Vying for Support: Lobbying a Legislator with Uncertain Preferences

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Abstract

We consider a dynamic model of lobbying with two opposing lobbyists vying for a legislator's support, whose preferences are uncertain. The results from the symmetric game show that the degree of uncertainty of legislator preferences has a direct effect on the bidding strategy of lobbyists. When the degree of uncertainty is low, lobbyists play in a one shot scenario. Conversely, we find that if the degree of uncertainty is high, the incentives of waiting outweigh its costs, and lobbyists proceed under a dynamic scenario. As the optimal policy function evolves as the state evolves, it is likely for lobbyists who start by bidding conservatively to end up in the one shot scenario. Interestingly, we also find multiplicity of equilibria when the degree of uncertainty is moderate. Under moderate levels of uncertainty, lobbyists can choose either to bid above or below the legislator's integrity threshold, as well as decide to end the game today or continue playing in the subsequent periods.

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1 Introduction

In this paper, we consider a dynamic model of lobbying wherein two opposing lobbyists vie for a legislator's support, given uncertain legislator preferences. We look beyond who wins the legislator's support, and focus on how the winning probabilities of each lobbyist evolve and how lobbyist behaviour changes as more information on legislator preference is acquired. Examining how lobbying proceeds behind closed doors could aid in drafting more effective lobbying regulations and provide constituents with an avenue to influence policy outcomes before issues hit legislature floors.

Lobbying is ubiquitous in most legislative systems. Other forms of lobbying, such as shadow lobbying, also exist outside legal bounds - influencing policy outcomes without oversight. The process in which lobbying influences legislation, however, is not yet fully understood. A large body of literature exists on the effects of lobbying on policy outcomes, either through information or transactional exchanges (Austen-Smith, 1993; Groseclose and Snyder, 1996; de Figueiredo, 2002; Hall and Deardorff, 2006). Both perspectives have been widely discussed, with empirical evidence suggesting that the effects of cash-for-favour lobbying activities are marginal (Grossman and Helpman, 2001; Ansolabehere et al., 2003; de Figueiredo and Richter, 2013). Despite this, public awareness on lobbying is centered largely on the perception of transactional lobbyist-legislator interactions.

Media reports on lobbying scandals have highlighted the prevalence of cash-for favour exchanges despite the apparent lack of direct impact in policy outcomes. The importance of focusing on these interactions goes back to the idea of the political agency introduced by Downs (1957). Politicians are less likely to seek rent where there is increased scrutiny. For example, the sectors with the highest levels of lobbying spending in the United States in the past five years do not include hot button issues such as abortion and gun laws (Center for Responsive Politics, 2017). The views of legislators are often undisclosed in these sectors, and this uncertainty in preferences provides politicians with opportunities for gain at the expense of the collective good. Furthermore, as public scrutiny is higher on cash-for-favour lobbying activities, politicians are more likely to take into account public preferences and move away from rent-seeking activities.

We approach the lobbying process as one where cash-for-favour exchanges occur as a means to obtain access to legislators for information. The model takes into account the degree of uncertainty on a non-strategic legislator's preference and the perceived advantages each lobbyist may have on their respective policies. We focus on the symmetric lobbyist case and look at the evolution of bids and the probabilities of winning.

The paper moves away from the sequential lobbying structure introduced by Groseclose and Snyder (1996). Instead, we adopt a simultaneous lobbying structure to capture how lobbying proceeds behind closed doors. Under shadow lobbying, where lobbyist- legislator interactions are kept private, the opportunity for lobbyists to counteroffer may not exist. The simultaneous lobbying approach takes this into account and retains focus on how interactions center on the uncertainty of legislator preferences.

Dekel et al. (2006) looked at vote buying and explored as an extension the presence of uncertainty in legislatures. They found that with a large enough body of legislators, one can predict who the winning lobbyist is. Buzard and Saiegh (2016) also looked at uncertainty in legislator preferences in a sequential vote buying model, specifically the allocation of bribes amongst legislators. The paper takes a closer look on how uncertainty in legislator preferences change lobbying dynamics. Understanding the lobbying dynamics may help provide additional insight on the lobbying's revolving-door phenomenon explored by i Vidal et al. (2012) where former staffers turned lobbyists use their connections to incumbents to push for legislation. Although dynamic models have previously been used to model the mechanics of lobbying in legislation, for example Wirl (1994), this is, to the best of our knowledge, the first paper to model uncertainty in lobbying dynamically.

The results from the symmetric game show how lobbyists interact depending on the degree of uncertainty on legislator preferences. We find that when the degree of uncertainty is low, lobbyists play in a one shot scenario. Payoffs lobbyists could obtain by learning more about the legislator's preference are negated by the cost of delaying the win and the increase in the bid of the following round, if it continues. Conversely, we find that if the degree of uncertainty is high, the incentives of waiting outweigh its costs, and lobbyists proceed under a dynamic scenario. As the optimal policy function evolves as the game proceeds, it is likely for lobbyists who start by bidding conservatively to end up in the one shot scenario. Interestingly, we also find multiplicity in equilibria when the degree of uncertainty is moderate. Under moderate levels of uncertainty, lobbyists can choose either to bid above or below the threshold. Depending on what lobbyists prioritize, one may choose to bid more aggressively to secure the vote early on, or conservatively to save on potential bidding costs.

We discuss the specifics of the model in section 2, run through the timing of the game in section 3, the probability computations in section 4 and the bias updating process in section 5. In sections 6 and 7, we analyse best responses and provide a full characterisation of the symmetric markov equilibrium. Finally, we illustrate the dynamics at work through an example in section 8 and provide a quick summary of the work done insofar in section 9.

2 The model

Two lobbyists, $j \in \{1, 2\}$, compete for the vote of one legislator for their policy j.

The legislator has a threshold $t \in \mathbb{R}_+$ to sell her vote and a preference $b \in \mathbb{R}$ for policy two over policy one. The bias is distributed $b \sim U(d_{min}, d_{max})$, where a positive b indicates a legislator bias towards policy two, and a negative b indicates a preference for policy one. In the symmetric case, the initial maximum is equal to the absolute value of the minimum, $d_{max} = -d_{min} = d$.

The legislator only considers a lobbyist's bid when it exceeds her threshold and provides a positive payoff. Both the threshold and the bids are bias adjusted to account for the legislator's preference. The legislator is non-strategic and will always accept the bid that surpasses the bias adjustment threshold and provides the highest positive payoff.

The legislator's utility is given by U_l,

$$U_l = \begin{cases} p_1 - t - b & \text{if lobbyist one wins,} \\ p_2 - t + b & \text{if lobbyist two wins,} \\ 0 & \text{if neither lobbyist wins} \end{cases}$$

where p_j is the bid of lobbyist j.

Both lobbyists do not know what the legislator's bias is, but are aware that it is has a distribution of $b \sim U(d_{min}, d_{max})$. The support of the distribution provides information on the maximum advantage each lobbyist may have in the bidding process. A large bias interval range implies that the degree of uncertainty on legislator preferences is high. A more negative bias indicates a stronger legislator preference for lobbyist one's proposed policy, while a more positive bias indicates a stronger legislator prefence for lobbyist two's proposed policy. As the minimum and maximum possible bias values are given by d_{min} and d_{max}, we call these values the maximum advantage, on in some cases the minimum disadvantage, for lobbyists one and two respectively.

Lobbyist's j receives w_j, conditional on winning the legislator's support. Both lobbyists are aware that have the same winning valuation, $w_j = w_{-j} = w$.

To win the legislator's support, the bid of each lobbyist, p_j, must reach the following thresholds:

$$\begin{cases} t_1 = t + b & \text{for } j = 1, \\ t_2 = t - b & \text{for } j = 2 \end{cases}$$

The utility of the lobbyists is given by :

$$U_1 = \begin{cases} w_1 - p_1 & \text{if } \ p_1 > p_2 + b \text{ and } p_1 \geq t_1 \\ 0 & \text{otherwise} \end{cases} ,$$

$$U_2 = \begin{cases} w_2 - p_2 & \text{if } \ p_2 > p_1 - b \text{ and } p_2 \geq t_2 \\ 0 & \text{otherwise} \end{cases}$$

The winning lobbyist is the lobbyist who reaches her bias adjusted threshold, and offers the higher bias adjusted bid. Lobbyists pay only when their bids are selected by the legislator. Since legislator preferences are not known, lobbyist have to calculate the probability that their offer will pass the threshold and exceed the bias adjusted bid of their opponent. On the other hand, if the bids of both lobbyist are below the adjusted threshold, then nobody wins the support of the legislator and the game continues to the next round.

At the end of each round, lobbyists find out the bids and the acceptance decision of the legislator. If no one wins the support of the legislator, and the game continues, lobbyist update their information set about legislator preferences.

3 Timing of the Game

1. Each lobbyist is made aware of the distribution of the legislator's preferences, b, on policy 1 over 2.
2. Two lobbyists approach the legislator, and submit the first set of sealed bids indicating what they are willing to pay in exchange for her support
3. Legislator will select the winner from all bids above her reservation value, t, adjusted with the corresponding bias, b.
4. Lobbyists find whether any of the submitted bids are accepted, and the bid values received by the legislator are made known to the lobbyists. The game ends when a bid is accepted, and bids are only paid out by the lobbyist who wins the legislator's support.
5. The game continues to the next round if no bids are accepted. Lobbyists will update their information on legislator preferences, taking the bid levels from the previous round in consideration.
6. The legislator selection process, the subsequent information updating on legislator preferences, and the bidding process will be repeated until a winning bid is selected.

4 Winning and continuation probabilities

To characterise the best response function of each lobbyist, we require the following two steps. First, we compute the winning and continuation probabilities and subsequently, we analyse how lobbyist update information on b or, equivalently, how the state space changes as the game continues for another round.

Let us start with the probability to win the support of the legislator.

Consider first lobbyist 1. She wins the support of the legislator if she passes the bias adjusted threshold of the legislator and offers a higher bias adjusted bid than her opponent, i.e.,

$$\Pr(1 \text{ wins}) = \Pr(p_1 \geq p_2 + b \text{ and } p_1 \geq t + b). \tag{1}$$

To compute (1) we consider the following two cases: $p_2 > t$ and $p_2 \leq t$.

If $p_2 > t$, then

$$\Pr(p_1 \geq p_2 + b \text{ and } p_1 \geq t + b) = \Pr(p_1 \geq p_2 + b) \tag{2}$$

since $p_1 \geq p_2 + b > t + b$.

Similarly, if $p_2 \leq t$, then

$$\Pr(p_1 \geq p_2 + b \text{ and } p_1 \geq t + b) = \Pr(p_1 \geq t + b) \tag{3}$$

since $p_1 \geq t + b \geq p_2 + b$.

Combining (2) and (3), we obtain

$$\Pr(p_1 \geq p_2 + b \text{ and } p_1 \geq t + b) = \begin{cases} \Pr(p_1 \geq p_2 + b) & \text{if } p_2 > t, \\ \Pr(p_1 \geq t + b) & \text{if } p_2 \leq t. \end{cases}$$

Using the assumption that b is uniformly distributed, we compute the probabilities explicitly. In particular,

$$\Pr(p_1 \geq p_2 + b) = \Pr(b \leq p_1 - p_2) = \frac{(p_1 - p_2) - d_{min}}{d_{max} - d_{min}}$$

and

$$\Pr(p_1 \geq t + b) = \Pr(b \leq p_1 - t) = \frac{(p_1 - t) - d_{min}}{d_{max} - d_{min}}.$$

For lobbyist two, we follow a similar argument. In particular,

$$\Pr(2 \text{ wins}) = \Pr(p_1 \geq p_2 - b \text{ and } p_1 \geq t - b)$$

and

$$\Pr(p_2 \geq p_1 - b \text{ and } p_2 \geq t - b) = \begin{cases} \Pr(p_2 \geq p_1 - b) & \text{if } p_1 > t, \\ \Pr(p_2 \geq t - b) & \text{if } p_1 \leq t. \end{cases}$$

Moreover,

$$\Pr(p_2 \geq p_1 - b) = \Pr(b \geq p_1 - p_2) = \frac{d_{max} - (p_1 - p_2)}{d_{max} - d_{min}}$$

and

$$\Pr(p_2 \geq t - b) = \Pr(b \geq t - p_2) = \frac{d_{max} - (t - p_2)}{d_{max} - d_{min}}.$$

Next, we compute the continuation probabilities.

If the offers of both lobbyists are below the adjusted thresholds, then the game continues to the next round. The probability can be computed as follows:

$$\begin{aligned} \Pr(p_1 < t + b \text{ and } p_2 < t - b) &= \Pr(p_1 - t < b < t - p_2) \\ &= \frac{t - p_2 - d_{min}}{d_{max} - d_{min}} - \frac{p_1 - t - d_{min}}{d_{max} - d_{min}} = \frac{2t - p_1 - p_2}{d_{max} - d_{min}}, \end{aligned}$$

where the last set of equalities follow from the assumption that b is uniformly distributed.

We obtain from the above a necessary condition for the game to continue. For the game to proceed, the average bid has to be below the integrity threshold. The probability of the game ending depends on the probabilities of each lobbyist winning. We compute the continuation probabilities as follows:

$$\Pr(neither\ win) = \begin{cases} \frac{2t-p_1-p_2}{d_{max}-d_{min}}, & \text{if } p_1 \leq t, p_2 \leq t \\ \frac{t-p_1}{d_{max}-d_{min}}, & \text{if } p_1 \leq t, p_2 > t \\ \frac{t-p_2}{d_{max}-d_{min}}, & \text{if } p_1 > t, p_2 \leq t \\ 0 & \text{otherwise} \end{cases}$$

5 Bias update

The game continues on to the next round when both current bids are less than their bias adjusted thresholds. From the rejected bids, lobbyists can observe that $b > p_1 - t$ and $b < t - p_2$. The support of the bias distribution is the updated from $[d_{min}, d_{max}]$ to $[d'_{min}, d'_{max}]$, where $d'_{min} = p_1 - t$ and $d'_{max} = t - p_2$. As bids are monotone increasing, it follows that the support of the distribution becomes narrower as more rounds are played. When $d_{min} > d_{max}$, the game ends in the current round.[1]

6 Best responses

We begin the analysis of the best responses with the value function of the lobbyists. For this section, we present the general best response functions without imposing symmetry conditions to better understand the breakdown of each lobbyist's best response.

The value function is given as follows:

$$V_j(d_{min}, d_{max}) = \max_{p_j} \{\Pr(j \text{ wins})(W_j - p_j) + \beta \Pr(p_1 < t_1 and p_2 < t_2) V_j(d'_{min}, d'_{max})\} \quad (4)$$

where $V_j(d_{min}, d_{max})$ denotes the continuation value of lobbyist j given the state variables d_{min}, and d_{max}.

[1]We denote all future values with ' , (*e.g.* p_j and p'_j are the present and future bids of lobbyist j, respectively).

The bidding process only ends when at least one of the lobbyists reaches the corresponding bias adjusted threshold. The continuation probability function is positive if the average bid, denoted by p_μ is less than the integrity threshold t. If this condition is not satisfied, the game ends in the current round. The winning probability for j depends on whether the opposing lobbyist's bid p_{-j} is above or below the threshold. From this, we find distinct value functions for the two possible scenarios under the symmetric framework, (1) when $p_{-j} > t$, $p_\mu \geq t$, and (2) when $p_{-j} < t$, and $p_\mu < t$.

Scenario 1: $p_{-j} > t$, $p_\mu \geq t$

$$EU_1 = \frac{p_1 - p_2 - d_{min}}{d_{max} - d_{min}} (w_1 - p_1)$$
$$EU_2 = \frac{d_{max} - (p_1 - p_2)}{d_{max} - d_{min}} (w_2 - p_2)$$

Scenario 2 $p_{-j} < t$, $p_\mu < t$

$$V_1(d_{min}, d_{max}) = \max_{p_1} \left\{ \frac{p_1 - t - d_{min}}{d_{max} - d_{min}} (w_1 - p_1) + \frac{2t - p_1 - p_2}{d_{max} - d_{min}} \beta V_1(p_1 - t, t - p_2) \right\}$$
$$V_2(d_{min}, d_{max}) = \max_{p_2} \left\{ \frac{d_{max} - (t - p_2)}{d_{max} - d_{min}} (w_2 - p_2) + \frac{2t - p_1 - p_2}{d_{max} - d_{min}} \beta V_2(p_1 - t, t - p_2) \right\}$$

The best responses are solved by finding the p_j that provides the maximum V_j for lobbyist j. The ease of finding the best responses vary depending on p_μ and t. When $p_\mu \geq t$, the lobbyists know that the game effectively becomes a one shot game. The identification of the best response in this case is straight forward, as the function to be maximized is simply the product of the probability of j winning and the corresponding payoff.

We begin with the best responses of both players when the opposing bid and the average bid are both above the threshold ($p_{-j} > t$, $p_\mu \geq t$),

$$p_1(p_2) = \frac{1}{2} (d_{min} + p_2 + w_1) \tag{5}$$

$$p_2(p_1) = \frac{1}{2}\left(-d_{max} + p_1 + w_2\right) \tag{6}$$

Here, we find that the threshold is not accounted for in the best responses of the lobbyists. As the opposing bid has already exceeded the baseline threshold and the game will end in the current round, lobbyist j will consider bidding above the opposing bid adjusted with the bias. The above best responses are effectively the average of each lobbyist's winning valuation and the opposing bid, adjusted with the lobbyist's corresponding maximum advantage.

In contrast to the one shot form when the average bid exceeds the threshold, the identification of the best responses for average bid values below the threshold involves the analysis of a dynamic structure. Although it is possible for the game to end when the average bid is below t, the probability of it continuing cannot be discounted as in the previous case. We look at the expected utility in a dynamic form, given by the Bellman function V_j. The guess and verify method is applied to solve for the best response functions for the lobbyists.

We look at the best responses of each lobbyist when the opposing bid and the average bid is below the threshold ($p_{-j} < t$, $p_\mu < t$):

$$p_1(p_2) = \frac{-\left(\sqrt{1-\beta}-1\right)d_{min} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta} - 1\right)w_1}{\beta} \tag{7}$$

$$p_2(p_1) = \frac{\left(\sqrt{1-\beta}-1\right)d_{max} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta} - 1\right)w_2}{\beta} \tag{8}$$

In contrast to the previous case, the best responses here do not take into account the opposing bid. As the threshold is higher than the opposing bid, and it is necessary for the threshold to be surpassed to be considered by the legislator, the opposing bid is excluded in the best response function. It follows that an increase in the threshold increases the bid values for lobbyist j, while an increase in the maximum advantage of the lobbyist decreases the bid value.

The best responses in this section are used to solve for the equilibria laid out in the next section.

7 Equilibrium

We now proceed to solve the symmetric Nash equilibria from the best responses above.

Although there are 16 potential action profiles, we rule out the scenarios where the p_μ conditions are mismatched (i.e. lobbyist j responds given $p_\mu \geq t$, and lobbyist $-j$ given $p_\mu < t$), as well as those where p_j and p_{-j} do not fulfill the p_μ condition (e.g. if p_j and p_{-j} are both greater than t, $p_\mu < t$ cannot be true). From the remaining action profiles, we find two possible outcomes - one where both lobbyists bid above the integrity threshold, and another where both lobbyists bid below.

We begin with the case where the game effectively becomes a static first price auction, $p_\mu \geq t$, and both lobbyists are best responding to $p_{-j} > t$. In this case, both lobbyists bid above the threshold t and the following bids are observed in equilibrium:

$$p_1^* = \frac{2d_{min} - d_{max} + 2w_1 + w_2}{3}$$
$$p_2^* = \frac{d_{min} - 2d_{max} + w_1 + 2w_2}{3}$$

From the equilibrium bids above, we see that each lobbyist is willing to pay at least two thirds of their wealth to win. As both lobbyists are aware that the game will end in the current round, it is expected to have both lobbyists bid more aggressively. It follows that the higher the value lobbyists place on winning the legislator's support, the higher their corresponding bids. Lobbyists will also adjust the bids with their maximum advantage, with lower bids given a higher maximum advantage. The opponent's winning valuation and maximum advantage also affect the equilibrium bid values in the same manner, albeit with half the weight individuals put on their own. Recall that best response function for opposing and average bids above the threshold are given by the average of lobbyist j's winning valuation and the adjusted opposing bid. As each opposing bid is a function of one's bid in equilibrium, it is expected for lobbyist j to put more weight towards her parameters compared to her opponent's.

Applying symmetric properties, the equilibrium bids are given as follows:

$$p_1^* = w - d \tag{9}$$
$$p_2^* = w - d \tag{10}$$

The lobbyist with the advantage is certain to surpass her bias adjusted threshold, increasing her probability of winning the legislator vote. However, as the legislator bias is only revealed at the end of the game, lobbyists can only speculate given the bias interval , $[-d, d]$.

We proceed with this analysis by looking at d_{min} and d_{max} values under which the above equilibrium is observed. Applying the restrictions $p_1, p_2 > t$ on the equilibrium bids above, we find that the equilibrium only holds when the bias interval satisfies the following conditions:

$$d_{min} > \frac{3t + d_{max} - 2w_1 - w_2}{2}$$
$$d_{max} < \frac{-3t + d_{min} + w_1 + 2w_2}{2}$$

For the symmetric case, the conditions can be further simplified as follows:

$$d_{min} > \frac{3t + d_{max} - 3w}{2}$$
$$d_{max} < \frac{-3t + d_{min} + 3w}{2}$$

We have established in Section 5 that the interval of the legislator bias distribution is always updated from $[d_{min}, d_{max}]$ to $[p_1 - t, t - p_2]$. It follows from the conditions above that both lobbyist tend towards bidding beyond the threshold as the information on the bias becomes more precise. A narrow bias distribution implies that the lobbyist does not have much to gain by updating the bias information further. The costs of delaying could then easily surpass the marginal benefit of learning more about the legislator bias, leading lobbyists to plays in a one shot scenario.

Now, we look at the case where both lobbyists bid below the threshold. We begin with looking at the best responses given in equations 10 and 11, and obtain the following equilibrium policy functions:

$$p_1^* = \frac{-\left(\sqrt{1-\beta}-1\right)d_{min} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta}-1\right)w_1}{\beta} \tag{11}$$

$$p_2^* = \frac{\left(\sqrt{1-\beta}-1\right)d_{max} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta}-1\right)w_2}{\beta} \tag{12}$$

When both bids are below the integrity threshold, the game continues until one of the bids exceeds its bias adjusted threshold. Lobbyists need to reach the bias adjusted threshold first, therefore focusing only on how best to reach the threshold while disregarding opponent actions. As before, each lobbyist knows that the opposing lobbyist is bidding below the threshold. By taking into account the threshold instead of the bids, one ensures that the bid will surpass the opposing lobbyist's adjusted bias bid. Lobbyists will bid towards meeting the adjusted bias threshold which they approximate with the threshold adjusted with their maximum advantage, taking into account their winning valuations.

Restricting equilibrium bids to values below the threshold, we obtain the following conditions:

$$d_{min} < \sqrt{1-\beta}\,(t-w)$$
$$d_{max} > \sqrt{1-\beta}\,(w-t)$$

Note that the term $\sqrt{1-\beta}$ is multiplied to the difference of lobbyists' winning valuation w_i and the threshold t to obtain the conditions for this equilibrium. Larger observed differences between winning valuations and the threshold increase the difficulty of fulfilling the above threshold. As the value of winning becomes larger with respect to t , the bids also tend to increase, decreasing the odds of lobbyists proceeding to the next round. Note that as β approaches, the proportion reduces, making it easier to satisfy the conditions on the last known legislator preference interval, $[d_{min}, d_{max}]$. This means that the more patient a lobbyist is the more willing she is to continue playing given a certain information interval.

Only the lobbyist preferred by the legislator can win in the current round per the rules of the game. If the bid is less than her bias adjusted threshold, the game continues. The one shot game will only be realized if the one with the advantage bids enough to satisfy her own bias adjusted threshold. The dynamic nature of the game allows for optimal policy functions to change depending on the current state of the game. It is unlikely for the information interval to stay within the conditions of this case, making the game more likely to proceed under the cases explained earlier. The transitions between cases are illustrated in the next section.

From the equilibrium conditions stated above, we can begin the characterization of the equilibrium.

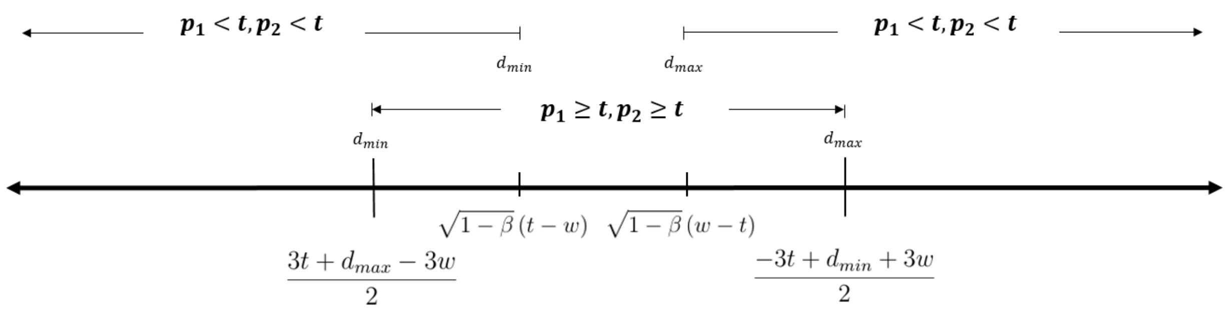

Figure 1: Symmetric Equilibrium Characterization

In Figure 1, we show the equilibria for each possible bias interval, $[d_{min}, d_{max}]$ under a symmetric game.

It can be observed that with a sufficiently narrow bias interval, lobbyists bid more aggressively and above the legislator threshold t.

In narrow intervals, the savings in accepted bid lobbyists may expect to find is small. The winning valuation decreases the longer the game continues. Lobbyists will consider the tradeoff between the potential savings from updating the interval and the loss in value from waiting, choosing only to wait when it is more profitable to do so. The one shot observation, where both lobbyists bid above the legislator threshold, occurs as a direct consequence of this.

The equilibrium where both lobbyists bid below the threshold occurs when the bias interval is sufficiently wide.

The level of precision in wider intervals is smaller, and overbidding may occur. Overbidding can be avoided by obtaining more information on legislator bias. Lobbyists become increasingly confident with their bias estimates as the interval narrows, increasing the probability of being the legislator bias for any given point within the interval. The updating occurs when no one wins the legislator's support, and the game continuous onto the next round. When both lobbyists are bidding below the threshold, the probability of the game ending is the smallest [2], providing the lobbyists the best odds to update their information on legislator bias.

There is however an area where both aforementioned equilibria intersect. When the bias interval is neither too wide nor too narrow, a lobbyist must weigh the potential savings if one wins in the current round, the additional expense that could be incurred if an extra round needs to be played, and the risk of losing the game.

We summarize the results above in the following propositions,

Proposition 1. *Under narrow bias intervals, lobbyists bid aggressively under a one shot scenario.*

Proposition 2. *Under wide bias intervals, lobbyists bid conservatively below the legislator's integrity threshold.*

Proposition 3. *Under moderate bias intervals, legislators can decide whether to play aggressively under a one shot scenario, or conservatively under a dynamic game.*

We explore the tradeoffs in more detail in the examples from the following section.

8 Example

Consider two lobbyists who stand to gain the same wealth in securing the legislator's support ($w_1 = w_2 = 2.5$). Both lobbyists know that the integrity threshold of the legislator, $t = 2$, is quite high in relation to their winning valuations. They are aware that they may not need to pay out the entire amount depending on the preference of the legislator, $b \sim [-d, d]$. Let

[2] Under this equilibrium, only the lobbyist with the preferred policy can win the legislator's support. The game ends only when the preferred lobbyist bids above her bias adjusted threshold

$\beta = 0.9$. We do not specify the bias in this example to fully explore the evolution of bids and winning probabilities in the symmetric game.

For illustration purposes, we look at three different bias intervals: [-0.1,0.1], [-1,1], [-0.4,0.4].

The bids can transition from one case to another, depending on the state of the game.

The interval [-0.1,0.1] is narrow enough for the one shot game to be played. Both lobbyists bid above the integrity threshold, with $p_1 = p_2 = 2.4$. As the game is symmetric, each lobbyist has an equal probability of winning. The winning lobbyist is the one with the highest price adjusted bid, given the legislator preference, b.

Under bias interval [-1,1], lobbyists both start with bidding below the integrity threshold and continue to do so until the final possible round of play where both bid above the threshold. The evolution of player bids and the corresponding probabilities of winning is shown in Figure 2.

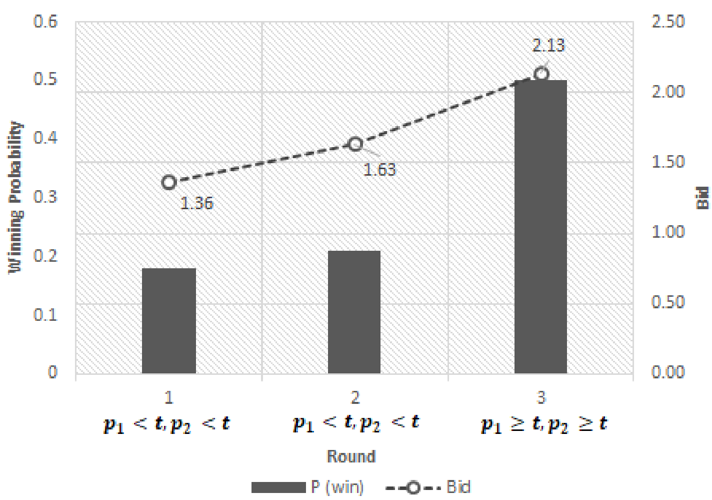

Figure 2: Evolution of Bids and Winning Probabilities ($d = 1$)

It can be observed that as the number of rounds increase, the bids increase as well. Recall that if the game continues onto the next round, both lobbyists have overestimated their advantage and would need to bid higher to reach the legislator's bias-adjusted integrity threshold. If a lobbyist who won in the first round proceeded to the third round, she would have had to spend fifty percent more in order to win in the third round. The potential savings when the bias interval is wide enough can compensate for a lower probability of winning. As bids are monotonic increasing, the intervals narrow for each subsequent round of play. This is reflected in the transition from the equilibrium where lobbyists bid below the integrity threshold in the second round, to the one shot equilibrium in the third round.

We move on to the bias interval [-0.4,0.4], where the interval is more moderate. The initial interval allows for either the equilibrium above or below the threshold to play out. Figure 3 shows how three possible games unfold, one where lobbyists bid above the threshold and the game ends immediately ($i.e.$ $p_j \geq t$) , one where lobbyists first bid less than then greater than threshold ($e.g.$ $p_j < t$, $p'_j \geq t$), and one where lobbyists choose to bid below the threshold while possible ($e.g.$ $p_j < t$, $p'_j < t$, $p''_j \geq t$). We denote the three cases in the graph below as One Shot, LG, and LLG respectively.

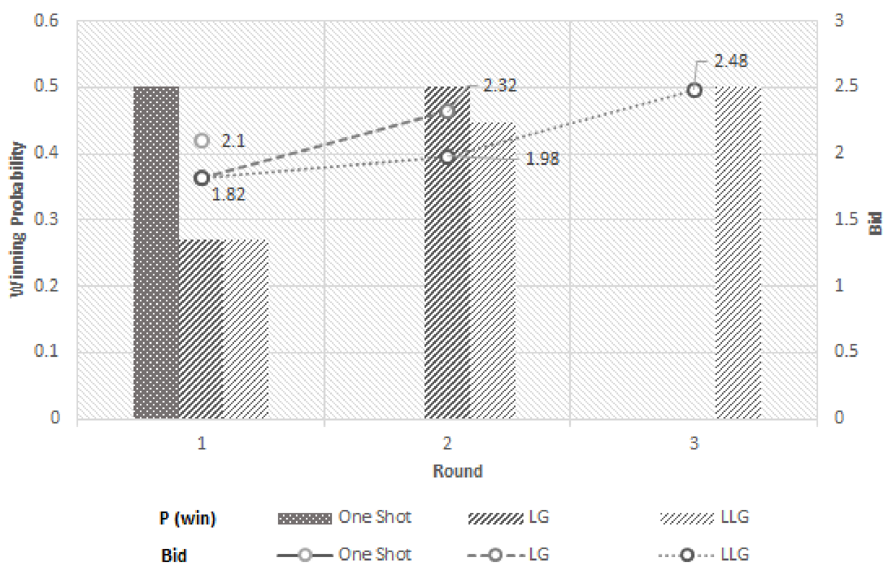

Figure 3: Evolution of Bids and Winning Probabilities ($d = 0.4$)

When the degree of uncertainty on legislator preferences is moderate, the payoff one can obtain from waiting can offset the cost of delaying the decision for another round. If one decides to bid below the threshold, learning more about the legislators when possible, the likelihood of ending in the third round is small at 4.87%. We compute this by finding the probability of the game reaching the third round, the product of the continuation probabilities for rounds 1 (45.9%) and 2 (10.6%). Waiting diminishes both the value of the wealth, via β, and increases the bidding thresholds. Although there are three viable routes on which the game could proceed, bidding above the integrity threshold on the outset provides the highest probability of winning at a reasonable bid, a route some lobbyists may take if they find the risk of diminishing payoffs in the future for a bargain bid today unattractive.

9 Comparative Statics

We have explored in the previous section the equilibria resulting from symmetric lobbyist interactions. To see how other model parameters affect equilibrium bids and winning probabilities, we perform comparative statics. From the first derivatives of the lobbyist bid in equilibrium p_j^* with respect to each parameter β, d, t, and w, we obtain the following:

Parameter Effects on Equilibrium Bids		
Parameters	When $p_j \geq t$	When $p_j < t$
$\uparrow\ \beta$	No Effect	$\downarrow\ p_j$
$\uparrow\ d$	$\downarrow\ p_j$	$\downarrow\ p_j$
$\uparrow\ t$	No Effect	$\uparrow\ p_j$
$\uparrow\ w$	$\uparrow\ p_j$	$\uparrow\ p_j$

A higher β will decrease lobbyist bids under $p_j < t$. As lobbyists become more patient, the urgency to secure the legislator's vote early decreases. The advantages of waiting to receive more information on legislator bias becomes more attractive, as the cost of delaying drops. This effect cannot be observed when $p_j \geq t$ as the game will end in one round , $\beta = 0$. The observed effects on bids from changes in the legislator's integrity threshold and winning valuation are as expected. When legislators become more honest, bids increase to reach the raised bias adjusted threshold. Similarly, an increase in the winning valuation will increase bids - the increase in winning valuation decreases the relative cost of the bid.

We also look at the effect of the degree of uncertainty, as observed through d, on the bids. Recall that $d_{max} = d$ and $d_{min} = -d$. Increasing d will widen the support of the bias distribution, and in turn the degree of uncertainty on legislator preference. We look at the effects of changing degree of uncertainty on legislator preference solely on the probability of winning first. High degrees of uncertainty on legislator preference provide lobbyists the opportunity to acquire more information on the bias. Lobbyists bid more conservatively and are more likely to bid below the threshold given this. If the degree of uncertainty increases, lobbyists will be bidding more aggresively at the same bid level. By retaining the same bid, lobbyists can increase the probability of winning. As each lobbyist tries to find a balance between higher winning probabilities and lower bid costs, the increase in the chances of winning from d will be taken as an opportunity to decrease bid costs, p_j.

Parameter Effects on Probabilities		
Parameters	When $p_j \geq t$	When $p_j < t$
$\uparrow\ d$	$\uparrow\ Pr(j\ wins)$	$\uparrow\ Pr(j\ wins)$
$\uparrow\ t$	No Effect	$\downarrow\ Pr(j\ wins)$
$\uparrow\ p_j$	$\uparrow\ Pr(j\ wins)$	$\uparrow\ Pr(j\ wins)$
$\uparrow\ p_{-j}$	$\downarrow\ Pr(j\ wins)$	No Effect

Note that for $p_j^* \geq t$, the game proceeds as a one shot game (i.e. $\beta = 0$) and the equilibrium bids do not consider the integrity threshold t. It follows then that both parameters will have no effect on the equilibrium bids, as shown in the table above. The condition for meeting the bias adjusted threshold $p_j > t_j$, is rendered unimportant as both lobbyists are aware that one of them has reached the bias adjusted threshold when both lobbyists bid above the threshold t.

10 Summary

From above, we find that degree of uncertainty of legislator preferences, given by the width of the bias intervals, has a direct effect on the bidding strategy of lobbyists in the symmetric game. We find that when intervals are narrow, that is uncertainty is low, lobbyists bid aggresively under a one shot game. Since both lobbyists would like to secure the legislator support immediately, both are willing to forgo the potential bias the legislator may have for their policy. Under this scenario, the saving lobbyists can gain from learning more about the legislator's preference are negated by the cost of delaying the win and the increase in the bid of the following round, if the game continues. Conversely, we find that if the bias interval is wide and uncertainty high, the incentives of waiting outweigh its costs, and lobbyists proceed under a dynamic scenario. Given the nature of the dynamic game, lobbyists who start by bidding conservatively may end up in the one shot scenario. We also find multiplicity in equilibria when the degree of uncertainty is neither too high nor too low. Under moderate levels of uncertainty, lobbyists can choose either to bid above or below the threshold. More specifically, lobbyists decide, under moderate degrees of uncertainty, whether to end the lobbying process and secure the vote at a good bid, or take a shot at a lower winning bid at the risk of eroding profits in the next rounds of play, if any.

The interactions above provide a snapshot on low lobbying may proceed behind closed doors. The paper provides valuable insights the public can use to engage with legislators at key points during the lobbying process. Results above imply that if issues are non-salient, and legislator positions are known, then lobbying may have already commenced for securing a legislator's support even before issues are due for legislative action. Political agents, however, may listen to constituent opinions and adjust their preference intervals accordingly, which may ultimately affect which lobbyist she lends her support to.

The results insofar look at what we find under the symmetric case. We are currently exploring outcomes for interactions under asymmetric cases, where one lobbyists bids above and another, below, the integrity threshold. Comparative statics will also be performed to see whether changes in the discount factor and the legislator's integrity threshold affect lobbyist behaviour.

References

David Austen-Smith. Information and influence: Lobbying for agendas and votes. *American Journal of Political Science*, 37(3):799–833, 1993. ISSN 00925853, 15405907. URL `http://www.jstor.org/stable/2111575`.

Tim Groseclose and James M. Snyder. Buying supermajorities. *American Political Science Review*, 90(2):303315, 1996. doi: 10.2307/2082886.

John M. de Figueiredo. Lobbying and information in politics. *Business and Politics*, 4(2): 125129, 2002. doi: 10.2202/1469-3569.1033.

Richard L. Hall and Alan V. Deardorff. Lobbying as legislative subsidy. *American Political Science Review*, 100(1):6984, 2006. doi: 10.1017/S0003055406062010.

Gene M. Grossman and Elhanan Helpman. Special interest politics, 2001.

Stephen Ansolabehere, John M. de Figueiredo, and James M. Snyder. Why is there so little money in u.s. politics? *The Journal of Economic Perspectives*, 17(1):105–130, 2003. ISSN 08953309. URL `http://www.jstor.org/stable/3216842`.

John M. de Figueiredo and Brian Kelleher Richter. Advancing the empirical research on lobbying. Working Paper 19698, National Bureau of Economic Research, December 2013.

Anthony Downs. An economic theory of political action in a democracy. *Journal of Political Economy*, 65(2):135–150, 1957.

Eddie Dekel, Matthew O. Jackson, and Asher Wolinsky. Vote buying i: legislatures and lobbying. (1434), 2006. URL `http://hdl.handle.net/10419/31254`.

Kristy Buzard and Sebastian Saiegh. Lobbying and legislative uncertainty, 2016.

Jordi Blanes i Vidal, Mirko Draca, and Christian Fons-Rosen. Revolving door lobbyists. *American Economic Review*, 102(7):3731–3748, 2012.

Franz Wirl. The dynamics of lobbying: A differential game. *Public Choice*, 80(3/4):307–323, 1994. ISSN 00485829, 15737101. URL `http://www.jstor.org/stable/30027087`.

A Derivation of Best Responses

Case 1. Symmetric Case, $p_j \geq t\ \forall j$.

We begin with the value function,

$$V_j(d_{min}, d_{max}) = \max_{p_j} \{\Pr(j \quad \text{wins})(W_j - p_j) + \beta \Pr(p_1 < t_1 \ \textit{and}\ p_2 < t_2) V_j(d'_{min}, d'_{max})\}$$

Recall that the function depends on p_{-j} through the calculation of the probabilities of winning. The probabilities are explained in detail in Section 4. The corresponding winning probabilities under $p_{-j} > t$ are,

$$\Pr(p_1 \ wins) = \frac{(p_1 - p_2) - d_{min}}{d_{max} - d_{min}}$$
$$\Pr(p_2 \ wins) = \frac{d_{max} - (p_1 - p_2)}{d_{max} - d_{min}}$$

As the average bid is greater than t, we fail to fulfill the necessary condition for the game to proceed the next round. The continuation probability, $\Pr(p_1 < t_1 \ \textit{and}\ p_2 < t_2)$, is equal to zero.

Applying the probabilities to the value function, we arrive at the following expected utility functions below:

$$EU_1 = \frac{p_1 - p_2 - d_{min}}{d_{max} - d_{min}} (w_1 - p_1)$$
$$EU_2 = \frac{d_{max} - (p_1 - p_2)}{d_{max} - d_{min}} (w_2 - p_2)$$

The utility functions are twice differentiable under the support $[d_{min}, d_{max}]$.

We verify that the second order necessary condition, $EU_j"(p_j) < 0$ for obtaining the maximum bids p_j is fulfilled, as the second derivatives of the expected utility functions given p_j, below, is always negative. [3]

$$\frac{\partial^2 EU_1}{\partial p_1} = \frac{\partial^2 EU_2}{\partial p_2} = \frac{2}{d_{min} - d_{max}} < 0$$

We obtain the first order necessary conditions for optimization below:

$$\frac{\partial EU_1}{\partial p_1} = \frac{d_{min} - 2p_1 + p_2 + w_1}{d_{max} - d_{min}} = 0$$
$$\frac{\partial EU_2}{\partial p_2} = -\frac{d_{max} - p_1 + 2p_2 - w_2}{d_{max} - d_{min}} = 0$$

The best responses, p_j, for each lobbyist j given the opponent's bid, p_{-j} are as follows:

$$p_1^*(p_2) = \frac{1}{2}(d_{min} + p_2 + w_1)$$
$$p_2^*(p_1) = \frac{1}{2}(-d_{max} + p_1 + w_2)$$

Case 2. Symmetric Case, $p_j < t \ \forall j$.

As with the previous case, we start with the value function,

$$V_j(d_{min}, d_{max}) = \max_{p_j} \{\Pr(j \quad \text{wins})(W_j - p_j) + \beta \Pr(p_1 < t_1 \ and \ p_2 < t_2) V_j(d'_{min}, d'_{max})\}$$

The corresponding winning probabilities under $p_{-j} \leq t$ are

[3] If $d_{min} > d_{max}$, uncertainty on legislator preferences disappear as the range of uncertainty values converge first to a single point before d_{min} exceeds d_{max}.

$$\Pr(p_1\ wins) = \frac{p_1 - t - d_{min}}{d_{max} - d_{min}}$$
$$\Pr(p_2\ wins) = \frac{d_{max} - (t - p_2)}{d_{max} - d_{min}}$$

When both lobbyists bid below the threshold t, the continuation probability is given by

$$\Pr(neither\ win) = \frac{2t - p_1 - p_2}{d_{max} - d_{min}}$$

Substituting the probabilities to the above value function, we get the following lobbyist Bellman equations:

$$V_1(d_{min}, d_{max}) = \max_{p_1} \left\{ \frac{p_1 - t - d_{min}}{d_{max} - d_{min}} (w_1 - p_1) + \frac{2t - p_1 - p_2}{d_{max} - d_{min}} \beta V_1(p_1 - t, t - p_2) \right\}$$
$$V_2(d_{min}, d_{max}) = \max_{p_2} \left\{ \frac{d_{max} - (t - p_2)}{d_{max} - d_{min}} (w_2 - p_2) + \frac{2t - p_1 - p_2}{d_{max} - d_{min}} \beta V_2(p_1 - t, t - p_2) \right\}$$

The guess and verify method is used in solving the Bellman equations. We propose a guess, $v_j(x, y)$, where

$$v_j(\mathrm{x}, \mathrm{y}) = \frac{a_j + b_j x + c_j x^2}{y - x},$$

where a_j, b_j, and c_j are undetermined coefficients.

Incorporating the guess to the Bellman equations, we obtain the following:

$$V_1(d_{min}, d_{max}) = \max_{p_1} \left\{ \frac{p_1 - t - d_{min}}{d_{max} - d_{min}} (w_1 - p_1) + \frac{\beta \left(a_1 + b_1 (p_1 - t) + c_1 (p_1 - t)^2\right)}{d_{max} - d_{min}} \right\}$$
$$V_2(d_{min}, d_{max}) = \max_{p_2} \left\{ \frac{d_{max} - (t - p_2)}{d_{max} - d_{min}} (w_2 - p_2) + \frac{\beta \left(a_2 + b_2 (t - p_2) + c_2 (t - p_2)^2\right)}{d_{max} - d_{min}} \right\}$$

Differentiating with respect to its choice variable p_j, we get the first order necessary conditions for the Bellman equations, and solve for the optimal p_j function :

$$\begin{aligned}
\frac{\partial V_1}{\partial p_1} &= \frac{w_1 - p_1}{d_{max} - d_{min}} - \frac{p_1 - t - d_{min}}{d_{max} - d_{min}} + \frac{\beta\left(b_1 + 2c_1\left(p_1 - t\right)\right)}{d_{max} - d_{min}} = 0 \\
0 &= w_1 - p_1 - (p_1 - t - d_{min}) + \beta(b_1 + 2c_1\left(p_1 - t\right)) \\
0 &= w_1 - 2p_1 + t + d_{min} + \beta b_1 + 2c_1\beta p_1 - 2c_1\beta t \\
p_1(2 - 2c_1\beta) &= w_1 + t + d_{min} + \beta b_1 - 2c_1\beta t \\
p_1^* &= \frac{w_1 + t + d_{min} + \beta b_1 - 2c_1\beta t}{2 - 2c_1\beta}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial V_2}{\partial p_2} &= -1 + \frac{w_2 - p_2}{d_{max} - d_{min}} + \frac{-d_{min} - p_2 + t}{d_{max} - d_{min}} + \frac{\beta\left(-b_2 - 2c_2\left(t - p_2\right)\right)}{d_{max} - d_{min}} = 0 \\
d_{max} - d_{min} &= w_2 - p_2 - d_{min} - p_2 + t + \beta(-b_2 - 2c_2(t - p_2)) \\
p_2(2 - 2c_2\beta) &= w_2 + t - b_2\beta - 2c_2\beta t - d_{max} \\
p_2^* &= \frac{w_2 + t - d_{max} - b_2\beta - 2c_2\beta t}{2 - 2c_2\beta}
\end{aligned}$$

To check if the second order necessary conditions for maximization are satisfied, the second derivative of V_j with respect to p_j twice has to be negative.

$$\begin{aligned}
\frac{\partial^2 V_1}{\partial p_1} &= \frac{2 - 2\beta c_1}{d_{min} - d_{max}} \\
\frac{\partial^2 V_2}{\partial p_2} &= \frac{2 - 2\beta c_2}{d_{min} - d_{max}}
\end{aligned}$$

whether $\frac{\partial^2 V_j}{\partial p_j} < 0$ depends on the value of c_j. If the value of $c_j < \frac{1}{\beta}$, the second order necessary condition will be fulfilled.

Before proceeding to the best response functions, the values of the undetermined variables, a, b, and c, have to be obtained for both Bellman equations. To do this, we substitute the optimal p_j function to V_j, and match these coefficients with the guess and verify expression, $v(x, y)$.

Substituting p_1^* and p_2^* into $V_1(d_{min}, d_{max})$ and $V_2(d_{min}, d_{max})$, respectively, we get

$$
\begin{aligned}
V_1^*(d_{min}, d_{max}) &= \frac{\beta\left(4a_1(1-\beta c_1)+\beta b_1^2\right)-2b_1\beta t+w_1(2b_1\beta-2t+w_1)+t^2+d_{min}^2}{4\left(d_{min}-d_{max}\right)\left(\beta c_1-1\right)}+ \\
&\quad \frac{2d_{min}\left(b_1\beta-2\beta c_1 t+w_1(2\beta c_1-1)+t\right)}{4\left(d_{min}-d_{max}\right)\left(\beta c_1-1\right)} \\
V_2^*(d_{min}, d_{max}) &= \frac{\beta\left(4a_2(1-\beta c_2)+\beta b_2^2\right)+2b_2\beta t-w_2(2b_2\beta+2t-w_2)+t^2+d_{max}^2}{4\left(d_{min}-d_{max}\right)\left(\beta c_2-1\right)}+ \\
&\quad + \frac{2d_{max}\left(b_2\beta+2\beta c_2 t-w_2(2\beta c_2-1)-t\right)}{4\left(d_{min}-d_{max}\right)\left(\beta c_2-1\right)}
\end{aligned}
$$

Matching the coefficients of $v(d_{min}, d_{max})$ to the above, we obtain the following a, b, and c values for both equations: [4]

For $V_1^*(d_{min}, d_{max})$

$$
\begin{aligned}
a_1 &= \frac{\beta\left(4a_1(1-\beta c_1)+\beta b_1^2\right)-2b_1\beta t+w_1\left(2b_1\beta-2t+w_1\right)+t^2}{-4(\beta c_1-1)} \\
b_1 &= \frac{2\left(b_1\beta-2\beta c_1 t+w_1(2\beta c_1-1)+t\right)}{-4(\beta c_1-1)} \\
c_1 &= \frac{1}{4(-1)(\beta c_1-1)}
\end{aligned}
$$

[4] Although there are two possible solutions for a, b, and c, we exclude the solution where the winning probabilities are negative. The proof is shown in Appendix B.

Solving for simultaneously for a_1, b_1, and c_1, we get,

$$a_1 = -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)^2}{2\beta}$$
$$b_1 = -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)}{\beta}$$
$$c_1 = \frac{1}{2\sqrt{1-\beta}+2}$$

For $V_2^*(d_{min}, d_{max})$

$$a_2 = -\frac{\beta(4a_2(1-\beta c_2)+\beta b_2^2)+2b_2\beta t-2w_2(b_2\beta+t)+t^2+w_2^2}{-4(\beta c_2-1)}$$
$$b_2 = -\frac{(2b_2\beta+t(4\beta c_2-2))-2w_2(2\beta c_2-1)}{4(-1)(\beta c_2-1)}$$
$$c_2 = \frac{1}{-4(\beta c-1)}$$

Solving for simultaneously for a_2, b_2, and c_2, we get,

$$a_2 = -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_2)^2}{2\beta}$$
$$b_2 = \frac{\left(\sqrt{1-\beta}-1\right)(t-w_2)}{\beta}$$
$$c_2 = \frac{1}{2\sqrt{1-\beta}+2}$$

We now substitute the coefficient values to the p_j^* to obtain the best responses of $p_j^*(p_{-j})$,

$$
\begin{aligned}
p_1^* &= \frac{w_1 + t + d_{min} + \beta b_1 - 2c_1\beta t}{2 - 2c_1\beta} \\
&= \frac{w_1 + t + d_{min} - \beta(\frac{(\sqrt{1-\beta}-1)(t-w_1)}{\beta}) - 2(\frac{1}{2\sqrt{1-\beta}+2})\beta t}{2 - 2(\frac{1}{2\sqrt{1-\beta}+2})\beta} \\
&= \frac{w_1 + t + d_{min} - (\sqrt{1-\beta}-1)(t-w_1) - \frac{\beta t}{\sqrt{1-\beta}+1}}{2 - \frac{\beta}{\sqrt{1-\beta}+1}} \\
&= \frac{w_1 + t + d_{min} - (\sqrt{1-\beta}-1)(t-w_1) + t(\sqrt{1-\beta}-1)}{1 + \sqrt{1-\beta}} \\
&= \frac{(w_1 + t + d_{min})(1-\sqrt{1-\beta}) + (\sqrt{1-\beta}-1)^2(t-w_1) - t(\sqrt{1-\beta}-1)^2}{(1+\sqrt{1-\beta})(1-\sqrt{1-\beta})} \\
&= \frac{(w_1 + t + d_{min})(1-\sqrt{1-\beta}) + (2-\beta-2\sqrt{1-\beta})(t-w_1) - t(2-\beta-2\sqrt{1-\beta})}{\beta} \\
&= \frac{-\left(\sqrt{1-\beta}-1\right)d_{min} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta}-1\right)w_1}{\beta}
\end{aligned}
$$

$$
\begin{aligned}
p_2^* &= \frac{w_2 + t - d_{max} - b_2\beta - 2c_2\beta t}{2 - 2c_2\beta} \\
&= \frac{w_2 + t - d_{max} - \beta(\frac{(\sqrt{1-\beta}-1)(t-w_2)}{\beta}) - 2(\frac{1}{2\sqrt{1-\beta}+2})\beta t}{2 - 2(\frac{1}{2\sqrt{1-\beta}+2})\beta} \\
&= \frac{w_2 + t - d_{max} - (\sqrt{1-\beta}-1)(t-w_2) - \frac{\beta t}{\sqrt{1-\beta}+1}}{2 - \frac{\beta}{\sqrt{1-\beta}+1}} \\
&= \frac{w_2 + t - d_{max} - (\sqrt{1-\beta}-1)(t-w_2) + t(\sqrt{1-\beta}-1)}{1 + \sqrt{1-\beta}} \\
&= \frac{(w_2 + t - d_{max})(1-\sqrt{1-\beta}) + (\sqrt{1-\beta}-1)^2(t-w_2) - t(\sqrt{1-\beta}-1)^2}{(1+\sqrt{1-\beta})(1-\sqrt{1-\beta})} \\
&= \frac{(w_2 + t - d_{max})(1-\sqrt{1-\beta}) + (2-\beta-2\sqrt{1-\beta})(t-w_2) - t(2-\beta-2\sqrt{1-\beta})}{\beta} \\
&= \frac{\left(\sqrt{1-\beta}-1\right)d_{max} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta}-1\right)w_2}{\beta}
\end{aligned}
$$

As the best responses do not depend on the opposing lobbyist's bid, the best response values are also the bids under Nash equilibrium for $p_{-j} < t$. We find the d_{min} and d_{max} conditions for $p_j^* < t$:

For $p_1^* < t$,

$$\begin{aligned}
\frac{-(\sqrt{1-\beta}-1)d_{min} - \sqrt{1-\beta}t + t + (\beta + \sqrt{1-\beta}-1)w_1}{\beta} &< t \\
-(\sqrt{1-\beta}-1)d_{min} - \sqrt{1-\beta}t + t + (\beta + \sqrt{1-\beta}-1)w_1 &< \beta t \\
(\sqrt{1-\beta} - (1-\beta))t - (\sqrt{1-\beta} - (1-\beta))w_1 &> (1 - \sqrt{1-\beta})d_{min} \\
(\sqrt{1-\beta})(t - w_1) &> d_{min}
\end{aligned}$$

For $p_2^* < t$,

$$\begin{aligned}
\frac{(\sqrt{1-\beta}-1)d_{max} - \sqrt{1-\beta}t + t + (\beta + \sqrt{1-\beta}-1)w_2}{\beta} &< t \\
(\sqrt{1-\beta}-1)d_{max} - \sqrt{1-\beta}t + t + (\beta + \sqrt{1-\beta}-1)w_2 &< \beta t \\
(\sqrt{1-\beta} - (1-\beta))t - (\sqrt{1-\beta} - (1-\beta))w_2 &> -(1 - \sqrt{1-\beta})d_{max} \\
(\sqrt{1-\beta})(w_2 - t) &< d_{max}
\end{aligned}$$

B Guess and Verify Solution Verification

In the previous section, we mentioned that there were two solutions from the Guess and Verify method. We show how the set of coefficients used in the derivation of best responses of each lobbyist is determined.

We begin again with matching the coefficients a_j, b_j, and c_j to $V_j^*(d_{min}, d_{max})$, as shown above.

$$a_1 = \frac{\beta\left(4a_1(1-\beta c_1) + \beta b_1^2\right) - 2b_1\beta t + w_1\left(2b_1\beta - 2t + w_1\right) + t^2}{-4(\beta c_1 - 1)}$$

$$
\begin{aligned}
b_1 &= \frac{2\left(b_1\beta - 2\beta c_1 t + w_1(2\beta c_1 - 1) + t\right)}{-4(\beta c_1 - 1)} \\
c_1 &= \frac{1}{4(-1)(\beta c_1 - 1)}
\end{aligned}
$$

Starting with c_1,

$$
\begin{aligned}
c_1 &= \frac{1}{4(-1)(\beta c_1 - 1)} \\
c_1(-4(\beta c_1 - 1)) &= 1 \\
4\beta c_1^2 - 4c_1 + 1 &= 0 \\
c_1 &= \frac{4 \pm \sqrt{16 - 16\beta}}{8\beta} \\
c_1 &= \frac{1 \pm \sqrt{1 - \beta}}{2\beta} \\
c_1 &= \frac{1}{\frac{2\beta}{1 \pm \sqrt{1-\beta}}} \\
c_1 &= \frac{1}{2(1 \mp \sqrt{1 - \beta})}
\end{aligned}
$$

We identify all coefficients based on c_{j+} (c_{j-}) with a $j+$($j-$) subscript.

$$
\begin{aligned}
c_{1+} &= \frac{1}{2(1 + \sqrt{1 - \beta})} \\
c_{1-} &= \frac{1}{2(1 - \sqrt{1 - \beta})}
\end{aligned}
$$

The two possible coefficient sets are based on c_{j+} or c_{j-}. We will show in full the proof of validity of the solution obtained from c_{j+}, and demonstrate why the same cannot be true for the solution from c_{j-}.

Solutions from c_{j+}

Substituting c_{1+} to b_1,

$$b_1 = \frac{2\left(b_1\beta - 2\beta(\frac{1}{2(1+\sqrt{1-\beta})})t + w_1(2\beta(\frac{1}{2(1+\sqrt{1-\beta})}) - 1) + t\right)}{-4(\beta(\frac{1}{2(1+\sqrt{1-\beta})}) - 1)}$$

$$b_1 = \frac{2\left(b_1\beta - (1-\sqrt{1-\beta})t - w_1\sqrt{1-\beta} + t\right)}{-2(1-\sqrt{1-\beta}) + 4}$$

$$b_1 = \frac{b_1\beta + \sqrt{1-\beta}t - w_1\sqrt{1-\beta}}{1+\sqrt{1-\beta}}$$

$$b_1(1-\beta+\sqrt{1-\beta}) = \sqrt{1-\beta}t - w_1\sqrt{1-\beta}$$

$$b_1 = \frac{t-w_1}{\sqrt{1-\beta}+1}$$

$$b_1 = -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)}{\beta}$$

Substituting c_{1+}, b_1 to a_1 ,

$$a_1 = \frac{\beta(4a_1(1-\beta(\frac{1}{2(1+\sqrt{1-\beta})})) + \beta b_1^2) - 2b_1\beta t + w_1(2b_1\beta - 2t + w_1) + t^2}{-4(\beta(\frac{1}{2(1+\sqrt{1-\beta})}) - 1)}$$

$$a_1(2+2\sqrt{1-\beta}) = \beta(2a_1(2-(1-\sqrt{1-\beta})) + \beta b_1^2) - 2b_1\beta t + w_1(2b_1\beta - 2t + w_1) + t^2$$

$$a_1 = \frac{\beta^2 b_1^2 - 2b_1\beta t + w_1(2b_1\beta - 2t + w_1) + t^2}{(1-\beta)(2+2\sqrt{1-\beta})}$$

$$a_1 = \frac{\beta^2(-\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)}{\beta})^2 - 2(-\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)}{\beta})\beta(t-w_1) + w_1(-2t+w_1) + t^2}{(1-\beta)(2+2\sqrt{1-\beta})}$$

$$a_1 = \frac{((\sqrt{1-\beta}-1)(t-w_1))^2 + 2(\sqrt{1-\beta}-1)(t-w_1)^2 + (t-w_1)^2}{(1-\beta)(2+2\sqrt{1-\beta})}$$

$$a_1 = \frac{((\sqrt{1-\beta}-1)^2 + 2(\sqrt{1-\beta}-\frac{1}{2}))(t-w_1)^2}{(1-\beta)(2+2\sqrt{1-\beta})}$$

$$a_1 = \frac{(1-\beta)(t-w_1)^2}{(1-\beta)(2+2\sqrt{1-\beta})}$$

$$a_1 = \frac{(t-w_1)^2}{(2+2\sqrt{1-\beta})}$$

$$a_1 = -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)^2}{2\beta}$$

We summarize the first set of solutions for lobbyist 1, a_{1+}, b_{1+}, and c_{1+}:

$$\begin{aligned}
a_{1+} &= -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)^2}{2\beta} \\
b_{1+} &= -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_1)}{\beta} \\
c_{1+} &= \frac{1}{2\sqrt{1-\beta}+2}
\end{aligned}$$

As the game is symmetric, the computations for lobbyist 2, a_{2+}, b_{2+}, and c_{2+} proceeds similary, obtaining the following set of coefficients:

$$\begin{aligned}
a_{2+} &= -\frac{\left(\sqrt{1-\beta}-1\right)(t-w_2)^2}{2\beta} \\
b_{2+} &= \frac{\left(\sqrt{1-\beta}-1\right)(t-w_2)}{\beta} \\
c_{2+} &= \frac{1}{2\sqrt{1-\beta}+2}
\end{aligned}$$

In order to verify if the solutions are valid, we look at whether the second order necessary condition is satisfied given c_{1+},

$$\begin{aligned}
\frac{\partial^2 V_1}{\partial p_1} &= \frac{2-2\beta c_1}{d_{min}-d_{max}} \\
\frac{\partial^2 V_1}{\partial p_1} &= \frac{2-2\beta(\frac{1}{2\sqrt{1-\beta}+2})}{d_{min}-d_{max}} \\
\frac{\partial^2 V_1}{\partial p_1} &= \frac{\sqrt{1-\beta}+1}{d_{min}-d_{max}} < 0
\end{aligned}$$

$$\frac{\partial^2 V_2}{\partial p_2} = \frac{2 - 2\beta c_2}{d_{min} - d_{max}}$$
$$\frac{\partial^2 V_2}{\partial p_2} = \frac{2 - 2\beta(\frac{1}{2\sqrt{1-\beta+2}})}{d_{min} - d_{max}}$$
$$\frac{\partial^2 V_2}{\partial p_2} = \frac{\sqrt{1-\beta}+1}{d_{min} - d_{max}} < 0$$

As $d_{max} > d_{min}$ for the game to proceed, the denominator above will always be negative, satisfying the second order necessary condition.

We also check if probability conditions hold under the bids p^*_{j+}. Recall that the bids are given as follows:

$$p^*_{1+} = \frac{-\left(\sqrt{1-\beta}-1\right)d_{min} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta} - 1\right)w_1}{\beta}$$
$$p^*_{2+} = \frac{\left(\sqrt{1-\beta}-1\right)d_{max} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta} - 1\right)w_2}{\beta}$$

To see if the probabilities of each player winning, and the continuation probability are greater than zero, the above bids are substituted into the probability functions

$$\begin{aligned}
Pr(1\ wins) &= \frac{-d_{min} + p_1 - t}{d_{max} - d_{min}} \\
&= \frac{-d_{min} + (\frac{-\left(\sqrt{1-\beta}-1\right)d_{min} - \sqrt{1-\beta}t + t + \left(\beta+\sqrt{1-\beta}-1\right)w_1}{\beta}) - t}{d_{max} - d_{min}} \\
&= \frac{(\sqrt{1-\beta} - (1-\beta))(w_1 - (t + d_{min}))}{\beta(d_{max} - d_{min})} > 0
\end{aligned}$$

Note that lobbyist can only win if the winning valuation w_j is at least the value of the adjusted integrity threshold, t_j. As $w_1 > t + d_{min}$, the probability of lobbyist 1 winning is always positive.

$$
\begin{aligned}
Pr(2\ wins) &= 1 - \frac{-d_{min} - p_2 + t}{d_{max} - d_{min}} \\
&= 1 - \frac{-d_{min} - (\frac{\left(\sqrt{1-\beta}-1\right)d_{max} - \sqrt{1-\beta}t + t + \left(\beta + \sqrt{1-\beta}-1\right)w_2}{\beta}) + t}{d_{max} - d_{min}} \\
&= \frac{(\sqrt{1-\beta} - (1-\beta))\,(w_2 - (t - d_{max})}{\beta\,(d_{max} - d_{min})} > 0
\end{aligned}
$$

Following a similar argument from above, the winning valuation of lobbyist two has to be greater than the adjusted integrity threshold for the lobbyist to participate, making the probability of lobbyist 2 winning always positive as well.

As the continuation probability is just 1 less the probabilities of each lobbyist winning, it will be positive when the conditions for d_{min} and d_{max} for $p_j^* < t$ are met.

We have shown that the set of coefficients $\{a_{j+}, b_{j+}, c_{j+}\}$ provide a valid solution for $V_j(d_{min}, d_{max})$.

Solutions from c_{j-}

We repeat the process of finding a_{j-} and b_{j-}, and arrive at the following set of coefficients for each lobbysit j:

$$
\begin{aligned}
a_{1-} &= \frac{\left(\sqrt{1-\beta}+1\right)(t - w_1)^2}{2\beta} \\
b_{1-} &= \frac{\left(\sqrt{1-\beta}+1\right)(t - w_1)}{\beta} \\
c_{1-} &= \frac{1}{2 - 2\sqrt{1-\beta}}
\end{aligned}
$$

$$
a_{2-} = \frac{\left(\sqrt{1-\beta}+1\right)(t - w_2)^2}{2\beta}
$$

$$b_{2-} = -\frac{\left(\sqrt{1-\beta}+1\right)(t-w_2)}{\beta}$$
$$c_{2-} = \frac{1}{2-2\sqrt{1-\beta}}$$

The second order necessary condition is satisfied, and the values are outlined below:

$$\frac{\partial^2 V_1}{\partial p_1} = \frac{2-2\beta c_1}{d_{min}-d_{max}}$$
$$\frac{\partial^2 V_1}{\partial p_1} = \frac{2-2\beta(\frac{1}{2-2\sqrt{1-\beta}})}{d_{min}-d_{max}}$$
$$\frac{\partial^2 V_1}{\partial p_1} = \frac{1-\sqrt{1-\beta}}{d_{min}-d_{max}} < 0$$

$$\frac{\partial^2 V_2}{\partial p_2} = \frac{2-2\beta c_2}{d_{min}-d_{max}}$$
$$\frac{\partial^2 V_2}{\partial p_2} = \frac{2-2\beta(\frac{1}{2-2\sqrt{1-\beta}})}{d_{min}-d_{max}}$$
$$\frac{\partial^2 V_2}{\partial p_2} = \frac{1-\sqrt{1-\beta}}{d_{min}-d_{max}} < 0$$

We now check if the bids under this coefficient set satisfies the probability conditions. The bids, p^*_{j-} are as follows [5]:

$$p^*_{1-} = \frac{\left(\sqrt{1-\beta}+1\right)d_1 + \left(\sqrt{1-\beta}+1\right)t + \left(\beta-\sqrt{1-\beta}-1\right)w_1}{\beta}$$

[5]The derivation follows the process outlined in Appendix A.

$$p^*_{2-} = \frac{-\left(\sqrt{1-\beta}+1\right)d_2 + \left(\sqrt{1-\beta}+1\right)t + \left(\beta - \sqrt{1-\beta}-1\right)w_2}{\beta}$$

$$\begin{aligned}
Pr(1\ \ wins) &= \frac{-d_{min} + p_1 - t}{d_{max} - d_{min}} \\
&= \frac{-d_{min} + (\frac{\left(\sqrt{1-\beta}+1\right)d_1 + \left(\sqrt{1-\beta}+1\right)t + \left(\beta-\sqrt{1-\beta}-1\right)w_1}{\beta}) - t}{d_{max} - d_{min}} \\
&= \frac{\left(\sqrt{1-\beta}+1-\beta\right)(t + d_{min} - w_1)}{\beta\,(d_{max} - d_{min})}
\end{aligned}$$

As stated before, the winning valuation of any lobbyist has to be greater than their adjusted integrity threshold for the lobbyist to participate. If lobbyist 1 is indeed bidding p^*_{i-}, then $w_1 > t + d_{min}$. From this, we find that the probability of winning, Pr(1 wins), is always negative, making the solution from the coefficient set $\{a_{j-}, b_{j-}, c_{j-}\}$ invalid.

We have proven in this section that only the set of coefficients $\{a_{j+}, b_{j+}, c_{j+}\}$ provide a valid solution for $V_j(d_{min}, d_{max})$.

C Equilibrium Computation

Symmetric Case, $p_j \geq t\ \forall j$**.**

From the best responses in the previous section, we compute the mutual best responses to find bids in Nash equilibrium:

$$\begin{aligned}
p^*_1(p^*_2) &= \frac{1}{2}(d_{min} + p^*_2 + w_1) \\
&= \frac{1}{2}(d_{min} + \frac{-d_{max} + p^*_1 + w_2}{2} + w_1) \\
2p^*_1 &= d_{min} + \frac{-d_{max} + p^*_1 + w_2}{2} + w_1
\end{aligned}$$

$$
\begin{aligned}
4p_1^* &= 2d_{min} - d_{max} + p_1^* + w_2 + 2w_1 \\
3p_1^* &= 2d_{min} - d_{max} + 2w_1 + w_2 \\
p_1^* &= \frac{1}{3}(2d_{min} - d_{max} + 2w_1 + w_2)
\end{aligned}
$$

$$
\begin{aligned}
p_2^*(p_1^*) &= \frac{1}{2}\left(-d_{max} + p_1^* + w_2\right) \\
&= \frac{1}{2}(-d_{max} + \frac{d_{min} + p_2^* + w_1}{2} + w_2) \\
2p_2^* &= -d_{max} + \frac{d_{min} + p_2^* + w_1}{2} + w_2 \\
4p_2^* &= -2d_{max} + d_{min} + p_2^* + w_1 + 2w_2 \\
3p_2^* &= d_{min} - 2d_{max} + w_1 + 2w_2 \\
p_2^* &= \frac{1}{3}\left(d_{min} - 2d_{max} + w_1 + 2w_2\right)
\end{aligned}
$$

Under the symmetric case, $w_1 = w_2 = w$, and $d_{max} = -d_{min} = d$. The following equilibrium bid values are observed.

$$
\begin{aligned}
p_1^* &= w - d \\
p_2^* &= w - d
\end{aligned}
$$

Applying the restriction $p_j \geq t$, we find the conditions under which the one shot equilibrium is observed:

$$
\begin{aligned}
d_{min} &> \frac{3t + d_{max} - 3w}{2} \\
d_{max} &< \frac{-3t + d_{min} + 3w}{2}
\end{aligned}
$$