Skewed Information Transmission

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Abstract

This paper analyzes strategic information transition between skewed agents. More concretely, we study the Crawford and Sobel (1982) setting in the case where agents are not biased, but they differ on the relative importance they put on avoiding "upward" or "downward" mistakes. We show that, even though the agents can communicate perfectly when the state is observed by the sender, the information transmission is significantly imprecise when there is a small noise in the observation. Hence, contrary to the previous perception, the absence of bias is not sufficient for precise equilibrium communication.

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1 Introduction

Since the seminal work in Crawford and Sobel (1982) (CS, henceforth) the cheap talk literature has studied strategic information transmission in different settings. The main focus typically is on analyzing how the difference in the objectives of a sender and a receiver (i.e., the bias) affects the equilibrium communication. An important insight is that a large bias makes the equilibrium information transmission coarse, while when the bias is small communication can be precise (see, for example, Spector (2000)).

This paper analyzes how skewness in the agents' preferences shapes strategic communication in a standard cheap talk setting. More concretely, we consider the case where the sender and the receiver agree on their ideal action, but disagree on the relative importance of avoiding "upward" or "downward" mistakes. We show that, in this instance, the information transmission in all equilibria with a finite or countable number of messages is significantly coarse. This contrasts with the conventional thought that a small bias and a large number of available messages allow precise information transmission.

We consider a setting analogous to the CS model. A sender observes a one-dimensional state of the world, sends a cheap talk message, and a receiver takes an action. Differently from the previous cheap talk literature, we assume that the relative bias between the agents is zero, so both the sender's and the receiver's ideal actions coincide in each state of the world. Nevertheless, agents are assumed to be differently skewed: they differ on the relative importance they put on avoiding "upward" or "downward" mistakes.

The main result in this paper is that, if the set of cheap talk messages available for communication finite or countable, the equilibrium information transmission is remarkably coarse. Put differently, even though the sender's and the receiver's ideal actions coincide in all states of the world, the equilibrium communication remains bounded away from full information transmission as the number of messages available for communication increases.

When the set of messages available for communication is uncountably infinite, our setting contains equilibria featuring full information transmission. Nevertheless, we show that such equilibria are fragile to, for example, the presence of a small cognitive cost of learning or using a language, or a small bias between the agents. We provide applications where our assumptions and result hold, such as communication between divisions in a firm or between members within a government.

We illustrate the main insights by first analyzing a simple and tractable example in Section 2, and then generalizing the main result in Section 3. Finally, Section 4 provides a discussion of the results, some examples of applications where agents have different skewness, and conclusions.

2 An Illustrating Example

To illustrate our main result, consider the following version of the CS model. There is a sender, $\theta = s$, and a receiver, $\theta = r$. First, nature draws a state of the world t using a uniform distribution in $[0, 1]$. The sender chooses a message m from a set M, which in this section is assumed to be $\mathbb{N}$. The receiver, after observing m but not t, decides an action $a \in [0, 1]$. If the realized state is t, the message sent is m, and the action is a, the payoffs of the sender and the receiver are, respectively,

$$u^s(t,a) = \begin{cases} -(1-k^s)(t-a) & \text{if } a<t, \\ -k^s(a-t) & \text{if } a \geq t, \end{cases} \quad \text{and} \tag{2.1}$$

$$u^r(t,a) = \begin{cases} -(1-k^r)(t-a) & \text{if } a<t, \\ -k^r(a-t) & \text{if } a \geq t, \end{cases} \tag{2.2}$$

where $k^r, k^s > 0$. Notice that both the sender's and the receiver's payoffs are maximized when $a = t$ (i.e., their relative bias is zero). When, instead, there is a mistake (so $a \neq t$), their payoff loss depends on its size and direction.

For agent $\theta \in \{s, r\}$, $\frac{k^\theta}{1-k^\theta}$ is a measure of the skewness of the payoff function of the θ-agent. It denotes the importance of making an upward mistake relative to making a downward mistake of the same size. If, for example, $k^r = \frac{1}{2}$, the receiver's payoff loss from making an upward mistake is the same from making a downward mistake. Similarly, if $k^s = \frac{1}{3}$ then the sender's payoff loss from a downward mistake is twice his payoff loss from an upward mistake of the same size. We use $\kappa \equiv \frac{k^s}{1-k^s} \Big/ \frac{k^r}{1-k^r}$ to denote the relative skewness of the sender with respect to the receiver. Also, without loss of generality (i.e., reversing the state space if necessary), we assume that $\kappa < 1$, that is, the receiver has a stronger preference than the sender for avoiding upward mistakes (relative to downward mistakes).[1]

A strategy of the sender is a map $\mu : [0, 1] \to M$, and a strategy for the receiver is a map $\alpha : M \to [0, 1]$. As in CS, we look for Bayesian Nash equilibria of this game. Without loss of generality, we focus our analysis on equilibria in pure strategies. It is easy to see that in our setting, similar to the CS model, all equilibria are essentially equivalent to partition equilibria, that is, equilibria where each message used with positive probability in equilibrium signifies an interval of the state space, so we focus our analysis on characterizing them.

Assume that there is a partition equilibrium with exactly J partition elements, for

[1] Reversing the state space is equivalent to replacing k^θ by $1 - k^\theta$ for both $\theta \in \{s, r\}$, which implies that the new value for κ is the inverse of its value before the reversion.

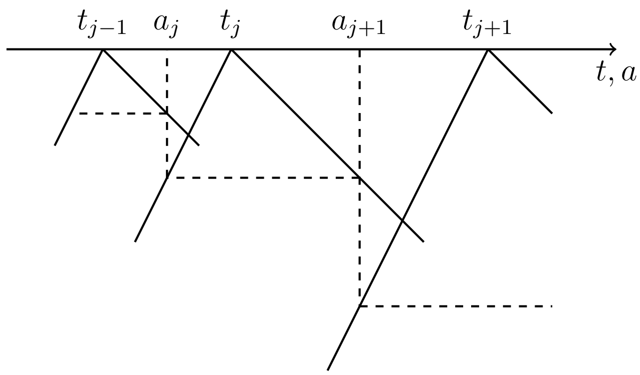

Figure 1: $u^r(t_{j-1},\cdot)$, $u^r(t_j,\cdot)$ and $u^r(t_{j+1},\cdot)$ for some equilibrium thresholds, in the case $k^s=\frac{1}{3}$ (sender's is negatively skewed) and $k^r=\frac{1}{2}$ (receiver is not skewed), so $\kappa=\frac{1}{2}$. Notice that the actions a_j and a_{j+1} are such that $u^r(t_j,a_j)=u^r(t_j,a_{j+1})$, and that $a_j=\frac{t_{j-1}+t_j}{2}$ and $a_{j+1}=\frac{t_j+t_{j+1}}{2}$, which implies that intervals are bigger for higher states.

some fixed $J \in \mathbb{N}$. We use $\{t_j^J\}_{j=0}^J$ to denote the boundaries of its partition elements, where $0=t_0<t_1<...<t_J=1$. For each $j=1,...,J$, we let m_j denote the message used by the sender when the state is in $[t_{j-1},t_j)$, and a_j the (uniquely determined) corresponding equilibrium action of the receiver.[2] Such an action satisfies

$$a_j \in \arg\max_a \int_{t_{j-1}}^{t_j} u^r(t,a)\,\mathrm{d}t \quad \Rightarrow \quad a_j = k^r\, t_{j-1} + (1-k^r)\, t_j \ . \tag{2.3}$$

Note that, if $k^r>\frac{1}{2}$, the receiver takes a relative low action within the interval to avoid an upward mistake.

In equilibrium, when the realized state is t_j, for $0<j<J$, then the sender is indifferent between sending the cheap talk messages m_j and m_{j+1}. This implies that it is necessarily the case that

$$(1-k^s)\,(t_j-a_j) = k^s\,(a_{j+1}-t_j) \ . \tag{2.4}$$

Now, if $k^s>\frac{1}{2}$, the sender is indifferent between the two messages only if the upper action a_{j+1} is closer to t_j than the lower action a_j.

We can use equations (2.3) and (2.4) to compare the size of two adjacent partition elements

$$t_{j+1}-t_j = \frac{t_j-t_{j-1}}{\kappa} \ . \tag{2.5}$$

[2]With some abuse of notation, we are going to use left-closed-right-open intervals to denote the (interval) elements of the equilibrium partition (notice that they are defined up to sets of measure zero).

If $\kappa < 1$ communication is more coarse for higher states. Figure 1 illustrates why. In the picture we have $k^r = \frac{1}{2}$, and therefore $a_j = \frac{t_j + t_{j+1}}{2}$ for all j. Furthermore, $k^s = \frac{1}{3}$ so, as we argued before, the sender's payoff loss from a downward mistake is higher than his payoff loss from an upward mistake of the same size. Hence, she is indifferent between two actions a_j and a_{j+1} only if t_j is closer to a_j than to a_{j+1}. Requiring that $t_0 = 0$ and $t_J = 1$, and using equation (2.5), we obtain that

$$t_j^J = \frac{\kappa^{J-j} \left(1 - \kappa^j\right)}{1 - \kappa^J} \; . \tag{2.6}$$

Hence, for each $J \in \mathbb{N}$, there is a unique partition equilibrium with J partition elements.

Assume now that there is an equilibrium with an infinite number of partition elements, sometimes called an "infinite equilibrium" (see Gordon (2010)). Equation (2.5) applies for each two adjacent partition elements. In fact, it is not difficult to see that the boundaries of the partition elements of the unique infinite equilibrium, denoted with some abuse of notation $\{t_{\infty-j}^{\infty}\}_{j=0}^{\infty}$, satisfy

$$t_{\infty-j}^{\infty} \equiv \lim_{J \to \infty} t_{J-j}^J = \kappa^j \; . \tag{2.7}$$

The following claim formalizes the previous analysis.

Claim 2.1. *For each $J \in \mathbb{N} \cup \{+\infty\}$ there is a unique equilibrium with J partitions.*

2.1 Comparative Statics

Use U_J^θ to denote the payoff of the θ-agent in the equilibrium with $J \in \mathbb{N} \cup \{+\infty\}$ messages, for $\theta \in \{s, r\}$. Simple algebra from the previous expressions shows that the following equations hold:

$$U_J^s = -\frac{k^r \left(1 - k^r\right)}{2} \frac{1 - \kappa}{1 + \kappa} \frac{1 + \kappa^J}{1 - \kappa^J}$$
$$U_J^r = -\frac{k^r \left(1 - k^s\right) \left(k^r + \left(1 - k^r\right) \kappa\right)}{2} \frac{1 - \kappa}{1 + \kappa} \frac{1 + \kappa^J}{1 - \kappa^J}$$

It is easy to see that each agent's payoff increases when the other agent's skewness parameter becomes closer to hers/his. Furthermore, as J increases, the payoff of both the sender and the receiver decreases. Nevertheless, their payoffs are bounded above away from 0: $\lim_{J \to \infty} U_J^\theta = U_\infty^\theta < 0$ for both $\theta \in \{s, r\}$. As Figure 2 illustrates, the size of the highest partition element in the infinite equilibrium (as well as in any other equilibrium, like the depicted equilibrium with $J = 6$ messages) is significantly large for each fixed $\kappa < 1$. The following claim formalizes these results:

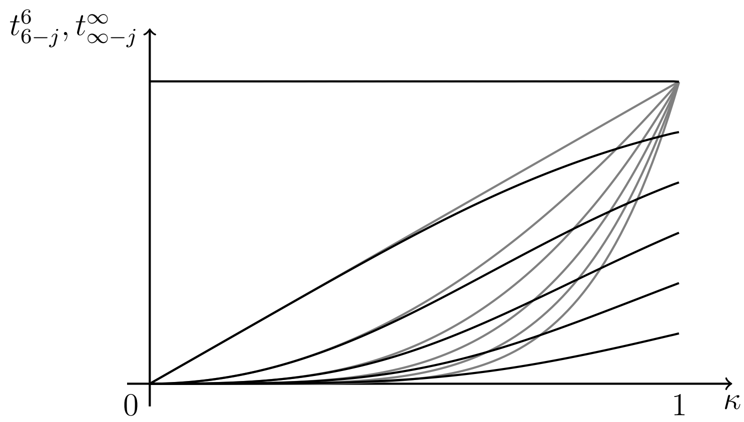

Figure 2: Black lines depict the values $\{t^6_{6-j}\}^6_{j=0}$, where high lines indicate low values of j ($t^6_{6-0} = 1$ and $t^6_{6-6} = 0$). Note that the closer κ is to 1, the closer is the communication strategy to the first best. Similarly, gray lines depict the values $\{t^\infty_{\infty-j}\}^6_{j=0}$, where again high lines indicate low values of j.

Claim 2.2. *The equilibrium payoff of both the sender and the receiver is strictly increasing in J. The size of the highest partition element is $\frac{1-\kappa}{1-\kappa^J} \geq 1-\kappa > 0$, that is, communication does not get arbitrarily precise as J increases.*

Claim 2.2 establishes that the information transmission is significantly coarse when the message space is countably finite. If, instead, we let the message space M to contain an uncountably infinite number of messages, there exist equilibria with precise information transmission (and, in particular, a fully informative equilibrium). In Section 4.1 we argue that restricting the focus to equilibria to with a finite number of used messages is sometimes reasonable, and that fully informative equilibria are fragile to adding a small bias between the sender and the receiver.

Pareto outcomes: We now compare the equilibrium outcomes of our model to efficient outcomes. To do this, fix the number of available cheap-talk messages at some $J \in \mathbb{N}$. We now obtain constrained-Pareto outcomes of our setting (constrained to the usage of J messages), that is, pairs of maps $\mu^J : [0,1] \to \{1,...J\}$ and $\alpha^J : \{1,...,J\} \to [0,1]$ that maximize

$$\mathbb{E}\big[\beta\, u^s(t, \alpha(\mu(t))) + (1-\beta)\, u^r(t, \alpha(\mu(t)))\big]$$

for some $\beta \in [0,1]$.

It is easy to see that any constrained-Pareto outcome partitions the state space in J equally-sized intervals, $\{[\frac{j-1}{J}, \frac{j}{J})\}^J_{j=1}$, and assigns the same action to all states of each element of the partition. Hence, differently from standard cheap-talk models, the sender and the receiver in our setting "agree" on the optimal communication strategy, but they disagree on the action that should be taken after each message.[3] In fact, the action that a constrained-Pareto outcome assigns to the sates in each interval $[\frac{j-1}{J}, \frac{j}{J})$ coincides with the ideal action of an agent with skewness parameter $k^\beta \equiv \beta\, k^s + (1-\beta)\, k^r$, that is, the action in equation (2.3) replacing k^r by k^β and t_j and t_{j-1} by, respectively, $\frac{j-1}{J}$ and $\frac{j}{J}$. This gives to the θ-agent a payoff equal to

$$-\frac{k^\theta\,(1-k^\theta) + (k^\beta - k^\theta)^2}{2\,J}\ .$$

As one can expect, the information transmission is precise in a constrained-Pareto outcome with a large number of messages J. Thus, for any sequence of constrained-Pareto outcomes as $J \to \infty$, the payoff of both the sender and the receiver decrease as J, while their equilibrium payoffs remain bounded away from zero.

3 General Case

We now generalize the result that communication with a finite set of messages is coarse when the agents are differently skewed. To do this, we present a general version of the setting introduced in Section 2.

As before, there is a sender, s, and a receiver, r. First, nature draws a state of the world t, now using a distribution with a continuous, strictly-positive density f in $[0, 1]$. The sender chooses a message m from a set M, which again is assumed to be $\mathbb{N}$. The receiver, after observing m but not t, decides an action $a \in [0, 1]$. If the realized state is t, the message sent is m, and the action is a, the payoffs of the sender and the receiver are, respectively, $u^s(t, a)$ and $u^r(t, a)$, which take the form

$$u^s(t,a) = \begin{cases} u^{s-}(t,a) & \text{if } a < t, \\ -u^{s+}(t,a) & \text{if } a \geq t, \end{cases} \quad \text{and} \tag{3.1}$$

$$u^r(t,a) = \begin{cases} u^{r-}(t,a) & \text{if } a < t, \\ -u^{r+}(t,a) & \text{if } a \geq t, \end{cases} \tag{3.2}$$

where u^{s-}, u^{s+}, u^{r-} and u^{r+} are strictly increasing and piecewise continuously-differentiable functions, so $u^r(t, \cdot)$ and $u^s(t, \cdot)$ are single peaked at t. Assume also that, for all $a_1, a_2, t_1, t_2 \in$

[3] As we can see in Figure 2, and using equation (2.6), the equilibrium elements become equally sized when $\kappa \to 1$. Hence, when agents are not relatively skewed, they use a constrained-optimal communication strategy in equilibrium.

$[0,1]$ with $a_1 < t_1 < t_2 < a_2$, we have[4]

$$u^s(t_1,a_1) \le u^s(t_1,a_2) \quad \Rightarrow \quad u^s(t_2,a_1) < u^s(t_2,a_2)\ ,$$
$$u^s(t_2,a_1) \ge u^s(t_2,a_2) \quad \Rightarrow \quad u^s(t_1,a_1) > u^s(t_1,a_2)\ .$$

The following theorem establishes that, as long as the agents are differently skewed at some state of the world, communication cannot be precise in an equilibrium with a finite number of used messages.

Theorem 3.1. *Assume that there exists some state $\bar{t} \in (0,1)$ such that $u_2^{s-}(\bar{t},\bar{t}^-)\, u_2^{r+}(\bar{t},\bar{t}^+) \neq u_2^{s+}(\bar{t},\bar{t}^+)\, u_2^{r-}(\bar{t},\bar{t}^-)$. Then, there exists some $\bar{\Delta} > 0$ such that, for all equilibria with a finite number of intervals, the minimum interval length is higher than $\bar{\Delta}$.*

Proof. Assume that $\bar{t} \in (0,1)$ is such that $u_2^{s-}(\bar{t},\bar{t}^-)\, u_2^{r+}(\bar{t},\bar{t}^+) \neq u_2^{s+}(\bar{t},\bar{t}^+)\, u_2^{r-}(\bar{t},\bar{t}^-)$. Assume also, for the sake of contradiction, that there is a strictly increasing sequence $(J_i)_{i=1}^{\infty}$ and a corresponding sequence of partition equilibria having, for each i, J_i intervals, such that the maximum length of a partition element, denoted Δ_i, is such that $\Delta_i \to 0$. Let $[t^i_{j_i-1}, t^i_{j_i})$ denote the partition element containing $\bar{t}$ in the i-th equilibrium.

By continuity of the derivatives of the payoff functions we have that for each $\varepsilon > 0$ there is some $\delta(\varepsilon) > 0$ such that

$$|u_2^{\theta x}(t,a) - u_2^{\theta x}(\bar{t},\bar{t})| < \varepsilon \quad \text{and} \quad |f(t) - f(\bar{t})| < \varepsilon$$

for all $x \in \{-,+\}$, $\theta \in \{s,r\}$ and $t,a \in [\bar{t} - \delta(\varepsilon), \bar{t} + \delta(\varepsilon)]$. Take a sequence $(\varepsilon_i)_i$ strictly decreasing towards 0 such that $\Delta_i < \delta(\varepsilon_i)^2$ for all i. Assume also, without loss of generality (i.e., rearranging the state space if necessary), that

$$k^s \equiv \frac{u_2^{s+}(\bar{t},\bar{t})}{u_2^{s-}(\bar{t},\bar{t}) + u_2^{s+}(\bar{t},\bar{t})} < \frac{u_2^{r+}(\bar{t},\bar{t})}{u_2^{r-}(\bar{t},\bar{t}) + u_2^{r+}(\bar{t},\bar{t})} \equiv k^r\ ,$$

which is equivalent, defining again $\kappa \equiv \frac{k^s}{1-k^s}\Big/\frac{k^r}{1-k^r}$, to the assumption that $\kappa < 1$ in our example in Section 2.[5] Hence, we have that the incentive compatibility for the receiver can

[4]This condition generalizes the standard condition $\frac{\partial^2}{\partial t \partial a} u^s(t,a) > 0$ to ensure that all equilibria are essentially equivalent to partition equilibria.

[5]Note that, since $u_2^{s-}(\bar{t},\bar{t}^-)\, u_2^{r+}(\bar{t},\bar{t}^+) \neq u_2^{s+}(\bar{t},\bar{t}^+)\, u_2^{r-}(\bar{t},\bar{t}^-)$, we have $u_2^{s-}(\bar{t},\bar{t}) > 0$ and $u_2^{r+}(\bar{t},\bar{t}) > 0$. The expressions below also apply to the case that $u_2^{s+}(\bar{t},\bar{t}) = 0$ or $u_2^{r-}(\bar{t},\bar{t}) = 0$ (or both), where $\kappa = 0$, which is not considered in our example in Section 2.

be written, as $i \to \infty$, as[6]

$$a_{j_i} = k^r\, t_{j_i-1} + (1-k^r)\, t_{j_i} + O(\varepsilon_i\,(t_{j_i} - t_{j_i-1})) + O((t_{j_i} - t_{j_i-1})^2)\ ,$$
$$a_{j_i+1} = k^r\, t_{j_i} + (1-k^r)\, t_{j_i+1} + O(\varepsilon_i\,(t_{j_i+1} - t_{j_i})) + O((t_{j_i+1} - t_{j_i})^2)\ .$$

Similarly, the indifference condition of the receiver is now given, as $i \to \infty$, by

$$\begin{aligned}(1-k_s)\,(t_{j_i} - a_{j_i}) &+ O(\varepsilon_i\,(t_{j_i} - t_{j_i-1})) + O((t_{j_i} - t_{j_i-1})^2)\\ &= k_s\,(a_{j_i+1} - t_{j_i}) + O(\varepsilon_i\,(t_{j_i+1} - t_{j_i})) + O((t_{j_i+1} - t_{j_i})^2)\ .\end{aligned}$$

Hence, we have that,

$$t_{j_i} - t_{j_i-1} = \kappa\left(t_{j_i+1} - t_{j_i} + O(\varepsilon_i\,(t_{j_i+1} - t_{j_i})) + O((t_{j_i+1} - t_{j_i})^2)\right) < \tilde{\kappa}\,(t_{j_i+1} - t_{j_i})\ ,$$

where $\tilde{\kappa} \equiv \frac{\kappa+1}{2} \in (\kappa, 1)$. Note that the size of the intervals decreases towards the left, at least, exponentially (with coefficient $\hat{\kappa}$). Hence, for all $j \in \{0, ..., j_i\}$,

$$t_j \geq t_{j_i} - \frac{1}{1-\tilde{\kappa}}(t_{j_i} - t_{j_i-1}) > \bar{t} - \frac{1}{1-\tilde{\kappa}}\,\Delta_i\ .$$

Since $\bar{t} > 0$, there exists a value of i such that $\bar{t} - \frac{1}{1-\tilde{\kappa}}\,\Delta_i > 0$. This is a contradiction since, in any equilibrium, $t_0 = 0$. □

Theorem 3.1 contrasts with the standard result that small bias allows precise information transmission (see, for example, Spector (2000)). The crucial assumption that drives such a result is that the payoff functions of the sender and the receiver are twice-differentiable and, as a result, locally symmetric around their ideal action. Indeed, Dilmé (2018b) shows that, under the assumption of twice-differentiability, when the size of a partition elements of an (putative) equilibrium is small, the relative sizes of neighboring partition elements are similar. An intuition for this result goes as follows. First, when a partition element is small, the action of the receiver is "close" to the center of the interval. This is the case because the distribution of states is approximately uniform in a small interval, and the payoff function of the receiver is approximately quadratic. Second, since sender's payoff function is also approximately quadratic, the equilibrium condition imposing that he is indifferent at the common boundary of two partition elements between their implied actions requires that their middle points are at approximately the same distance of their common boundary. As a result, the size of two consecutive small intervals is

[6] The "error terms" appear because the derivatives u_2^{s-} and u_2^{s+}, as well as the density function, have small variations (of order $\varepsilon\,(t_{j_i} - t_{j_i-1})$)

approximately the same.[7]

The same argument cannot be used when agents are differently skewed. In small intervals, where the distribution of states approximately uniform and the payoff functions are approximately piecewise linear, equation (2.5) approximately holds. The different skewness of the agents implies that either the action is not close to the middle of the partition element, or the actions are not at a similar distance from the threshold state. This implies that the size of small partition elements changes approximately exponentially. The proof of Theorem 3.1 uses that, in the direction where the size of the partition elements decreases exponentially, there is an accumulation point, and uses this to reach a contradiction.

4 Discussion, Applications and Conclusions

4.1 Discussion

As we clarify in the previous analysis, our setting contains equilibria with full revelation of information when the set of messages has an uncountably infinite number of messages. The existence of this kind of equilibria is, however, fragile to many aspects of the model. We discuss here two of them.

First, even though the assumption that messages are "cheap" seems plausible when their number is not too large, this may not be the case when there are many of them. This is typically in standard cheap talk models where a non-zero bias makes the communication coarse.[8] When the number of messages is large, instead, it seems more plausible that communication is costly *per se*, either because its complexity or length (see Hertel and Smith (2013)). For example, in communication models with no bias (such as Ćremer et al. (2007), Jäger et al. (2011), Sobel (2015) and Dilmé (2018a)), it is commonly argued that agents face cognitive costs of learning or using a language, so the number of messages that can be used for communication agents is limited and therefore precise communication is not possible. Our result highlights that precise communication is, in some settings, very fragile to the presence of small cognitive costs.

Another rationale for the finiteness of the message space is the existence of a small bias

[7] Dilmé (2018b) allows for a payoff function which is not symmetric. The characterization of the equilibrium communication for a small bias shows that the third derivative of the payoff functions is relevant for determining the coarseness of communication in different regions of the state space, but does not prevent the equilibrium information transmission from being very precise.

[8] Even when infinite equilibria exist, they are coarse, and their partition elements can be approximated by equilibria with a finite number of messages.

between the sender and the receiver. To illustrate this, consider the case where the receiver has the same payoff function as in equation (3.2), but now the sender's payoff is given by

$$u^s(t,a) = \begin{cases} -(1-k^s)\,(\phi\, t - a) & \text{if } a < \phi\, t, \\ -k^s\,(a - \phi\, t) & \text{if } a \geq \phi\, t, \end{cases}$$

with $\phi > 1$.[9] Note that now the ideal action of the sender when the state is $t \in (0,1]$ is $a = \phi\, t > t$. It is then not difficult to see that, in this case, for each $\phi > 1$, only equilibria with coarse information transmission exist (even if M contains an uncountably infinite number of elements). In fact, there is a unique infinite equilibrium, with the thresholds of its partition elements being $\{t^{\infty}_{\infty - j} \equiv \hat{\kappa}^j\}_{k=0}^{\infty}$ (similar to equation (2.7)), where $\hat{\kappa} \in (0,1)$ solves

$$\hat{\kappa} = \kappa - (\phi - 1)\,\frac{\hat{\kappa}}{1-\hat{\kappa}}\,\frac{1}{k^r\,(1-k^s)}\ .$$

In this case, the additional bias of the sender towards high states worsens communication (since $\hat{\kappa} < \kappa$). As the bias disappears (i.e., as $\phi \to 1$), the limit of the thresholds of the partition elements of the (unique) infinite equilibrium satisfy equation (2.7). Thus, in this case, only (and all) equilibria with a finite or countably-infinite number of partition elements of the case with no bias can be approximated by equilibria in the model with small bias.

4.2 Applications

Settings where agents are differently skewed appear naturally when they have a common (production/spending) target, but they disagree on the relative cost of missing the target upwards or downwards. Examples include communication between owners/divisions within a firm (see below), or between members of a government (such as the tax and prime minister) when they have different opinion on the political cost of running budget deficits.

Consider, for example, two agents owning (or divisions belonging to) a firm producing some good. Assume, for simplicity, that the price of the good is 1 and they equally share the profits. Owner "s" is an expert on estimating the demand t (suppose she does it perfectly, for simplicity), distributed uniformly in $[0,1]$, and communicates it to owner "r". Owner r decides the production $a \in [0,1[$. The cost of the production materials is $c \in (0,1)$, and

[9] This specification is convenient to simplify expressions. Indeed, as Melumad and Shibano (1991) show, in the CS model with uniform distribution and payoff functions $\hat{u}^r(t,a) = -(t - a)^2$ and $\hat{u}^s(t,a) = -(\phi\, t - a)^2$, with $\phi > 1$, there is an infinite equilibrium where partition elements satisfy an equation analogous to (2.7).

the entrepreneur r incurs an extra opportunity cost $c' \in (0, \frac{1-c}{2})$ per unit produced. Thus, their payoffs if a units are produced and the demand is t are

$$u^s(t,a) = \frac{1}{2}(\min\{a,t\} - c\,a) \;\text{ and }\; u^r(t,a) = \frac{1}{2}(\min\{a,t\} - (c{+}2\,c')\,a) \ .$$

It is not difficult to see that, in this example, both owners prefer a production $a = t$ above any other production level. Simple algebra shows that, in this case,[10]

$$k^s = c \;\text{ and }\; k^r = c + 2c' \ .$$

Hence, as in the example in Section 2, $\kappa < 1$. As a result, if there is a finite number of messages available for communication, the equilibrium information transmission is going to be coarse. While low demand realizations will be communicated precisely, the communication of high demand realizations will be significantly coarse.

4.3 Conclusions

This paper shows that when agents are differently skewed, even if they agree in the ideal action for each state of the world, equilibrium communication between them will tend to be coarse. This is likely to be the case when they share a common goal, but they disagree on the relative cost of making upward of downward mistakes. Hence, relatively small departures from the usual assumptions on the shape of the payoff functions may have significant effect on some equilibrium predictions.

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[10] Differently from our example in Section 2, now the payoff functions of the sender and the receiver have a "level" that depends on the realized state. Nevertheless, their incentives and the equilibrium construction do not depend on the levels, so our analysis also applies in this case.

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