approximately) $b_2$. Then its expected payoff is $\Pr(v_1 < b_2) \cdot (v_2 - b_2) = b_2(v_2 - b_2)$. Thus its bidding strategy is $b_2(v_2) = v_2/2$. Then the seller revenue is always $v_2/2$.

We now compare the performances of these 3 auction mechanisms. In the primary dealer system, the seller expected revenue is 1/4 and the variance is 1/48. Although the revenue is 25% lower but the variance is reduced to 1/48th compared with the direct bidding system\textsuperscript{12}. The variance reduction in this example comes from 2 sources. First, after observing bids by an indirect bidder, the primary dealer bids match the bids by the indirect bidder. Second, knowing this, the indirect bidders reduce its bids. This bid shading also reduces the variance.

To put this idea in the context of the US Treasury auctions, we proceed in the

\textsuperscript{11}This equilibrium strategy has the property that the distribution of the primary dealer’s bids are identical to that of the uninformed bidder as in Engelbrecht-Wiggans, Milgrom, and Weber (1982). This bidding pattern can be consistent with the observation by Hortacsu and Kastl (2012) in the Canadian Treasury auctions data that, after observing a relatively low bid by a customer, the dealer submits a new bid that is weakly lower below its original bid.

\textsuperscript{12}The loss of revenue from the primary dealer system in this example is due to the fact that there is only one primary dealer. The US Treasury auctions system has 21 primary dealers and competition among bidder will lead to more aggressive bids (Section 5.B).
following 5 steps. First, we develop a theoretical model of multiple unit auctions with the primary dealer system in the economic environment of independent private values and discrete units of the security. This modelling strategy is different from the previous models of Hortacsu, Kastl, and Zhang (2014) but allows us to capture the characteristics of the economic environments where the seller issues finite and indivisible units of securities in the auction. We derive 3 results for this model: the existence of a mixed strategy perfect equilibrium (Lemma 1), bidder first order conditions (Proposition 2 and 8), and the asymptotic analysis of equilibrium bidding behavior (Proposition 3). Second, based on the first order condition, we develop a bootstrap estimator of bidder private values consistent with the observed market outcome (Section 4.A). Third, based on the asymptotic analysis, we derive bounds on the bidder strategic response to

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13 Previous Treasury auction models employ the framework of the “share auctions” model of Wilson (1979) that assumes continuous quantity and prices (Hortacsu (2002), Hortacsu and Kastl (2012), Kastl (2011), Kastl (2012), and Hortacsu, Kastl, and Zhang (2014)). But this paper considers the alternative discrete unit modelling approach in auctions (Reichert (1968), Milgrom (1981), Milgrom and Weber (1982a), Hendricks and Porter (1988), Rustichini, Satterthwaite, and Williams (1994), Jackson and Swinkels (2005), Reny and Perry (2006), Fudenberg, Mobius, and Szeidl (2007), McAdams (2008), Hortacsu and McAdams (2010), and Kazumori (2013)) based on the recent progresses in the academic research. (1) Kremer and Nyborg (2004) show that underpricing in the “share auctions” model will vanish with a discrete demand schedule and argue that the model with discrete set of bids can explain the empirical fact that underpricing does not materialize in practice. (2) Kastl (2011) develops a model of uniform auctions of a perfect divisible good in which the bidders submit discrete bid points. But Kastl (2012) notes that the equilibrium may not exist with uniform price auctions because of ties: “since ties cannot be ruled out in the uniform price auctions with restricted strategy sets, even with independent types, the technique of obtaining existence as in Reny and Zamir (2004) or Jackson and Swinkels (2005) cannot be applied. In contrast, the model here establishes an asymptotic result which bounds the maximum loss of one bidder from using restricted strategies.” (Kastl (2012)). As another example, Reguant (2013) states: "(we) make(s) the assumption that ties with other firms or with different units within a given firm do not happen with positive probability, which allows me to avoid the problems that arise in the presence of ties (see Kastl (2012), for example).” (3) The asymptotic analysis in the discrete models allow us to partially identify bidder strategic responses to different auction systems. As such, these two approaches complement each other.

14 An intuition of a perfect equilibrium is that the equilibrium is robust to uncertainties about other bidders’ strategies. Perfect equilibrium has been used in the auction literature (Milgrom (1981) and Abraham, Athey, Babaioff, and Grubb (2012), for example).

15 An idea is that, when a bidder places a bid, it must be that the bidder does not wish to bid higher, and it implies that the value cannot be too high. Also since the bidder does not wish to bid lower, the value should not be too low. The structural estimation method follows a theory of inferences on partially identified models Andrews, Berry, and Jie (2004), Chernozhukov, Hong, and Tamer (2007), McAdams (2008), Ciliberto and Tamer (2009), Hortacsu and McAdams (2010), and Pakes, Porter, Ho, and Ishii (2014). Also a perfect equilibrium provides a natural justification of the use of the smoothed bootstrap (Tibshirani (1988) and Hall, DiCiccio, and Romano (1989)). Wolak (2007) also uses a smoothing approach to construct a differentiable approximation of the profit function.
the auction systems (Proposition 4)\textsuperscript{16} and use these bounds to forecast and simulate the outcome from counterfactual policies. Then, we study the bidder allotment and bidding method during the period 2008-2013 using publicly available Treasury auction data, and conduct numerical counterfactual analysis of the primary dealer system.

Our main findings are the following 5 points: Using the publicly available TreasuryDirect data, we find that, during the period of financial crisis, (1) primary dealers’ allotment shares have decreased especially in securities with short term maturities, (2) investment funds shares have increased in securities with longer term maturities, and (3) the share of the direct bidding has increased. To understand the implications for the policy, we conduct the counterfactual analysis to show that (3) the primary dealer system has the lowest volatility in comparison with the same level of auction prices in comparison with other distribution systems, (4) the dealers’ allocation in the primary dealer system is slightly higher than one with the direct bidding system and has significantly lower standard deviations than other systems, and (5) the primary dealer profits are higher with the primary dealer system in comparison with the ones in the direct bidding system but its magnitude is much smaller than the one in the Canadian Treasury auctions estimated by Hortacsu and Kastl (2012)\textsuperscript{17}. These results suggest that, during the period of financial crisis where the primary dealer’s capacities and demands are limited and investment funds looked for assets with safety and risklessness,

\textsuperscript{16} An idea is that, since a bidder will bid close to the true value of the security as the number of bidders increases, the bid should be bounded within some range of the true value. These bounds on equilibrium bids are based on Rustichini, Satterthwaite, and Williams (1994), McAdams (2008), and Kazumori (2014). Furthermore, the bounds in this paper is consistent with the recent result in Satterthwaite, Williams, and Zachariadis (2014) that is based on asymptotic expansion of the first order condition of uniform price auctions. These bounds, or how much discretion a bidder has to strategically play the bids depends on how the auctions are organized. Economically, these bounds take into account of the following 4 effects: (1) the number of bidders in the auction will affect competitiveness of the auctions, thus affecting the amount that a bidder can strategically change the bid, (2) an indirect bidder needs to expect that its primary dealer observes the bids and chooses its bids conditioning on this information, (3) a primary dealer can observe bids by indirect bidders and estimate the order flow, and (4) a primary dealer takes into account of the fact that bids by indirect bidders limit the room for strategic discretions. An important precursor of this approach is Hong and Shum (2004) that compare the rates at which the price aggregates information depends on the structure of the common value uniform price auctions. There have been previous literatures in auction theory that characterize bounds on equilibrium strategies when the exact characterization is not available (e.g., Maskin and Riley (2000)). Manski (2007) studies the prediction of counterfactual choice behavior based on partially identified choice probabilities. (Manski (2014) is an application to income tax policy.)

\textsuperscript{17} This result is consistent with a recent finding in Hortacsu, Kastl, and Zhang (2014). Hortacsu and Kastl (2012) define the information advantage as a measure of how much extra profit a dealer make when updating the bids after observing the indirect bidders’ bids. Here this paper compares the dealer’s profits under the primary dealer system with the one under the direct bidding system.
the primary dealer system would have reduced the interest rate risks, enhanced the demand for Treasury securities, and contributed to the financial stability\textsuperscript{18}.

The economic reasonings behind these results are as follows: (1) during the period of the financial crisis, the prime dealer’s capacity and demand for short-term Treasuries for hedging has decreased because of the balance sheet effects, (2) also during this period, the funds are attracted to liquid Treasury securities, (3) a primary dealer mirrors indirect bidder bids, which reduces volatility of bids compared with the direct bidding system, and the joint bidding system has less bidders and competition\textsuperscript{19}, (4) a primary dealer has more precise information on the residual supply curve via observing indirect bidders’ bids, that leads to a higher payoff and less volatility in the profits in comparison with other category of bidders\textsuperscript{20}, and (5) the dealer’s additional knowledge of indirect bidders’ bids will increase the dealer profits over the one in direct bidding systems, but competition among bidders is more intense in the US auctions\textsuperscript{21}.

The main economic contributions of the paper are that this paper develops a new rationale for the primary dealer system to achieve these policy objectives. The previous view on the primary dealer system focuses on its liquidity provision effect to the Treasury markets. This paper goes one step further to show that the primary dealer system reduces the interest rate risk given the same level of the participation in the auction when the integrity of the Treasury auction system is maintained through the verification of the indirect bidders’ bids. Indeed, the simulation results suggest that the main difference among the auction systems is in terms of risk rather than in terms of the expected prices (yields). This risk reduction would be important especially during the period of financial crisis since it will attract investors who value liquidity and

\textsuperscript{18}These results are consistent with the market microstructure literature that shows importance of the market making system (Huang and Stoll (1996), Tse and Zabotina (2004), and Venkataraman and Waisburd (2007)).

\textsuperscript{19}An idea is that, start from the direct bidding system and consider allowing dealers to observe indirect bidders’ bids. Then dealers will have more accurate estimate of the residual supply curve. Then dealers will bid more aggressively and have less uncertainty in bidding. Then, given dealer’s aggressive bidding behavior, direct and indirect bidders, who have less information than the dealers, will reduce the bids, that will further reduce the volatility of bids.

\textsuperscript{20}This observation - more informed bidders bid more aggressively - is consistent with the findings in the OCS auctions (Hendricks and Porter (1988) and Hendricks, Porter and Wilson (2004)). That is, the information advantage of the primary dealers in the Treasury auctions can be related to the information advantage of firms owning adjacent wildcat leases in the OCS auctions, but more intense competitions in the Treasury auctions limit the impact of these asymmetries.

\textsuperscript{21}In the US market, the number of primary dealers in the US auctions is about the twice of the number of primary dealers in the Canadian auctions, leading to a higher level of competition among primary dealers.
safety of the US Treasury securities (Krishnamurthy and Vissing-Jorgensen (2013))\textsuperscript{22} and contribute to the financial stability\textsuperscript{23}.

For the rest of the paper, we would proceed as follows. Section 2 summarizes the institutional features of the US Treasury auctions. Section 3 defines a model of uniform price auctions with the primary dealer system. Section 4 develops a method to partially identify the effect from alternative policies from observed market outcomes. Section 5 applies the framework to study the US Treasury auctions. Section 6 concludes. Appendix 7 provides proofs of Propositions and additional details.


We begin by describing the institutional features of the US Treasury auctions.

A. The Primary Dealer System.

The primary dealer system organizes the Treasury auctions around primary dealers\textsuperscript{24}.

Primary Dealers. The primary dealers serve as trading counterparties of the Federal Reserve Bank of New York in its implementation of monetary policy. In addition, these primary dealers serve the following 4 crucial roles in the US Treasury auctions: (1) Bid in every auction, for, at a minimum, an amount of securities representing its pro rata share, based on the number of primary dealers at the time of the auction, of the offered

\textsuperscript{22}See McAfee and McMilan (1987), Bajari and Hortacsu (2003), and Athey, Levin, and Seila (2011) for theoretical and empirical analysis of entry in auctions. Our results are in line with the practitioners’ comments such as Bartolini and Cottarelli (1994). Longstaff (2004) estimates importance of liquidity in Treasury markets.

\textsuperscript{23}In other words, the results of this paper can be summarized as follows: “Hortacsu, Kastl, and Zhang (2014) show that under the assumption that the mechanism is achieving an approximately efficient allocation, the total bidder surplus in these auctions amounts to about 3 basis point and thus the 3 basis point is the upper bound that one can hope from a redesign of the auction mechanism. This paper goes one step further and conducts policy counterfactuals comparing the primary dealer system, direct bidding system, and the joint bidding system. The result confirms their findings. Furthermore, we find that the key difference among these auction systems is in terms of interest rate risks, and that the primary dealer system reduces the risk of the Treasury auction system. Then, the conclusions of this paper is that the primary dealer system contributes to the debt management objective and the financial stability when the integrity of the Treasury auction system is ensured with the objective verifications of indirect bidder bids.”

\textsuperscript{24}The Federal Reserve conducts its purchases and sales of securities chiefly through transactions with primary dealers (“open market operations”) rather than directly with the Treasury. As such, the Federal Reserve purchases most of the Treasury securities in the secondary market.
amount. (2). Its bid prices should be reasonable when compared to the range of rates trading in the when-issued market, taking into account of market volatility and other risk factors. (3). Submit bids from indirect bidders directly to the Treasury. (4). Also in the quarterly refunding process, the Treasury solicits advice and views from the private sector through questions to primary dealers and interview half of the primary dealers each quarter through face-to-face meetings. In 1988 there were 46 primary dealers and today there are 21.

**Indirect Bidders.** Indirect bidders place bids through a direct submitter\(^{25}\). Indirect bidders are investment funds (money-market funds and hedge funds) and Foreign and International Monetary Authorities who bid through the Federal Reserve Bank of New York. The Federal Reserve engages in spot-checking of customer bids in Treasury auctions to verify the legitimacy of large winning bids submitted for customer accounts. In addition, the Treasury and the FRBNY implement a formal system that requires customers who make large winning bids through dealers or depository institutions to verify their bids in writing to the Federal Reserve prior to the settlement date \(\text{(US Treasury et al. (1992))}\)^{26}.

**Direct Bidding.** Direct bidding refers to a submission of bids by an entity directly to the Treasury or Federal Reserve rather than through an intermediary such as a bank or a securities dealer. Direct bidders include, but are not limited to, primary dealers, other brokers and dealers, various types of investment funds, insurance companies, depository institutions, foreign and international entities, the Federal Reserve \(\text{(System Open Market Account)}\)^{27}.

\(^{25}\)In 1992 the Treasury announced that all government securities brokers and dealers registered with the SEC would be eligible to submit bids for customers in Treasury auctions. Previously, only primary dealers and depository institutions had this privilege. The Treasury is not imposing any limitation on the combined amount awarded to a dealer and the customers for whom the dealer has placed bids. The model of the paper allows a wider interpretation of “primary dealer system” where bidders other than the primary dealer can route the bids from indirect bidders.

\(^{26}\)This development is in response to Salmon Brothers submission of unauthorized customer bids in December 1990, February 1991, April 1991, and May 1991 auctions.

\(^{27}\)Government Accounting Office \(\text{(2010)}\) notes that there were 825 investors making use of the auction system that allows direct bidding in 2004.
B. Single Price Auctions.

The Treasury department conducts auctions at the Federal Reserve Bank of New York and the Bureau of Public Debt in Washington DC\textsuperscript{28}.

Public Announcements. The public announcement provides information on the amount of the security being offered, auction date, issue date, maturity date, terms and conditions of the offering, and noncompetitive and competitive bidding close times. Bids may be submitted at any time from the opening of the bidding until the public deadline.

Noncompetitive Bidding. Bidders can submit a competitive bid or a noncompetitive bid\textsuperscript{29}. A noncompetitive bid is mostly employed by smaller, less sophisticated bidders, such as corporations and individuals. A noncompetitive bid is capped to $5 million for individuals and entities, and $100 million for sovereign central banks. A noncompetitive bidder is not permitted to have a position in the security being auctioned in the when-issued, futures, or forward markets prior to the auction. Noncompetitive bidding closes before the close of competitive bidding.

Competitive Bidding. A competitive bid specifies the rate, yield, or discount margin that is acceptable. The Treasury changed the rule to receiving bids on the basis of yields rather than prices in 1983. The minimum price increments are 0.5 basis points for thirteen, twenty-six, fifty-two week, and cash management bills, and 0.1 basis points for all other securities. Competitive bid is limited to 35 percent of the amount offered to the public\textsuperscript{30}. A bidder is not permitted to submit both competitive and noncompetitive bids in a single auction (US Treasury et al. (1992)). The competitive closing time for bills is normally 11:30 a.m and normally 1:00 p.m. for notes, bonds, FRNs, and TIPS.

A Single Price Auction. After the close of an auction, after it has been determined that bids have complied with the Treasury requirements, Treasury awards all noncompetitive bids and then accepts competitive bids in the ascending order of their yields (lowest to highest) until the quantity of awarded bids reaches the offering amount. All bidders will receive the same yield at the highest accepted bid\textsuperscript{31}.

\textsuperscript{28}The Federal Reserve Act of 1913 provides that the Federal Reserve Banks will act as fiscal agents and depositories when required to do so by the Secretary of the Treasury.

\textsuperscript{29}A provision of noncompetitive bids was first introduced in 1967 for bill auctions.

\textsuperscript{30}The Treasury adopts ceilings of the maximum award per bidder of 35 percent of the public offering since 1962. A bidder must report its net long position along with its auction bids if the sum of its net long position in when-issued, forward, and futures contracts and its bids exceeds 35 percent of the offering.

\textsuperscript{31}The UK gilt auctions have an additional Post Auction Option Facility where all bidders are offered
Auction Results Announcement. After the conclusion of the auction, the auction results are posted on the TreasuryDirect website as a press release.

Settlement. On the day of issuance, the settlement (the exchange of securities for funds) takes place either on the books of a depository institution or between depository institutions through the Federal Reserve’s Fedwire securities transfer system.


We now develop a theoretical model of Treasury auctions characterized by the primary dealer system and the single price auctions to frame empirical analysis\textsuperscript{32}.

A. The Economic Environment.

A seller offers multiple units of security to buyers. The economic environment is parameterized by the offering amount $M$, the numbers of primary dealers, indirect bidders, and direct bidders $N_{PD}$, $N_{ID}$, and $N_{D}$, and their distributions of private values $F_{PD}$, $F_{I}$, $F_{D}$.

The Seller. There is a single seller who offers multiple units of the security. Let $M \in \mathbb{N}_{++}$ be the offering amount of the securities in the auction.

Bidders. There are 3 categories of bidders: primary dealers, indirect bidders, and direct bidders\textsuperscript{33}. Let $I_{PD}$, $I_{ID}$, and $I_{D}$ be a set of primary dealers, indirect bidders, and direct bidders. Let $I = I_{PD} \cup I_{ID} \cup I_{D}$ be the collection of all bidders. Let us further denote $N_{PD} \in \mathbb{N}_{+}$, $N_{ID} \in \mathbb{N}_{+}$, and $N_{D} \in \mathbb{N}_{+}$ as the number of primary dealers, indirect bidders, and direct bidders. Then there are $N_{PD} + N_{D} + N_{ID}$ bidders in the auction. For each indirect bidder $i \in I_{ID}$, let $PD(i) \in I_{PD}$ be the primary dealer through which indirect bidder $i$ submits a bid in the auction. For each primary dealer $i \in I_{PD}$, let $\{j \in I_{ID} : PD(j) = i\}$ be a collection of indirect bidders connected the right to purchase up to an additional percentage of the gilt they were allocated at the published prices.

\textsuperscript{32}The model of the paper is based on the models of uniform price auctions of discrete units by Reichert (1968), Milgrom (1981), Rustichini, Satterthwaite, and Williams (1994), Pesendorfer and Swinkels (1997), Jackson and Swinkels (2005), Cripps and Swinkels (2006), Reny and Perry (2006), and Kazumori (2013). We extend these models to incorporate the primary dealer system. This model take a different approach from “the share auction models” of Hortacsu (2002), Hortacsu and Kastl (2012), Kastl (2011), Kastl (2012), and Hortacsu, Kastl, and Zhang (2014).

\textsuperscript{33}We simplify the model by not including noncomptitive bids explicitly in the model. Empirically, the share of noncompetitive bids in the auction is small.
to dealer $i$ and let $N_{ID,i}$ be the number of indirect bidders connected to the primary dealer $i$.

*Bidder’s Private Value for the Security.* Let us denote bidder $i$’s monetary value of the $m = 1, ..., M$th unit of the security offered by $v_{i,m} \in V_{i,m} = [0, \pi] \subseteq \mathbb{R}_+$. Here $\pi \in \mathbb{R}_+$ is the maximum value that any bidder can have for a unit of security. Then, bidder $i$’s value profile is an $M$-dimensional vector $\overrightarrow{v}_i \equiv (v_{i,1}, ..., v_{i,M})$. We assume that a bidder’s marginal values are nonincreasing: $v_{i,1} \geq ... \geq v_{i,M}$. Then the set of possible value profiles for a bidder is denoted by $V_i = V_{i,m} \times ... \times V_{i,m}$. 

*Distribution of Values among Bidders.* We assume a category $i = PD, D, ID$ bidder’s value is an independent draw according to a distribution $F_i : \mathbb{R}_+^M \rightarrow [0, 1]$ with an atomless and uniformly bounded (from below and from above) density $f_i : \mathbb{R}_+^M \rightarrow \mathbb{R}_{++}$ with its common support $V_i$.

*Bidder Preferences.* We assume that a bidder’s ex post payoff is additively separable across units and is quasi-linear. Suppose bidder $i \in I$ has a private value $\overrightarrow{v}_i \in V_i$. Let $q_{i,m} \in \{0, 1\}$ be bidder $i$’s allocation for the $m = 1, ..., M$th unit from the auction: $q_{i,m} = 1$ if bidder $i$ wins the $m$th unit of the security and 0 otherwise. Then, bidder $i$’s allotment is an $M$-dimensional vector $\overrightarrow{q}_i \equiv (q_{i,1}, ..., q_{i,M}) \in \{0, 1\}^M$. In a single price auction, every bidder pays the same price $p$ for each unit of the security. Thus bidder $i$’s ex post payoff function is given by $u(v_i, p, q_i) \equiv \sum_{m=1}^{M} q_{i,m}(v_{i,m} - p)$. We assume that a bidder is risk-neutral.

*Auction Systems.* In the primary dealer system, there are primary dealers who route bids from indirect bidders and also direct bidders who submit bids directly to auctionner. In the direct bidding system, every bidder has a direct link to the auctionner, instead of only submitting indirect bids through primary dealers, and places a bid to maximize the profits from auctions. As an example of the joint bidding system, municipal bond issuers select a syndicate of underwrites to distribute bonds to the public. An issuer will select an underwriter through competitive bidding process\textsuperscript{34}. We make a simple assumption where a primary dealer and indirect bidders form a consortium where a primary dealer and indirect bidders share the private values and

\textsuperscript{34}Another example of the joint bidding system is bidding consortia in auctions for offshore leases where participating firms may reveal their bidding intentions on all the tracts under discussion (Porter (1995)). The joint bidding system can also be interpreted as a situation where the primary dealer and indirect bidders submit bids jointly instead of primary dealer routing the bids by indirect bidders with spot checking and verifications.
submit competitive bids to maximize the profit for the consortium\footnote{Indeed, a key characteristics of the joint bidding system is that it will reduce the number of bidders and thus competition in the auction, and this effect would be robust to a detail of interactions among bidders within the consortium.}

\textit{Scaling the Auction Environments.} We conduct an asymptotic analysis as the number of bidders increase. In this process, indexed by \( n \), the seller auctions \( n \cdot M \) units of securities among \( n \cdot N_{PD} \) primary dealers, \( n \cdot N_{D} \) direct bidders, and \( n \cdot N_{ID} \) indirect bidders. That is, in the \( n \)th environment, there are \( n \cdot (N_{PD} + N_{D} + N_{ID}) \) bidders in this auction. Each bidder demands a fixed \( M \in \mathbb{N}++ \) units of securities\footnote{This formulation follows Pesendorfer and Swinkels (1997). To be precise, we further classify primary dealers and indirect bidders into following subcategories. Let \( PD_l \subseteq I_{PD}, l = 0, 1, \ldots \) be the collection of primary dealers connected to \( l \) indirect bidders. Let \( |PD_l| \in \mathbb{N}_+ \) denote the number of primary dealers with \( l \) indirect bidders connected. For each \( n = 1, 2, \ldots \)th economic environment, we assume that there are \( n \cdot |PD_l| \) primary dealers each connected to \( l \) indirect bidders.}. Each bidder demands a fixed \( M \in \mathbb{N}++ \) units of securities\footnote{This paper assumes that the bid is in terms of the security price instead of yield to simplify the calculation of a bidder payoff. Since there is one-to-one relations between the bond price and the yield, this assumption is without loss of generality.}. For each bidder

\[ M \]

of possible (unit) bids for the bid (option) that ensures the zero payoff regardless of other bidders’ bids. The Single Price Auction Mechanism.

Let \( PD \) be the set of primary dealers with \( PD \) bidders. That is, in the first building block is to define the single price auction mechanism.

\textit{Bids.} Bidder \( i \) places a bid \( b_{i,m} \in \mathbb{R}_+ \) for each \( m \in \{1, \ldots, M\} \)th unit of the security\footnote{Technically, this assumption ensures that a bidder will not play dominated strategies in the auction. This assumption is common in the literature (Athey (2001) and Reny and Zamir (2004), for example).}. Let \( \bar{b} \in \mathbb{R}_+ \) be the maximum recognized bid. Let \( \bar{b} < 0 \) be a nonparticipation bid (option) that ensures the zero payoff regardless of other bidders’ bids\footnote{The next building block is to define the single price auction mechanism.}. The set of possible (unit) bids for the \( m \)th unit is denoted by \( B_{i,m} \equiv \bar{b} \cup [0, \bar{b}] \subseteq \mathbb{R} \). Bidder \( i \)’s bid profile is an \( M \)-dimensional vector \( \overrightarrow{b}_i \equiv (b_{i,1}, \ldots, b_{i,M}) \in \mathbb{R}^M \). The set of possible bids for a bidder is \( B_i \equiv B_{i,m} \times \ldots \times B_{i,m} \). A profile of bids by all bidders is an \( M \)-dimensional vector \( \overrightarrow{b} \equiv (\overrightarrow{b}_{1}, \ldots, \overrightarrow{b}_{n \cdot (N_{PD} + N_{D} + N_{ID})}) \).

For each bidder \( i \), let \( b_{i,j} \) denote an \( M \cdot \{n \cdot (N_{PD} + N_{D} + N_{ID}) - 1\} \) dimensional vector \( b_{i,j} = (\overrightarrow{b}_1, \ldots, \overrightarrow{b}_{i-1}, \overrightarrow{b}_{i+1}, \ldots, \overrightarrow{b}_{n \cdot (N_{PD} + N_{D} + N_{ID})}) \) that are bids by bidders other than \( i \).

\textit{Order Statistics of Bids.} Given a collection of \( M \cdot n \cdot (N_{PD} + N_{D} + N_{ID}) \) bids \( \overrightarrow{b} \), we can reorder them so that \( b_{M \cdot n \cdot (N_{PD} + N_{D} + N_{ID}) \cdot M \cdot n \cdot (N_{PD} + N_{D} + N_{ID})} \leq \ldots \leq b_{M \cdot n \cdot (N_{PD} + N_{D} + N_{ID}) \cdot 1} \).

Let \( b_{M \cdot n \cdot (N_{PD} + N_{D} + N_{ID}) \cdot t} \) be the \( t \)th highest order statistics out of \( M \cdot n \cdot (N_{PD} + N_{D} + N_{ID}) \) bids.
The Market Clearing Price. Let us denote $p(b): \frac{B_i \times \ldots \times B_i}{M \cdot n \cdot (N_{PD} + N_D + N_{ID}) \text{bids}} \to B_{i,m}$ the market clearing price function when bidders submitted bids $\vec{b}$. In a single price auction, the price is set to be the lowest accepted bid: $p(b) = b_{M \cdot n \cdot (N_{PD} + N_D + N_{ID}) \cdot M \cdot n}$. 

The Allocation. Let $q_m(\vec{b}_i, \vec{b}_{-i}): \frac{B_i \times \ldots \times B_i}{\text{bider } i\text{'s bids}} \to \frac{B_i \times \ldots \times B_i}{n \cdot (N_{PD} + N_D + N_{ID}) - 1 \text{ times}: \text{bids by bidders other than } i}$ be the allocation of bidder $i$ for the $m$th unit when bidder $i$’s bid is $\vec{b}_i$ and other bidders’ bids are $\vec{b}_{-i}$. In the single price auction, $q_m(\vec{b}_i, \vec{b}_{-i}) = 1$ if $b_{i,m} > p(b)$, $= 0$ if $b_{i,m} < p(b)$, and the ties are broken using the uniform tie-breaking rule.\(^{39}\)

C. Bayesian Games.

We now define a Bayesian auction market game $\Gamma(n)$ where there are $n \cdot (N_{PD} + N_{ID} + N_D)$ bidders and the economic environment satisfies the assumptions in Section 3.A.\(^{40}\)

Bidder’s Information Set. Bidder $i$ places a bid $b_i \in B_i$ based on its private value $v_i \in V_i$. In addition, an auction system might allow some bidders to obtain additional information about other bidders before submitting their bids. Formally, let $\vec{z}_i$ be an information variable that represents the information available to bidder $i \in I$ before the submission of bids. Let $Z_i$ be the set of possible information that would be available to bidder $i$. Primary dealer $i$ has indirect bidders $\{j \in I_{ID}: PD(j) = i\}$ who will bid through primary dealer $i$. Then the primary dealer’s information additional its private value is $\vec{z}_i = \{\vec{b}_j\}_{j \in I_{ID}: PD(j) = i}$. A direct bidder and an indirect bidder will not observe other bidders’ bids. For such bidder $i$, $\vec{z}_i = \emptyset$. A primary dealer in the joint bidding system observes the private values of indirect bidders connected to the primary dealer.

Bidder’s Strategy. Bidder $i$’s (behavioral) strategy is an $M$-dimensional random variable $\vec{\beta}_i(\vec{v}_i, \vec{z}_i): V_i \times Z_i \to B_i$ over the set of possible bid profiles. Let $g_i(b_i|v_i, z_i): B_i \times V_i \times Z_i \to \mathbb{R}_+$ be the probability density function of a bid schedule $\vec{b}_i \in B_i$ when bidder $i$ bids according to the strategy $\vec{\beta}_i(\vec{v}_i, \vec{z}_i)$.

Bayesian Nash Equilibrium. A strategy profile $\vec{\beta}_n^* = (\vec{\beta}_{n,PD}^*, \vec{\beta}_{n,ID}^*, \vec{\beta}_{n,D}^*)$ is a
type-symmetric Bayesian Nash equilibrium (Athey, Levin, and Seila (2011)) if (i) every primary dealer chooses a strategy $\vec{\beta}^*_{PD,n}$, every indirect bidder chooses a strategy $\vec{\beta}^*_{ID,n}$, every direct bidder chooses a strategy $\vec{\beta}^*_{D,n}$, and, (ii) for each bidder $i \in I$, for its private value $\vec{v}_i \in V_i$ almost everywhere, for almost every realization of information $\vec{z}_i \in Z_i$, any bid $\vec{b}_i \in B_i$ that the bidder chooses with a positive probability according to $\vec{\beta}_i(\vec{v}_i, \vec{z}_i)$ maximizes the expected payoff of bidder $i$ when other bidders submit bids according to $\vec{\beta}^*_n$.

**Perfect Equilibrium.** A perfect equilibrium (Selten (1975)) is a Bayesian Nash equilibrium that is robust to noncredible threats. To define a perfect equilibrium, first consider a “smoothed game” $\Gamma(\varepsilon, n)$ where each bidder, with a given positive probability $\varepsilon > 0$, chooses a bid for each unit nonstrategically according to the uniform distribution on $B_{i,m}$, independently from a bidder’s private value, from bids for other units, and from other bidders$^{41}$. Let $\vec{\beta}^*_{\varepsilon,n}$ be a type-symmetric Bayesian Nash equilibrium of $\Gamma(\varepsilon, n)$. A type-symmetric perfect equilibrium of $\Gamma(n)$ is a type-symmetric Bayesian Nash equilibrium of $\Gamma(n)$ that is also a limit of a sequence of a type-symmetric Bayesian Nash equilibria $\{\vec{\beta}^*_{\varepsilon,n}\}_\varepsilon$ of smoothed games $\{\Gamma(\varepsilon, n)\}_\varepsilon$ as $\varepsilon \to 0$\textsuperscript{42}.

**Equilibrium Existence.** Given the private value of bidders, there exists a perfect equilibrium of $\Gamma(n)^{43}$.

Lemma 1: For each $n \in \mathbb{N}_{++}$ and the auction market game $\Gamma(n)$, there exists a perfect equilibrium $\vec{\beta}^*_n$.

The proof is in Appendix 7.A. Due to heterogeneity among bidders, there can exist multiple trembling hand perfect equilibria$^{44}$ and these equilibria may not have a close form expression. Thus we formulate the first order condition in Section 3.D to characterize equilibrium bids.

$^{41}$Technically, these perturbations and nonstrategic bids serve as a smoothing kernel of the equilibrium bids distributions. We make assumptions of uniform distributions to simplify the presentation. Other examples used in statistics are Epaneshnikov, Gaussian, and Triangular. Empirical examples of such non profit-maximizing bids could be foreign official investor’s inelastic direct or indirect bids driven by these dollar reserves (Krishnamurthy and Vissing-Jorgensen (2013)) and financial institutions’ bids to purchase Treasury securities to satisfy the regulatory requirements.

$^{42}$The notion of convergence is convergence in distribution (i.e., weak convergence).

$^{43}$Kastl (2012) discusses existence of a Bayesian Nash equilibrium in divisible goods uniform price auctions with a finite set of bids points.

$^{44}$Rustichini, Satterthwaite, and Williams (1994) numerically calculate multiple equilibria in single unit double auctions.
D. Bidder First Order Conditions.

The first order conditions quantify the trade-off faced by bidders between the benefits of submitting a higher bid (which increases the probability of winning the security) weighted against the cost of doing so (that of increasing the market clearing price). Consider an auction market game \( \Gamma(\varepsilon, n) \) and assume that bidders behavior are characterized by a Bayesian Nash equilibrium strategy profile \( \overrightarrow{\beta}^*_{\varepsilon,n} \). Take a bidder \( i \in I \) with a private value \( \overrightarrow{v}_i \in V_i \), and let \( \overrightarrow{b}^*_{\varepsilon,i,n} \in B_i \) be a bid profile that bidder \( i \) places with a positive probability according to the equilibrium strategy \( \overrightarrow{\beta}^*_{\varepsilon,i,n}(\overrightarrow{v}_i) \).

Residual Supply Curves. Bidder \( i \) will choose the bids to maximize his expected payoffs in competition with other bidders’ bids. These bids by other bidders can be summarized by the residual supply curve that will provide the available units for bidder \( i \) at each price. Since a bidder does not know bids by other bidders, the bidder can only estimate the residual supply curve based on the information available to the bidder. Let \( R_{\varepsilon,i,n} \) be a random variable that represents the residual supply curve for bidder \( i \). The information that a bidder can use to estimate the residual supply curve is different depending on whether the bidder is a direct bidder, an indirect bidder, or a primary dealer. A direct bidder does not observe bids by other bidders. Thus the direct bidder estimates the residual supply curve based on the information that other bidders’ private values are from the distribution functions \( F_i, i = PD, D, \) and \( ID \), and that these bidders bid according to an equilibrium bidding strategy \( \overrightarrow{\beta}^*_{\varepsilon,n} = (\overrightarrow{\beta}^*_{\varepsilon,PD,n}, \overrightarrow{\beta}^*_{\varepsilon,D,n}, \overrightarrow{\beta}^*_{\varepsilon,ID,n}) \). An indirect bidder also does not observe bids by other bidders. But indirect bidder \( i \) is aware that its primary dealer observes the bids and estimates the residual supply curve based on the information \((\overrightarrow{\beta}^*_{\varepsilon,n}, \beta_j(\cdot, \overrightarrow{b}^*_{\varepsilon,i,n} \cdot)_{j \in I_{PD:j=PD(i)}})\). A primary dealer observes the realization of its indirect bidders’ bids. Thus the primary dealer \( i \)’s information is \((\overrightarrow{\beta}^*_{\varepsilon,n}, \{\overrightarrow{b}^*_{\varepsilon,j,n} \}_{j \in I_{ID:j=ID(i)}})\). 45

First Order Condition. We first consider an “upward deviation” where bidder \( i \in I \) increases the bid for an \( m = 1, \ldots, M \)th unit by \( \Delta \in \mathbb{R}_{++} \) while keeping the bids for other units constant. Let \( \overrightarrow{b}'_{\varepsilon,i,n} \in B_i \) be the new bid profile of bidder \( i \) after the change in the bids. There are two events where this change in bids affects the outcome of the auction for bidder \( i \): (1) \( \overrightarrow{b}'_{\varepsilon,i,n} \) does not win the \( m \)th unit but \( \overrightarrow{b}'_{\varepsilon,i,n} \) wins the \( m \)th unit,

45In other words, \( R_{\varepsilon,i,n} \) encodes information that a bidder in a different category can have, allowing a unified treatment of the first order condition of bidders.
denoted by $\overline{W}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})$, and (2) $\overline{b}^*_{\varepsilon,i,n}$ is the pivotal bid, wins the $m$th unit, and an increase in the bid will increase the auction price, denoted by $\overline{L}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})$. Let $\Delta P(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}, \overline{b}_{\varepsilon,-i,n})$ be the change in the market clearing price when other players’ bids are $\overline{b}_{\varepsilon,-i,n}$. Then, the first order condition for an equilibrium bidding is that the gain from winning the $m$th unit obtained by increasing the bid is less than the cost from increasing the market clearing prices and payments so that the bidder does not change the bid from $\overline{b}^*_{\varepsilon,i,n}$.

Proposition 2: Consider bidder $i \in I$ with the private value $\overline{v}_i \in V_i$. Take $\overline{b}^*_{\varepsilon,i,n} \in B_i$ to be a bid that bidder $i$ places with a positive probability according to the equilibrium bidding strategy $\overline{\beta}^*_{\varepsilon,i,n}(\overline{v}_i)$. Consider an “upward deviation” where bidder $i$ increases the bid for an $m \in \{1, \ldots, M\}$th unit by $\Delta > 0$ while keeping the bids for other units constant. Let $\overline{b}'_{\varepsilon,i,n} \in B_i$ be the new bid profile of bidder $i$ after the change in the bid. Then,

$$\Pr_i(\overline{W}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n}))$$

$$\leq \Pr_i(\overline{W}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n}))$$

$$\cdot \left( v_{i,m} - \mathbb{E}_i[\Delta P(\overline{b}'_{\varepsilon,i,n}, \overline{b}^*_{\varepsilon,i,n}, \overline{b}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})] \right)$$

$$\cdot \left( (m-1) \cdot \mathbb{E}_i[\Delta P(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}, \overline{b}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})] \right)$$

$$+ \Pr_i(\overline{L}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n}))$$

$$\cdot \left( m \cdot \mathbb{E}_i[\Delta P(\overline{b}'_{\varepsilon,i,n}, \overline{b}^*_{\varepsilon,i,n}, \overline{b}_{\varepsilon,-i,n})|\overline{L}_{i,m}(\overline{b}^*_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})] \right)$$

The proof is in Appendix 7.B. The first order condition for the “downward deviation” where bidder $i \in I$ decreases the bid for the $m \in \{1, \ldots, M\}$th unit by $\Delta > 0$ while keeping the bids for other units constant can be derived using the same method (Proposition 8 in Appendix 7.F).

The first order conditions are different from Hortacsu and McAdams (2010) since this paper considers single unit auctions. Also they are different from Hortacsu, Kastl, and Zhang (2014) since this paper takes into account of indivisibility of securities offered.
E. Asymptotic Analysis

The next step of the analysis is to understand bidding behavior from the first order conditions. Given the complexity of the economic environment and the auction system, it would be difficult to solve the first order condition (1) for an equilibrium bids completely except for a simple case as in the Introduction. Then we take an asymptotic approach to show that, when there are a large number of bidders as in the US Treasury auctions, the discretions that each bidder can have to strategically manipulate bids will be small, and bidders’ bids will be close to the price taking behavior\(^{47}\).

Proposition 3: Consider a sequence of auction market games \(\{\Gamma(n)\}_{n=1,2,\ldots}\). Then, for any sequence of perfect equilibrium strategy profiles \(\{\beta^*_n\}_{n=1,2,\ldots}\) of \(\Gamma(n)\), there exists a subsequence \(\{n'\}\) such that, for each bidder \(i \in I\), for each private value \(v_i \in V_i\), for each \(m = 1,\ldots,M\)th unit, an equilibrium bidding strategy \(\{\beta^*_{i,m,n'}(v_i)\}_{n'=1,2,\ldots}\) converges to the price taking behavior as \(n' \to \infty\), and the rate of convergence is \(O(1/n)\).

The formal proof is in Appendix 7.C\(^{48}\). Proposition 3 applies to a primary dealer, a direct bidder, and an indirect bidder in each auction system. Proposition 3 will be used to derive bounds on equilibrium bids in Proposition 4.


Based on the theoretical results in Section 3, we develop a bootstrap method to estimate bidder private values from the observed market outcome, identify the bounds on bidder strategic responses to the auction mechanisms, and combine these two to forecast the outcomes from counterfactual policies.

\(^{47}\)We define that bidder \(i\) adopts the price taking behavior for the \(m\)th unit if bidder \(i\) bids the true value of the \(m\)th unit of the security \(v_{i,m}\).

\(^{48}\)Cripps and Swinkels (2006) show that it is possible to obtain the convergence of bids in private value double auctions (without primary dealers) with weaker conditions of distribution of bids and values using the large deviation theory. This result does not seem to have a counterpart in the divisible goods auction model.
A. The Bootstrap Estimator of Bidder Private Values.

Given the observed market outcome, we estimate the set of private values consistent with it using a bootstrap method\textsuperscript{49}.

**Bidder First Order Condition.** Consider bidder $i \in I$ with its private value $v_i \in V_i$ at $\Gamma(\varepsilon, n)$. Take its first order condition for the “upward deviation” (1). By solving the first order condition, we obtain an upper bound of $v_{i;m}$:

$$v_{i;m} \leq \mathbf{E}_i[P(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,i,n})]$$

$$+ (m - 1) \cdot \mathbf{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,-i,n})|\overline{W}_{i,m}(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,i,n})]$$

$$+ \frac{\Pr_i(\overline{L}_{i,m}(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,i,n}))}{\Pr_i(\overline{W}_{i,m}(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,i,n}))} \cdot m \cdot \mathbf{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,i,n})|\overline{L}_{i,m}(\overline{b}_{\varepsilon,i,n}; \overline{b}_{\varepsilon,i,n})].$$

Let us denote the upper bound (the right hand side of the inequality) by $\overline{G}_{\varepsilon,i,m,n}(\overline{b}_{\varepsilon,i,n}) \in \mathbf{R}_+$. Similarly, the first order condition for the “downward deviation” provides a lower bound $\underline{G}_{\varepsilon,i,m,n}(\overline{b}_{\varepsilon,i,n}) \in \mathbf{R}_+$. Then, the identified set of private values for the $m$th unit consistent with the observed bid is $\{v_{i,m} \text{ such that } \underline{G}_{\varepsilon,i,m,n}(\overline{b}_{\varepsilon,i,n}) \leq v_{i,m} \leq \overline{G}_{\varepsilon,i,m,n}(\overline{b}_{\varepsilon,i,n}) \subseteq V_{i,m}\}$\textsuperscript{50}. Proposition 2 ensures that these upper and lower bounds converge to $v_{i,m}$ as $n \to \infty$.

**Bootstrap Estimator of Bounds.** It would not be possible to derive the lower bound and the upper bound analytically. Then the inference is based on the empirical analogue using the smoothed bootstrap\textsuperscript{51}. Consider bidder $i$ with a bid $b^*_{\varepsilon,i,m,n} \in B_{i,m}$ for an $m = 1, ..., M$th unit. To calculate the bounds, we first estimate the residual supply. The residual supply is based on direct bidders’ bids, primary dealers’ bids, and indirect bidders’ bids. We consider subcategories where each one consists of primary dealers connected with the same number of indirect bidders. With the assumption of type symmetric independent distributions of private values and equilibrium bidding strategies,

\textsuperscript{49}The estimation using empirical analogue of the first order condition is developed in Guerre, Perrigne, and Vuong (2000), Hortacsu (2002), Tamer (2003), Chernozhukov, Hong, and Tamer (2007), Ciliberto and Tamer (2009), and Hortacsu and McAdams (2010). Since the model takes a discrete unit approach, the estimation procedure is different from the ones in Hortacsu and Kastl (2012), Kazumori and Tchuindjo (2014), and Hortacsu, Kastl, and Zhang (2014).

\textsuperscript{50}In case we cannot explicitly solve for the upper bound and the lower bound function from the moment conditions, we follow the criterion function approach of Chernozhukov, Hong, and Tamer (2007).

\textsuperscript{51}Another approach could be to approximate them with the first order asymptotic theory.
bids by a primary dealer and by connected indirect bidders within each subcategory are iid. Thus we can apply the standard bootstrap method (Bickel and Freedman (1981)) to estimate the distribution of equilibrium bids for each subcategory. Then, putting them together, we can estimate the residual supply that bidder $i$ faces. Second, using the estimated distribution of the residual supply, we can calculate the probabilities of the events that the change in the bid will affect the outcome and the price impacts in each case. An $\varepsilon$-smoothing can reduce the variance of quantile estimators. Combining them, one obtains an estimate of the bounds. The bootstrap approximation to the $1 - \alpha$ level critical points can be used in the construction of confidence intervals. The details of the estimation procedure are described in Appendix 7.D.

B. Partial Identification of Bidder Strategic Responses to Counterfactual Policies

The previous section developed the identification set for bidder private values. The next step is to characterize a bidder’s strategic responses to counterfactual auction systems given the bidder private values from the first order condition$^{52}$.

The First Order Condition. Consider an auction market game $\Gamma(\varepsilon, n)$ and suppose that bidders bidding behavior are characterized by an equilibrium bidding strategy profile $\beta^*_\varepsilon, n$. Consider bidder $i \in I$ with the private value $v_i \in V_i$. From the first order condition for the “upward deviation” (1) we have:

\[
v_{i,m} - (m - 1) \cdot E_i[\Delta P(b^*_\varepsilon,i,n, b^*_\varepsilon,i,n, b^*_\varepsilon,\varepsilon,-i,n) | W_{i,m}(b^*_\varepsilon,i,n, b^*_\varepsilon,i,n)]
- (E_i[\Delta P(b^*_\varepsilon,i,n, b^*_\varepsilon,\varepsilon,-i,n) | W_{i,m}(b^*_\varepsilon,i,n, b^*_\varepsilon,i,n)] - b^*_\varepsilon,i,m,n)
- \frac{Pr_i(L_{i,m}(b^*_\varepsilon,i,n, b^*_\varepsilon,i,n))}{Pr_i(W_{i,m}(b^*_\varepsilon,i,n, b^*_\varepsilon,i,n))} \cdot m \cdot E_i[\Delta P(b^*_\varepsilon,i,n, b^*_\varepsilon,\varepsilon,-i,n) | L_{i,m}(b^*_\varepsilon,i,n, b^*_\varepsilon,i,n)]
\leq b^*_\varepsilon,i,m,n.
\]

Intuitively, a bidder bids below the true value of a security as long as lowering a bid reduces the price and payment without losing the security, but not more than that. Let us denote this lower bound (i.e., the left hand side of the inequality) by $H_{\varepsilon,i,m,n}(v_i) \in$.

$^{52}$Recently, Satterthwaite, Williams, and Zachariadis (2014) consider the buyer’s bid double auctions and derive the asymptotic analysis of the first order condition to obtain an approximate solution of the offset strategy. The formula is similar to the one in Proposition 3 and contained in the interval in Proposition 3. One can conduct an alternative analysis using Satterthwaite, Williams, and Zachariadis (2014) and can derive the qualitatively same result.
Given that a bidder does not employ a dominated bid, the identified set of bids consistent with the strategic behavior is \( \{ b_{\varepsilon,i,m} : H_{\varepsilon,i,m,n}(\overline{v}_i) \leq b_{\varepsilon,i,m} \leq v_{i,m} \} \subseteq B_{i,m} \).

**Bounds on Equilibrium Bids.** We now employ Proposition 8 to derive the lower bound under various auction systems:

**Proposition 4:** Consider an auction market game \( \Gamma(\varepsilon,n) \). Consider a bid increment \( \Delta \in \mathbb{R}_{++} \). In the primary dealer system, bidder \( i \in I \) with the private value \( v_{i,m} \in V_{i,m} \) for an \( m \in \{1, ..., M\} \)th unit has a bid from an interval \([v_{i,m} - \Delta - (2m - 1) \cdot \varepsilon^{-M/(n(N_{PD} + N_{D} + N_{ID})-1)+1}, v_{i,m}] \subseteq B_{i,m} \) if the bidder is a direct bidder or an indirect bidder, and an interval \([v_{i,m} - \Delta - (2m - 1) \cdot \min\{\varepsilon^{-M/(n(N_{PD} + N_{D} + N_{ID})-1)+1}, \{ b_{\varepsilon,j,m,i \in PD(j), m=1,...,M; b_{\varepsilon,j,m} \geq v_{i,m} \} \} \leq v_{i,m} \}] \subseteq B_{i,m} \) if the bidder is the primary dealer. In the direct bidding system, bidder \( i \in I \) will bid from an interval \([v_{i,m} - \Delta - (2m - 1) \cdot \varepsilon^{-M/(n(N_{PD} + N_{D} + N_{ID})-1)+1}, v_{i,m}] \). In the joint bidding system, primary dealer \( i \in I_{PD} \) will bid, representing a consortium, from an interval \([v_{i,m} - \Delta - (2m - 1) \cdot \varepsilon^{-M/(n(N_{PD} - 1)+1)}, v_{i,m}] \).

A formal proof of Proposition 4 is in Appendix 7.E. These lower bounds do not depend on equilibrium selections. Intuitively, the degree that a bidder will reduce the bid depends on the level of competition in the auction. For example, \( \varepsilon^{-M/(n(N_{PD} + N_{D} + N_{ID})-1)+1} \) is the amount that a bidder can increase the bid to win over the smoothing bids while keeping the price impact minimum. This amount will decrease as the number of bidders \( n \) increases. The term \( \min\{b_{\varepsilon,j,m,i \in PD(j), m=1,...,M; b_{\varepsilon,j,m} \geq v_{i,m} \} \} \) is the amount that the primary bidder can increase the bid to win over the indirect bidders with the minimum price impacts\(^{53}\). We would also take the view that these intervals are the set of bids that the auctioneer can expect from a bidder given the private value. Thus the width of the interval represents the auctioneer’s uncertainty about the bidder’s bids, thus is some indication of a risk of an auction.

**Comparison between the Primary Dealer System and the Direct Bidding System.** The key difference between the primary dealer system and the direct bidding system comes from the primary dealer’s observation of indirect bidders’ bids. In the primary dealer system, the dealer observe indirect bidders’ bids and mirror them. In direct bidding system, the primary dealer does not observe indirect bidders’ bids and reduce

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\(^{53}\)When there are sufficiently many bidders in the auction, according to Proposition 3, bids by direct bidders, indirect bidders, and primary dealers bids are approximated by the price taking behavior with the rate \( O(1/n) \). As such, including the bounds from these bids would not affect the comparison among auction systems in the first order.
bids to match (approximately) their expected bids. When the realization of indirect bidders’ bids are higher than the expected value, the dealer shades the bids less in the primary dealer system. But if there are intense competitions, when the realized indirect bidders’ bids are low, the primary dealer would not be able to shade the bids much since it will lose the security. In other words, an observation of indirect bidders’ bids, that is, more accurate information on the residual supply, can limit the dealer’s bid shading and make its bid more aggressive.\footnote{We now illustrate this point using a very simple example. Suppose that the seller has two units to sell and the seller employs a uniform price auction. Consider a dealer with the private value 10 for the security. Let $b_{PD}$ be the bid by the dealer. Suppose, in addition, that the dealer is on the margin, i.e., there is another bidder whose bid is above 10 (for sure) and that the dealer is pivotal. Suppose that there are $n$ indirect bidders whose bids are iid from uniform $[0, 1]$. Then the primary dealer intends to reduce the bid till it matches the highest bid from indirect bidders. Let $b_{ID,\text{max}}$ be the highest bid from indirect bidders.}

Empirically, Office of Debt Management (2012a) finds that the primary dealers place the most aggressive bids over indirect and direct bidders (after taking out the bids submitted to fulfill their pro-rata bidding requirement) that matches the pattern of bids.\footnote{Similarly, in OCS lease auctions, Hendricks, Porter and Wilson (2004) that the distribution of the informed bidders’ bids stochastically dominates the distribution of uninformed bidders’ bids. In the timber auctions, Athey, Levin, and Seila (2011) find that mills (large firms with manufacturing capacity) submit high bids and win disproportionately.}

Comparison between the Direct Bidding System and the Indirect Bidding System.

Let us compare the amounts that the dealer would shade the bid under the primary dealer system and the direct bidding system. In the primary dealer system where the dealer can observe indirect bidders’ bids, the dealer would shade the bid to match the highest bid among indirect bidders and the dealer can actually observe the highest bid. In the direct bidding system, the dealer does not observe any of indirect bidders’ bids and has to make an inference. Note that the distribution of the maximum of $n$ bids among indirect bidders is $F_{b_{ID,\text{max}}}(b_{ID,\text{max}}) = b_{ID,\text{max}}^n$. The primary dealer wins only when its bid is higher than $b_{ID,\text{max}}$ and in that case it pays $b_{PD}$ for the security. Thus the dealer chooses the bid $b_{PD}$ to maximize the expected profit $\Pr(b_{PD} > b_{ID,\text{max}}) \cdot (1 - b_{PD}) = b_{PD}^n \cdot (1 - b_{PD})$. The first order condition is $n \cdot b_{PD}^{n-1} - (n + 1) \cdot b_{PD}^n = 0$. Thus $b_{PD} = \frac{n}{n + 1}$. In other words, in the direct bidding system, the dealer shades the bid to match the expected highest bids from indirect bidders.

Now consider the probability that the amount that the dealer shades the bid in the primary dealer system is less than the amount that the dealer shades in the direct bidding system:

$$\Pr \left( \frac{n}{n + 1} \right) = 1 - \left( \frac{n}{n + 1} \right)^n \to 1 \text{ as } n \to \infty.$$
The difference between these two systems come from the fact that the joint bidding system has the lowest level of competition among \( n \cdot N_{PD} \) consortium. It will give a bidding consortium more discretion to shade their bids\(^{56}\).

Counterfactual Policy Experiments. Combining the identification of bidder private values and the above identification of bidder strategic responses to counterfactual policies allows us to simulate the auction outcome. We first sample bidder private values from the identification intervals obtained from the bootstrap estimation. For each sampled private value, for each auction system, we can obtain the interval of equilibrium bids. By sampling bids from this interval, we can simulate the outcome of the auction. This procedure will provide confidence intervals of the market outcomes for each auction system. We interpret that one auction system can be more volatile than the other if the identified set of outcomes from one auction system is wider than the other. The next section will quantify these effects.

5. The Primary Dealer System in the US Treasury Auctions.

This section studies the US Treasury auctions based on the framework developed in Section 3 and 4.

A. Changes in the Primary Dealers Behavior during the Financial Crisis.

We first examine the change in the economic environment during the period of financial crisis using the publicly available data.

Data. The data used in this section comes from Treasury Securities Offering Announcement, Treasury Auction Result, and Investor Class Auction Allotments. The Treasury Auction Results describe the distribution of securities among primary dealers, indirect bidders, direct bidders, and noncompetitive bids. The Investor Class Auction Allotment data show the distributions of securities among depository institutions, deal-

\(^{56}\)Cheng (2015) consider the model where the winner in the secondary market can resell the good to the buyers in the secondary market and shows that including a bidder from the secondary market to the primary market can increase the revenue for the seller since it would reduce the rents of the intermediary.
ers and brokers, investor funds, foreign and international, individuals, and others. We consider records of 1584 auctions from April 1, 2008 to December 19, 2013. We then compare the results with Fleming (2007) who studied Treasury auctions during the period between May 5, 2003 and December 28, 2005.

**Decreases in the Primary Dealer Shares.** In 2009-13, primary dealers account for 56.6% of securities, down from 70.9% in 2001-05 (see Table 1). Primary dealers have bought as little as 14.0%, in contrast to 33.6% of an issue in previous years. Figure 2 shows that the dealer shares for cash management bills are 69% in contrast to 93.1% in 2001-05, 61.7% in contrast to 84.7% for 4 week bills, 60.9% in contrast to 66.3% for thirteen-week bills, and 52.1% in contrast to 63.1% for 26-week bills. That is, primary dealers have higher shares in securities with short term maturity, and their shares have dropped more for these short maturity securities.

**Increases in the Investment Fund Shares.** Table 3 from the Investor Class Data shows that Investment funds now have 18.0%, up from 6.5%. Investment fund shares tend to increase with maturity, with an average of 16.2% (up from 3.2%) for cash management bills and 27% (up from 13.7%) for ten-year notes.

**Increase in the Direct Bidding.** Direct bidders other than primary dealers buy 9.1%, an increase from 2.4%. Direct bidders have bought as much as 45.4% instead of a previous 31.6% of an issue. Table 4 shows that increases in direct bidding comes from: depository institutions who get the allotment from direct bidding and indirect bidding evenly, dealers and brokers other than primary dealers who get 80% of the allocation from direct bidding instead of 48% in Fleming (2007), pension and retirement funds get 30% of allocation from direct bidding that is also an increase from Fleming (2007),

57 “Depository Institutions” include banks, savings and loan associations, credit unions, and commercial bank investment accounts. “Individuals” include individuals, partnerships, personal trusts, estates, non-profit and tax-exempt organizations, and foundations. “Dealers and Brokers” include primary dealers, other commercial bank dealer departments, and other non-bank dealers and brokers. “Pension and Retirement Funds and Insurance Companies” include pension and retirement funds, state & local pension funds, life insurance companies, casualty and liability insurance companies, and other insurance companies. “Investment Funds” include mutual funds, money market funds, hedge funds, money managers, and investment advisors. “Foreign and International” include private foreign entities, non-private foreign entities placing tenders external of the Federal Reserve Bank of New York (FRBNY), and official foreign entities placing tenders through FRBNY.

58 For an earlier period, US Treasury et al. (1992) report that during the period of January 1990 through Setember 1991, primary dealers were awarded about 72 percent, customers were awarded 5 percent, and noncompetitive awards accounted 20 percent of privately awarded Treasury securities. The top 10 firms (out of primary dealers and customers) combined 50 percent of total private awards in bill auctions and 66 percent in note and bond auctions. This pattern is consistent with the one in Fleming (2007).
investment funds who get 16% which is an increase from 3% in Fleming (2007), and Foreign and international who get about 10% of allocation also a significant increase. Figure 6 finds that, first, for securities with short and medium term maturities, the dealer and brokers have majority of direct bidding, although the share of investment funds have increased. Second, for securities with longer maturities, the investment funds are often the majority of direct bidding, in contrast to the previous periods when the dealers and brokers had majorities.

**Effects of Financial Crisis.** These changes are related to the financial crisis. First, the decrease in primary dealers share for securities with shorter maturities is consistent with the view that the primary dealers have less capability for hedging because of the balance sheet effect. Second, for investment funds, Vayanos (2004) presents a mechanism that investment fund managers subject to withdrawals when fund performance falls below a threshold has preference for liquidity that is time-varying and increasing with volatility. Empirically, Beber, Brandt, and Kavajecz (2009) show that the fund flows in Euro-area government bond markets are driven in liquidity at the time of financial stress.

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59 Figure 5 shows that, first, for securities with a short period of maturity, the share of investment funds have increased and the shares of dealers and brokers for indirect bidding have decreased. Second, for securities with the medium maturity, the foreign and international purchases play a large role as in the previous years, but the share of investment funds have increased. Finally, for securities with long maturity, investment funds play a large role in contrast to the previous period when Foreign and International were majorities.

60 The chart is based on regressions of 3 bidder categories on 6 investor classes with other dealers and brokers excluding the primary dealer, and without individuals. We conduct regression of direct bidding shares on investor classes while excluding individuals. The qualitative results are robust to other specifications with different categories for bids and investors.

61 Graveline and McBrady (2011) show that dealers’ demand for Treasury securities to hedge their inventories of the corporate bond and mortgage-backed securities have been reduced after the financial crisis. Fontaine and Garcia (2011) find that increases in funding liquidity lower risk premia for Treasury securities. Lou, Yan, and Zhang (2011) find that the price impact of Treasury auctions is stronger when the total risk expected by the primary dealers is higher. Cassola, Hortacsu, and Kastl (2012) study the ECB auctions for one week loans and find that the bank’s bids reflect the funding costs from the interbank markets (and strategic responses to these situations). Adrian, Burke, and McAndrews (2009) describe the Federal Reserve’s Primary Dealer Credit Facility to provide overnight loans to primary dealers.

62 Another possibility is that the Treasury provided direct bidding status to investors to enhance participation in Treasury auctions. In a press article, Fliter (2011) reports that the US Treasury provided the People’ Bank of China a direct computer link to its auction system in 2011.
B. Counterfactual Analysis of Auction Systems.

As is seen in the Introduction, these changes in the bidding environments during the financial crisis have led to a question concerning the optimal distribution mechanism of Treasury securities. We now examine this question by conducting a counterfactual analysis comparing the primary dealer system, the direct bidding system, and the joint bidding system. As an example, we consider the estimated private values from one auction of 10 year notes held in 2012\(^{63}\). We set \(\Delta = 5\) (in terms of prices) and \(\varepsilon = 0.005\(^{64}\). We simulated bids based on the procedure explained in Section 4 and then calculate the distribution of market prices.

**Prices.** Figure 8, 9, and 10 show that the primary dealer system has the highest mean and the lowest volatility of the market prices. Also the primary dealer system has the lowest CaR (the maximum interest cost that can be realized in a given auction at a given confidence level\(^{65}\)) among all the auction systems.

**Allocations.** The allotment for the primary dealer is 2% higher than with the direct bidding system with the primary dealer system. But the standard deviation of the primary dealer allotment under a primary dealer system is 27% lower than under the direct bidding system and 40% lower in comparison with the joint bidding system. This result is consistent with the view that the primary dealer has a better estimate of the residual supply under the primary dealer system that leads to less uncertainty.

**Information Advantage of the Primary Dealers.** The information advantage of primary dealers is much lower in the US Treasury auctions than in Canadian Treasury auctions. This result is consistent with a recent finding in Hortacsu, Kastl, and Zhang (2014). The standard deviation of the primary dealer profits is 22% lower than the direct bidding system.

6. Conclusion.

US Treasury securities markets play a crucial role in the US and world economy. The research question of this paper is to examine the role of the primary dealer system

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\(^{63}\)The estimation was conducted in December 2013 in preparation for the presentation at the Winter Meeting of the Econometric Society in January 2014. Hortacsu, Kastl, and Zhang (2014) also conduct an estimation with a single auction to deal with auction heterogeneity.

\(^{64}\)It will correspond to about 10 basis point in terms of yields in these auctions. The quantitative results presented in this paper are robust to various parameter values.

\(^{65}\)For example, Risbjerg and Holmlund (2004).
in the primary market auction. This question is important since recent developments in the economic environment after the financial crisis have led to the reconsideration of the security distribution method.

The main idea of the paper is that the primary dealer system mirrors the bids by indirect bidders in the process of routing the bids and reduce the volatility of the bids and selling prices. The main results are that the primary dealer system contributes to the debt management objective of minimizing the cost of financing over time by realizing the balance of competition and price stability when the integrity of the Treasury auction system is maintained by the objective verification of the indirect bidders’ bids. The counterfactual analysis shows that the primary dealer system achieves the lowest price volatilities in comparison with the policy alternatives of the joint bidding system and the direct bidding system. At the time of the financial crisis where the primary dealers’ capacity and demands are limited due to the balance sheet effects, the primary dealer system could have reduced the interest rate risk, enhanced the demand from investment funds who value risklessness and safety of Treasury securities, and contributed to the financial stability.

The contributions of the paper are that this paper develops a full equilibrium model of the primary dealer system with uniform price auctions of discrete units and quantifies the effectiveness of the primary dealer system to the public debt management objective.

The analysis of this paper further could be extended to study the following policy questions. The first question is “Does the optimality of the primary dealer system hold in more general environment?” To answer this question, it would be helpful to examine the allocation, profits, and bid distributions of the primary dealer, direct bidder, and indirect bidders in the auction\footnote{It would be meaningful to compare these results with the ones in OCS auctions in Hendricks and Porter (1988) and timber auctions in Athey, Levin, and Seila (2011).} and conduct counterfactual analysis for a large class of auctions. The second policy question is to “What are the determinant of bidder participation and demands in the Treasury auctions?” Previously Krishnamurthy and Vissing-Jorgensen (2013) find that investors value the liquidity and safety of U.S. Treasury bonds. It would be interesting to examine the economic determinants of primary dealers and other bidders’ bidding behavior and the bid-to-cover ratio. It is hoped that these researches contribute to the debt management objectives.
REFERENCES


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### 7. Appendix.

This Appendix provides proofs for Proposition 1-8 and the description of the bootstrap estimation procedure. This Appendix is not intended for publications.

#### A. Proof of Proposition 1.

We proceed in following steps: we first consider a smoothed auction game \( \Gamma(\varepsilon, n) \) where \( \varepsilon > 0 \) is a smoothing parameter. We then develop a finite approximation \( \Gamma(\Delta, \varepsilon, n) \) where \( \Delta > 0 \) is the minimum bid grid size and multiples. We construct a Bayesian Nash equilibrium \( \overrightarrow{\beta}_{\Delta, \varepsilon, n}^* \) of \( \Gamma(\Delta, \varepsilon, n) \). By taking the bid grid size \( \Delta \to 0 \), we show existence of a Bayesian Nash equilibrium \( \overrightarrow{\beta}_{\varepsilon, n}^* \) of a smoothed game \( \Gamma(\varepsilon, n) \). Then we take \( \varepsilon \to 0 \) and it will lead to existence of a perfect equilibrium \( \overrightarrow{\beta}_n^* \).

*Smoothed Auction Game* \( \Gamma(\varepsilon, n) \). We would like to prove existence of a perfect equilibrium \( \overrightarrow{\beta}_n^* \) of \( \Gamma(n) \). To apply the definition of a perfect equilibrium, we consider a sequence of perturbations \( \{\varepsilon_l\}_{l=1,2,...} \in \mathbb{R}_{++} \) such that \( \varepsilon_l \to 0 \) as \( l \to \infty \). Then, for each \( \varepsilon_l > 0 \), let \( \Gamma(\varepsilon_l, n) \) be an auction market game where, with probability \( \varepsilon_l \), a bid is chosen according to the uniform distribution on \( B_i \).
Finite Approximation of $\Gamma(\varepsilon_l, n)$. Fix $\varepsilon_l \in \mathbb{R}_{++}$. Consider a smoothed game $\Gamma(\varepsilon_l, n)$. At $\Gamma(\varepsilon_l, n)$, a bidder’s profit function is not continuous when a bidder’s bid is tied with other bidders’ bids. Thus we cannot apply Glicksberg (1952) Theorem 1 to show existence of a mixed strategy Bayesian equilibrium in $\Gamma(\varepsilon_l, n)$. Therefore, we first define a finite approximation $\Gamma(\Delta, \varepsilon_l, n)$ with the bid grid size $\Delta > 0$. At $\Gamma(\varepsilon_l, n)$, each bidder submits a bid for each unit from a set $B_{i,m} = \mathbb{B} \cup [0, \bar{b}]$. Consider a finite approximation of $B_{i,m}$ by $B_{\Delta,i,m} = \mathbb{B} \cup \{0, \Delta, 2\Delta, ..., \bar{b}\}$ where $\Delta > 0$ is the minimum bid grid size. Then, for each $\Delta > 0$, let $\Gamma(\Delta, \varepsilon_l, n)$ be a finite approximation of $\Gamma(\varepsilon_l, n)$ where each bidder submits a bid for each unit from a set $B_{\Delta,i,m}$. Since $\Gamma(\Delta, \varepsilon_l, n)$ has a finite set of actions, by Glicksberg (1952) Theorem 1, there exists a mixed strategy Bayesian Nash equilibrium $\overline{\beta}^*_{\Delta,l, n}$ for $\Gamma(\Delta, \varepsilon_l, n)$.

Bayesian Nash Equilibrium of $\Gamma(\varepsilon_l, n)$. Consider a sequence of perturbations $\{\Delta_k\}_{k=1,2,...} \in \mathbb{R}_{++}$ with $\Delta_k \to 0$ as $k \to 0$. For each $\Delta_k > 0$, consider a finite approximation $\Gamma(\Delta_k, \varepsilon_l, n)$ and its Bayesian Nash equilibrium $\overline{\beta}^*_{\Delta_k,l, n}$. Then, by Prohorov’s theorem, there exists a subsequence $\{\Delta_{k'}\}_{k'=1,2,...}$ and a limit strategy profile $\overline{\beta}^*_{l, n}$ such that $\overline{\beta}^*_{\Delta_k,l, n} \to \overline{\beta}^*_{l, n}$ weakly as $\Delta_{k'} \to 0$. Suppose $\overline{\beta}^*_{l, n}$ involves a tie with a positive probability: that is, there exists a positive set of private values such that bidders with such private values places the same bid for some units with positive probabilities. Then, among such bidders, since no bidder will place a dominated bid, it would be that a bidder with a higher private value will prefer to increase the bid by some amount to win the tie. This deviation will be a profitable deviation from $\overline{\beta}^*_{\Delta_k,l, n,i}$ for $\Gamma(\Delta_k, \varepsilon_l, n)$ for sufficiently small $\Delta_k$. Therefore, it cannot be that $\overline{\beta}^*_{l, n}$ involves a tie with a positive probability. Then since $\overline{\beta}^*_{\Delta_k,l, n}$ is an equilibrium of $\Gamma(\Delta_k, \varepsilon_l, n)$, by taking a limit of the equilibrium conditions, $\overline{\beta}^*_{l, n}$ is a Bayesian Nash equilibrium of $\Gamma(\varepsilon_l, n)$.

Perfect Equilibrium of $\Gamma(n)$. Now consider a sequence of perturbations $\{\varepsilon_l\}_{l=1,2,...} \in \mathbb{R}_{++}$ with $\varepsilon_l \to 0$ as $l \to \infty$. For each $\Gamma(\varepsilon_l, n)$, consider its Bayesian Nash equilibrium $\overline{\beta}^*_{\varepsilon_l, n}$ that is shown to exist through an analysis in the previous paragraph. Then, by Prohorov’s theorem, there exists a subsequence $\{\varepsilon_{l'}\}_{l'=1,2,...}$, and a strategy profile $\overline{\beta}^*_{n}$ such that $\varepsilon_{l'} \to 0$ and that $\overline{\beta}^*_{\varepsilon_{l'}, n} \to \overline{\beta}^*_{n}$ weakly as $\varepsilon_{l'} \to 0$. Then, by following the argument in the previous paragraph, one can show that $\overline{\beta}^*_{n}$ does not involve ties with a positive probability. Then, also by applying the argument in the previous paragraph,

\footnote{See Reny and Zamir (2004), Jackson and Swinkels (2005), Kazumori (2013) for a complete argument regarding the resolution of ties in the private value model.}
is a Bayesian Nash equilibrium of \( \Gamma(n) \). Then, \( \tilde{\beta}^*_n \) satisfies the requirement for a perfect equilibrium of \( \Gamma(n) \).

**B. Proof of Proposition 2.**

Consider an auction market game \( \Gamma(\varepsilon, n) \) and suppose that bidders’ behavior are described by a Bayesian Nash equilibrium strategy profile \( \tilde{\beta}^*_{\varepsilon,n} \). Let us consider a decision problem of bidder \( i \) with private value \( \overline{V}_i \). Bidder \( i \) would be choosing a bid according to an equilibrium bidding strategy \( \tilde{\beta}^*_{i,n}(\overline{V}_i) \). Let \( \tilde{b}^*_{i,n} \) be a bid bidder \( i \) places with a positive probability according to a bidding strategy \( \tilde{\beta}^*_{i,n}(\overline{V}_i) \). Then \( \tilde{b}^*_{i,n} = (\tilde{b}^*_{i,1,n}, \ldots, \tilde{b}^*_{i,M,n}) \) where \( \tilde{b}^*_{i,m,n} \) would denote the bid for the \( m \)-th unit.

Consider an “upward deviation” where bidder \( i \) increases the bid for the \( m \)-th unit by \( \Delta > 0 \) while keeping the bids for other units constant. Let \( \tilde{b}'_{i,n} \) be a new bid profile after the upward deviation. That is, \( \tilde{b}'_{i,n} = (\tilde{b}^*_{i,1,n}, \ldots, \tilde{b}^*_{i,m-1,n}, \tilde{b}^*_{i,m,n} + \Delta, \tilde{b}^*_{i,m+1,n}, \ldots, \tilde{b}^*_{i,M,n}) \). Then there are 2 cases that increasing the bid to \( \tilde{b}'_{i,n} \) will affect the outcome for bidder \( i \). The first case is the case that bidder \( i \) is able to win the \( m \)-th unit of the security after the increase of the bid from \( \tilde{b}^*_{i,n} \) to \( \tilde{b}'_{i,n} \). Let \( \tilde{W}_{i,m}(\tilde{b}^*_{i,n}, \tilde{b}'_{i,n} | R_{i,n}) \) be the event where other bidders’ private values are such that the above situation will take place when bidder \( i \) has information \( R_{i,n} \). When the event \( \tilde{W}_{i,m}(\tilde{b}^*_{i,n}, \tilde{b}'_{i,n} | R_{i,n}) \) takes place, the bidder obtains profits from winning the \( m \)-th unit, but the bidder might need to pay a higher price for the security since the upward deviation may increase the market clearing price. We now calculate these terms one by one.

**Payoff from Winning the \( m \)-th Unit.** First, let us denote \( P_i(\tilde{W}_{i,m}(\tilde{b}^*_{i,n}, \tilde{b}'_{i,n} | R_{i,n})) \) as the probability of the event \( \tilde{W}_{i,m}(\tilde{b}^*_{i,n}, \tilde{b}'_{i,n} | R_{i,n}) \). When bidder \( i \) wins the \( m \)-th unit, bidder \( i \) collects the value \( v_{i,m} \) from the \( m \)-th unit. In this case, the price that bidder \( i \) pays is, if other bidders’ bids are \( \tilde{b}^*_{-i,n} \), by definition, \( P(\tilde{b}'_{i,n}, \tilde{b}^*_{-i,n}) \) where \( P(\tilde{b}'_{i,n}, \tilde{b}^*_{-i,n}) \) function, defined in Section 3A, returns the market clearing price when the profile of the bids is \( (\tilde{b}'_{i,n}, \tilde{b}^*_{-i,n}) \). But of course, bidder \( i \) does not observe the bids by other bidders, and the probability that the other bidders’ bids is

---

\(^{68}\)We note that the probability \( P_i(\tilde{W}_{i,m}(\tilde{b}^*_{i,n}, \tilde{b}'_{i,n} | R_{i,n})) \) depends on the bidder’s identity (categories, to be exact) since the estimates of the residual demand curve will be different depending on whether the bidder is a primary dealer, direct bidder, or an indirect bidder. Furthermore, we would like to note that the probability does not depend on the realization of bidder \( i \)'s private value. It is because of the assumption that the distribution of private values among bidders is independent: a bidder’s estimates of the other bidders’ values do not depend on the specific realization of a bidder’s private value.
\( \overline{b}^{*}_{\varepsilon,-i,n} \) depends on the distribution of the residual demand curve conditional on the event \( W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n}) \). Consequently, the expected price that bidder \( i \) would pay is \( E_i[P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,-i,n})|W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})] \). Then, given the assumption of quasi-linear preferences, bidder \( i \)'s expected payoff from the \( m \)th unit is\(^{69} \)

\[
P_i(W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})) \\
\cdot (v_{i,m} - E_i[P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})|W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})].
\]

**Higher Payment for Other \( m - 1 \) Units of Security.** When bidder \( i \) wins the \( m \)th unit as a result of placing a higher bid for the \( m \)th unit, bidder \( i \) may need to pay higher prices for the other \( m - 1 \) units of security that bidder \( i \) would be allotted. We now calculate the change in bidder \( i \)'s payment. First, the probability of the event is, as in the previous case, \( P_i(W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})) \). Let us define the change in the market clearing price from increasing the bid from \( \overline{b}^{*}_{\varepsilon,i,n} \) to \( \overline{b}'_{\varepsilon,i,n} \) when other bidders' bids are \( \overline{b}^{*}_{\varepsilon,-i,n} \) to be \( \Delta P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}, \overline{b}^{*}_{\varepsilon,-i,n}) = P(\overline{b}'_{\varepsilon,i,n}, \overline{b}^{*}_{\varepsilon,-i,n}) - P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}^{*}_{\varepsilon,-i,n}) \). Since bidder \( i \) is not informed of the bids by other bidders at the time of bidding and bidder \( i \) is risk-neutral, bidder \( i \) estimates its expected payment conditional on the event \( W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n}) \) by \( E_i[\Delta P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}, \overline{b}^{*}_{\varepsilon,-i,n})|W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})]. \) By assumption of nonincreasing bids, bidder \( i \) would need to pay for the increases in the price for the other \( m - 1 \) units. Thus the change in the expected payment is

\[
(m - 1) \cdot E_i[\Delta P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}, \overline{b}^{*}_{\varepsilon,-i,n})|W_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})].
\]

**The Payment Increase in the Second Case.** We now consider the second case where increasing the bid to \( \overline{b}'_{\varepsilon,i,n} \) changes the outcome of the auction in a following situation: bids by other bidders are such that bidder \( i \) already wins the \( m \)th unit with the bid \( \overline{b}^{*}_{\varepsilon,i,n} \), thus increasing the bid will not affect the allocation but will increase the market clearing price, thus bidder \( i \) has to pay more. Let \( T_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n}) \) be the event such that the other bidders' private values are such that the above case takes place given bidder \( i \)'s information \( R_{\varepsilon,i,n} \). Then, following the reasoning in the previous case, the change in the payment is

\[
m \cdot E_i[\Delta P(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}, \overline{b}^{*}_{\varepsilon,-i,n})|T_{i,m}(\overline{b}^{*}_{\varepsilon,i,n}, \overline{b}'_{\varepsilon,i,n}|R_{\varepsilon,i,n})].
\]

\(^{69}\)Here, again, given bidder \( i \)'s bids, the expected value does not depend on the specific realization of bidder \( i \)'s private value by the assumption of independent private values.
Putting All Together. By combining (2), (3), and (4), we get the first order condition (1). We note that this formulation applies to bidders in all categories (because different information that a bidder in a different category has is encoded in the residual supply curve $R_{\varepsilon,i,n}$).

C. Proof of Proposition 3.

Outline of the Proof. Our objective is to show that, when bidders employ perfect equilibrium strategies, then, as the number of bidders in the market increases, the bid will converge to the price taking behavior where a bidder’s bid is its private value for the security. Our approach is first to approximate a perfect equilibrium with Bayesian Nash equilibria of the smoothed games. The reason we introduce a perturbation is that it will smooth the residual supply curve. This smoothness, that is, absence of gaps in the residual supply curve, will allow us to bound the bidder strategic behavior as the number of market participants grows large. Then, as the number of market participants increases, the price impact will vanish and a bidder’s bid will converge to the price taking behavior as the number of market participants increases. Then, since the limit strategy profile is the price taking behavior for each perturbation, as the probability of perturbation goes to zero, the equilibrium bids converge to the price taking behavior.

Interchanging the Order of Limits. In the above argument, we first increase the number of market participants with the fixed probability of perturbations and then take the probabilities of perturbation to zero. But in the perfect equilibrium, we first take the probability of perturbations to zero and then increase the number of bidders in the auction. Thus we need to make sure that the order of limits can be interchanged. The following condition (Pringsheim convergence) will provide a sufficient condition.

Definition 5: A sequence $\{a_{n,m}\} \in \mathbb{R}$ indexed by $(n,m)$ converges to $L \in \mathbb{R}$ in the sense of Pringsheim (in terms of $n$) if, for any $\varepsilon > 0$ and $m \in \mathbb{N}_{++}$, there exists $N(\varepsilon, m) \in \mathbb{N}_{++}$ such that, for any $n > N(\varepsilon, m)$, $|a_{n,m} - L| < \varepsilon$.

This condition says, for every index $m \in \mathbb{N}_{++}$, for a sequence $\{a_{n,m}\}_{n=1,2,...}$ for $m$ fixed, if we take $n$ sufficiently large, then, $a_{n,m}$ will be sufficiently close to the same limit $L$. In the context of the auction model, $n$ is the number of bidders in the auction and $m$.

Pringsheim (1897) defines a condition for convergence in double sequences. This definition is a slight modification (where the limit as $n \rightarrow \infty$ is the same for all fixed $m$). I thank Gabriel Carroll for a conversation on this definition.
is an index for the probability of the perturbation. This condition would be satisfied if, for each positive probability of perturbation, equilibrium bids converge to the same price taking behavior as the number of bidders in the auction increases. Next, we show that, once this condition is satisfied, one can interchange the order of limits:

Lemma 6: Suppose (1) \( \{a_{n,m}\} \in \mathbb{R} \) converges to \( L \in \mathbb{R} \) in the sense of Pringsheim (in terms of \( n \)), and (2) for each \( n \in \mathbb{N}_{++} \), \( \{a_{n,m}\}_{n=1,2,...} \) for fixed \( n \) converges to \( a_n \in \mathbb{R} \) as \( m \to \infty \), then, \( \lim_{n \to \infty} \lim_{m \to \infty} a_{n,m} = L \).

Proof: By assumption (2), for each \( n \), \( \lim_{m \to \infty} a_{n,m} \) exists and \( a_n = \lim_{m \to \infty} a_{n,m} \). Then we want to show that, for each \( \varepsilon > 0 \), there exists \( N \in \mathbb{N}_{++} \) such that, for each \( n > N \), \( |a_n - L| < \varepsilon \). By approximating \( a_n \) by \( a_{n,m} \), we have \( |a_n - L| \leq |a_n - a_{n,m}| + |a_{n,m} - L| \). Then, since \( \lim_{m \to \infty} a_{n,m} = a_n \), for each \( \varepsilon/2 \), there exists \( m \) such that \( |a_n - a_{n,m}| < \varepsilon/2 \). By assumption (1), for such \( m \), for \( \varepsilon/2 \), there exists \( N(\varepsilon/2, m) \) such that for every \( n > N(\varepsilon/2, m) \), \( |a_{n,m} - L| < \varepsilon/2 \). Then, we have \( |a_n - L| < \varepsilon \) for \( n > N(\varepsilon/2, m) \).

Thus, if a double sequence \( \{\beta_{\varepsilon,n}^*\} \) satisfies the condition in Definition 5, then, we can interchange the order of limits, which implies that a sequence of perfect equilibria will converge to the price taking behavior. Then, to prove Proposition 3, it remains to show that \( \{\beta_{\varepsilon,n}^*\} \) satisfies the condition of Definition 5.

Approximation by Smoothed Games. Consider a sequence of perfect equilibria \( \{\beta_n^*\}_{n=1,2,...} \) of \( \{\Gamma(n)\}_{n=1,2,...} \). Then, by the definition of a perfect equilibrium, for each \( n \), for each \( \beta_n^* \), there exists a sequence of Bayesian Nash equilibria \( \{\beta_{\varepsilon,n}^*\}_\varepsilon \) of \( \{\Gamma(\varepsilon, n)\}_\varepsilon \) such that \( \beta_{\varepsilon,n}^* \to \beta_n^* \) weakly as \( \varepsilon \to 0 \). Let us now fix \( \varepsilon > 0 \), and consider a sequence of equilibria \( \{\beta_{\varepsilon,n}^*\}_{n=1,2,...} \) of smoothed games \( \{\Gamma(\varepsilon, n)\}_{n=1,2,...} \) as \( n \to \infty \). By Prohorov’s theorem, there exists a subsequence \( \{\beta_{\varepsilon,n'}^*\}_{n'=1,2,...} \) and a limit strategy profile \( \beta_{\varepsilon}^* \) such that \( \beta_{\varepsilon,n'}^* \to \beta_{\varepsilon}^* \) as \( n' \to \infty \). Then, by following the steps of the argument developed in Lemma 1, we can show that \( \beta_{\varepsilon}^* \) is an equilibrium of the limit game \( \Gamma(\varepsilon) \).

Asymptotic Analysis of the First Order Condition. From the first order condition (1), since the event \( \tilde{W}_{i,m} (\tilde{b}_{\varepsilon,i,n}^*, \tilde{b}_{\varepsilon,i,n}' | R_{\varepsilon,i,n}) \) takes place with a positive probability
given $\varepsilon$-smoothing, we have

$$v_{i,m} - \mathbb{E}_i[P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})] = \mathbb{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$$

(A)

$$\leq (m - 1) \cdot \mathbb{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$$

(B)

$$+ \frac{\mathbb{P}_i(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})}{\mathbb{P}_i(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})}$$

(C)

$$\cdot \mathbb{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$$

We then evaluate each term one by one.

**Evaluation of (A).** Note that $\mathbb{E}_i[P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$ is the expected market clearing price when a previous bid $\overline{b}_{\varepsilon,i,m}$ did not win the $m$th unit of security but a higher bid $\overline{b}_{\varepsilon,i,m}$ did. Therefore, the market clearing price conditional on $\overline{W}_{i,m}(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})$ is bounded below by $\overline{b}_{\varepsilon,i,n}^*$ and bounded above by $\overline{b}_{\varepsilon,i,n}^*$. Consequently, as $\Delta \to 0$, $\mathbb{E}_i[P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})] \to \overline{b}_{\varepsilon,i,n}^*$.

**Evaluation of (B).** $\mathbb{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})] \mathbb{W}_{i,m}(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$ is the expected price impact when bidder $i$ increases the bid from $\overline{b}_{\varepsilon,i,m}$ to $\overline{b}_{\varepsilon,i,m}$. Given the pricing rule, the market clearing price is $b_{M-n(N_{PD}+N_{ID})}\cdot m$. Then, when bidder $i$ increases the bid from $\overline{b}_{\varepsilon,i,n}^*$ to $\overline{b}_{\varepsilon,i,n}^*$, the expected change in the market clearing prices is bounded by

$$\mathbb{E}_i[\Delta P(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*)] \mathbb{W}_{i,m}(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$$

$$\leq \mathbb{E}_i[b_{M-n(N_{PD}+N_{ID})}\cdot m - b_{M-n(N_{PD}+N_{ID})}\cdot m | \mathbb{W}_{i,m}(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$$

$$\leq \mathbb{E}_i[\text{distance between 2 consecutive bids by perturbation} | \mathbb{W}_{i,m}(\overline{b}_{\varepsilon,i,n}^*, \overline{b}_{\varepsilon,i,n}^*, R_{\varepsilon,i,n})]$$

$$\approx \varepsilon \cdot n \cdot (N_{PD} + N_{ID}) \cdot M + 1.$$
\(b_{M-n}(N_{PD}+N_{D}+N_{ID})_{n-M-1}-b_{M-n}(N_{PD}+N_{D}+N_{ID})_{n-M}\). For the second inequality, \(b_{M-n}(N_{PD}+N_{D}+N_{ID})_{n-M-1}-b_{M-n}(N_{PD}+N_{D}+N_{ID})_{n-M}\) is the distance between consecutive 2 order statistics among \(M \cdot n \cdot (N_{PD}+N_{D}+N_{ID})\) bids. Most of these bids are placed strategically, but some of them, approximately \(\varepsilon \cdot M \cdot n \cdot (N_{PD}+N_{D}+N_{ID})\), are placed by smoothed bids. Then, the distance \(b_{M-n}(N_{PD}+N_{D}+N_{ID})_{n-M-1}-b_{M-n}(N_{PD}+N_{D}+N_{ID})_{n-M}\) is less than the distance between two consecutive order statistics among \(\varepsilon \cdot M \cdot n \cdot (N_{PD}+N_{D}+N_{ID})\) bids placed by perturbations. For the third equality, when there are \(\varepsilon \cdot M \cdot n \cdot (N_{PD}+N_{D}+N_{ID})\) bids placed independently according to the uniform distribution on \(B_{i,m}\), according to the standard result on the distribution of order statistics from the uniform distribution, the expected value of the distance between two order statistics is \(\bar{b}/(\varepsilon \cdot M \cdot n \cdot (N_{PD}+N_{D}+N_{ID})+1)\).

Then, for fixed \(\varepsilon\), as \(n \to \infty\), \(E_i[\Delta P(\overrightarrow{b}_{\varepsilon,i,n}; \overrightarrow{b}_{\varepsilon,i,n}, \overrightarrow{b}_{\varepsilon,-i,n})|\overrightarrow{W}_{i,m}(\overrightarrow{b}_{\varepsilon,i,n}; \overrightarrow{b}_{\varepsilon,i,n}|R_{\varepsilon,i,n})] \to 0\) for the rate at \(O(1/n)\).

**Evaluation of (C).** The objective is to show that, for each fixed \(\varepsilon > 0\), the term (C) is bounded below and above by bounds independent of \(n\). Let us first consider an event \(\overrightarrow{W}_{i,m}(\overrightarrow{b}_{\varepsilon,i,n}; \overrightarrow{b}_{\varepsilon,i,n}|R_{\varepsilon,i,n})\). When this event \(\overrightarrow{W}_{i,m}(\overrightarrow{b}_{\varepsilon,i,n}; \overrightarrow{b}_{\varepsilon,i,n}|R_{\varepsilon,i,n})\) takes place, bidder \(i\) wins the \(m\)th unit as a result from increasing the bid to \(b_{\varepsilon,i,m,n}+\Delta\). Then, first, consider the case where there are \(n \cdot M\) bids above \(b_{\varepsilon,i,m,n}\). Second, there is one bid between \(b_{\varepsilon,i,m,n}\) and \(b_{\varepsilon,i,m,n}+\Delta\) by bidders other than \(i\). Thus,

<table>
<thead>
<tr>
<th>bidder (i)</th>
<th>(m) bids above (b_{\varepsilon,i,m,n})</th>
</tr>
</thead>
<tbody>
<tr>
<td>bidder (i')</td>
<td>1 bid between (b_{\varepsilon,i,m,n}) and (b_{\varepsilon,i,m,n}+\Delta)</td>
</tr>
<tr>
<td>bidder other than (i) and (i')</td>
<td>(n \cdot M - m - m' + 1) bids above (b_{\varepsilon,i,m,n})</td>
</tr>
<tr>
<td>total</td>
<td>(n \cdot M + 1) above (b_{\varepsilon,i,m,n})</td>
</tr>
</tbody>
</table>

where bidder \(i\) can have at most \(n \cdot M - m + 1\) bids above \(b_{\varepsilon,i,m,n}\). Thus this event can

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\(^{71}\) This is the case where there is 1 bid by bidders other than \(i\) between \(b_{\varepsilon,n,i,m}\) and \(b_{\varepsilon,n,i,m}+\Delta\). The case where there are more than 1 bids in this interval can be studied following a similar procedure to show that they are bounded from below and from above uniformly for \(n\).
be represented by.\footnote{To be precise, since a primary dealer observes bids by indirect bidders, bids by the primary dealer and bids by indirect bidders are not independent. But our bounds only depend on \(\varepsilon\)-smoothing that take place independently across bidders. Thus this formulation is legitimate to calculate the bounds from such independent \(\varepsilon\)-smoothings.}

\[
\begin{align*}
\overline{W}_{i,m}(\overrightarrow{b}^*,\overrightarrow{b}'_{i,n}|R_{\varepsilon,i,n})
&= \left(\text{bidder } i \text{ has } m \text{ bids above } b^*_{\varepsilon,i,n,m}\right) \\
&\cap \bigcup_{i' \neq i} \bigcup_{m'=1,\ldots,M-m+1} \left(\text{bidder } i' \text{ has an } m'\text{th bid between } b^*_{\varepsilon,i,n,m} \text{ and } b'_{\varepsilon,i,n,m}\right) \\
&\cap \left(\text{bidders other than } i \text{ and } i' \text{ has } n \cdot M - m - m' + 1 \text{ bids above } b^*_{\varepsilon,i,n,m}\right).
\end{align*}
\]

Next consider an event \(\overline{L}_{i,m}(\overrightarrow{b}^*,\overrightarrow{b}'_{i,n}|R_{\varepsilon,i,n})\). \(\overline{L}_{i,m}\) is the event that bidder \(i\) wins the \(m\)th unit with the bid by \(b^*_{\varepsilon,i,n,m}\) and increasing the bid will increase the market clearing price. Thus \(b^*_{\varepsilon,i,n,m}\) has to be the \(M \cdot n\)th highest bid. As in the previous case, bidder \(i\) has \(m\) units of bids above \(b^*_{\varepsilon,i,n,m}\). Thus,

\[
\begin{array}{|c|c|}
\hline
\text{bidder } i & \text{ }m\text{ bids above } b^*_{\varepsilon,i,n,m} \\
\hline
\text{bidder } i' & \text{ }m'\text{ bids in total above } b^*_{\varepsilon,i,n,m} \\
\hline
\text{bidder other than } i \text{ and } i' & \text{ }n \cdot M - m - m' \text{ bids above } b^*_{\varepsilon,i,n,m} \\
\hline
\text{total} & n \cdot M \text{ above } b^*_{\varepsilon,i,m,n} \\
\hline
\end{array}
\]

Then, this case can be represented by:

\[
\begin{align*}
\overline{L}_{i,m}(\overrightarrow{b}^*,\overrightarrow{b}'_{i,n}|R_{\varepsilon,i,n})
&= \left(\text{bidder } i \text{ has } m \text{ bids above } b^*_{\varepsilon,i,n,m}\right) \\
&\cap \bigcup_{i' \neq i} \bigcup_{m''=0,\ldots,M-m} \left(\text{another } i' \text{ has a } m'' \text{ bid above } b^*_{\varepsilon,i,n,m}\right) \\
&\cap \left(\text{bidders other than } i \text{ and } i' \text{ has } n \cdot M - m - m' \text{ bids above } b^*_{\varepsilon,i,n,m}\right).
\end{align*}
\]

where bidder \(i'\) has at most \(n \cdot M - m\) bids above \(b^*_{\varepsilon,i,m,n}\). Note it is possible that bidder \(i'\) does not have any bid above \(b^*_{\varepsilon,i,m,n}\) as long as other bidders “make up.” Thus the index for \(m''\) starts from 0. Then, since the distribution of private values is independent (and we consider independent smoothing), we have

\[72\]
To find the bounds, we use the following simple result from Milgrom (1979):

Lemma 7: Let \( \{a_n\}, \{b_n\}, \{c_n\}, \) and \( \{d_n\} \) be sequences of nonnegative real numbers. Suppose that there exist some constants \( e, f, g, h \in \mathbb{R}_+ \) such that \( e \leq \frac{a_n}{b_n} \leq f \) and \( g \leq \frac{c_n}{d_n} \leq h \) for every \( n \). Then we have \( eg \leq \frac{\sum a_n c_n}{\sum b_n d_n} \leq fh \).

Then we can understand the difference between \( W_{i,m} \) and \( L_{i,m} \) as follows. For each bidder \( i \), when bidders other than \( i \) and \( i' \) have \( n \cdot M - m - m' \) units of bids above \( b_{\varepsilon,i,n,m}^* \), (1) if bidder \( i' \) has \( m' \) bids above \( b_{\varepsilon,i,n,m}^* \), then, there will be \( n \cdot M \) bids above \( b_{\varepsilon,i,n,m}^* \) and \( b_{\varepsilon,i,n,m}^* \) is pivotal, thus the event \( L_{i,m} \) will take place, and (2) if bidder \( i' \) bids more aggressively and place one more bid between \( b_{\varepsilon,i,n,m}^* \) and \( b_{\varepsilon,i,n,m}^* + \Delta \), then \( b_{\varepsilon,i,n,m}^* \) will not win and the event \( W_{i,m} \) takes place.\(^{73}\)

Then, from the above Lemma 7, it will be suffice to show that, for every bidder \( i \in I, i' \in I \) with \( i \neq i' \), and for each \( m \in \{1, \ldots, M\} \),

\[
\Pr_i \left( \frac{\sum \Pr_i \left( \text{bidder } i' \text{ has } 1 \text{ bid between } b_{\varepsilon,i,n,m}^* \text{ and } b_{\varepsilon,i,n,m}^* + \Delta \right)}{\Pr_i \left( \text{bidder } i' \text{ places a bid between } b_{\varepsilon,i,n,m}^* \text{ and } b_{\varepsilon,i,n,m}^* + \Delta \right)} \right)
\]

\[= \frac{\sum \Pr_i \left( \text{bidder } i' \text{ has } 1 \text{ bid between } b_{\varepsilon,i,n,m}^* \text{ and } b_{\varepsilon,i,n,m}^* + \Delta \right) \cdot \Pr_i \left( \text{bidders other than } i \text{ and } i' \text{ have } n \cdot M - m - \sum \Delta' \right)}{\Pr_i \left( \text{bidder } i' \text{ has an } m \text{ bid above } b_{\varepsilon,i,n,m}^* \right) \cdot \Pr_i \left( \text{bidders other than } i \text{ and } i' \text{ have } n \cdot M - m - \sum \Delta' \right)} \]

\(^{73}\)In other words, there have to be a bidder who bids more aggressively.
is bounded from below and above by constants independent of \( n \) and \( m \). Then we will show each of (C1), (C2), and (C3) is bounded from below and above by constants independent of \( n \) and \( m \). An intuition is that, since a distribution of bids is uniformly bounded because of \( \varepsilon \)-smoothing, there are uniform lower and upper bounds of these probabilities.

**Evaluating (C1).** When bidder \( i' \) places \( m \) bids above \( b_{\varepsilon,i,n,m}^* + \Delta \), the probability of this event is highest when bidder \( i' \)'s bidding strategy \( \beta_{\varepsilon,n,i,m}^* \) puts all the weights above \( b_{\varepsilon,i,n,m}^* + \Delta \), and lowest when \( \beta_{\varepsilon,n,i,m}^* \) puts none of the weights above \( b_{\varepsilon,i,n,m}^* + \Delta \). For the upper bound, then the probability that each bid has a bid above \( b_{\varepsilon,i,n,m}^* + \Delta \) is all the probability except for smoothing. In this case, bidder \( i' \) places a unit bid above \( b_{\varepsilon,i,n,m}^* + \Delta \) with the probability \( (1 - \frac{b_{\varepsilon,i,n,m}^* + \Delta}{\Delta}) \). Thus an upper bound for (C1) is \( (1 - \frac{b_{\varepsilon,i,n,m}^* + \Delta}{\Delta})^m \). This bound is maximized at \( m = 1 \) and the maximum is \( (1 - \frac{b_{\varepsilon,i,n,m}^* + \Delta}{\Delta}) \). We note that this upper bound is effective for all \( n \) and \( m \). Next, the lower bound is the case where bidder \( i \) places a bid above \( b_{\varepsilon,i,n,m}^* + \Delta \) only through smoothing. In this case, bidder \( i \) places a bid above \( b_{\varepsilon,i,n,m}^* + \Delta \) with the probability \( \varepsilon \cdot (\frac{b_{\varepsilon,i,n,m}^* + \Delta}{\Delta}) \). Then a lower bound for (A) is given by \( \left\{ \varepsilon \cdot (\frac{b_{\varepsilon,i,n,m}^* + \Delta}{\Delta}) \right\}^M \). This lower bound is also independent of \( n \) and \( m \).

**Evaluating (C2).** The bid for the \( m + 1 \)st unit is not independent from the bid for the 1st unit to the \( m \)th unit. But given the assumption that an \( \varepsilon \)-smoothing of bids for the \( m + 1 \)st unit takes place independent of bids for other units, the maximum probability that the \( m + 1 \)st unit takes place between \( b_{\varepsilon,i,n,m}^* + \Delta \) is the case that the conditional distribution of bids puts all the weights on the interval, and in this case, the probability is \( (1 - \varepsilon) + \varepsilon \cdot \frac{\Delta}{\Delta} \). On the other hand, the minimum probability that the bid will be in the interval is the case where an equilibrium strategy puts none of the weights on the interval. In this case, the probability is \( \varepsilon \cdot \frac{\Delta}{\Delta} \). In both cases, the lower and upper bounds are independent of \( n \) and \( m \).
Evaluating (C3). The probability \( \Pr(\text{bidder } i' \text{ places } m \text{ bids above } b_{\varepsilon,i,n,m}) \) can be bounded following the steps used in (C1) and these bounds are uniform and independent of \( n \) and \( m \).

Then, since (C1)-(C3) will be uniformly bounded, from Lemma 2, (C) is also uniformly bounded. Then, Proposition 2 follows by combining (A), (B), and (C).

D. The Estimation Procedure.

Consider a smoothed auction market game \( \Gamma(\varepsilon, n) \) where there are \( n \cdot N_{PD} \) primary dealers, \( n \cdot N_{ID} \) indirect bidders, and \( n \cdot N_D \) direct bidders in the auction and also each bidder’s bids will be smoothed with probability \( \varepsilon \). Suppose bidders’ bidding behavior are described by a Bayesian equilibrium \( \tilde{\beta}^*_{\varepsilon,n} = (\tilde{\beta}^*_{\varepsilon,n,PD}, \tilde{\beta}^*_{\varepsilon,n,D}, \tilde{\beta}^*_{\varepsilon,n,ID}) \) where each primary dealer uses a bidding strategy \( \tilde{\beta}^*_{\varepsilon,n,PD} \), a direct bidder employs a bidding strategy \( \tilde{\beta}^*_{\varepsilon,n,D} \), and each indirect bidder uses a bidding strategy \( \tilde{\beta}^*_{\varepsilon,n,ID} \). Then consider bidder \( i \) with a private value \( \tilde{v}_i \) and a bid \( \tilde{b}^*_{\varepsilon,i,n} \). Then the objective is estimate bounds \( G_i \) and \( \bar{G}_i \) using the bootstrap.

Smoothing. In a smoothed game, bidder \( i \) chooses an equilibrium strategy \( \tilde{\beta}^*_{\varepsilon,n,i} \) with probability \( 1 - \varepsilon \) and a bid is chosen nonstrategically from a uniform distribution on \( B_i \) with probability \( \varepsilon \). In other words, each bid is additively augmented with independent random errors from underlying kernel density. That is, these samples are obtained as \( (1 - \varepsilon) \tilde{\beta}^*_{\varepsilon,n,i} + \varepsilon \tilde{Z}_i \) with an independent iid sample \( \tilde{Z}_i \) from a uniform distribution.

Classify Bids into Categories. We now classify bids into following categories: (1) bids by direct bidders (Category D). (2) bids by primary dealers who do not have any indirect bidder who bids through that indirect bidder (Category PD\( _0 \)), (3) bids by primary dealers who have only one indirect bidder and its indirect bidder’s bids (Category PD\( _1 \)),….. Then, in each category that involves a primary dealer, we consider a profile of bids by a primary dealer and bids by indirect bidders who bid through the primary dealer. For example, if we consider a category with a primary dealer and one indirect bidder we consider a vector of 2\( M \) bids for each pair. Then for each category, bids (or a vector of bids) are iid.

Conduct Bootstrap within Each Category. As an example, consider a category PD\( _1 \). In this category, a pair of a primary dealer’s bids and an indirect bidder’s bids is a 2\( M \) dimensional bids. For each \( i \in PD\_1 \), let \( \tilde{b}^*_{\varepsilon,i,n} = (\tilde{b}^*_{\varepsilon,i,n}; \tilde{b}^*_{\varepsilon,j,n;i=PD(j)}) \) denote a vector of primary dealer \( i \)’s bids and an indirect bidder’s bids. Let \( |PD\_1| \) be the number
of primary dealers in $PD_1$. Let $F_{\varepsilon,PD(1)}(\tilde{b}_{\varepsilon,1,n}^*)$ be the distribution function of a $2M$ dimensional bids by a primary dealer and an indirect bidder. Then, $\tilde{\beta}_{\varepsilon,1,n}^*, \ldots, \tilde{\beta}_{\varepsilon,|PD(1)|,n}^*$ are independent random variables with a common distribution function $F_{\varepsilon,PD(1)}$. Let $F_{\varepsilon,PD(1),n}$ be the empirical distribution of $\tilde{b}_{\varepsilon,1,n}^*, \ldots, \tilde{b}_{\varepsilon,|PD(1)|,n}^*$, putting mass $1/|PD(1)|$ on each bid profile. Then, in the resampling, the data $\tilde{b}_{\varepsilon,1,n}^*, \ldots, \tilde{b}_{\varepsilon,|PD(1)|,n}^*$ are treated as a population with the distribution function $F_{\varepsilon,PD(1),n}$. Given $(\tilde{b}_{\varepsilon,1,n}^*, \ldots, \tilde{b}_{\varepsilon,|PD(1)|,n}^*)$, let $\tilde{b}_{\varepsilon,1,n}^*, \ldots, \tilde{b}_{\varepsilon,n,n}^*$ be a conditionally independent sample with the common distribution $F_{\varepsilon,PD(1),n}$. Then the idea of bootstrap is that a statistics from a resample approximates the original distribution $F_{\varepsilon,PD(1)}$.

**Convergence in the Mallows metric.** Following Bickel and Freedman (1981), we consider the Mallows metric between 2 distributions. Let $S_2$ be the set of distributions functions $F$ with the finite second moments. In the construction of this paper, when there are sufficiently many bidders in the auction, these bidding strategies will be close to the price taking behavior, thus the distribution of equilibrium bids will have finite second moments. Then, for two distributions $F'$ and $F''$, consider the Mallows metric $\rho^2(F', F'')$ as in Mallows (1972). The convergence of a sequence of distribution function $\{F_n\}$ to $F$ in terms of the Mallow metric is equivalent to $F_n \to F$ weakly and the convergence of the second moments. Then, Bickel and Freedman (1981) and De Martini and Rapallo (2008) show that, for multidimensional distributions, $F_{\varepsilon,PD(1),n}^*$ converges to $F_{\varepsilon,PD(1)}$ in the Mallows metric as $n \to \infty$.

**Convergence of the Bid Distributions.** The discussion in the above paragraph shows that, for each category of bids, bootstrap distributions of bids converge to the distribution of equilibrium bids in the Mallows metric. The distribution of bids in the auction is a mixture of distribution of bids from each category. Then, the bootstrap distributions of bids obtained above converge to the distributions of bids generated by $\tilde{\beta}_{\varepsilon,n}^*$ in the Mallows metric.

**Estimation of bounds.** Since the convergence in the Mallows metric implies convergence in the distribution, the quantile of the bootstrap distribution converges to the quantile of the distribution of equilibrium bids according to $\tilde{\beta}_{\varepsilon,n}^*$. This implies consistency of the bootstrap estimates of $\bar{W}$, $\bar{W}$, $\bar{L}$, and $\bar{L}$ as $n \to \infty$. Furthermore, convergence in distribution implies convergence of (A). Consequently, the above bootstrap estimates will lead to consistent estimates of bounds. Asymptotic normality follows from the argument of Bickel and Freedman (1981) Theorem 3.1.
E. Proof of Proposition 4.

Since a bidder can use a nonparticipation bid \( b \), the bidder will not bid more than its private value \( v_{i,m} \). We will derive the lower bound of equilibrium bids for each category of bidders in each auction system.

**Primary Dealer System.** We first consider a primary dealer system, where primary dealers, direct bidders, and indirect bidders place bids. We first consider direct bidders. We have

\[
H_{\varepsilon,i,m,n}^{PD}(v_{i,m}) = v_{i,m}
\]

\[-(m - 1) \cdot E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}', b_{\varepsilon,-i,n}^*)]\]

\[-(E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')|W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')] - b_{\varepsilon,m,i,m})\]

\[-\frac{Pr_i(L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')|W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}'))}{Pr_i(W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}'))} \cdot m \cdot E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}', b_{\varepsilon,-i,n})|L(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')]\]

We study each term one by one. For (A), we have

\[
E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}', b_{\varepsilon,-i,n})|W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')] \leq \frac{\varepsilon \cdot M \cdot \{n(N_{PD} + N_D + N_{ID}) - 1\} + 1}{\bar{b}}
\]

This bound is derived by nonstrategic bids that are independent of bidders’ equilibrium strategies, a bidder’s information set, and its private values. For (B), for any value of \( b_{\varepsilon,n,i,m}^* \), (B) is an expected increase in the market clearing price when bidder \( i \) increases the bid by \( \Delta \). As such, (B) is bounded above by \( \Delta \). For (C), we have

\[
0 < \frac{Pr_i(L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')}}{Pr_i(W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}'))} < 1.
\]

For (D), as in (A), we have

\[
E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}', b_{\varepsilon,-i,n})|L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}')] \leq \frac{\varepsilon \cdot M \cdot \{n \cdot (N_{PD} + N_D + N_{ID}) - 1\} + 1}{\bar{b}}.
\]
Putting them together, we have

\[ H_{P,m}^{D} (v_{i,m}) \geq v_{i,m} - \Delta - (2m - 1) \cdot \frac{\bar{b}}{\varepsilon \cdot M \cdot \{ n \cdot (N_{PD} + N_{D} + N_{ID}) - 1 \} + 1}. \]

An indirect bidder chooses its best response using the first order condition. Proposition 2 shows that bids by direct bidders, indirect bidders, and primary dealers will be close to the price taking behavior as the auction markets become large. Thus we employ the same bound with the direct bidder.

For a primary dealer, his ability to shade its bid is limited by the bids by indirect bidders. That is, even if there are no nonstrategic bids that limit the primary dealers’ ability to affect the price, if the primary dealer shades its bid below the indirect bidders’ bids, then the primary dealer will lose its bids. Thus, the primary dealer takes into account of indirect bidders’ bids when estimating the price impact. Primary dealer \(i\) has bids \(\{b_{\varepsilon,n,j}:PD(j)=i\}\) from indirect bidders connected to the primary dealer \(i\). Then a bid by the indirect bidders closest to the primary dealer’s bid \(b_{\varepsilon,i,m,n}^{*}\) is denoted by \(\min_{b_{\varepsilon,j,m,n}}: PD(j) = i \text{ and } b_{\varepsilon,n,j,m} \geq b_{\varepsilon,i,m,n}^{*} b_{\varepsilon,n,j,m}^{*}\). That is, when primary dealer \(i\) increases the bid from \(b_{\varepsilon,n,j,m}^{*}\) to \(b_{\varepsilon,n,j,m}^{*} + \Delta\), the increase in the market clearing price may come from surpassing the indirect bidders’ bids or the nonstrategic bid. That is,

\[ E_{i}[\Delta P(\bar{b}_{\varepsilon,i,n}, \bar{b}_{\varepsilon,i,n}, \bar{b}_{\varepsilon,m,n})|W_{i,m}(\bar{b}_{\varepsilon,i,n}, \bar{b}_{\varepsilon,i,n})] = \min\{ \min_{b_{\varepsilon,j,m,n}: PD(j) = i} \text{ and } b_{\varepsilon,n,j,m} \geq b_{\varepsilon,i,m,n}^{*} b_{\varepsilon,j,m,n}^{*} \} \]

\[ \frac{\bar{b}}{\varepsilon \cdot M \cdot \{ n \cdot (N_{PD} + N_{D} + N_{ID}) - 1 \} + 1}. \]

(bidders excluding the primary bidder and the bidder connected to that bidder

**Direct Bidding System.** In the direct bidding system, every bidder has a direct link to its auctioneer. Thus a bidder’s bound is the same with the one in the direct bidder’s bid in the primary dealer system. In other words, in the direct bidding system, there are no strategic considerations involving primary dealers and indirect bidders.

**Joint Bidding System.** In the joint bidding system, we assume that a primary dealer submits bids that represents a consortium. In this competitive bidding process, the
auction has only $n \cdot N_{PD}$ bidders. Thus its bid is bounded by

$$H^{PD}_{\varepsilon,i,m,n}(v_{i,m}) \geq v_{i,m} - \Delta - (2m - 1) \cdot \frac{\bar{b}}{\varepsilon \cdot M \cdot n \cdot N_{PD} + 1}.$$ 

That is, since the joint bidding system, only the consortium bids in the auctions, that will lead to less competitive markets.

F. The Statement and Proof of Proposition 8.

We now consider the downward deviation where bidder $i$ decreases the bids for the $m$th unit from $b^*_{\varepsilon,i,m,n}$ to $b^*_{\varepsilon,i,m,n} - \Delta$. We follow the steps similar to the ones used in the upward deviation case. Let $\bar{b}^{i}_{\varepsilon,i,n}$ be the new bid profile after bidder $i$ decreases the bid for the $m$th unit. That is, $\bar{b}^{i}_{\varepsilon,i,n} = (\bar{b}^*_{\varepsilon,i,1,n}, \ldots, \bar{b}^*_{\varepsilon,i,m-1,n}, \bar{b}^*_{\varepsilon,i,m,n} - \Delta, \bar{b}^*_{\varepsilon,i,m+1,n}, \ldots, \bar{b}^*_{\varepsilon,i,M,n}).$ Then there are 2 cases that reducing the bid will affect the outcome for bidder $i$. In the first case, after lowering the bid, bidder $i$ will lose the $m$th unit of the security. Let $L_{i,m}(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n})$ be the event where other bidders bids are such that bidder $i$ will no longer be allocated the $m$th unit of the security as a result of lowering the bids. In the second case, bidder $i$ will still secure the $m$th unit after lowering the bids, and lowering the bid will save the payment for bidder $i$. Let $W_{i,m}(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n})$ be an event such that both bid $\bar{b}^*_{\varepsilon,i,m,n}$ and $\bar{b}'_{\varepsilon,i,m,n} - \Delta$ would be allocated the $m$th unit.

Proposition 8: Consider a “downward deviation” where bidder $i \in I$ with the private value $v_i \in V_i$ decreases the bid for the $m = 1, ..., M$th unit by $\Delta > 0$ while keeping the bids for other units constant. Let $\bar{b}'_{\varepsilon,i,n}$ be the new bid profile of bidder $i$ after the deviation. Then,

$$\Pr(L_{i,m}(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n}))$$

$$= (m - 1) \cdot \Pr(L_{i,m}(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n}))$$

$$+ \Pr(W_{i,m}(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n}))$$

$$\cdot m \cdot E_i[\Delta P(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n}, \bar{b}^*_{\varepsilon,-i,n}) | W_{i,m}(\bar{b}^*_{\varepsilon,i,n}, \bar{b}'_{\varepsilon,i,n} | R_{\varepsilon,i,n})].$$

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We now provide the proof of Proposition 8 below.

The Payoff from the First Case. When the event $L_{i,m}$ takes place, the bidder loses the payoffs that the bidder used to obtain from the $m$th unit, but the bidder would be able to pay less for other units of security by lowering the market clearing price. We now calculate the payoffs one by one. First, when bidder $i$ use to employ the bid $b_{\varepsilon,i,m;n}$, the bidder is allocated the $m$th unit. In this case, bidder $i$ will derive value $v_{i,m}$ from obtaining the $m$th unit of the security. The payment in this case is $P(b_{\varepsilon,i,m;n}, b_{\varepsilon,i,m;n})$ when other bidders employ the bids $b_{\varepsilon,i,m;n}$. Then the payoff bidder $i$ used to derive is

$$\Pr_i(L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|R_{\varepsilon,i,n}))$$

$$\cdot (v_{i,m} - E_i[P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|R_{\varepsilon,i,n})])].$$

Second, lowering the bid to $b_{\varepsilon,i,n}^*$ may affect the market clearing price, thus the payment for other $m - 1$ units of the security that bidder $i$ will be allocated. Let us denote the savings in the market clearing price by $\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*)$. Then, the expected reduction in the payment is

$$\Pr_i(L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|R_{\varepsilon,i,n}))$$

$$(m - 1) \cdot E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|L_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|R_{\varepsilon,i,n})]].$$

The Payoff from the Second Case. We now consider the payoff changes when lowering the bid from $b_{\varepsilon,i,n}^*$ to $b_{\varepsilon,i,n}^*$ will not affect the allocation but reduces the payment for bidder $i$. Let us denote the saving in the market clearing price in this case by $\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*)$. Then, the expected change in the payment is

$$\Pr_i(W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|R_{\varepsilon,i,n}))$$

$$m \cdot E_i[\Delta P(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|W_{i,m}(b_{\varepsilon,i,n}^*, b_{\varepsilon,i,n}^*|R_{\varepsilon,i,n})]].$$

Putting It All Together. Since a bid $b_{\varepsilon,i,n}^*$ is a best response for bidder $i$, bidder $i$ will not benefit from lowering the bid from $b_{\varepsilon,i,n}^*$ to $b_{\varepsilon,i,n}^*$. It implies that the expected monetary loss from the $m$th unit is larger than the possible saving from the lower market price. Then, combining (6)-(8), we get (5).
Table 1 Bidder Category Purchase Shares for All Treasury Securities, 2008-13.

<table>
<thead>
<tr>
<th>Category</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary Dealer</td>
<td>56.6%</td>
<td>12.9%</td>
<td>14.0%</td>
<td>100.0%</td>
</tr>
<tr>
<td>Direct Bidder</td>
<td>9.1%</td>
<td>6.1%</td>
<td>0.0%</td>
<td>45.4%</td>
</tr>
<tr>
<td>Indirect Bidder</td>
<td>32.0%</td>
<td>12.3%</td>
<td>0.0%</td>
<td>70.9%</td>
</tr>
<tr>
<td>Noncompetitive</td>
<td>2.3%</td>
<td>2.4%</td>
<td>0.0%</td>
<td>12.4%</td>
</tr>
<tr>
<td>Sample Size</td>
<td>1584</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: Treasury Auction Results available at TreasuryDirect. Note: This table reports descriptive statistics of bidder category purchase shares for 1585 Treasury auctions April 1, 2008 and December 19, 2013. The numbers in the parenthesis are from Fleming (2007).

Table 2 Bidder Category Purchase Shares by Issue Type, 2008-13

Source: Treasury Auction Results available at TreasuryDirect. Note: This chart plots average bidder category purchase shares by issue type for 1585 Treasury auctions between April 1, 2008 and December 19, 2013.
Table 3 Investor Class Allotment Shares for All Treasury Securities, 2008-13

<table>
<thead>
<tr>
<th>Investor Class</th>
<th>Mean</th>
<th>SD</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depository Institutions</td>
<td>0.5% (0.5%)</td>
<td>1.3% (2.6%)</td>
<td>0.0% (0.0%)</td>
<td>24.2% (31.6%)</td>
</tr>
<tr>
<td>Individuals</td>
<td>1.7% (0.5%)</td>
<td>1.8% (1.2%)</td>
<td>0.0% (0.0%)</td>
<td>10.2% (16.3%)</td>
</tr>
<tr>
<td>Dealers and Brokers</td>
<td>65.1% (75.4%)</td>
<td>13.6% (14.6%)</td>
<td>26.0% (34.5%)</td>
<td>100.0% (100.0%)</td>
</tr>
<tr>
<td>Private and Brokers</td>
<td>0.2% (0.0%)</td>
<td>0.8% (0.4%)</td>
<td>0.0% (0.0%)</td>
<td>11.0% (8.5%)</td>
</tr>
<tr>
<td>Investment Funds</td>
<td>18.0% (6.5%)</td>
<td>11.5% (7.6%)</td>
<td>0.0% (0.0%)</td>
<td>57.1% (46.1%)</td>
</tr>
<tr>
<td>FIMA</td>
<td>13.8% (12.5%)</td>
<td>11.6% (9.7%)</td>
<td>0.0% (0.0%)</td>
<td>61.1% (38.6%)</td>
</tr>
<tr>
<td>Other</td>
<td>0.8% (4.6%)</td>
<td>2.2% (4.2%)</td>
<td>0.0% (0.0%)</td>
<td>27.3% (20.3%)</td>
</tr>
</tbody>
</table>

Source: Investor Class Auction Allotment data. Note: This table reports descriptive statistics of bidder category purchase shares for 1567 Treasury auctions April 1, 2008 and December 19, 2013. The numbers in the parenthesis are from Fleming (2007).

Table 4 Bidder Category Purchase Shares in Issue Type, 2008-13.

Source: Investor Class Auction Allotment data. Note: The chart plots average investor class allotment shares by issue type for 1567 Treasury auctions between April 1, 2008 and December 19, 2013.
Table 5 Results of Regressions of Bidder Category Purchase Shares on Investor Class Allotment Shares

<table>
<thead>
<tr>
<th>Investor class</th>
<th>Bidder Category</th>
<th>Primary Dealer</th>
<th>Direct</th>
<th>Indirect</th>
<th>Noncompetitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depository Institutions</td>
<td>0.000</td>
<td>0.515</td>
<td>0.484</td>
<td>0.001</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.000232)</td>
<td>**(0.0941)</td>
<td>**(0.0963264)</td>
<td>(0.0202167)</td>
<td></td>
</tr>
<tr>
<td>Individuals</td>
<td>0.002</td>
<td>-0.647</td>
<td>0.415</td>
<td>1.230</td>
<td></td>
</tr>
<tr>
<td></td>
<td>**(0.000166)</td>
<td>**(0.0675)</td>
<td>**(0.0690)</td>
<td>**(0.0145)</td>
<td></td>
</tr>
<tr>
<td>Primary Dealers</td>
<td>1.000</td>
<td>-0.021</td>
<td>0.019</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td></td>
<td>**(1.256e-05)</td>
<td>**(0.00509)</td>
<td>**(0.00522)</td>
<td>(0.00109)</td>
<td></td>
</tr>
<tr>
<td>Dealers</td>
<td>0.000</td>
<td>0.803</td>
<td>0.185</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>Brokers other than PD</td>
<td>(6.461e-05)</td>
<td>**(0.0262)</td>
<td>**(0.0268)</td>
<td>*(0.00563)</td>
<td></td>
</tr>
<tr>
<td>Pension Retirement fund</td>
<td>-0.001</td>
<td>0.305</td>
<td>0.718</td>
<td>-0.022</td>
<td></td>
</tr>
<tr>
<td></td>
<td>**(0.000393)</td>
<td>(0.1593)</td>
<td>**(0.163)</td>
<td>(0.0342)</td>
<td></td>
</tr>
<tr>
<td>Investment Funds</td>
<td>0.000</td>
<td>0.156</td>
<td>0.858</td>
<td>-0.014</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.133e-05)</td>
<td>**(0.00866)</td>
<td>**(0.00886)</td>
<td>**(0.00186)</td>
<td></td>
</tr>
<tr>
<td>Foreign</td>
<td>0.000</td>
<td>0.096</td>
<td>0.883</td>
<td>0.021</td>
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<tr>
<td></td>
<td>(2.293e-05)</td>
<td>**(0.00930)</td>
<td>**(0.00952)</td>
<td>**(0.002)</td>
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</tr>
<tr>
<td>Other</td>
<td>0.000</td>
<td>0.026</td>
<td>0.926</td>
<td>0.049</td>
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<tr>
<td></td>
<td>(0.000135)</td>
<td>(0.0546)</td>
<td>**(0.0559)</td>
<td>**(0.0117)</td>
<td></td>
</tr>
<tr>
<td>$R^2$ Adjusted</td>
<td>1.000</td>
<td>0.814</td>
<td>0.980</td>
<td>0.912</td>
<td></td>
</tr>
</tbody>
</table>

Source: Treasury Auctions Result Data and Investor Class Auction Allotment Data. Note: This table reports results of regressions of bidder category purchase shares on investor class allotment shares for 1567 Treasury auctions between April 1, 2008 and December 19, 2013. Table 5 is the regression of bidder category shares over 4 bidders - primary dealer, direct, indirect, and noncompetitive on the investor class allotment shares over 8 classes - depository, individual, primary dealers, other dealer and brokers (excluding PD), private pension funds, investment funds, foreign and international, and finally others. Note that this result is different from Fleming (2007) in two ways: (1) the regression includes primary dealers as an independent variable and (2) the estimated shares are based on the regressions run together for bills and coupon securities. Qualitative results of these regressions are robust to other specifications.
Table 6 Estimated Breakdown of Indirect Bid by Issue Type, 2008-11.

Source: Treasury Auctions Result Data and Investor Class Auction Allotment Data. Note: This chart plots the average estimated breakdown of indirect bidder purchases by issue type for 1567 Treasury auctions between April 1, 2008 and December 19, 2013 based on the previous regressions.

Table 7 Estimated Breakdown of Direct Bid by Issue Type, 2008-13

Source: Treasury Auctions Result Data and Investor Class Auction Allotment Data. Note: This chart plots the average estimated breakdown of indirect bidder purchases by issue type for 1567 Treasury auctions between April 1, 2008 and December 19, 2013.
Table 8 The Distribution of Simulated Market Prices under the Primary Dealer System

This table is based on the simulation results for 5000 samples of auction results for the primary dealer system. The simulation results use only the estimated private values.
Table 9: The Distribution of Simulated Market Prices Under the Direct Bidding System

This table is based on the simulation results for 5000 samples of auction results for the direct bidding system. The simulation results use only the estimated private values.
Table 10 The Distribution of Simulated Market Prices Under the Joint Bidding System

This table is based on the simulation results for 5000 samples of auction results for the joint bidding system. The simulation results use only the estimated private values.