The Performance of Core-Selecting Auctions: An Experiment*

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Abstract

Combinatorial auctions, in particular core-selecting auctions, have increasingly attracted the attention of academics and practitioners. We experimentally analyze core-selecting auctions under incomplete information and find that they perform better than the Vickrey auction. The proportions of efficient allocations are similar in both types of auctions, but the proportions of stable (core) allocations and the revenue are higher in the core-selecting auctions. This is in particular true for an independent private values setting in which theory does not predict this better performance of the core-selecting auction. We trace the causes of the performance differences back to patterns in bids. The core-selecting auctions provide incentives for overbidding the own valuation and – under certain conditions – also for bid-shading, which can hamper performance. In the experiment, bidders react in the predicted direction to these incentives, though less pronouncedly than predicted.

JEL classification: D44, C72, D82, C92
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1 Introduction

Combinatorial auctions are widely applied for the procurement or sale of multiple heterogeneous services or goods like bus services, school meals, and the sale of telecommunications spectrum licenses (e.g. Ausubel and Baranov, 2014; Cantillon and Pesendorfer, 2006; Lunander and Lundberg, 2012; Olivares et al., 2012). As a distinguishing feature they allow bidders to express complex preferences through bids on subsets of items. Allowing for such a comprehensive bidding space provides new challenges for auction design.

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A prevalent objective of auction designs is to achieve an efficient allocation at competitive prices, which is typically subsumed by the aim of achieving a stable allocation, i.e., an allocation that is in the core of the associated cooperative game (e.g. Ausubel and Milgrom, 2005).[1] This requirement has led theorists to suggest a new class of combinatorial auctions – the bidder-optimal core-selecting auctions – which are being increasingly adopted by practitioners.[2] Core-selecting auctions select an allocation that is in the core based on reported values, i.e., bids. Reporting the valuations truthfully is, however, usually not in the interest of bidders and as a result the equilibrium allocation may not be in the core.[3] Under complete information, misreporting in equilibrium may still result in an allocation in the core (Day and Milgrom, 2008). However, this is typically not the case under incomplete information (Beck and Ott, 2013; Goeree and Lien, 2016). Motivated by these theoretical findings, we address the following empirical research question: How do core-selecting auctions perform under incomplete information?

In a laboratory experiment, we investigate efficiency, (allocative) stability, and revenue properties of a core-selecting auction under incomplete information and, as a benchmark, of the Vickrey auction.[4] For two different informational settings, we test specific hypotheses on the performance of these auctions based on the respective unique Bayesian Nash equilibrium in undominated strategies. We furthermore relate observed differences from predicted outcomes to systematic deviations in bidding behavior from equilibrium behavior.

We concentrate on a specific bidder-optimal core-selecting auction. This constitutes the only core-selecting auction for which Bayesian Nash equilibria in undominated strategies have been characterized for an independent private values setting with bids allowed on all subsets of items (by Beck and Ott, 2013) and which exhibits some additional advantageous features for an experimental investigation (for details, see Section 2.1). In addition, we choose the Vickrey auction as a natural benchmark because its presumed deficiencies have motivated scholars to develop core-selecting auctions as an alternative (Ausubel and Milgrom, 2002).[5] It

[1]An allocation is in the core if there is no group of bidders and the seller in which everybody could be made better off by reallocating objects and payments within this group. We therefore term core allocations stable.

[2]Core-selecting payment rules have been used since 2005 worldwide in combinatorial clock auctions for telecommunications spectrum, e.g., in Australia, Austria, Canada, the Netherlands, and the UK (Ausubel and Baranov, 2014; Cramton, 2013). The payment rule is applied in the final (proxy bidding) round of this auction.

[3]Whereas the core-selecting auction cannot guarantee stable allocations, it nevertheless assures that based on reported valuations (i.e. bids), allocations are in the core. This does not prevent incentives for renegotiations, but it might increase non-successful bidders' acceptance of auction outcomes because the opponents' joint payments are never below their reported values.

[4]The Vickrey auction is a Vickrey-Clarke-Groves mechanism.

[5]A central concern is the ability of the Vickrey auction to generate stable allocations. Instability of the Vickrey auction arises because it may generate low revenue in relation to the truthfully reported valuations, i.e., non-competitive prices. Even zero revenue may occur, and did occur for example in the recent Incentive

also provides a weakly dominant strategy to bid the valuation (and thus bidding is robust to changes in the informational environment).

Both core-selecting and Vickrey auctions assign items to bidders by maximizing the sum of (package) bids. In the Vickrey auction, each bidder pays his externality on the other bidders and the seller. In a core-selecting auction, each group of bidders pays in sum at least its externality on the remaining bidders and the seller, and no one pays more than his reported valuation. In a bidder-optimal core-selecting auction, in addition, the price vector is not Pareto-dominated by any other price vector that fulfills these requirements.[6]

To keep the experimental design simple, we focus on auctions of two items with one global and two local bidders. One local bidder is only interested in buying the first item, the other local bidder is only interested in the second item,[7] and the global bidder is only interested in the bundle of both items.[8] Valuations (for desired items) are independently drawn. A bidder may submit a multi-dimensional bid that involves reported values for individual items and for the bundle.

The local bidders' prevalent problem in a core-selecting auction is the threshold problem and, in particular, the free-rider aspect that it involves. In order to each win an item, the sum of their bids (on their respective desired item) must exceed the bid (the threshold) of the global bidder for the bundle. However, if their bids for the items are successful, they jointly have to pay the global bidder's bid and they each want to free ride on the other's high payment. A peculiarity of the core-selecting auction is that a local bidder can reduce her payment by increasing the other local bidder's payment (Beck and Ott, 2013). She can increase the other's payment by increasing his externality, which tightens the constraint on his payment. She can increase his externality by shading her bid for her desired item below her valuation and by a bid spread, i.e., by a positive difference between her bid for the bundle and her bid for the desired item. Bid shading decreases her reported value for the realized trade. The bid spread may increase the reported most valuable trade between her, the global bidder, and the seller.

The information held by local bidders affects their exposure to the threshold problem and therefore the equilibrium predictions for the core-selecting auction.[9] In our first informational setting, a standard independent private values setting, bid shading involves the risk of not

Auction for reallocating spectrum in the US (Ausubel et al., 2017, Section 8.2).

[6]When there are only two items, these bidder-optimal price vectors minimize revenue in the reported core.

[7]Our local bidders may complement each other to outbid the global bidder. If no bidders are complementary, truthful bidding is an equilibrium in both the bidder-optimal core-selecting auctions and the Vickrey auction.

[8]This setting captures for example local and global geographic interests of mobile communication providers.

[9]Bidders may know more than their private valuation, e.g., because local bidders might have identical business models, or because, depending on the rules on eligible bids in the previous rounds, bidders in the final auction round of a combinatorial clock auction may have learned about their opponents' interests and the possible final assignments (e.g., Ausubel and Cramton, 2011).

receiving the item (by missing the threshold). Furthermore, the optimal bid spread even amounts to overbidding (the own valuation) on the bundle and comprises the risk of receiving the bundle instead of the item at a higher price. In the second informational setting, which we call the semi-private values setting, local bidders also learn each other's valuations, which allows them to correctly infer the other local bidder's equilibrium bid. This additional information allows them to circumvent the risks that arise with independent private values. As a consequence, an ex-post equilibrium exists and a local bidder's equilibrium bid does not involve bid shading. She bids her valuation for her desired item and fully free-rides on the other local bidder by bidding with a large bid spread that depends on the other local bidder's (known) bid.

Due to the misreporting of valuations, the core-selecting auction is predicted to perform worse than the Vickrey auction in terms of efficiency and stability in the independent private values setting. Bid shading and bid spread cause inefficient assignments of items. Inefficiency or low payments of the global bidder (as a result of a local bidder's bid shading) prevent core allocations. With semi-private values, truthful bids for the desired item render the core-selecting auction efficient (like the Vickrey auction) and the large bid spread assures that payments are sufficiently high to generate core allocations and a higher revenue than the Vickrey auction.

The main experimental findings are as follows:

1. The core-selecting auction achieves higher revenue and higher proportions of stable allocations than the Vickrey auction in either informational setting. As another favorable property, it avoids zero-revenue outcomes, which are prevalent in the Vickrey auction.

2. In the independent private values setting, the core-selecting auction performs better than predicted with respect to stability and revenue. With semi-private values, it performs similarly to the first setting but fails to achieve the predicted level of revenue and full stability.

3. Bid spread and bid shading are more prevalent in the core-selecting auction than in the Vickrey auction. However, as compared to predictions, bid spreads are less frequent and their size is smaller in either informational setting. Furthermore, bid shading is not predicted but is common with semi-private values, and with independent private values it is common but its size is smaller than predicted.

In the Vickrey auction, average revenue and proportions of efficient allocations are largely in line with theoretical predictions but stable allocations are less frequent than predicted in both informational settings. Bidders often deviate from truthful bidding, as is commonly observed in experiments,[10] but they bid their valuation more frequently than in the core-selecting auction.

In sum, the observation that with independent private values bids deviate less from the valuations than predicted can explain the good performance of the core-selecting auction. If bids equal valuations, these auctions achieve their desirable (and eponymous) properties. Bidding closer to valuations than predicted results in a better than predicted performance in the independent private values setting, and engaging in bid shading instead of focusing on the bid spread results in a similar (but worse than predicted) performance in the semi-private values setting.

Literature Review

Much of the existing experimental research on combinatorial auctions is motivated by spectrum auctions. These studies typically perform "wind tunnel" tests and compare outcomes of different auction formats, e.g., mechanisms considered by the Federal Communications Commission of the USA for spectrum auctions, in settings chosen to closely resemble real-life settings. They investigate the impact of the chosen auction format on efficiency, revenue, and bidding behavior (without adhering to any equilibrium analyses). Among the mechanisms studied without hypotheses based on equilibrium predictions are mechanisms with and without package bids or restrictions on possible packages as well as dynamic and static auctions (Banks et al., 2003; Brunner et al., 2010; Bichler et al., 2013; Goeree and Holt, 2010; Kazumori, 2010; Kagel et al., 2010, 2014; Kwasnica et al., 2005; Ledyard et al., 1997; Porter, 1999; Porter et al., 2003; Scheffel et al., 2012). Some of these apply core-selecting rules in the final proxy-bidding phase of the combinatorial clock auction (Kazumori, 2010; Bichler et al., 2013). Experiments that are based on an equilibrium analysis investigate combinatorial sealed-bid Vickrey auctions (Chen and Takeuchi, 2010; Isaac and James, 2000) as well as combinatorial sealed-bid pay-as-you-bid and Dutch auctions (Chernomaz and Levin, 2012; Kokott et al., 2017). The experiment on core-selecting auctions closest to ours is by Marszalec (2014). An important difference is that he disallows multi-dimensional bids, which we consider an essential element of combinatorial auctions. With this restriction, equilibrium predictions exist for the payment rules that he implements, the so-called Vickrey-nearest and reference payment rules (Ausubel and Baranov, 2010; Goeree and Lien, 2016). He compares several sealed-bid auctions: the Vickrey auction, the pay-as-bid auction, and two bidder-optimal core-selecting auctions in an independent private values setting with two local bidders and one global bidder. His subjects

[10] In combinatorial Vickrey auctions, bidders tend to bid below their valuations in the study by Chen and Takeuchi (2010), who classify 7% of their bidders as truthful bidders, 73% as underbidders, and 20% as overbidders. Isaac and James (2000) do not find evidence of a mean deviation of the bid from the valuation. For an overview for single-unit Vickrey auctions, see Kagel and Levin (2016).

submit a one-dimensional bid for their desired item or bundle, which disallows the use of bid spreads.[11] If bidders were restricted to such bids in our core-selecting auction with independent private values, the predicted revenue, total surplus, and payoff of our subjects would be lower and the proportion of stable allocations would be higher than without this restriction (see Beck and Ott, 2013). In the setup of Marszalec (2014), the Vickrey auction is predicted to perform better than the core-selecting auctions with respect to revenue and efficiency but the data provide no evidence of differences between these auctions (whereas the pay-as-bid auction outperforms them). He finds indications of tacit collusion in his recurring Vickrey auctions by local bidders who submit high bids that decrease each other's payments.

2 Theoretical Considerations and Hypotheses

This section briefly summarizes the theoretical findings that underlie our experimental setup. For a more comprehensive presentation, including proofs, see Beck and Ott (2013). We restrict our attention to Bayesian Nash equilibrium in undominated strategies. Our hypotheses follow directly from the theoretical predictions.

2.1 Setup and Theory

A seller offers two heterogeneous items, A and B, to three bidders. Bidder 1 (a local bidder) is only interested in item A and assigns zero marginal value to item B, (local) Bidder 2 is only interested in item B and assigns zero marginal value to item A, and (global) Bidder 3 is only interested in the bundle AB of both items and single items have zero value to him. We denote bidders' valuations and bids for the possible packages of items according to Table 1 and refer to Bidder 2 as female and to Bidders 1 and 3 as male, to facilitate comprehension.

We consider two informational settings. In all settings the distribution of valuations and the valuation structure (i.e., the bidders' zero valuations for certain items) is common knowledge. We assume that $v_1 \sim U[0, 100]$, $v_2 \sim U[0, 100]$, and $v_3 \sim U[0, 200]$. Before submitting bids, in the *independent private values* setting, each Bidder i learns only his or her valuation v_i. In the *semi-private values* setting, local Bidders 1 and 2 both learn v_1 and v_2 (but not v_3), and the global Bidder 3 learns only v_3.

[11] Further differences to our setting are: we implement two informational settings whereas he focuses on independent private values; we do not severely restrict bids in the Vickrey auction whereas he restricts subjects either to bid at most their valuation or gives an explicit hint of potential losses in the case of overbidding; his core-selecting auctions use different payment rules; our subjects bid for thirty rounds in one auction format and do not interact with other human bidders (which excludes repeated-game effects) whereas his subjects bid in each of the four auction types for ten rounds with randomly assigned role as local or global bidder in each round, and they interact repeatedly with random rematching within the room after each round.

Table 1: Valuations of the three bidders and notation for their bids

Package	$\emptyset$	A	B	AB	Package	A	B	AB
Valuation of Bidder 1	0	v_1	0	v_1	Bid b_1 of Bidder 1	b_1^A	b_1^B	b_1^{AB}
Valuation of Bidder 2	0	0	v_2	v_2	Bid b_2 of Bidder 2	b_2^A	b_2^B	b_2^{AB}
Valuation of Bidder 3	0	0	0	v_3	Bid b_3 of Bidder 3	b_3^A	b_3^B	b_3^{AB}

In all auctions, bidders submit non-negative bids $b_i = (b_i^A, b_i^B, b_i^{AB})$. These are exclusive (XOR) bids, that is, at most one bid of a bidder is successful. Successful bids are determined by maximizing the sum of bids over feasible assignments of items to bidders, i.e., by:

$$\arg\max\{b_1^A + b_2^B, b_1^A + b_3^B, b_2^A + b_1^B, b_2^A + b_3^B, b_3^A + b_1^B, b_3^A + b_2^B, b_1^{AB}, b_2^{AB}, b_3^{AB}\}.$$

In the case of a tie, awarding individual items is prioritized over awarding the bundle, and remaining ties are broken randomly.

The payoff π_i of Bidder i who receives a package of items at a price p_i equals his valuation for the awarded package minus the payment p_i. A bidder who receives no item has a payoff of zero (unsuccessful bidders incur zero payments). The seller obtains the revenue $\pi_0 = p_1 + p_2 + p_3$. Each bidder maximizes his or her expected payoff. Denote the successful bid of Bidder i by $\hat{b}_i$, with $\hat{b}_i \in \{0, b_i^A, b_i^B, b_i^{AB}\}$ and $\hat{b}_i = 0$ if i is not awarded any item.

Vickrey auction Each Bidder i pays his or her reported externality on the other bidders:

$$p_i = p_i^V = \max\{b_j^A + b_k^B, b_k^A + b_j^B, b_j^{AB}, b_k^{AB}\} - \hat{b}_j - \hat{b}_k \text{ for } i, j, k \in \{1, 2, 3\}, i \neq j \neq k.$$

Both with independent and with semi-private values, the equilibria in undominated strategies are given by $b_1 = (v_1, 0, b_1^{AB})$, $b_2 = (0, v_2, b_2^{AB})$, and $b_3 = (0, 0, v_3)$, with $b_1^{AB} \leq v_1$ and $b_2^{AB} \leq v_2$. All these equilibria are outcome equivalent and in the salient equilibrium, bids equal valuations.

Core-selecting auction In a bidder-optimal core-selecting auction, payments have to fulfill

$$\begin{aligned} &\min p_1 + p_2 + p_3 \\ \text{s.t.}\quad & p_1 + p_2 + p_3 \geq 0 \\ & p_i + p_j \geq \max\{b_k^A, b_k^B, b_k^{AB}\} - \hat{b}_k \text{ for } i, j, k \in \{1, 2, 3\}, i \neq j \neq k \\ & p_i \geq p_i^V \text{ for } i \in \{1, 2, 3\}. \end{aligned}$$

Thus, each bidder and each pair of bidders have to pay at least their reported externality on the remaining bidders. The conditions ensure that bidders that are not awarded any item pay nothing, a bidder does not pay more than his valuation for the item or bundle he receives, and a bidder who wins both items pays $p_i = p_i^V$.

If two bids are successful, the above constraints do not uniquely pin down the feasible payments. There are infinitely many bidder-optimal core-selecting auctions that differ in the payment rule. Our selection of the payment rule is based on the following criteria. First, an equilibrium for an unrestricted bidding space has been identified for the incomplete information settings. Second, two out of three bidders have simple undominated or weakly dominant strategies. They can therefore easily be computerized, and the strategies can easily be explained to the subjects in the experiment. Third, subjects' equilibrium bids comprise interesting features like bid shading and bid spread (see below). Fourth, the subjects' strategies differ from those of the computerized bidders and, therefore, it is not optimal to imitate their bidding behavior. Already each of the first two criteria leaves us with a so-called favored-bidder rule (Beck and Ott, 2013). We apply the favored-bidder payment rule such that Bidder 1 pays $p_1 = p_1^V$.[12] With any payment rule, the threshold problem exists and at least one bidder can reduce the payment by bid shading or a bid spread. In the semi-private values case it holds that in any equilibrium with any payment rule bid spreads occur and the equilibrium strategies with our payment rule remain mutual best responses. Therefore, we are confident that our experimental analysis provides insights that hold beyond a specific choice of the payment rule.

In our auction, Bidder 1's undominated strategies are $b_1(v_1) = (v_1, 0, b_1^{AB})$ with $b_1^{AB} \leq v_1$, which are all outcome equivalent (for any b_2 and b_3). Bidder 3 has a weakly dominant strategy to bid his valuation, i.e., $b_3(v_3) = (0, 0, v_3)$. Bidder 2 never wants to win item A, and we concentrate w.l.o.g. on equilibria with $b_2^A(v_2) = 0$ (all other equilibria are outcome equivalent). Given $b_1(v_1) = (v_1, 0, v_1)$, $b_3(v_3) = (0, 0, v_3)$, and $b_2^A(v_2) = 0$, Bidder 2's payment is:

$$p_2 = \begin{cases} 0 & \text{if } \hat{b}_2 = 0 \\ \max\{v_1, v_3\} & \text{if } \hat{b}_2 = b_2^{AB} \\ \max\{v_3 - p_1^V, p_2^V\} = \max\{\min\{v_3, v_3 - b_2^{AB} + b_2^B, b_2^B\}, 0\} & \text{if } \hat{b}_2 = b_2^B. \end{cases}$$

For the independent private values setting, in equilibrium, Bidder 2's strategy involves $b_2^B(v_2)$ and $b_2^{AB}(v_2)$ as depicted in Figure 1. With semi-private values, Bidder 2 can condition her bid on v_1 and v_2. Bidding according to $b_1(v_1, v_2) = (v_1, 0, v_1)$, $b_2(v_1, v_2) = (0, v_2, v_1 + v_2)$,

[12] Bidder 1 is called "favored" because he pays the lowest possible payment that fulfills the core constraints. Note that with this favored-bidder payment rule, the (computerized) equilibrium strategies of Bidders 1 and 3 with independent private values are the same as with semi-private values.

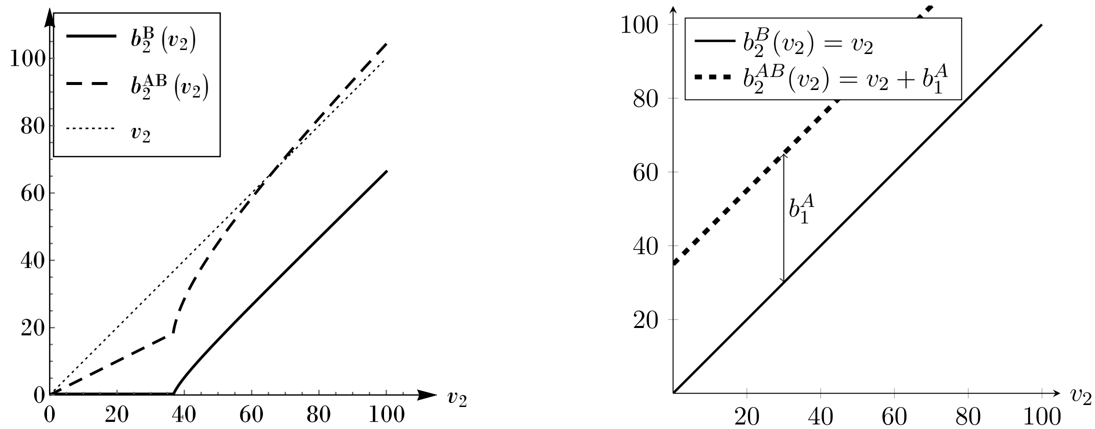

Figure 1: Equilibrium bidding functions $b_2^B(v_2)$ and $b_2^{AB}(v_2)$ in the core-selecting auction with independent private values (LHS) and semi-private values (RHS) (Beck and Ott, 2013).

and $b_3(v_3) = (0, 0, v_3)$ is an equilibrium (and all other equilibria are outcome equivalent).[13]

Equilibrium allocations An allocation is *efficient* if the items are awarded to maximize the sum of values. An allocation is in the *core* (or *stable*) if and only if it is efficient and payments satisfy

$$p_i + p_j \geq \max\{v_k^A, v_k^B, v_k^{AB}\} - \hat{v}_k \qquad \text{for } i, j, k \in \{1, 2, 3\}, i \neq j \neq k$$
$$\hat{v}_i \geq p_i \geq \max\{v_j^A + v_k^B, v_k^A + v_j^B, v_j^{AB}, v_k^{AB}\} - \hat{v}_j - \hat{v}_k \qquad \text{for } i, j, k \in \{1, 2, 3\}, i \neq j \neq k,$$

where $\hat{v}_i$ is Bidder i's valuation for his or her awarded package, $\emptyset$, A, B, or AB, in the efficient allocation.

Table 2 shows the probability of an efficient allocation $(\pi_0, \pi_1, \pi_2, \pi_3)$, the probability of an allocation in the core (a stable allocation), and the expected revenue π_0. The abbreviations C-IP, V-IP, C-SP, and V-SP refer to the core-selecting auction (C) or Vickrey auction (V) and the informational setting independent private values (IP) or semi-private values (SP).

Efficiency: The core-selecting auction by design generates allocations in the core, and, thus, efficient allocations, if bidders bid their valuations. However, with independent private values, Bidder 2 does not bid her valuation and the auction might inefficiently assign the bundle AB to Bidder 3 or Bidder 2. As a result, theory predicts that an efficient allocation occurs only with probability 0.84. Equilibrium allocations in C-SP, V-IP, and V-SP are efficient.

Stability: In equilibrium with independent private values, the core-selecting auction selects

[13]Note that our equilibrium with semi-private values is also an ex-post equilibrium.

Table 2: Equilibrium predictions for $v_1 \sim U[0, 100]$, $v_2 \sim U[0, 100]$, and $v_3 \sim U[0, 200]$

Auction and Informational Setting	C-IP	C-SP	V-IP & V-SP
Probability of an Efficient Allocation	0.84	1	1
Probability of a Stable Allocation	0.31	1	0.5
Revenue $E[\pi_0]$	59.6	79.2	58.3

a core allocation with probability 0.31. In 98.5% of these core allocations, Bidders 1 and 2 are efficiently awarded items A and B and pay at least their externality on Bidder 3. With semi-private values, the equilibrium allocation is always in the core. Allocations are efficient and payments are high enough to satisfy the core constraints because the bids on desired items are truthful. Notably, the core-selecting auction achieves an allocation in the core either always (for semi-private values) or predominantly in cases where Bidders 1 and 2 win (for independent private values), while the allocation from the Vickrey auction is in the core if and only if Bidder 3 wins AB,[14] which happens with probability 0.5.

Revenue: The predicted expected revenue in the Vickrey auction is 58.3, which is slightly below the level in C-IP (59.6) and clearly below the level in C-SP (79.2). Notably, for every realization of v_1, v_2 and v_3, the predicted revenue in C-SP is higher than in V-SP.

Note that in the Vickrey auction it is possible that bidders receive an item at a price of zero. The probability of zero revenue in equilibrium is 0.17. In the core-selecting auction, zero revenue cannot occur given non-zero bids of Bidders 1 and 3. The maximum revenue in equilibrium of the Vickrey auction is 200, which exceeds that of C-IP and equals that of C-SP.

2.2 Hypotheses

An auctioneer can usually choose the auction format, but not the informational setting. Thus, we focus on comparing C-IP with V-IP as well as C-SP with V-SP. All hypotheses are based on the unique equilibrium allocations and the equilibrium bidding strategies given in Table 2 and Section 2.1.

Auction performance We compare three key performance indicators of auctions: the proportion of efficient allocations, the proportion of stable (core) allocations, and the revenue.

Hypothesis 1 (Efficiency). In the independent private values setting, the core-selecting auction generates a lower proportion of efficient allocations than the Vickrey auction.

[14]There is one exception: If $v_1 + v_2 = v_3$, then the allocation is in the core although Bidders 1 and 2 win.

Hypothesis 2 (Stability). In the independent private values setting, the core-selecting auction generates a lower proportion of core allocations than the Vickrey auction.

Hypothesis 3 (Stability). In the semi-private values setting, the core-selecting auction generates a higher proportion of core allocations than the Vickrey auction.

Hypothesis 4 (Revenue). In the semi-private values setting, the core-selecting auction generates a higher revenue than the Vickrey auction.

Bidding behavior In both informational settings, the core-selecting auction provides incentives to submit a higher bid for the bundle than for the single item, i.e., for a *bid spread*, $b_2^{AB} - b_2^B > 0$, for all valuations v_2.[15] With independent private values, the equilibrium bids comprise *bid shading*, $b_2^B < v_2$, for all v_2. In the Vickrey auction, in contrast, bid spread and bid shading do not occur in equilibrium.

Hypothesis 5 (Bid spread and bid shading). The proportion of bids with a bid spread is higher in the core-selecting auction than in the Vickrey auction in both settings. With independent private values, the proportion of bids with bid shading is higher in the core-selecting auction than in the Vickrey auction.

3 Experimental Design

We implement five treatment conditions: two different auction rules (Vickrey auction, core-selecting auction) in two informational settings (independent private values, semi-private values) plus one dynamic variant of an auction rule in one informational setting (core-selecting auction with two stages and semi-private values). Each subject experiences one treatment (between-subjects design). The first two auctions are described in Section 2.1 whereas the auction with two stages is described below. We are mainly interested in the performance of the core-selecting auction. We implement the Vickrey auction as our point of comparison for the performance of the core-selecting auction in the two informational settings.

In each of 30 identical rounds, the subject had the role of Bidder 2 and bid against two automated Bidders 1 and 3 that bid according to their respective weakly dominant strategy. The valuation of Bidder 2 and the bids of Bidders 1 and 3 were drawn randomly at the beginning of each round. The subject knew the distribution of bids of the other bidders, and in the

[15]Note, a bid spread can lead to Bidder 2 winning the bundle AB. For given bids of the opponents, her payment if she wins AB in the core-selecting auction is weakly higher than if she wins B ($\max\{v_1, v_3\} \geq \max\{\min\{v_3, v_3 - b_2^{AB} + b_2^B, b_2^B\}, 0\}$). The incentive for the bid spread arises due to its reducing influence on the payment in the case of winning the item B.

Table 3: Bidder 2's information in the informational settings

Informational setting	Information of Bidder 2
Independent private values	Realization of $v_2 \sim U\{1,2,\ldots,100\}$ Distribution of $b_1^A = b_1^{AB} \sim U\{1,2,\ldots,100\}$, $b_1^B = 0$ Distribution of $b_3^{AB} \sim U\{1,2,\ldots,200\}$, $b_3^A = b_3^B = 0$
Semi-private values	Realization of $v_2 \sim U\{1,2,\ldots,100\}$ Realization of $b_1^A = b_1^{AB} \sim U\{1,2,\ldots,100\}$, $b_1^B = 0$ Distribution of $b_3^{AB} \sim U\{1,2,\ldots,200\}$, $b_3^A = b_3^B = 0$

semi-private values setting also the bid of Bidder 1 (see Table 3).[16] The automation of Bidders 1 and 3 allowed us to keep as much control as possible of the subjects' bidding environment, to eliminate their strategic uncertainty, and to exclude repeated game and behavioral effects of human interaction. This enables subjects to remain focused on the auction rules and allows instructions to be kept as short and clear as possible while retaining the inherent complexity of the auction rules. Bidder 2 had to decide on b_2^B and b_2^{AB}, while b_2^A was set to zero.[17] For bids b_2^B and b_2^{AB} either no number (empty entry box) or any integer between 0 and 999 could be chosen. At the end of each round, subjects learned their awarded item(s), their payment, and their payoff. We provide translated instructions in Appendix D.

The payment rule of the core-selecting auction is complex and therefore we add a fifth treatment, with semi-private values, designed to facilitate a subject's problem of finding an optimal strategy. In the fifth treatment, we allow Bidder 2 to revise her bids upwards after she has learned which item(s) she has been assigned with her (preliminary) bids. We expect that this two-stage version of the core-selecting auction (denoted C-SP-2) helps Bidder 2 by allowing her to think separately about how to reduce payments in a second step.[18]. More specifically, in the two-stage version of the core-selecting auction in the first stage, Bidders 1 and 3 submit final bids and Bidder 2 submits preliminary bids b_2^B and b_2^{AB}. Then she learns which item or bundle she would be awarded (none, B, or AB) based on the preliminary bids. In the second stage, Bidder 2 has the opportunity to increase her bids. The final outcome is

[16]The instructions do not mention valuations of Bidders 1 and 3, only bids. This may reduce potential (symmetric) imitation of Bidder 1 by Bidder 2. Note, however, that our asymmetric design of the core-selecting auctions prevents that imitating Bidder 1 (assuming that he bids his valuation) is a best response for Bidder 2 to the bids of Bidders 1 and 3.

[17]Bidder 2 will never want her bid b_2^A to be successful because $v_2^A = 0$. Allowing her to decide on her bid b_2^A would have complicated the instructions and therefore we decided to set $b_2^A = 0$.

[18]Further motivation for this two-stage treatment is provided by the fact that most implementations of core-selecting auctions have multiple bidding stages and at least partially allow bidders to deduce information concerning the feasible assignments in the final sealed-bid stage (Ausubel and Baranov, 2014)

Table 4: Auctions, information, and numbers of subjects in the two sessions of each treatment

Treat.	Auction	Informational Setting	# Subjects
C-IP	Core-selecting	Independent private values	34 (18+16)
V-IP	Vickrey	Independent private values	32 (16+16)
C-SP	Core-selecting	Semi-private values	35 (17+18)
C-SP-2	Core-selecting, two stages	Semi-private values, award information	31 (18+13)
V-SP	Vickrey	Semi-private values	31 (17+14)

determined based on the second stage bids. All equilibria in C-SP-2 are outcome equivalent to that of the sealed-bid auction in C-SP. In equilibrium, Bidders 1 and 3 bid as in the sealed-bid auction. Bidder 2's best response must lead to the same payoff as in the sealed-bid (single-stage) version, because this is the maximum that she can achieve (and she could achieve this payoff in the two-stage auction by bidding $b_2(v_1, v_2) = (0, v_2, v_1 + v_2)$ in stage one). However, she has alternate best responses in which she bids less than $(0, v_2, v_1 + v_2)$ in stage one and, if she is not awarded B, bids $b_2(v_1, v_2) = (0, v_2, v_1 + v_2)$ in stage two, whereas if she is awarded B in stage one, she bids $b_2(v_1, v_2) = (0, b_2^{B\prime}, v_1 + b_2^{B\prime})$ with $b_2^{B\prime} \in [b_2^B, v_2]$, where b_2^B is her first-stage bid on B. We therefore predict that the two-stage core-selecting auction outperforms the Vickrey auction with respect to stability and revenue (as the one-stage version does).

Hypothesis 3′*.* With semi-private values, the two-stage version of the core-selecting auction generates a higher proportion of core allocations than the Vickrey auction.

Hypothesis 4′*.* With semi-private values, the two-stage version of the core-selecting auction generates a higher revenue than the Vickrey auction.

While not affecting equilibrium outcomes, introducing a second bidding stage might influence human bidding behavior. We predict that the two-stage version guides Bidder 2 towards her best response strategy and hence that, if there is a difference to the one-stage version, then the experimental outcome will be closer to the equilibrium predictions of all allocations being in the core and the high equilibrium revenue that is due to Bidder 2's aggressive bid. Based on this behavioral argument, we expect the two-stage version of the core-selecting auction to outperform the sealed-bid auction with respect to all three variables.

Hypothesis 6*.* With semi-private values, the two-stage version of the core-selecting auction generates more efficient allocations, core allocations, and revenue than the sealed-bid version.

For each of the five treatments, we conducted two experimental sessions in the AiXperiment laboratory in Aachen using z-Tree (Fischbacher, 2007). For details on the sessions, see Table 4. The 163 participants were recruited from a pool of students with a technical background using ORSEE (Greiner, 2015). In each session, 18 subjects were invited and all of those that showed up participated in the experiment. Subjects received written instructions that were then read out aloud. Then, for 10 minutes, subjects could enter bids in a split screen (such that they could enter two sets of bids in parallel and directly compare the outcomes, see the screen shot in Appendix D.2) of a practice program, which then calculated the corresponding outcome and repeated the explanations from the instructions on how winners and payments were determined. Next, subjects had to answer 22 or 23 questions (depending on the treatment) about the rules, followed by another period with the practice program (of at least five minutes and up to 10 rounds of entering two sets of bids). After that, the part of the experiment with 30 rounds of auctions began.

At the end of the sessions, subjects were asked to fill out a questionnaire (comments, age, gender, degree program). Sessions lasted for 1.5 to 2.5 hours. Subjects were paid according to their cumulated earnings plus a lump-sum in currency units (CU) plus a guaranteed show-up fee of five euros. In C-IP, C-SP/C-SP-2, and V-IP/V-SP the lump sum payments were 100, 125, and 160 CU, and the conversion rates were 0.05, 0.04, and 0.03125 euro/CU. The mean, minimum, and maximum total earnings were 26.4 euros, 5 euros, and 42.8 euros.

4 Results

Findings on efficiency, stability, and revenue precede an analysis of bidding behavior, including an investigation of the driving forces behind our findings.

4.1 Aggregate Results on Efficiency, Stability, and Revenue

We present the results for independent private values (Hypotheses 1 and 2) and for semi-private values (Hypotheses 3, $3'$, 4, $4'$, and 6) separately. For the data analysis in this subsection, we aggregate the 30 observations of each individual bidder and consider the mean observation of each subject. These are independent, because each subject interacts only with two automated bidders and not with other subjects. Sample sizes are given in Table 4. The descriptive statistics in Tables 5 and 7 provide overviews of the experimental outcomes in the treatments with independent private values and semi-private values, respectively. Histograms showing the distributions of proportions of efficient allocations, of proportions of stable allocations, and of revenues are given in Appendix A.1.

Tables 6 and 8 display the results of Wilcoxon-Mann-Whitney (WMW) tests (with continuity correction) on between-treatment comparisons.[19] The equilibrium predictions based on

[19]The WMW test requires equal distributions of the parameter in the populations. If this assumption is

Table 5: Summary statistics for the IP treatments: Proportions of efficient allocations and stable (core) allocations and mean revenue (with one observation per subject). Equilibrium predictions are based on realized valuations.

Variable	Treatment	Equilibrium	Mean	Median	Std. Dev.	Min.	Max.
Efficiency	C-IP	0.79	0.88	0.90	0.10	0.63	1
	V-IP	1	0.89	0.93	0.11	0.53	1
Stability	C-IP	0.30	0.50	0.47	0.13	0.20	0.80
	V-IP	0.50	0.33	0.33	0.18	0	0.60
Revenue	C-IP	61.1	69.0	67.5	8.1	52.6	87.9
	V-IP	61.4	63.6	63.5	10.1	43.9	89.5

Table 6: Wilcoxon-Mann-Whitney tests for IP treatments.

Variable	Hyp.	H_0	H_1	U	p-value
Efficiency	1	$\text{Eff}_{\text{C-IP}} = \text{Eff}_{\text{V-IP}}$	$\text{Eff}_{\text{C-IP}} < \text{Eff}_{\text{V-IP}}$	445	0.1005
Stability		$\text{Stab}_{\text{C-IP}} = \text{Stab}_{\text{V-IP}}$	$\text{Stab}_{\text{C-IP}} \neq \text{Stab}_{\text{V-IP}}$	272.5	0.0000
Revenue		$\text{Rev}_{\text{C-IP}} = \text{Rev}_{\text{V-IP}}$	$\text{Rev}_{\text{C-IP}} \neq \text{Rev}_{\text{V-IP}}$	372	0.0277

the realized valuations, given in Tables 5 and 7, serve as further points of comparison. The results of sign-tests on within-treatment comparisons of predicted and observed performances are given in Table 11 in Appendix B. For all tests, we apply a significance level of 5%.

4.1.1 Independent Private Values

The core-selecting auction performs better than the Vickrey auction. We cannot find sufficient evidence that the core-selecting auction is less likely to result in an efficient outcome than the Vickrey auction. Thus, our data do not support Hypothesis 1. Moreover, in contrast to Hypothesis 2, the core-selecting auction generates significantly more stable allocations than the Vickrey auction. The number of core allocations in C-IP is 51% higher than in V-IP. Comparing revenues, we find that the mean revenue in C-IP is 8% higher than in V-IP, whereas predicted revenues are of similar size. One reason for the lower revenue in the Vickrey auction is the high number (15%) of rounds with zero revenue, which is in line with the theoretical

violated, rejecting the null hypothesis does not allow to conclude on a location shift (i.e., a difference in mean or median). We therefore also conduct one-tailed tests of stochastic equality with H_0: $P(X_1 > X_2) = P(X_1 < X_2)$ suggested by Schlag (2008) that do not require any assumption on distributions. In each case (i.e., for our tests of Hypotheses 1 to 6), the conclusions based on these tests are identical to those based on WMW tests. The conclusions on Hypotheses 1 to 6 also hold if we only consider the last 20 auction rounds (based on the WMW test and the test by Schlag (2008), where the latter test supports Hypothesis 4 not at the 5% but at the 10% significance level).

Table 7: Summary statistics for the SP treatments: Proportions of efficient allocations and stable (core) allocations and mean revenue (with one observation per subject). Equilibrium predictions are based on realized valuations.

Variable	Treatment	Equilibrium	Mean	Median	Std. Dev.	Min.	Max.
Efficiency	C-SP	1	0.89	0.90	0.07	0.77	1
	C-SP-2	1	0.92	0.93	0.06	0.77	1
	V-SP	1	0.93	0.93	0.08	0.73	1
Stability	C-SP	1	0.53	0.50	0.16	0.23	1
	C-SP-2	1	0.57	0.53	0.13	0.37	0.90
	V-SP	0.46	0.33	0.37	0.17	0.00	0.67
Revenue	C-SP	78.5	67.1	66.5	8.8	50.2	84.5
	C-SP-2	78.1	69.7	71.1	8.0	49.8	83.4
	V-SP	59.6	60.6	62.1	11.0	36.5	81.5

prediction (16%). The distributions of all observed revenues in the two auctions in Appendix A.2 visualizes the difference between the auctions.

The core-selecting auction performs better than predicted and the Vickrey auction performs worse than predicted. In particular, the core-selecting auction produces 11% (or 9 percentage points) more efficient allocations and 67% (or 20 percentage points) more stable allocations than predicted, and its revenue is 13% (or 8 CU) higher than predicted. In contrast, the Vickrey auction achieves only 89% of the predicted 100% efficiency and generates 34% (or 17 percentage points) fewer stable allocations than predicted. Its revenue is approximately at its predicted level.

4.1.2 Semi-Private Values

The core-selecting auctions perform similarly to each other, and both perform better than the Vickrey auction. The two-stage version of the core-selecting auction produces slightly more efficient allocations, slightly more core allocations, and a slightly higher revenue for the seller than its sealed-bid counterpart, but none of these differences is statistically significant (and therefore Hypothesis 6 cannot be confirmed). Comparing the core-selecting auctions with the Vickrey auction, we observe similar levels of efficiency (of 89 to 93%). The data support Hypotheses 3, $3'$, 4, and $4'$, that is, the core-selecting auctions perform better than the Vickrey auction with respect to stability and revenue. The proportion of stable allocations in the core-selecting auctions is by more than 60% (or 20 percentage points) higher than in the Vickrey auction. Revenues in C-SP and C-SP-2 are by 11% and 15% higher than in the

Table 8: Wilcoxon-Mann-Whitney tests on Hypotheses 3 to 6 for SP treatments.

Variable	Hyp.	H_0	H_1	U	p-value
Efficiency	6	$\text{Eff}_{\text{C-SP-2}} = \text{Eff}_{\text{C-SP}}$	$\text{Eff}_{\text{C-SP-2}} > \text{Eff}_{\text{C-SP}}$	416.5	0.0513
Stability	6	$\text{Stab}_{\text{C-SP-2}} = \text{Stab}_{\text{C-SP}}$	$\text{Stab}_{\text{C-SP-2}} > \text{Stab}_{\text{C-SP}}$	429	0.0725
	3	$\text{Stab}_{\text{C-SP}} = \text{Stab}_{\text{V-SP}}$	$\text{Stab}_{\text{C-SP}} > \text{Stab}_{\text{V-SP}}$	207	0.0000
	3′	$\text{Stab}_{\text{C-SP-2}} = \text{Stab}_{\text{V-SP}}$	$\text{Stab}_{\text{C-SP-2}} > \text{Stab}_{\text{V-SP}}$	121	0.0000
Revenue	6	$\text{Rev}_{\text{C-SP-2}} = \text{Rev}_{\text{C-SP}}$	$\text{Rev}_{\text{C-SP-2}} > \text{Rev}_{\text{C-SP}}$	444.5	0.1052
	4	$\text{Rev}_{\text{C-SP}} = \text{Rev}_{\text{V-SP}}$	$\text{Rev}_{\text{C-SP}} > \text{Rev}_{\text{V-SP}}$	353	0.0076
	4′	$\text{Rev}_{\text{C-SP-2}} = \text{Rev}_{\text{V-SP}}$	$\text{Rev}_{\text{C-SP-2}} > \text{Rev}_{\text{V-SP}}$	240.5	0.0004

Kruskal-Wallis tests reject that $\text{Stab}_{\text{C-SP}}$, $\text{Stab}_{\text{C-SP-2}}$, and $\text{Stab}_{\text{V-SP1S}}$ stem from the same population ($p = 0.001$) and reject the same for $\text{Rev}_{\text{C-SP}}$, $\text{Rev}_{\text{C-SP-2}}$, and $\text{Rev}_{\text{V-SP}}$ ($p = 0.002$). Adjusting the p-values with Holm correction, taking into account that we conducted three comparisons on stability and on revenue, does not make a difference for our conclusions.

Vickrey auction. This lower revenue of the Vickrey auction is caused by 15% of rounds with zero revenue (as visualized by the distributions of revenues in Appendix A.2), in line with the predicted 18%.

None of the auctions performs better than predicted. They do not achieve the predicted 100% efficiency but allocate the goods efficiently in 89 to 93% of the cases. Both core-selecting auctions fail to result in stable allocations in almost half the cases, contrasting theoretical predictions, and achieve only 85% (C-SP) and 89% (C-SP-2) of the predicted revenue (and these differences are statistically significant). In the Vickrey auction, the number of stable allocations is 28% lower than predicted, but revenue is on the predicted level.

4.2 Individual Bidding Behavior

We first examine how (and to what extent) individual bidding behavior deviates from equilibrium behavior. We then identify properties of actual bidding that can be linked to the findings in Section 4.1.

4.2.1 Equilibrium Bids and Observed Bids

Our main observation is that subjects do react to the incentives that the core-selecting auctions provide. The data provide support in favor of Hypothesis 5. In all three core-selecting treatments, bid spreads are significantly more frequent than in the respective Vickrey treatment with the same information structure.[20]

[20] Kruskal-Wallis test on C-SP, C-SP-2, and V-SP: $p = 0.0047$. Data points: $\#(b_2^{AB} > b_2^{B})$ of a subject. Three one-tailed WMW tests with continuity correction on H_0: $\#(b_2^{AB} > b_2^{B})_{\text{T1}} = \#(b_2^{AB} > b_2^{B})_{\text{T2}}$. T1=C-IP vs. T2=V-IP: $U = 187$, $p = 0.000$; T1=C-SP vs. T2=V-SP: $U = 323.5$, $p = 0.002$; T1=C-SP-2 vs. T2=V-SP:

Table 9: Proportions of bids with relevant properties. Equilibrium predictions in parentheses.

Property	C-IP	V-IP	C-SP	C-SP-2	V-SP
$b_2^B < b_2^{AB}$	0.58 (1)	0.22 (0)	0.46 (1)	0.37 (1)	0.19 (0)
$b_2^B < b_2^{AB}$ & $b_2^B < v_2$	0.51 (1)	0.08 (0)	0.34 (0)	0.32 (0)	0.10 (0)
$b_2^B < b_2^{AB}$ & $b_2^B = v_2$	0.004 (0)	0.02 (0)	0.06 (1)	0.02 (1)	0.05 (0)
$b_2^B \geq b_2^{AB}$ & $b_2^B = v_2$	0.11 (0)	0.22 (1)	0.09 (0)	0.09 (0)	0.39 (1)
$b_2^B < v_2$	0.75 (1)	0.29 (0)	0.73 (0)	0.75 (0)	0.30 (0)
$b_2^B = v_2$	0.12 (0)	0.24 (1)	0.15 (1)	0.12 (1)	0.44 (1)
$b_2^B > v_2$	0.13 (0)	0.46 (0)	0.12 (0)	0.13 (0)	0.26 (0)

Subjects in the core-selecting private-values treatments correctly react to the incentive for bid shading in 75% of all auction rounds. In 51%, they correctly combine bid shading with a bid spread (see Table 9).[21] These bidding patterns are qualitatively in line with predictions, but not quantitatively. Bids on item B are closer to subjects' valuations than predicted. The median (mean) difference between predicted and observed bid shading (conditional on observed bid shading) is 12 (9) CU.

Subjects in the semi-private values treatments also shade bids, which is in contrast to predictions. In C-SP and C-SP-2, bid shading occurs in 73% and 75% of all bids and bid spreads occur in 46% and 37% of all bids (see Table 9). Some subjects follow the equilibrium strategy,[22] and 6% and 2% of all bids combine truthful bidding on B with some bid spread, $b_2^B = v_2 < b_2^{AB}$. Conditional on applying a bid spread, the observed bid spreads are positively correlated with the bid b_1^A, i.e., with the optimal bid spread. This implies that bidders use the relevant information b_1^A. The average bid spread is smaller than the optimal bid spread b_1^A. (See the regression results in Table 12 in Appendix B.)

The observed (less pronounced than predicted) bid-shading in the independent private values environment could be explained by risk-aversion. This explanation, though, is incon-

$U = 286$, $p = 0.003$. This result still holds when we eliminate observations with a bid spread and suboptimal overbidding $b_2^{AB} > b_2^B > v_2$ (which occurs in 7%, 6%, and 2% of all rounds in C-IP, C-SP, and C-SP-2, and in 9% of all rounds in V-IP and V-SP (see Table 13, case 5)).

[21] 24% of the observed bids fulfill the assumptions of the theoretical analyses by Ausubel and Baranov (2010) and Goeree and Lien (2016) for risk-neutral bidders and by Schneider et al. (2015) for risk-averse bidders, as implemented experimentally by Marszalec (2014) (see Table 13, case 6, bids with bid shading and no bid spread, which correspond to one-dimensional bids on the preferred item).

[22] In C-SP, one subject bid $b_2 = (0, v_2, v_2 + b_1^A)$ in 28 of the 30 rounds (and in the two other rounds deviated by one), one subject did so in five rounds, and one subject in one round. In C-SP-2, one subject bid according to equilibrium in eight rounds and one in five rounds.

sistent with the observed bidding behavior in the semi-private values environment for which equilibrium bids on B equal valuations and in particular do not depend on a bidder's risk attitude.

In the Vickrey auction it is optimal to bid according to $b_2^B = v_2 \geq b_2^{AB}$. Such bids occur in 22% and 39% of all auction rounds in V-IP and V-SP (see Table 9). Deviations from truthful bidding, although dominated, are commonly observed in experiments with Vickrey auctions. The mean deviation $b_2^B - v_2$ is 12 CU and 1 CU in V-IP and V-SP (and medians are zero). Bid shading occurs in about 30% of the rounds in each of the two settings and overbidding on B in 46% and 26% of all rounds with private and semi-private values, respectively. Thus, the lower proportion of truthful bidding on B in V-IP, as compared to V-SP, comes with a higher proportion of overbidding.

4.2.2 Relating Auction Performance to Individual Bidding Behavior

The core-selecting auction performs better than predicted with respect to efficiency and stability in the independent private values setting, whereas its proportion of stable outcomes is worse than predicted in the semi-private values setting. The Vickrey auction is less stable than predicted in either setting.

In equilibrium, allocations are stable in the core-selecting auctions whenever Bidders 1 and 2 each win an item (in particular, equilibrium allocations with semi-private values are always stable) and in the Vickrey auction whenever Bidder 3 wins.[23] This dichotomy suggests dividing the sample according to the identity of the winner(s). Table 10 shows the proportions of efficient allocations and stable allocations in the five treatments calculated separately for the two cases.[24] According to this table, differences between predictions and observations mainly

[23] Recall that a stable allocation is an efficient allocation with sufficiently high payments (that are still below the valuation; see Section 2.1). In a core-selecting auction, if Bidders 1 and 2 win, the auction rules assure sufficiently high payments if there is no overbidding on B. In the core-selecting auction with semi-private values and in the Vickrey auction, if Bidder 3 wins, truthful bidding by Bidders 1 and 2 on A and B assures that the payment is sufficiently high. In detail, the conditions for an allocation to be in the core are the following. In all auctions, if Bidder 2 wins AB or an item is not assigned, the allocation is inefficient and therefore not stable. Bid shading on B by Bidder 2 threatens stability as it may cause Bidder 3 to inefficiently win, while overbidding on B may cause Bidders 1 and 2 to inefficiently win. If Bidder 3 efficiently wins AB, bid shading by Bidder 2 for B causes a violation of stability if Bidder 3's payment $p_3 = \max\{b_2^{AB}, v_1 + b_2^B\}$ is below $v_1 + v_2$. If Bidders 1 and 2 efficiently win in a core-selecting auction, truthful bidding by Bidders 1 and 3 assures that $p_2 \geq \max\{0, v_3 - v_1\}$ and $p_1 + p_2 \geq v_3$, but the condition $p_1 \geq \max\{0, v_3 - v_2\}$ can be violated due to overbidding on B because the auction rules enforce $p_1 = \max\{0, v_3 - b_2^B, b_2^{AB} - b_2^B\}$. If Bidders 1 and 2 win in a Vickrey auction, the sum of their payments will be too low for a stable allocation if $p_1 + p_2 = \max\{v_3 - b_2^B, b_2^{AB} - b_2^B, 0\} + \max\{v_3 - v_1, 0\}$ is less than v_3, which holds unless there is a tie or a large bid spread.

[24] We do not consider the cases in which Bidder 2 is awarded the bundle AB or Bidder 1 is awarded A while B is not sold, because then the allocation is neither efficient nor stable. In C-IP, C-SP, C-SP-2, V-IP, and V-SP, Bidders 1 and 2 are awarded A and B in 40%, 44%, 45%, 52%, and 51% of all auctions; Bidder 3 is awarded

Table 10: Proportions of efficient allocations and stable allocations given the successful bids. Equilibrium predictions are provided in parentheses.

Successful bids	Efficient	Stable	Efficient	Stable	Efficient	Stable
	C-IP		C-SP		C-SP-2	
b_1^A and b_2^B	0.98 (1)	0.91 (1)	0.96 (1)	0.91 (1)	0.97 (1)	0.93 (1)
b_3^{AB}	0.88 (0.80)	0.24 (0.01)	0.90 (1)	0.25 (1)	0.94 (1)	0.30 (1)
	V-IP		V-SP			
b_1^A and b_2^B	0.88 (1)	0.04 (0)	0.96 (1)	0.07 (0)		
b_3^{AB}	0.95 (1)	0.67 (1)	0.95 (1)	0.63 (1)		

occur if Bidder 3 wins. We therefore focus on this case.

Efficiency (and therefore also stability) is in all auctions impaired by bid shading of Bidder 2 if it leads to an inefficient assignment of the bundle to Bidder 3. Stability can be negatively affected by bid shading even if efficiency is not. Whereas a small degree of bid shading has no impact on efficiency if $v_1 + v_2 > v_1 + b_2^B \geq v_3$, stability is destroyed by any degree of bid shading (if also $b_2^{AB} < v_1 + v_2$) in the core-selecting and the Vickrey auctions. This is because Bidder 3's payment $v_1 + b_2^B$ then drops below the lower bound $v_1 + v_2$ required by stability (see Footnote 23).

Bid shading is prevalent in the core-selecting auctions and indeed Table 10 does show lower proportions of efficient allocations and of stable allocations in these auctions than in the Vickrey auction. In the independent private values setting, bid shading in the core-selecting auction is predicted for all bids. Because bid shading occurs less often and is less pronounced than predicted, it affects efficiency and stability less than predicted. In the core-selecting auctions with semi-private values, no bid shading (and no instability) is predicted; however, the observed proportion of bid shading is similar to that in the independent private values setting and therefore results in similar proportions of stable allocations.[25] In the Vickrey auction, bid shading occurs for approximately 30% of bids and is less prevalent than in the core-selecting auctions; it contributes to the observed 33% and 37% instable allocations (instead of the predicted 0%).[26]

AB in 55%, 53%, 52%, 46%, and 46% of all auctions; Bidder 2 is awarded AB in 5%, 3%, 3%, 2%, and 3% of all auctions. Bidder 1 is awarded A when B is not sold in one auction in V-SP.

[25] Stable allocations when Bidder 3 wins and Bidder 2 uses bid shading (i.e., $v_3 \geq b_2^{AB} \geq v_1 + v_2 > v_1 + b_2^B$) are rare and occur in C-IP in 2%, in C-SP in 0.1%, and in C-SP-2 in 0% of all bids where Bidder 3 wins and Bidder 2 uses bid shading.

[26] Truthful bids and bids above the value on B contribute 29/44 and 36/18 percentage points to the 67%/63% stable allocations in V-IP/V-SP. Truthful bids and bids above the value on B contribute 12/16/15 and 10/8/15 percentage points to the 24%/25%/30% stable allocations in C-IP/C-SP/C-SP-2. Stability hinges on the bid

To sum up, all core-selecting auctions generate similar proportions of efficient and of stable allocations, and instable allocations result mainly if Bidder 3 wins. The observed instability is mainly caused by bid shading, which is predicted in the independent private values setting but not in the semi-private values setting. There is less than predicted bid shading with independent private values and more than predicted with semi-private values, resulting in the overall good performance of C-IP and the bad performance of C-SP and C-SP-2 as compared to the equilibrium predictions with respect to stability and revenue. In the Vickrey auction, as predicted, instable allocations are frequent when Bidders 1 and 2 win. In contrast to predictions, they also occur (to a lower extent) if Bidder 3 wins due to bid shading by Bidder 2. V-IP and V-SP perform similarly with respect to stability, although the higher proportion of overbidding on B in the independent private values setting (46% vs. 26%, see Table 9) causes a higher proportion of inefficient allocations.

5 Conclusion

We provide experimental evidence in support of a core-selecting auction when information is incomplete. First, it performs better than the Vickrey auction if revenue or stability properties are a concern to the auctioneer. Efficiency is high in all auctions that we analyze, but proportions of core allocations and revenue are higher in the core-selecting auction. In particular, the latter does not exhibit zero revenue outcomes, which are prevalent in the Vickrey auction. Second, in an independent private values environment, the core-selecting auction generally performs well; at least it performs better than predicted with respect to efficiency, stability, and revenue. The core-selecting auction performs similarly well in our semi-private values environment, though worse than predicted in equilibrium. For practical applications, it is worthwhile stressing that individual bidding behavior is less effected by the informational setting – independent private values or semi-private values – than predicted by theory. In particular, bids are closer to valuations than is predicted in the independent private values setting (for risk-neutral bidders).

Although we derived our hypotheses based on the assumption of risk-neutral bidders who play Bayesian Nash equilibrium strategies, our hypotheses, to some extent, remain valid under different assumptions. First, our hypotheses are unaffected by the bidders' risk attitudes in the semi-private values setting, in which ex-post equilibria exist and in which therefore the risk attitude does not affect optimal bidding behavior. Second, due to the experimental setup in which subjects only encounter computerized opponents, our hypotheses remain valid if subjects

b_2^{AB} for 0 to 2 of the percentage points.

have social preferences but are unaffected by payoffs of non-human players. Third, because the bidding behaviors of the other bidders are known to subjects (who know the distribution of the bids and not of the valuations of computerized players) our hypotheses do not require assumptions about subjects' beliefs of their opponents' strategies or any other assumptions about strategic interaction.

In our setting with two items, free riding is easier than in larger settings or in settings in which complementary bidders have to coordinate on which items they are aiming for. Given that bidders do not fully use their strategic opportunities in the simple setting, core-selecting auctions might also perform well compared to Vickrey auctions in such more complex environments.

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A Visualization of Data

A.1 Proportions of Efficient and of Stable Allocations and Mean Revenues

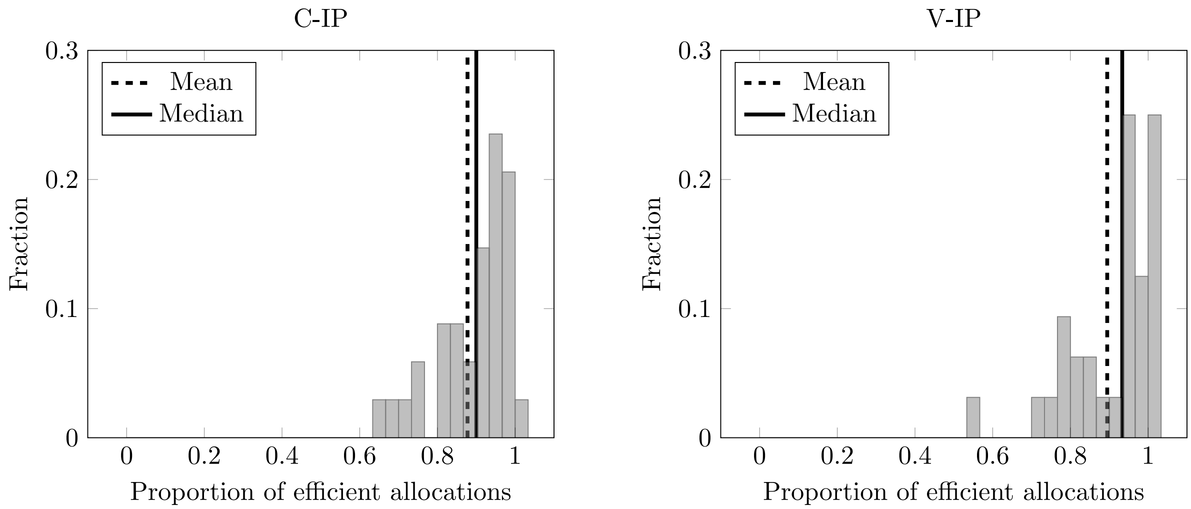


Figure 2: Proportion of efficient allocations per subject and their relative frequency in the C-IP and the V-IP treatments (31 bins of width 1/30).

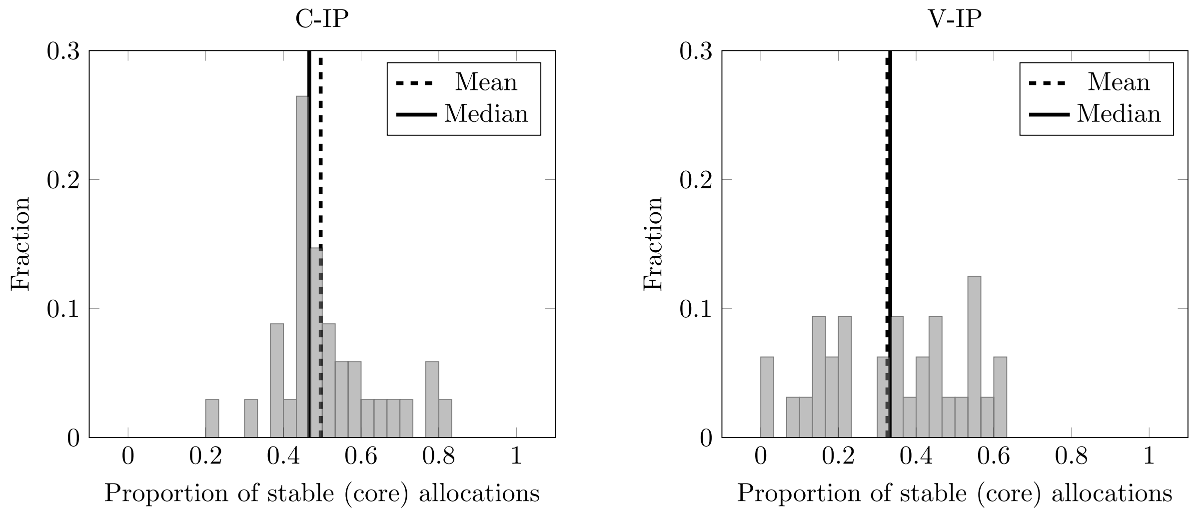


Figure 3: Proportion of stable (core) allocations per subject and their relative frequency in the C-IP and the V-IP treatments (31 bins of width 1/30).

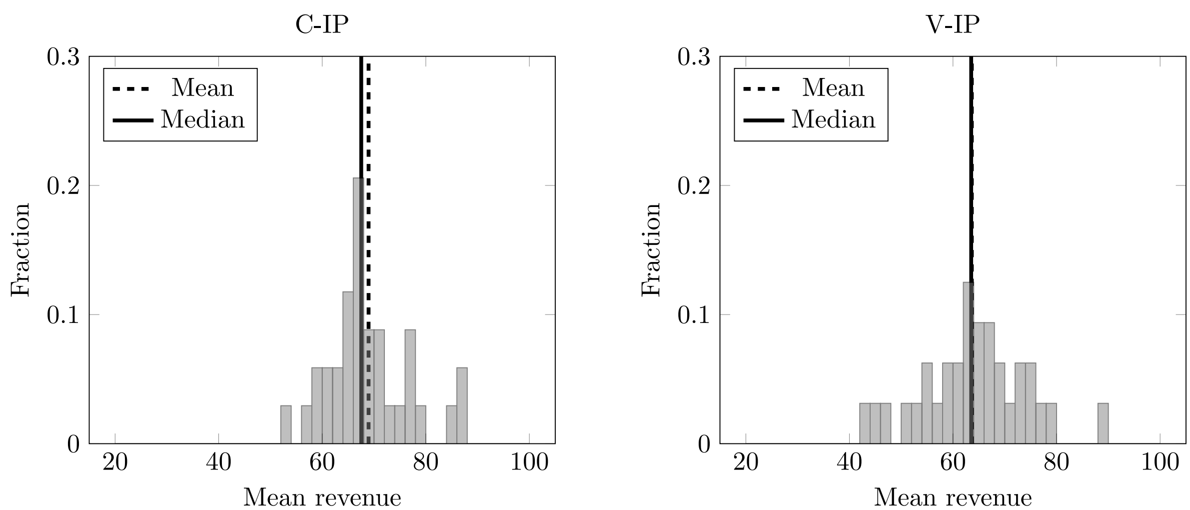


Figure 4: Mean revenue per subject and their relative frequencies in the C-IP and the V-IP treatments (bins of width two).

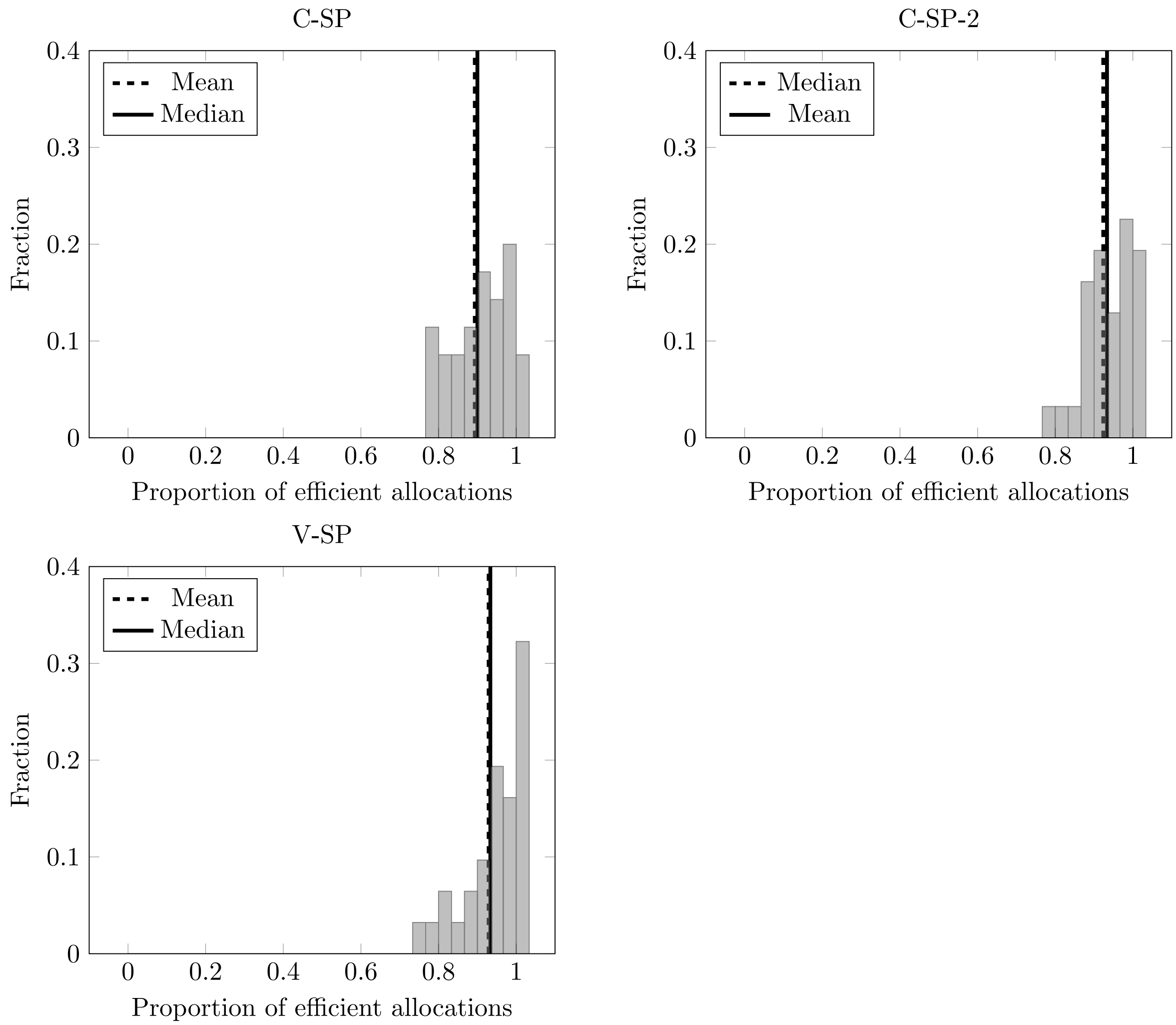


Figure 5: Proportion of efficient allocations per subject and their relative frequency in the C-SP, C-SP-2, and V-SP treatments (31 bins of width 1/30).

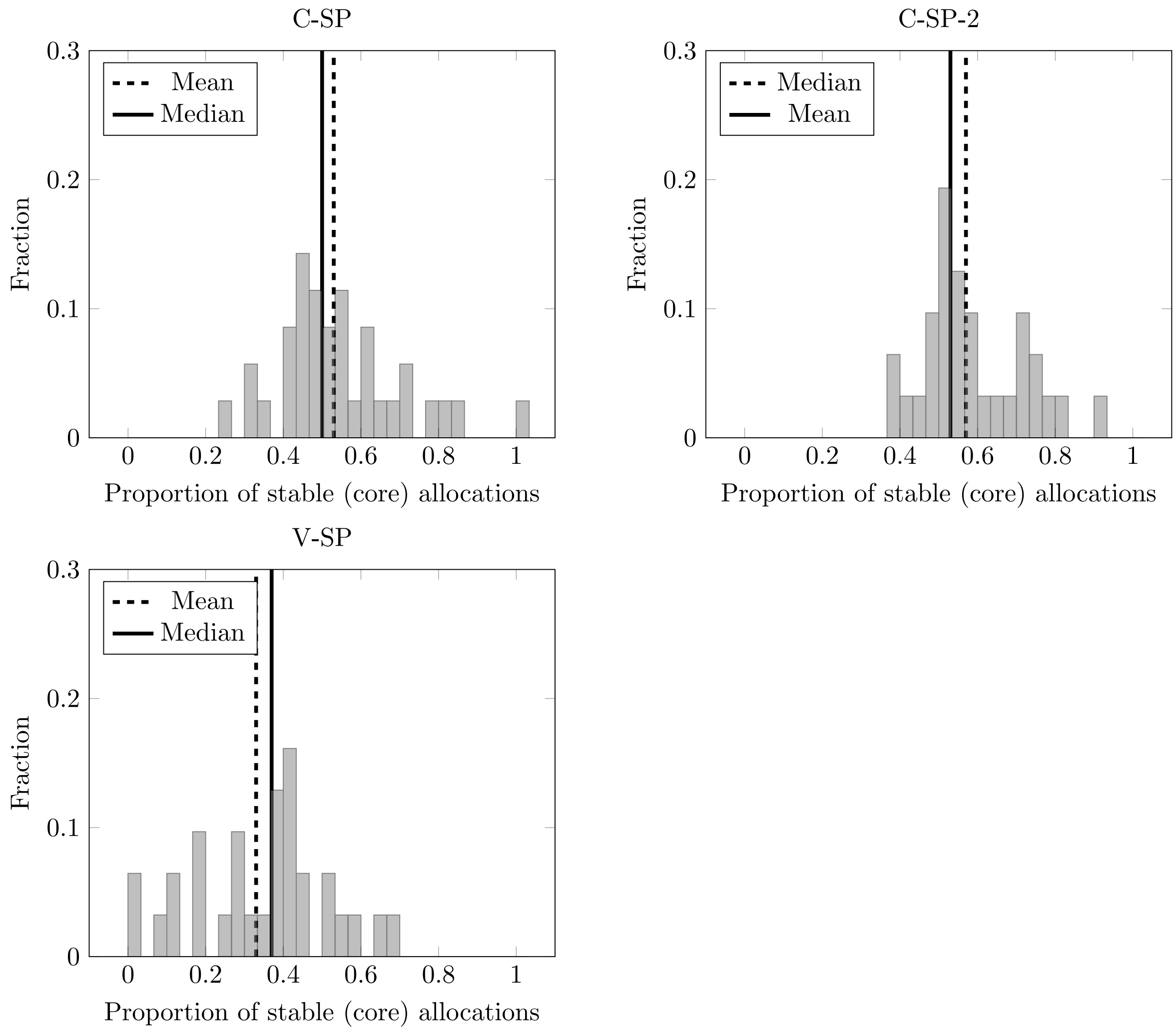


Figure 6: Proportion of stable (core) allocations per subject and their relative frequency in the C-SP, C-SP-2 and V-SP treatments (31 bins of width 1/30).

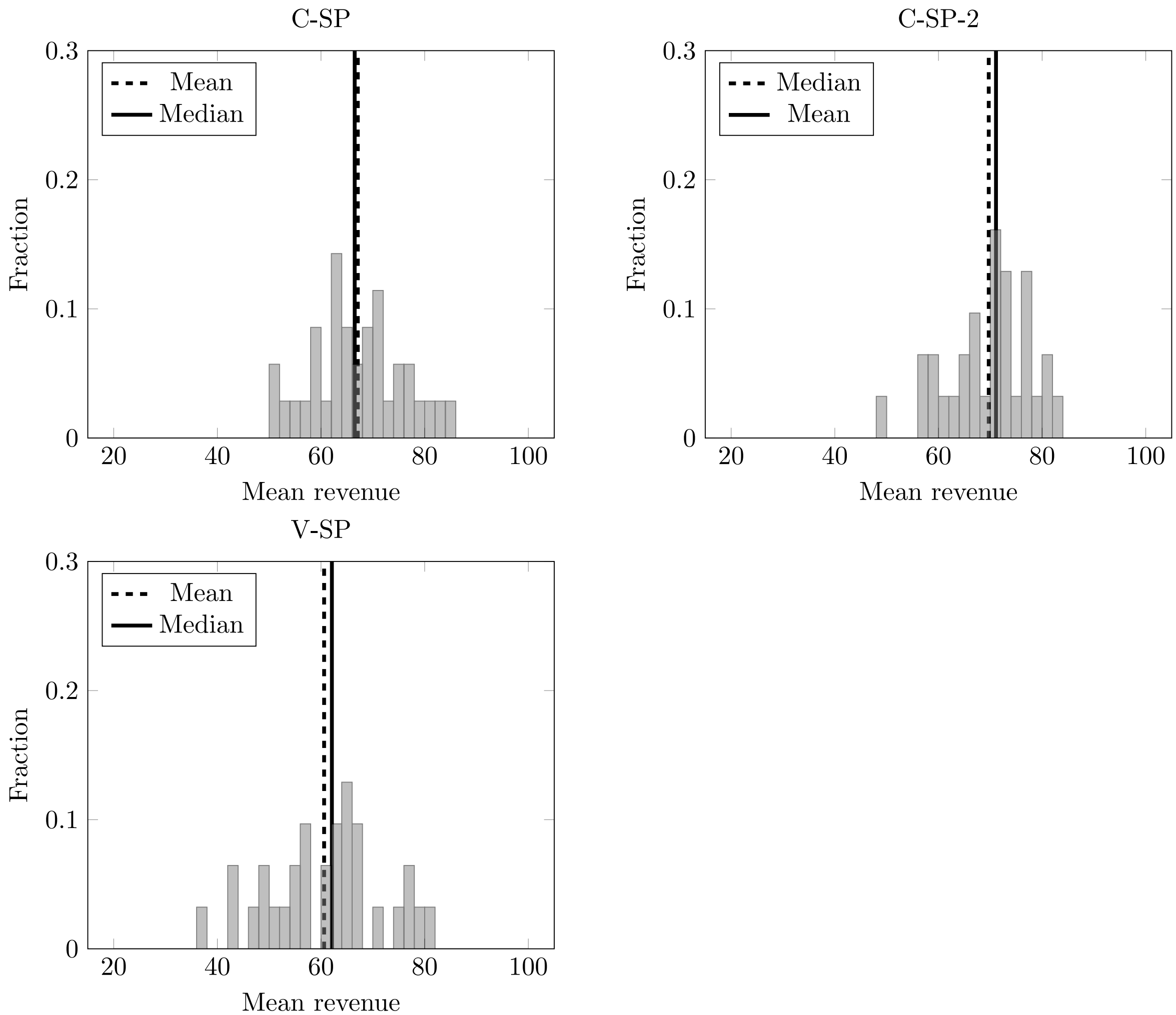


Figure 7: Mean revenue per subject and the revenues' relative frequencies in the C-SP, C-SP-2 and V-SP treatments (bins of width two).

A.2 Revenue Distribution

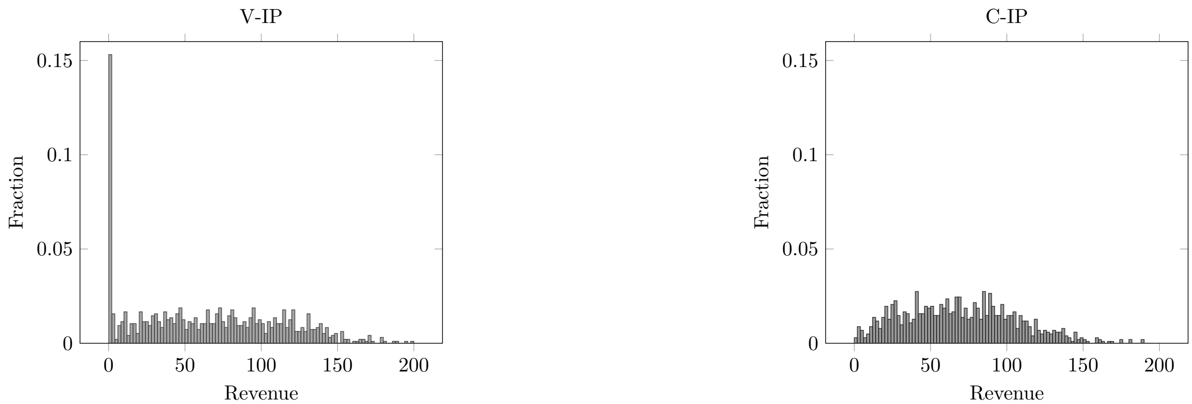


Figure 8: Distribution of the revenue in the V-IP and C-IP (30 observations per subject, bins of width two).

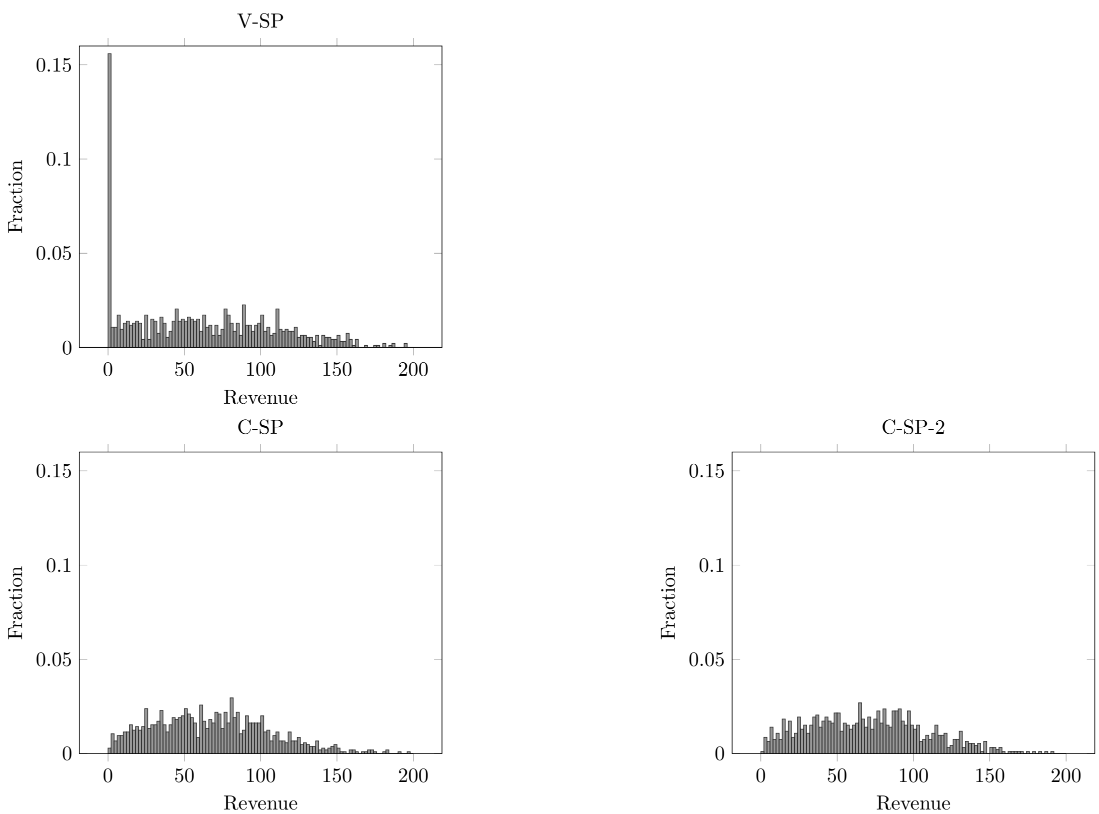


Figure 9: Distribution of the revenue in V-SP, C-SP, and C-SP-2 (30 observations per subject, bins of width two).

B Results of Statistical Tests

Table 11: Two-tailed sign-tests for IP treatments and for SP treatments.

Independent private values treatments		
Variable	H_0	p-value
Efficiency	$\text{Eff}_{\text{C-IP}} = 0.79$	0.0000
Stability	$\text{Stab}_{\text{C-IP}} = 0.30$	0.0000
	$\text{Stab}_{\text{V-IP}} = 0.50$	0.0070
Revenue	$\text{Rev}_{\text{C-IP}} = 61.1$	0.0000
	$\text{Rev}_{\text{V-IP}} = 61.4$	0.1102

Semi-private values treatments		
Variable	H_0	p-value
Stability	$\text{Stab}_{\text{V-SP}} = 0.46$	0.0009
Revenue	$\text{Rev}_{\text{C-SP}} = 78.5$	0.0000
	$\text{Rev}_{\text{C-SP-2}} = 78.1$	0.0000
	$\text{Rev}_{\text{V-SP}} = 59.6$	0.4731

Table 12: Dependency of bid spreads on b_1^A in C-SP and in CS-SP-2.
Data: Bids with $b_2^{AB} - b_2^B > 0$. Regression with clustered standard errors (per subject).
Model: $\left(b_2^{AB} - b_2^B\right)^{ij} = \beta_0 + \beta_1 \left(b_1^A\right)^{ij} + \varepsilon^{ij} + \delta^i$, i: subject, j: round.

C-SP ($N = 482$)					
	Coefficient	SE	t	p	95% confidence interval
b_1^A	0.260	0.084	3.09	0.004	$[0.088, 0.433]$
Constant	17.244	3.455	4.99	0.000	$[10.177, 24.310]$
	$F(1, 29) = 9.53$, $p = 0.004$, $R^2 = 0.09$				

C-SP-2 ($N = 348$)					
	Coefficient	SE	t	p	95% confidence interval
b_1^A	0.450	0.089	5.08	0.000	$[0.269, 0.631]$
Constant	5.213	2.484	2.10	0.044	$[0.141, 10.286]$
	$F(1, 30) = 25.82$, $p = 0.000$, $R^2 = 0.27$				

C Partition of Bids

Table 13: Partition of observed bids. Equilibrium predictions of shares in parentheses. Figure 10 illustrates the partition.

Property	C-IP	V-IP	C-SP	C-SP-2	V-SP
① $b_2^{AB} \leq b_2^B = v_2$	0.11 (0)	0.22 (1)	0.09 (0)	0.09 (0)	0.39 (1)
② $b_2^{AB} > b_2^B = v_2$	0.004 (0)	0.02 (0)	0.06 (1)	0.02 (1)	0.05 (0)
③ $b_2^{AB} \leq v_2 < b_2^B$	0.02 (0)	0.23 (0)	0.04 (0)	0.08 (0)	0.07 (0)
④ $v_2 < b_2^{AB} \leq b_2^B$	0.04 (0)	0.14 (0)	0.03 (0)	0.03 (0)	0.09 (0)
⑤ $v_2 < b_2^B < b_2^{AB}$	0.07 (0)	0.09 (0)	0.06 (0)	0.02 (0)	0.09 (0)
⑥ $b_2^{AB} \leq b_2^B < v_2$	0.24 (0)	0.21 (0)	0.39 (0)	0.43 (0)	0.20 (0)
⑦ $b_2^B < b_2^{AB} \leq v_2$	0.45 (0.65)	0.04 (0)	0.26 (0)	0.25 (0)	0.06 (0)
⑧ $b_2^B < v_2 < b_2^{AB}$	0.06 (0.35)	0.04 (0)	0.08 (0)	0.08 (0)	0.04 (0)

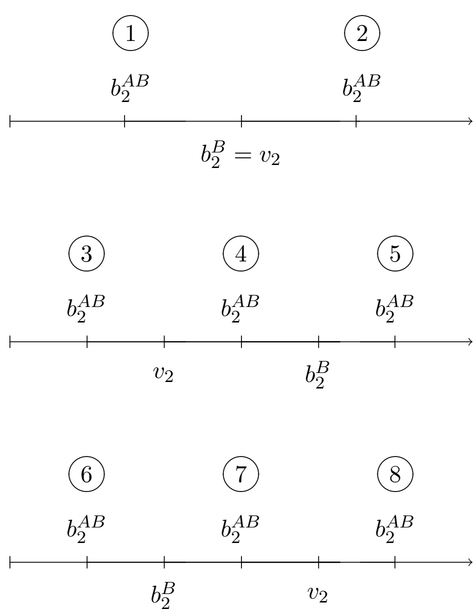


Figure 10: Complete partition of the strategy space with respect to the value and the bid.

D Translation of the Experimental Materials

The experimental materials comprise translations of the experimental instructions, a plot of the translated practice software, and a translation of the quiz questions.

D.1 Instructions

We provide a translation of the instructions to treatment C-IP. The adjustments for the other treatments are given in brackets.

Welcome!

You participate in an experiment on decision-making. In this experiment, you can earn cash. You will receive a payment of 5 euros for sure. How much you will earn in addition depends on your decisions. In the experiment the unit of measurement are so-called currency units [CU]. The amount of CU that you earn during then experiment will be converted into euro, added to the guaranteed payment, and paid out to you in cash. 20 CU [25 CU in C-SP and C-SP-2, 32 CU in V-IP and V-SP] are converted into 1 euro. Your decisions and inputs will be recorded anonymized. You make your decisions isolated from the others at your computer terminal. Communication between the participants is not allowed. The instructions will now be read aloud. Please listen carefully and, if you want, use your printed copy and read along. If you will have any questions at the end of this introduction please raise your hand. An experimental assistant will come to your seat and clarify your question. In what follows, "he" and "his" will be used to simplify the presentation. Please consider this neutral language.

Instructions to the Experiment

You will participate in 30 identical rounds. In each round, two different objects, A and B, will be sold in an auction.

In the auction, there are three bidders, called Bidder 1, Bidder 2, and Bidder 3. In the auction, a bidder can receive Object A, Object B, the bundle AB consisting of both objects, or no object. Note that each object can be awarded only once.

You are Bidder 2. You are only interested in Object B. Object B and the bundle AB have the same value for you. Receiving only Object A or no object has no value for you. Bidders 1 and 3 are automated bidders. Below you learn how the two will bid.

Description of a round

At the beginning of the round, you are assigned a **value** W, which is shown to you at your screen. You value W specifies by how much you value the Object B and the bundle AB. You value W is randomly chosen from the integers from 1 to 100, such that each of the hundred integers has the same probability of being chosen.

Then, the **auction** is conducted. In this auction, you can submit bids. Your bids and the bids of Bidders 1 and 3 determine which object or bundle you receive and which price you have to pay. [C-SP-2: Then, the **auction** is conducted. It has two stages. In stage 1, you can submit bids. Based on your bids and the bids of Bidders 1 and 3 the the objects or the bundle are assigned to the bidders. Your assignment of stage 1 i shown to you on your screen. In stage 2 you have the opportunity to adjust your bids upwards once. The bids of Bidders 1 and 3 remain the same. The bids on stage 2 are final bids. Based on these final bids, it will be determined which object or bundle you receive and which price you have to pay.]

This determines your **round payoff**. If you do not receive any object, your round payoff is **zero**. If you receive an object or the bundle, your round payoff equals your value W minus your price:

$$\text{round payoff} = (\text{value for the received object or bundle}) - (\text{price})$$

After the auction, your received object or bundle, your price, and your round payoff is shown to you. Then, a new round begins.

Auction rules

Bids: In the auction, bids for Object A, for Object B, and for the bundle AB can be submitted. Bids can be equal to or greater than zero.

The table shows the bids that the respective bidder is allowed to submit. Here, e.g., g_1^A denotes the bid of Bidder 1 for the Object A.

	A	B	AB
Bidder 1	g_1^A		g_1^{AB}
Bidder 2 (you)		g_2^B	g_2^{AB}
Bidder 3			g_3^{AB}

- Bidder 1 is automated. He bids the same amount for A and for AB, $g_1^A = g_1^{AB}$. This bid is randomly chosen from the integers from 1 to 100, such that each integer is equally likely. You do *not* learn this bid. [V-SP, C-SP, and C-SP-2: You learn this bid before you submit your bid.]
- You as Bidder 2 may submit bids for B and AB. You therefore submit no, one, or two bids of your choice.
- Bidder 3 is automated. He only submits a bid for the bundle AB. His bid is randomly chosen from the integers from 1 to 200, such that each integer is equally likely. You do *not* learn this bid.

Assignment: To determine who receives which object or bundle, the three bids for the bundle AB are compared with the sum of the bid of Bidder 1 for A and the bid of Bidder 2 for B. Thus, for the assignment g_1^{AB}, g_2^{AB}, g_3^{AB} and $g_1^A + g_2^B$ are compared. The assignment is determined as follows:

- If the sum of the bid for A and the bid for B is equal to or greater than the highest of the bids for AB, then Bidder 1 receives Object A and Bidder 2 receives Object B. Bidder 3 receives no object.
- Otherwise, the bidder with the highest bid for the bundle AB receives both objects and the other two bidders receive no object. If the highest bid for AB was submitted by two or three bidders, then one of these is randomly and with equal probability chosen to receive the bundle.

If you do not bid for an object or bundle then you cannot receive it. If you bid neither for B nor for AB, then you receive nothing. If you do not bid for B, then only g_1^A, g_1^{AB}, g_2^{AB}, and g_3^{AB} are compared. If you do not bid for AB, then only bids g_1^{AB}, g_3^{AB} and $g_1^A + g_2^B$ are compared.

Your price: When according to the assignment rule we have determined who receives which object or bundle, your price is calculated. If you receive nothing, you pay nothing. If you receive an object or bundle, your price is calculated as follows.

- **If you receive the bundle AB**, then your price (p_2) equals the bid for AB that would just be sufficient to match the highest of the bids of Bidders 1 and 3. Thus, it holds that

$$p_2 = \text{maximum } \{g_1^A, g_1^{AB}, g_3^{AB}\}.$$

- **If you receive only the Object B**, then your price is determined in two steps. In step 1, the price of Bidder 1 (p_1) and your preliminary price (p_2^v) are determined. In step 2, this is used to determine your price (p_2).
  Step 1: The price of Bidder 1 (p_1) equals the bid for A that would just be sufficient to match together with your bid for B the highest bid of you and Bidder 3. Thus, it holds that

$$p_1 + g_2^B = \text{maximum } \{g_2^B, g_2^{AB}, g_3^{AB}\},$$

  or, rearranged,

$$p_1 = \text{maximum} \left\{g_2^B,\ g_2^{AB},\ g_3^{AB}\right\} - g_2^B. \qquad (*)$$

  Your preliminary price (p_2^v) equals the bid for B that would just be sufficient to match together with the bid of Bidder 1 for A the highest bid of Bidders 1 and 3. Thus, it holds that

$$p_2^v + g_1^A = \text{maximum } \{g_1^A, g_1^{AB}, g_3^{AB}\},$$

  or, rearranged,

$$p_2^v = \text{maximum } \{g_1^A, g_1^{AB}, g_3^{AB}\} - g_1^A.$$

Step 2: We check whether your preliminary price (p_2^v) together with the price of Bidder 1 (p_1), which has been calculated in ($*$), exceeds the bid of Bidder 3 for AB (g_3^{AB}) and then determine the price as follows.

a) If it holds that $p_2^v + p_1 \geq g_3^{AB}$, then your price equals the preliminary price:

$$p_2 = p_2^v.$$

b) If it holds that $p_2^v + p_1 < g_3^{AB}$, then your price is adjusted upwards, such that your price and the price of Bidder 1 (p_1) together just match the bid of Bidder 3 for AB (g_3^{AB}):

$$p_2 + p_1 = g_3^{AB},$$

or, rearranged,

$$p_2 = g_3^{AB} - p_1.$$

[V-IP, V-SP: **If you receive only the Object B**, then your price (p_2) equals the bid for B that would just be sufficient to match together with the bid of Bidder 1 for A the highest bid of Bidders 1 and 3. Thus, it holds that

$$p_2 + g_1^A = \text{maximum } \{g_1^A, g_1^{AB}, g_3^{AB}\},$$

or, rearranged

$$p_2 = \{g_1^A, g_1^{AB}, g_3^{AB}\} - g_1^A.]$$

In these calculations, a bid that has not been submitted is treated like a bid equal to zero. [C-SP-2: Your price is only calculated on stage 2.]

Practice software

To familiarize you with the assignment and price rules, you will be provided with a practice software before the 30 auction rounds begin. In this software, you can enter bids for Bidder 1, Bidder 2, and Bidder 3. The practice software then calculates for you according to the auction rules who receives which object or bundle and what Bidder 2 has to pay. In the software, you may enter two sets of bids in parallel. Your inputs to the practice software are not relevant for your payoff.

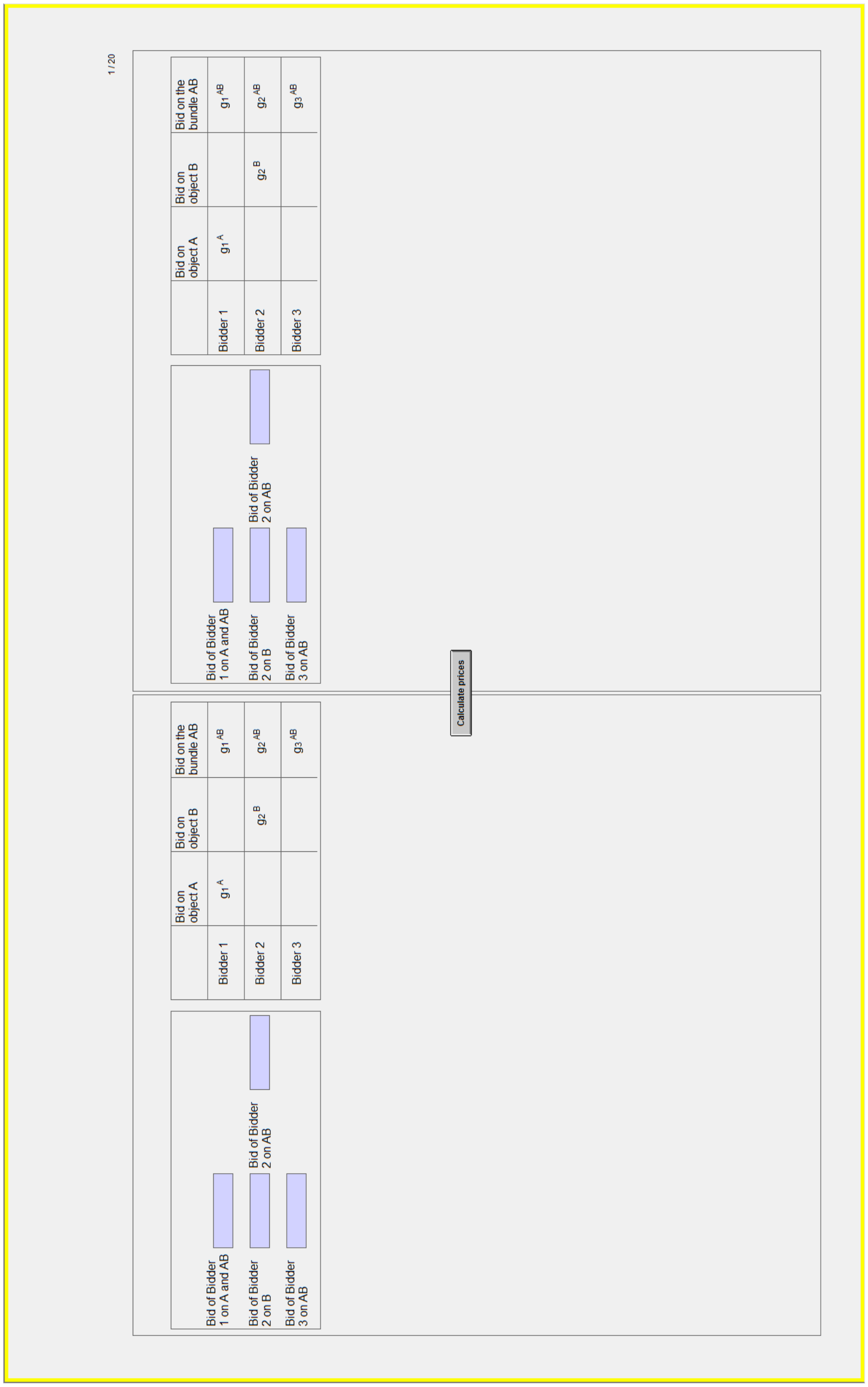
Payoff

You receive a starting credit of 100 CU [125 CU in C-SP and C-SP-2, 160 CU in V-IP and V-SP]. Your total payoff in CU is the sum of your starting credit and your 30 round payoffs in CU. Your total payoff in CU will be converted into euros at the end of the experiment. Immediately after the experiment, your total payoff and your guaranteed payoff of 5 euros will be paid out to you. The payout will be individually and anonymously in the room next door.

Temporal sequence

The reading out of the instructions is followed by 10 minutes, in which you can use your printed copy of the instructions and the practice software at your screen to become familiar with the auction rules. Then, a quiz follows, in which you will answer several questions on your screen about the contents of these instructions. Then, the practice software is again available to you for up to 10 rounds. This practice phase ends at the lasted five minutes after all participants have answered all questions correctly. The experimental assistant will then ask you to end the practice software by pressing the OK-button until the software ends. Then the 30 auction rounds begin for all participants at the same time. If you have a question, please raise your hand. Your question will then be answered at your seat.

D.2 Screen Shot of the Practice Software



D.3 Quiz Questions

The correct answers to the questions are given in brackets or marked in bold. We indicate the questions or answers that differ between treatments.

1. How many rounds do you participate in? [30]
2. How many objects are sold in each auction? [2]
3. How many bidders participate in each auction? [3]
4. How many of those three bidders are automated? [2]
5. Which bidder number do you have? I am bidder [1] **[2]** [3].
6. How much is object A worth to you?
   (a) Nothing.
   (b) It has a value W for me, which lies between 1 and 100 and which I learn at the beginning of the round.
7. How much is object B worth to you?
   (a) Nothing.
   (b) It has a value W for me, which lies between 1 and 100 and which I learn at the beginning of the round.
8. How much is the bundle AB worth to you?
   (a) Nothing.
   (b) It has a value W for me, which lies between 1 and 100 and which I learn at the beginning of the round.
9. Do the object B and the bundle AB have the same value for you?
   (a) Yes.
   (b) No, the value of B and the value of AB can differ.
10. How is your value for the object B and the bundle AB determined?
   (a) At the beginning of each round, it is drawn randomly from the integers between 1 and 100.
   (b) At the beginning of each round, it is drawn randomly from the integers between 1 and 200.
   (c) It is known that it lies between 1 and 100, but it is unknown how it is determined.

11. Do you learn the bid of Bidder 1 for the object A or the bid of Bidder 3 for the bundle AB?

   (a) I learn the bids of Bidder 1 and Bidder 3.

   (b) I learn the bid of Bidder 1. I only know the distribution of the bid of Bidder 3. [in C-SP, C-SP-2, V-SP]

   (c) I learn the bid of Bidder 3. I only know the distribution of the bid of Bidder 1.

   (d) I do not learn the bids of Bidder 1 and Bidder 3. I only know their distributions. [in C-IP, V-IP]

12. How is the bid of Bidder 1 determined?

   (a) He bids the same on A and AB. He does not bid on B. His bid on A and AB is drawn randomly from the integers between 1 and 100.

   (b) He bids the same on A and AB. He does not bid on B. Nothing is known about how his bid is determined.

   (c) He bids only on AB. He does not bid on A and B. His bid on AB is drawn randomly from the integers between 1 and 200.

13. How is the bid of Bidder 3 determined?

   (a) He bids the same on A and AB. He does not bid on B. His bid on A and AB is drawn randomly from the integers between 1 and 100.

   (b) He bids the same on A and AB. He does not bid on B. Nothing is known about how his bid is determined.

   (c) He bids only on AB. He does not bid on A or B. His bid on AB is drawn randomly from the integers between 1 and 200.

14. Who is awarded the objects or the bundle? Mark **all** correct statements.

   (a) If the sum of the bids on A and B is greater than or equal to the highest bid on AB, Bidder 1 is awarded object A and Bidder 2 is awarded object B.

   (b) The assignment is always randomly determined.

   (c) The bidder with the highest bid always gets an object or bundle.

   (d) If the sum of the bids on A and B is smaller than the highest bid on AB, one of the bidders with the highest bid on the bundle AB gets this bundle.

15. Assume you are awarded the bundle AB. Mark all statements that are true for sure.

   (a) My bid on B plus the bid of Bidder 1 on A in sum are smaller than my bid on AB.

   (b) I bid on AB as least as much as the other two bidders.

   (c) My price p_2 is equal to the highest bid of the other two bidders.

16. [Only in C-IP, C-SP, C-SP-2] Assume you are awarded the object B. Mark all statements that are true for sure.

(a) My bid on B plus the bid of Bidder 1 on A in sum are greater than or equal to each bid on AB.

(b) My price p_2 is equal to the highest bid of the other two bidders.

(c) To determine my price p_2, I first need to determine the price p_1 of Bidder 1.

(d) If my preliminary price p_2^v plus the price p_1 of Bidder 1 does not exceed the bid of Bidder 3 on AB, I have to pay the difference in addition.

(e) The sum of my price p_2 plus the price p_1 of Bidder 1 is at least as high as the bid of Bidder 3 on AB.

16. [Only in V-IP, V-SP] Assume you are awarded the object B. Mark all statements that are true for sure.

(a) My bid on B plus the bid of Bidder 1 on A in sum are greater than or equal to each bid on AB.

(b) My price p_2 is equal to the highest bid of the other two bidders.

(c) p_2 equals the bid on B that would just be enough to meet, in sum with the bid of Bidder 1 on A, the highest bid of the Bidders 1 and 3.

17. Who is awarded the objects or the bundle in the following example?

	Bid on object A	Bid on object B	Bid on the bundle AB
Bidder 1	3		3
Bidder 2		88	74
Bidder 3			147

(a) Bidder 1 gets A and Bidder 2 gets B.

(b) Bidder 1 gets AB.

(c) Bidder 2 gets AB.

(d) Bidder 3 gets AB.

18. Who is awarded the objects or the bundle in the following example?

	Bid on object A	Bid on object B	Bid on the bundle AB
Bidder 1	41		41
Bidder 2		56	78
Bidder 3			67

(a) Bidder 1 gets A and Bidder 2 gets B.

(b) Bidder 1 gets AB.

(c) Bidder 2 gets AB.

(d) Bidder 3 gets AB.

19. Calculate the price p_2 of Bidder 2 in the following example.
$p_2 =$ [45] in C-IP, C-SP, C-SP-2; $p_2 =$ [26] in V-IP, V-SP

	Bid on object A	Bid on object B	Bid on the bundle AB
Bidder 1	41		41
Bidder 2		56	78
Bidder 3			67

20. Calculate the price p_2 of Bidder 2 in the following example.
$p_2 =$ [34] in C-IP, C-SP, C-SP-2; $p_2 =$ [23] in V-IP, V-SP

	Bid on object A	Bid on object B	Bid on the bundle AB
Bidder 1	49		49
Bidder 2		34	12
Bidder 3			72

21. Calculate the price p_2 of Bidder 2 in the following example. $p_2 =$ [25]

	Bid on object A	Bid on object B	Bid on the bundle AB
Bidder 1	8		8
Bidder 2		13	36
Bidder 3			25

22. [Only in C-SP-2] Which statements are true for stage 2 of the auction? Mark all correct statements.

(a) My price p_2 is only calculated in stage 2.

(b) The bids of Bidders 1 and 3 from stage 1 do not change in stage 2.

(c) I can arbitrarily change my bids.

(d) I learn the bids of Bidder 1 and Bidder 3.

(e) I learn the assignment based on the bids submitted in stage 1.

23. How is the round payoff computed?

(a) It is equal to the value W.

(b) It is equal to the price p_2.

(c) It is equal to the value W minus the price p_2, if I am awarded an object or the bundle and zero, otherwise.