Organizations and Coordination in a Diverse Population*

Liang Dai[†] and Ming Yang[‡]

Shanghai Jiao Tong University and Duke University

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Abstract

We study the role of organizations in coordinating actions of diverse individuals with strategic complementarity and incomplete information. An organization obligates its members to take collective actions and thus mitigates strategic uncertainty caused by informational frictions. But it also compels its members to take the collective action not in their favor and makes them reluctant to join ex ante. In light of this tradeoff, we identify strategic complementarity and preference heterogeneity as the key determinants of whether organizations are desirable in the sense of welfare improvement, and whether they are sustainable in the sense of incentive compatibility for members to join ex ante. If preference heterogeneity dominates strategic complementarity, organizations could be desirable, but are not sustainable. Otherwise, organizations are desirable, but there is an upper bound for the size of sustainable organizations. The bound increases in the degree of strategic complementarity and decreases in the degree of preference heterogeneity. In all equilibria with organizations, welfare increases with size of organizations. Finally, perturbation to the preference distribution can have non-monotonic impact on the size limit of sustainable organizations, offering a novel perspective to study the impact of migration or polarization on existing social order.

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[†]Antai College of Economics and Management, Shanghai Jiao Tong University. liangdai@sjtu.edu.cn

[‡]Fuqua School of Business, Duke University. ming.yang@duke.edu

"Each of us puts his person and all his power in common under the supreme direction of the general will, and, in our corporate capacity, we receive each member as an indivisible part of the whole.

'The problem is to find a form of association which will defend and protect with the whole common force [a] person and goods of each associate, and in which each, while uniting himself with all, may still obey himself alone, and remain as free as before.' This is the fundamental problem of which the Social Contract provides the solution..." —Jean Jacques Rousseau, The Social Contract

1 Introduction

Situations where diverse individuals must coordinate their actions are ubiquitous, and such coordination plays a key role in human society. However, such coordination is often difficult when those individuals are uncertain of actions that others would take. We refer to such uncertainty as *strategic uncertainty*.[1] Examples abound:

1) In a presidential election, citizens with similar political ideologies would like to concentrate their votes on one of the few presidential candidates from their camp. But a citizen is often uncertain of which candidate other citizens from his camp will vote for;

2) In an industry consisting of firms with different productivities, it is profitable for a firm to raise the price for its product if so do its opponents, but the firm is often uncertain of the pricing decisions of its opponents;

3) Often times countries with similar national interest would like to coordinate their national defence policies. But they may not know perfectly the stands of each other for a particular event.

As a result, various institutions, in particular, organizations, emerge in human society to facilitate coordination. As a defining characteristic, members of an organization choose an action through a prespecified collective decision rule, and are subsequently all obliged to take that action. In the examples above,

1) A political party consisting of those citizens runs a primary election or a caucus to choose one candidate, and obligates its members to vote for that candidate by only nominating him/her for the subsequent presidential election;

[1] The impact of strategic uncertainty has been well studied in the literature, especially that on global games. See, for example, (Morris and Shin 2003) for a survey and subsequent works on macroeconomics and finance.

2) A cartel consisting of those firms determines whether to raise the price together through a negotiation process following its charter, and obligates its members to follow the decision by its enforcement scheme. OPEC can be viewed as a special example;

3) A military alliance formed by those countries requires them to come to the aid of any member state subject to an armed attack.[2] NATO and Warsaw Pact are two particular examples.

From the perspective of an individual, when joining an organization, he "puts his person and all his power in common under the supreme direction of the general will". On one hand, he enjoys the "power of resistance" that is "greater than the resources at the disposal of each individual"[3], as it is guaranteed that his action is at least aligned to all members of the organization, which is in turn based on the private information of all members in addition to his own. On the other hand, "as an indivisible part of the whole", he binds himself to the organization and loses his "natural liberty" of choosing his action based on his own preference and information. Moreover, it is his freedom of choosing ex ante whether or not to join the organization that makes him "still obey himself alone, and remain as free as before". The tradeoff that he faces is precisely an information-based microfoundation of the "fundamental problem" proposed by Rousseau in the foreword of the paper.

In light of this tradeoff, in this paper, we study when organizations featuring the aforementioned defining characteristic are sustainable in the sense that it is incentive compatible for their members to join the organizations ex ante, and when they are desirable in the sense of welfare improvement. For this purpose, we borrow the modeling tool from the literature on global games to generate strategic uncertainty desired.

To fix idea, we consider a coordination game with a continuum of players. Each player observes a private signal of the state of nature (henceforth "fundamental"), and faces a binary choice between action 1 ("invest") and action 0 ("not invest"). His payoff of taking action 0 is normalized to 0, and that of taking action 1 is a benefit that increases with fundamental and the mass of other players who also take action 1 ("investors"), net of an exogenous investment "cost" that varies across players. The dispersion of the invest-

[2] For instance, Article 5 of North Atlantic Treaty states that "[t]he Parties agree that an armed attack against one or more of them in Europe or North America shall be considered an attack against them all and consequently they agree that, if such an armed attack occurs, each of them, in exercise of the right of individual or collective self-defence recognised by Article 51 of the Charter of the United Nations, will assist the Party or Parties so attacked by taking forthwith, individually and in concert with the other Parties, such action as it deems necessary, including the use of armed force, to restore and maintain the security of the North Atlantic area."

[3] This is also taken from Rousseau's "The Social Contract".

ment cost represents preference heterogeneity of players. Knowing his own investment cost but before seeing his private signal, each player decides whether to join an organization. Once in an organization, he is obliged to follow its collective decision. A player staying outside the organizations takes an action based on his private signal simultaneously with others. Then fundamental is realized and each player receives his payoff accordingly. To hightlight the economic insights, we base our analysis on the investment game, a leading example commonly employed in the literature on coordination games. We show in Section 6.2 that the insights obtained go through in regime change games, another popular example of coordination games.

As in a standard global game model, there are two types of informational frictions faced by each player in ours. In addition to strategic uncertainty, a player is also uncertain of the fundamental, which we refer to as *fundamental uncertainty*. Fundamental uncertainty is exogenously generated by the noise of each player's private signal of fundamental. This reflects the inability of individuals in reality to perfectly understand the world. Strategic uncertainty arises from the independence of noises across players. This reflects the difference of models and information sources across individuals in the society. For our purpose, to highlight the role of strategic uncertainty, following the convention of the literature on global games, we focus on the limiting case when fundamental uncertainty vanishes.[4] Hereafter, we use "private signals become infinitely precise" and "fundamental uncertainty vanishes" interchangeably.

In such an environment, there are potentially two sources of welfare loss. The first one is the conflict between social welfare and individual interest. That is, due to positive externality of investment, a social planner can increase social welfare by forcing players with high investment costs to invest when they do not want to. The second source of welfare loss is miscoordination due to strategic uncertainty. We identify strategic complementarity and preference heterogeneity as the key determinants of whether organizations are sustainable

[4] Another reason why we consider such limiting case is as follows. Although the model that we consider is a one-shot game, the joining decision should be understood as a long-term one in the sense that the one-shot investment decision in the model is an average of a series of decisions to be made after a player makes up his mind whether to join an organization. In the long run, with the continuous accumulation of knowledge and experience and improvement of methodology, private knowledge of individuals becomes more and more precise, and thus fundamental uncertainty that they face vanishes. On the contrary, as well known in the literature on global games, strategic uncertainty remains huge no matter how small fundamental uncertainty is, providing that fundamental uncertainy is not zero. In reality, due to the persistent difficulty for individuals to eliminate this difference through communication, such independence is long-lasting even in the long run. As a result, strategic uncertainty is persistent and thus becomes the essential cause of mis-coordination. Hence, the limiting case that we consider captures the realistic sharp contrast as such between fundamental uncertainty and strategic uncertainty in the long run.

in the sense of incentive compatibility for members to join ex ante, and whether they are desirable in mitigating the aforementioned welfare loss.

When preference heterogeneity is large enough to dominate strategic complementarity, welfare loss due to informational frictions disappears asymptotically as fundamental uncertainty vanishes, and thus there is no need for organizations to resolve it. Intuitively, since players all face the same investment opportunity in equilibrium, those with lower investment costs are more willing to invest. That is, when a player is indifferent between investing or not at some cutoff value of his signal, he knows that players with costs lower than him are willing to invest at that cutoff. If the players' cutoffs converge to different values as fundamental uncertainty vanishes, strategic uncertainty also vanishes, because any player indifferent between investing or not is almost sure that players with cost lower (higher) than him are (are not) investing. Consequently, the information friction does not leads to mis-coordination. In this case, no organizations is sustainable either, as individuals are so different that they are reluctant to follow the collective action mandated by any organization.

When preference heterogeneity is dominated by strategic complementarity, informational frictions does lead to strategic uncertainty. Because in this case, players' incentive to coordinate their actions are so strong that they asymptotically choose the same investment strategy despite their different preferences. Thus, when a player is indifferent between investing or not, so are others with large probability, making him uncertain of how many of them are investing. Therefore, in this case, organizations can help mitigate welfare loss due to informational frictions by mechanically resolving strategic uncertainty within them. However, while a larger organization mitigates welfare loss better, its members are more heterogeneous and thus more reluctant to follow its collective action. This is exactly the "fundamental problem" proposed by Rousseau aforementioned, and draws an upper bound for the size of sustainable organizations. We also establish that in any equilibrium with organizations, all individuals take asymptotically the same action whether or not they are members of any organization.

Organizations can mitigate welfare loss due to informational frictions. If a social planner can force players to form organizations, they may also mitigate welfare loss due to conflict of interest as well. In particular, we show that the organization that consists of all players can perfectly resolve both types of welfare loss as long as preference heterogeneity is not too strong relative to strategic complementarity. But this does not imply that a social planner can always restore efficiency by doing so, specifically when preference heterogeneity of players are so large that it is not socially optimal to enforce collective actions.

The largest sustainable organization in our model, which consists of the largest possible sub-population obedient to collective decision making, can be interpreted as the scope of social order. As an application, we use our model to study how shocks to the distribution of individual preferences affects the scope of existing social order. In reality, such shocks can be generated by migration or polarization. We find that inflow/outflow of population of different types of players with the same magnitude can have non-monotonic impact on the scope of social order, and so can the increase/decrease of social inequality.

The rest of the paper is organized as follows. The rest of this section reviews the relevant literature. Section 2 introduces the primitives of the model and the benchmarks without organizations. Section 3 probes into the organized games with incomplete information and discusses the desirability and the sustainability of organizations. Section 4 discusses the welfare implications of organizations. Section 5 discusses how perturbation to the distribution of preferences affects the boundary of the largest sustainable organization. Section 6 examines the developed economic insights with some extensions. All proofs are relegated to the appendix unless otherwise specified.

1.1 Literature Review

Our work is related to the literature on global games, pionnered by (Carlsson and van Damme 1993) and elegantly summarized by (Morris and Shin 2003). In particular, our work is most closely related to the literature on global games with heterogeneous players. As a few examples, (Corsetti, Dasgupta, Morris, and Shin 2004) characterize the impact of a large trader on a population of small ones; (Frankel, Morris, and Pauzner 2003) prove equilibrium uniqueness for a large class of these games; (Guimaraes and Morris 2007) allow for payoff heterogeneity in a model of currency attacks and find that risk-averse speculators have a surprisingly high influence on the attack; (Sákovics and Steiner 2012) provide a criterion that can be used to find the optimal targets for a variety of interventions in regime change games with heterogeneous agents. Our work differs from theirs in two aspects. First, we have different focuses. While these papers mainly focus on identifying who matters most in a coordination game, ours mainly probes into when organizations can arise to mitigate mis-coordination. Second, although both the investment game and the regime change game are commonly studied in this strand of literature, they do not make a qualitative difference in existing models. In our paper, as shown in Section 6.2, the desirability and the sustainability of organizations in the two games rely heavily on the strength of strategic complementarity relative to the degree of preference heterogeneity. Recently, based on (Sákovics and Steiner 2012), (Drozd and Serrano-Padial 2017) discusses credit enforcement cycle driven by collective default of borrowers, and (Leister, Zenou, and Zhou 2017) studies strategic interaction in networks. While our work also deals with coordination games with heterogeneous players, the investment game setup allows us to explicitly highlight the role of strategic complementarity and preference heterogeneity in generating strategic uncertainty. In addition, instead of studying a finance question or strategic interaction in a network structure, our main focus is on when organizations are desirable and sustainable in reducing informational frictions and coordinating people's actions, in particular, the interaction between organizations and players' preference heterogeneity.

Our work is also related to the comparison between weak and strong complementarity in (Angeletos and Pavan 2004). While they focus on different impacts of the precision of public and private information in these two environments, our focus is on when organizations can arise to facilitate coordination.

We borrow the modeling tools to analyze the aggregate behavior of organization from the literature on information aggregation in large elections, in particular, (Feddersen and Pesendorfer 1997). But we focus on different aspects of collective decision making. While their focus is on how large elections aggregate individuals' private information, our focus is on whether the collective decision making mechanism can exist to facilitate coordination.

Our work is also related to the vast literature on organizational economics, well surveyed by (Gibbons and Roberts 2012). Two strands of literature are of particular relevance. One is theories of firms, for example, (Grossman and Hart 1986), (Hart and Moore 1990), (Holmström and Milgrom 1994), and (Holmström and Tirole 1991), as surveyed by (Gibbons 2005). Our work complements this literature with a different perspective: organizations arise from coordination of large population and alleviate fundamental and strategic uncertainty faced by each member. We also identify key factors and provide conditions under which organizations function well for the purpose. The other strand studies coordination-adaptation tradeoffs in shaping organizational forms, as exemplified by (Dessein and Santos 2006), (Alonso, Dessein, and Matouschek 2008), (Rantakari 2008), (Brunnermeier, Bolton, and Veldkamp 2013) and (Dessein, Galeotti, and Santos 2016). We instead fix the internal structure of organizations and focus on desirability and sustainability of organizations in a coordinative environment with informational frictions.

Our work is also related to the literature on the number and size of nations, e.g., (Alesina and Spolaore 1997), and on clubs, well surveyed by (Scotchmer 2008). Our work focuses on different microfoundations. In their models. the benefit from joining an organization/entity is the *physical* increasing return to scale, e.g., due to nonrivalry of public goods, size of markets, and better hedge of idiosyncratic risks. In our model, the benefit stems from alleviation of information frictions.

Lastly, our work is also related to the literature on coalition formation, well surveyed by (Ray and Vohra 2015). As exemplified by (Acemoglu, Egorov, and Sonin 2008) and (Acemoglu, Egorov, and Sonin 2012), existing models in this literature are typically based on complete information setup and abstract from microfoundations of values of different coalitions. Microfounded on coordination games with both incomplete information and preference heterogeneity, our model highlights the pros and cons of organizations in such environments — reduction of informational frictions versus conflict of interest between an organization and its constituents. Moreover, our modeling approach is also different. While existing literature adopts either blocking approach under cooperative game framework or bargaining approach under noncooperative game framework, our model features simultaneous move under noncooperative game framework in both organization formation stage and investment stage.

2 Primitives

Our analysis is based on global games with preference heterogeneity, featuring 1) strategic complementarity, 2) incomplete information, and 3) preference heterogeneity. This section lays out its primitives and provides some preliminary analysis before the introduction of organizations in Section 3.

There is a continuum of players of mass 1. Each player i simultaneously faces a binary choice $a_i \in \{0, 1\}$. His payoff of choosing $a_i = 0$ is normalized to 0, and that of choosing $a_i = 1$ is a benefit $u(\theta, l)$ net of a cost $c_i \in [\underline{c}, \bar{c}]$. The benefit is common to all players who choose $a_i = 1$. It depends on fundamental $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$ and the mass l of players also choosing action 1.

Fundamental θ captures the impact of aggregate variables (e.g. macroeconomic condition). Each player i starts with a common prior of θ with continuous and bounded density $h(\cdot)$ that is strictly positive everywhere, and subsequently observes a private signal $x_i = \theta + \sigma\varepsilon_i$, where ε_i is a continuous random variable with mean 0 and variance 1, and distributed with density $g(\cdot)$. This implies strict monotone likelihood ratio property (SMLRP). We assume that the support of h, $\left[\underline{\theta}, \bar{\theta}\right] \supset [\underline{c}, r + \bar{c}]$. This is to guarantee that the support of h covers both dominance regions: when θ is close to $\underline{\theta}$ $(\bar{\theta})$, no player (every player) wants to invest even if everyone (nobody) else does.

Strategic complementarity among players is built in the benefit $u(\theta, l)$ of taking action 1, captured by $\frac{\partial u}{\partial l} > 0\ \forall l$. That is, the more of players other than i chooses action 1, the more attractive action 1 is to player i. This can be interpreted as positive externalities, for example, due to network effect, technology spillover, etc. To fix idea, we mainly consider the investment game commonly discussed in the literature, where $u(\theta, l) = \theta - r(1 - l)$, with $r > 0$. That is, strategic complementarity is constant with uniform magnitude r. In later discussion, we also interchangeably refer to r as cost of mis-coordination. To illustrate the critical role of strategic complementarity, in Section 6.2 we consider the regime change game, with $u(\theta, l) = 1_{\{l \geq 1-\theta\}}$. that is, players care about whether "the regime is changed" (i.e. the event $l \geq 1 - \theta$) in a discontinuous manner.

The investment cost c_i can be viewed as a preference parameter that captures local variables and information, for example, idiosyncratic components of a firm's productivity. We assume that c_i follows the uniform distribution on $[\underline{c}, \bar{c}]$, and is independent and identically distributed across players. Hence, $\bar{c} - \underline{c}$ captures the heterogeneity of players' preference. The distribution of investment costs is common knowledge of all players. Player i knows the realization of his own c_i, but not those of others.

The primitives of the model follow the convention of the global game literature. The only difference is the heterogeneity in the investment cost c_i. As will be clear later, two key parameters shape our main results: 1) the strength of strategic complementarity r, and 2) the heterogeneity of players' preference $\bar{c} - \underline{c}$.

We are interested in symmetric equilibria and social welfare, defined as follows:

Definition 2.1 *A symmetric equilibrium in an unorganized game (UG) with incomplete information is a strategy profile in which each player follows strategy* $a : (x, c) \mapsto \{0, 1\}$ *such that*

$$a(x, c) \in \arg\max_{a \in \{0,1\}} a \cdot \mathbf{E}\left[u(\theta, l(\theta)) - c | x\right]$$

and

$$l(\theta) = \frac{1}{\bar{c} - \underline{c}} \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a(x, c)\,|\theta\right] dc$$

for all $x \in \mathbb{R}$ and $c \in [\underline{c}, \bar{c}]$.

Definition 2.2 *Social welfare in an unorganized game (UG) when players use symmetric strategy a is*

$$W^{UG}(a) = \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a(x, c)\,|\theta\right] \cdot \left[u(\theta, m(\theta)) - c\right] \frac{dc}{\bar{c} - \underline{c}} h(\theta)\, d\theta \ , \tag{1}$$

where

$$l(\theta) = \frac{1}{\bar{c} - \underline{c}} \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a(x, c)\,|\theta\right] dc \ .$$

2.1 Complete Information Benchmark

We first analyze the benchmark with complete information, where $\sigma = 0$, or equivalently, $x_i = \theta\ \forall i$. That is, θ is common knowledge of all players. It turns out that the characteristics of equilibria differ significantly when strategic complementarity dominates preference heterogeneity (more precisely, $r \geq \bar{c} - \underline{c}$) and when the former is dominated by the latter ($r < \bar{c} - \underline{c}$). When strategic complementarity dominates preference heterogeneity, the result resembles that of an ordinary coordination game with homogeneous players. That is, when fundamental θ falls in the non-dominance region, there are multiple Pareto-ordered symmetric Nash equilibria. When strategic complementarity is dominated by preference heterogeneity, in contrast, there is a unique Nash equilibrium.

Proposition 2.1 *Every symmetric equilibrium is characterized by a cutoff function $\hat{\theta}(\cdot)$, such that a player with cost c invests if and only if $\theta \geq \hat{\theta}(c)$.*

i) If $r < \bar{c} - \underline{c}$, then there is a unique symmetric equilibrium: $\hat{\theta}(c) = \frac{r\bar{c}}{\bar{c} - \underline{c}} + \left(1 - \frac{r}{\bar{c} - \underline{c}}\right) c$;

ii) If $r \geq \bar{c} - \underline{c}$, then there are three symmetric equilibria: 1) $\hat{\theta}(c) = \bar{c}\ \forall c$; 2) $\hat{\theta}(c) = \underline{c} + r$ $\forall c$; and 3) $\hat{\theta}(c) = \frac{r\bar{c}}{\bar{c} - \underline{c}} + \left(1 - \frac{r}{\bar{c} - \underline{c}}\right) c$; In addition, 1) yields higher social welfare than 2) and 3).

A standard coordination game with homogeneous players can be thought of as the case $\bar{c} - \underline{c} = 0$, so that we always have multiple equilibria if $r > \bar{c} - \underline{c}$, and the possibility that players coordinate on inefficient actions. The complete information benchmark in this section shows that such inefficiency can be resolved by introducing sufficient preference heterogeneity to it, and provides a clear characterization of the necessary extent of such sufficiency: whether preference heterogeneity is large enough to dominate strategic complementarity among players (i.e., $r < \bar{c} - \underline{c}$). Note that in the knife edge case $r = \bar{c} - \underline{c}$, $\bar{c} = \underline{c} + r = \frac{r\bar{c}}{\bar{c}-\underline{c}} + \left(1 - \frac{r}{\bar{c}-\underline{c}}\right) c$ for all c, so the three equilibria in ii) coincide.

As will be clear later, first, the same threshold for such sufficiency of preference heterogeity in resolving mis-coordination also works when incomplete information is further introduced; Second, preference heterogeneity is the main obstable of organization formation, and thus plays a key role in determining the desirability and sustainability of organizations.

2.2 Unorganized Games with Incomplete Information

Now we analyze the case of incomplete information, that is, $\sigma > 0$. Incomplete information creates two kinds of informational frictions. First, *fundamental uncertainty* arises, as a player cannot perfectly infer the fundamental θ from his private signal x. Second, since everyone can only base his action on his signal realization, in order to infer others' actions, a player has to infer others' signal realizations from his own, others' inference of his inference, and so on. This makes him uncertain of actions others would take. Such uncertainty is called *strategic uncertainty.*

As in a standard global game model, fundamental uncertainty is exogenously generated by the noise of each player's private signal of fundamental. This reflects the inability of individuals in reality to perfectly understand the world. Strategic uncertainty arises from the independence of noises across players. This reflects the difference of models and information sources across individuals in the society. For our purpose, to highlight the role of strategic uncertainty, following the convention of the literature on global games, we focus on the limiting case when fundamental uncertainty vanishes throughout the paper.

Although the model that we consider is a one-shot game, the one-shot investment decision in the model should be understood as a long run average of a series of decisions to be made. In the long run, with the continuous accumulation of knowledge and experience and improvement of methodology, private knowledge of individuals becomes more and more precise, and thus fundamental uncertainty that they face vanishes. On the contrary, as well known in the literature on global games, strategic uncertainty remains huge no matter how small fundamental uncertainty is, providing that fundamental uncertainy is not zero. In reality, due to the persistent difficulty for individuals to eliminate this difference through communication, such independence is long-lasting even in the long run.[5] As a result, strategic uncertainty is persistent and thus becomes the essential cause of mis-coordination. Hence, the limiting case when fundamental uncertainty vanishes also captures the realistic sharp contrast as such between fundamental uncertainty and strategic uncertainty in the long run.

As common in this literature, we will focus on symmetric equilibria in which all players are playing switching strategies. That is, a player with cost c and signal x will take action

$$a(x,c) = \begin{cases} 1 \text{ if } x \geq \widehat{x}\,(c) \\ 0 \text{ if } x < \widehat{x}\,(c) \end{cases}.$$

An equilibrium is characterized by the cutoff function $\widehat{x}\,(c)$. In particular, we write $\widehat{x}^{\sigma}\,(c)$ in order to indicate that the equilibrium is obtained given fundamental uncertainty σ. It is shown in the appendix that when fundamental uncertainty vanishes, it is without loss of generality to focus on this class of symmetric equilibria. The following proposition characterizes the equilibrium.

Proposition 2.2 *Let $\widehat{x}^{\sigma} : [\bar{c}, \underline{c}] \to \mathbb{R}$ characterize an equilibrium of the investment game given fundamental uncertainty σ. For any $\varepsilon > 0$, there exists a $\overline{\sigma} > 0$ such that for all $\sigma \in (0, \overline{\sigma})$,*

i) if $r < \bar{c} - \underline{c}$, then for all $c \in [\underline{c}, \bar{c}]$,

$$\widehat{x}^{\sigma}\,(c) \in \left(\widehat{\theta}\,(c) - \varepsilon, \widehat{\theta}\,(c) + \varepsilon\right),$$

where $\hat{\theta}(c) = \frac{r\bar{c}}{\bar{c}-\underline{c}} + \left(1 - \frac{r}{\bar{c}-\underline{c}}\right) c$;

ii) if $r \geq \bar{c} - \underline{c}$, then for all $c \in [\underline{c}, \bar{c}]$,

$$\widehat{x}^{\sigma}\,(c) \in \left(\widehat{x}_{inv} - \varepsilon, \widehat{x}_{inv} + \varepsilon\right),$$

where

$$\widehat{x}_{inv} = \frac{r}{2} + \mathbf{E}c$$

[5] The noise x that is independent across players captures the subtle differences in the understanding of the world among individuals that are not communicable. For example, economists have different models of the economy. Other than the existence, the differences across these models may not be known across economists, and cannot be precisely communicated even if they are known. Such differences persist in the long run although the models become more and more precise. Another example is differences in subjective feelings or emotions that affect people's understanding of the world and are hard to convey in words. Although people can be more and more rational in the long run, such differences cannot be completely eliminated.

and $\mathbf{E}c = \frac{\bar{c}+\underline{c}}{2}$ *is the expectation of* c.

Note that $\widehat{\theta}(c)$ has the same expression as in Proposition 2.1.

As in Section 2.1, a clear bifurcation is drawn with respect to whether $r \leq \bar{c} - \underline{c}$:

When $r > \bar{c} - \underline{c}$, that is, when preference heterogeneity is too low to dominate strategic complementarity, players with different costs choose the same cutoff $\widehat{x}_{inv}$ asymptotically when fundamental uncertainty vanishes. When a player receives the cutoff signal realization $\widehat{x}_{inv}$ and is thus indifferent between investing or not, as fundamental uncertainty vanishes, he is almost sure that fundamental θ equals $\widehat{x}_{inv}$. Yet, he is also sure that signal realizations of others are close to $\widehat{x}_{inv}$ as well. Therefore, he is uncertain of the mass of investors. This is precisely strategic uncertainty, which is originated from fundamental uncertainty, but still persists when the latter vanishes. This is in line with exisiting global game models without heterogeneity of players' preferences, as the special case of $\bar{c} - \underline{c} = 0$ also falls into this category. Strategic uncertainty results in welfare loss, as indicated by

$$\begin{aligned} \lim_{\sigma \to 0} \widehat{x}^{\sigma}(c) &= \frac{r}{2} + \mathbf{E}c \\ &> \bar{c}. \end{aligned}$$

Recall from Proposition 2.1 that $\bar{c}$ is the common switching point for all players in the welfare-maximizing equilibrium of the corresponding complete information benchmark. Therefore, for almost all types of players, the switching cutoffs of their strategies are inefficiently high even though fundamental uncertainty vanishes.

When $r < \bar{c} - \underline{c}$, that is, when preference heterogeneity is sufficiently high to dominate strategic complementarity, a player's cutoff $\widehat{x}^{\sigma}(c) \to \widehat{\theta}(c)$ in the limit, which strictly increases with cost c. This means that strategic uncertainty is eliminated: when a player receives his cutoff signal realization $\widehat{x}^{\sigma}(c)$ and is thus indifferent between investing or not, he is almost sure that players with cost lower (higher) than his own are investing (not investing). As a result, the equilibrium converges to that of the complete information benchmark in Section 2.1.

In the knife edge case $r = \bar{c} - \underline{c}$, $\widehat{\theta}(c) = \bar{c} = \widehat{x}_{inv}$ for all c, so strategic uncertainty is still present as the case $r > \bar{c} - \underline{c}$. However, strategic uncertainty in this special case does not result in welfare loss, because we still have $\widehat{x}^{\sigma}(c) \to \widehat{\theta}(c)$ as in the case $r < \bar{c} - \underline{c}$.

3 Organizations and Their Sustainability

This section discusses the sustainability of organizations, that is, whether it is incentive compatible for members of an organization to join it. As a preview of the main results, no organization is sustainable if strategic complementarity is weakly dominated by preference heterogeneity, i.e., $r < \bar{c} - \underline{c}$, and there is an upper limit for total mass of organizations otherwise.

Although the model that we consider is a one-shot game, the joining decision should be understood as a long-term one in the following sense. The investment cost c_i captures the primitive preference of each individual i that is independent of the state of the world and known to each player before he decides whether to join an organization. The difference between fundamental θ and a player's private signal x_i reflects the persistent imperfection of an individual's knowledge of the world. And the one-shot investment decision in the model should be interpreted as an average of a series of decisions to be made after a player makes up his mind whether to join an organization. Such interpretation is valid in situations in reality where it is not too easy for an individual to change his joining decision over time.

In order to highlight the main tension, in this section we focus on equilibria with *at most one organization*. Section 6.1 discusses the senario with multiple organizations.

This section is organized as follows: Section 3.1 articulates the behavioral assumption we make for organizations. Section 3.2 shows that no organization is asymptotically sustainable when preference heterogeneity dominates strategic uncertainty (i.e., when $r < \bar{c} - \underline{c}$). The next subsections focus on the complementary case when preference heterogeneity is dominated by strategic uncertainty (i.e., when $r \geq \bar{c} - \underline{c}$). Section 3.3 establishes that sustainable organizations must be downward-exhaustive, and that social welfare strictly increases with the size of the organization when fundamental uncertainty vanishes. Lastly, Section 3.4 characterizes the upper bound for the size of asympotically sustainable organizations.

3.1 Our Notion of Organizations

As mentioned at the beginning of the introduction, a (generic) organization is a group of people who choose an action through a prespecified collective decision rule, and are subsequently all obliged to take that action. In this paper, in order to focus on the desirability and sustainability of organizations, following the convention of political economy literature, we make the following behavioral assumption: The collective action of an organzation is determined by the optimization of its median voter, whose cost of investment equals the median of all its members. The median voter (hypothetically) has mass equal to that of the organization and knows perfectly fundamental θ. He chooses his favorite action, internalizing the impact of her mass. Formally, we define this notion of organization considered in this paper as follows.

Definition 3.1 *An organization consists of a set of players with positive mass whose cost of investment c are distributed according to a measure μ on $[\underline{c}, \bar{c}]$ such that 1) all these players ("members") are obliged to take a collective action and that 2) the collective action is*

$$b(\theta) \in \arg\max_{b \in \{0,1\}} b \cdot \left[u\left(\theta, \lambda + l_{1-\lambda}\left(\theta\right)\right) - c_{med}\left(\mu\right)\right] \ ,$$

where

$$\lambda = \int_{\underline{c}}^{\bar{c}} \mu(dc) > 0$$

is the mass of members,

$$l_{1-\mu}\left(\theta\right) = \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a\left(x, c\right)|\theta\right] \left(\frac{dc}{\bar{c} - \underline{c}} - \mu(dc)\right)$$

is the mass of investors who are not members, and the median voter's cost of investment $c_{med}\left(\mu\right)$ is such that

$$\frac{\int_{c_{med}(\mu)}^{\bar{c}} \mu(dc)}{\lambda} = \frac{1}{2} \ .$$

The obligation that every member has to follow the action chosen by a (hypothetical) representative member may seem dictatoral and unreasonable at a first glance, but in the spirit of (Feddersen and Pesendorfer 1997), it can be microfounded as the result of an election within the organization following the simple majority rule (i.e., all the members invest if and only if more than half of them vote for it).[6] In such an election, because the payoff of investment for each member strictly decreases with the cost of investment,

[6] We consider this particular decision rule for three reasons. First, it is widely used in reality, including supernational organizations like United Nations, congresses of many countries at different levels, shareholders meeting of companies, administrative committees of various organizations such as universities, government agencies, etc. Second, this decision rule is also widely discussed in the literature, especially that on political economy, among which (Romer 1975), (Meltzer and Richard 1981), (Feddersen and Pesendorfer 1997), (Myerson 1998) are some classic examples. Lastly, it provides technical convenience for our analysis. Because under this decision rule, the two actions are symmetric in the sense that there is no status quo to be implemented if the number of supporters of its alternative fails to reach a majority different from 50%. Moreover, under this decision rule, the power of each member in an organization is symmetric, so that we do not need to keep track of the (potentially endogeneous) distribution of power in addition to that of investment cost.

in equilibrium the organization with mass λ invests if and only if the (real) median voter votes for it. As a result, to an outsider, the organization as a whole behaves identically to a (hypothetical) single player with cost of investment $c_{med}(\mu)$ but with a positive mass equal to that of the organization, λ. In addition, as mentioned in the second paragraph of the introduction, the election with simple majority rule (conditional on a member being pivotal) effectively aggregates private information of all its members about fundamental θ, and thus fundamental uncertainty a member faces vanishes as the number of members approaches a continuum. Therefore, to an outsider, the (hypothetical) median voter behaves as if he knows fundamental θ perfectly. [7]

We are interested in the following two issues. First, given an organization, the behavior of it and its outsiders. Second, whether a particular organization is sustainable in the sense that both its insiders and outsiders are willing to join and not join it ex ante, respectively. The following equilibrium concept combines both these issues.

Definition 3.2 *A voluntary organizational equilibrium consists of*

1) an organization defined by measure μ;

2) a symmetric strategy $a : (x, c) \longmapsto \{0, 1\}$ for players not in the organization; and

3) a strategy for the organization $b : \theta \mapsto \{0, 1\}$,

such that:

a) given b and μ, for all $x \in \mathbb{R}$ and $c \in [\underline{c}, \bar{c}]$,

$$a(x, c) \in \arg\max_{a'} a' \cdot \{\mathbf{E}[u(\theta, \lambda b(\theta) + l_{1-\lambda}(\theta)) | x] - c\} \ ,$$

where

$$l_{1-\mu}(\theta) = \int_{\underline{c}}^{\bar{c}} \mathbf{E}[a(x, c) | \theta] \left(\frac{dc}{\bar{c} - \underline{c}} - \mu(dc) \right)$$

and

$$\lambda = \int_{\underline{c}}^{\bar{c}} \mu(dc) > 0 \ ;$$

b) given a and μ, for all $\theta \in [\underline{\theta}, \bar{\theta}]$,

$$b(\theta) \in \arg\max_{a' \in \{0,1\}} a' \cdot [u(\theta, \lambda + l_{1-\lambda}(\theta)) - c_{med}(\mu)] \ ,$$

[7] Once again, in order to accomodate the additive noise structure that is more convenient for the analysis of our games with incomplete information (in the spirit of global games), we change the setup of (Feddersen and Pesendorfer 1997) and generalize their results to allow the support of private signals to be a continuum. Details of the derivation of such generalization are available upon request.

where $c_{med}(\mu)$ is such that

$$\frac{\int_{c_{med}(\mu)}^{\bar{c}} \mu(dc)}{\lambda} = \frac{1}{2} \;;$$

c) for players not in the organization

$$\begin{aligned} & \mathbf{E}\left\{a(x,c)\left(\mathbf{E}\left[u\left(\theta, \lambda b\left(\theta\right) + l_{1-\lambda}\left(\theta\right)\right)|x\right] - c\right)\right\} \\ \geq \quad & \mathbf{E}\left\{b(\theta)\left(\mathbf{E}\left[u\left(\theta, \lambda b\left(\theta\right) + l_{1-\lambda}\left(\theta\right)\right)|x\right] - c\right)\right\} \;; \end{aligned} \tag{2}$$

and

d) for players in the organization

$$\begin{aligned} & \mathbf{E}\left\{a(x,c)\left(\mathbf{E}\left[u\left(\theta, \lambda b\left(\theta\right) + l_{1-\lambda}\left(\theta\right)\right)|x\right] - c\right)\right\} \\ \leq \quad & \mathbf{E}\left\{b(\theta)\left(\mathbf{E}\left[u\left(\theta, \lambda b\left(\theta\right) + l_{1-\lambda}\left(\theta\right)\right)|x\right] - c\right)\right\} \;. \end{aligned} \tag{3}$$

In the definition, requirement a) reads that every player outside the organization maximizes his payoff given the strategies of all other outsiders and of the organization. Requirement b) reads that the organization maximizes the payoff of the (hypothetical) median voter who internalizes its mass, taking as given the strategy of all outsiders. Requirement c) and d) are participation constraints that players in (outside) the organization prefer joining (not joining) the organization to not joining (joining) it. This means that the organization is sustainable. In the rest of the paper, when we say that an organization μ is sustainable, we mean that there is a voluntary organizational equilibrium supporting it. As in Section 2.2, to highlight the role of strategic uncertainty, we focus on the limiting case when fundamental uncertainty vanishes ($\sigma \to 0$). If an organization μ is sustainable in this limiting case, we say it is *asymptotically sustainable*.

Again, as in unorganized games in Subsection 2.2, we focus on symmetric equilibria in which all players are playing switching strategies. That is, in an equilibrium with fundamental uncertainty σ, $b^{\sigma}(\theta) = 1_{\theta \geq \hat{\theta}^{\sigma}}$ and $a^{\sigma}(x,c) = 1_{x \geq \hat{x}^{\sigma}(c)}$ for some switching points $\hat{\theta}^{\sigma}$ for the organization and $\hat{x}^{\sigma}(c)$ for the unorganized player with cost c. It is also shown in the appendix that this is without loss of generality when fundamental uncertainty vanishes. We use $\hat{\theta} \equiv \lim_{\sigma \to 0} \hat{\theta}^{\sigma}$ and $\hat{x}(\cdot) \equiv \lim_{\sigma \to 0} \hat{x}^{\sigma}(\cdot)$ to denote the limiting equilibrium switching points for the organization and the unorganized players respectively of a sequence of equilibria $\left\{\mu, \hat{\theta}^{\sigma}, \hat{x}^{\sigma}(\cdot)\right\}$. The existence of the limits is proved in the appendix.

Note that the definition of an organization does not restrict the "shape" of it. We start with a simplification in this respect: an organization must be a continuous interval in equilibrium.

Lemma 3.1 *In a voluntary organizational equilibrium, if players c_1 and $c_2 > c_1$ are both in an organization, then so are all players $c \in [c_1, c_2]$.*

Intuitively, since $c \in [c_1, c_2]$, the preference of player c is more aligned with the median voter c_{med} than either player c_1 or c_2. If both c_1 and c_2 prefer to join the organization, so would player c. As a result, it is without loss of generality to focus on organizations that are intervals. That is, each organization consists of all players whose cost of investment are in $[c_1, c_2]$ for some $c_1 < c_2$. Note that the end point types c_1 and/or c_2 may be indifferent between joining an organization or not, and between joining two different organization in Section 6.1. This may make their organization membership indeterminate. But since their total mass is zero, their membership does not affect our analysis. And thus for notational convenience, we use closed intervals to denote organizations hereafter.

3.2 Unsustainability of Organizations under Large Preference Heterogeneity

We first discuss the case in which strategic complementarity is dominated by preference heterogeneity (i.e., $r < \bar{c} - \underline{c}$). The following proposition states the conclusion.

Proposition 3.1 *If $r < \bar{c} - \underline{c}$, no organization is asymptotically sustainable.*

An argument analogous to Proposition 2.2 shows that if and only if $r < \bar{c} - \underline{c}$, there is no strategic uncertainty among unorganized players, that is, their limiting switching points $\hat{x}(c)$ when fundamental uncertainty σ vanishes strictly increasing in c. Since $\hat{x}(\cdot)$ is strictly increasing, the mass $\eta(\theta)$ of investors out of the unorganized players must be a continuous and strictly increasing function of θ. As illustrated in Figure 1, this property of $\eta(\theta)$ makes the equilibrium benefit of investment u as a function of θ similar in shape to that in the case of all-inclusive organization, and so is the argument for non-sustainability of the organization.

To see this, we first determine the limiting strategy of the organization in the corresponding exogenous organizational equilibrium. Whe the organization invests, the benefit

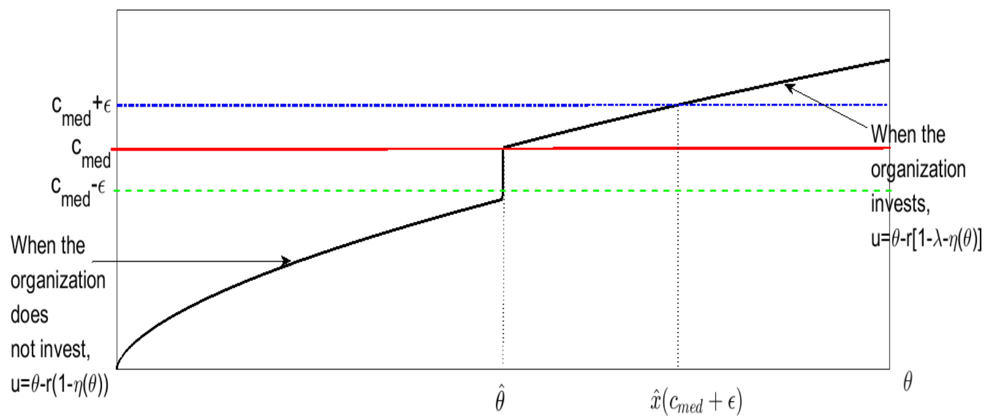

Figure 1: Organization under Large Preference Heterogeneity

of investment is

$$\begin{aligned} u(\theta, l) &= \theta - r\left(1 - \lambda - \eta(\theta)\right) \\ &= \theta + r\eta(\theta) - r(1-\lambda). \end{aligned}$$

Therefore, it invests if and only if $\theta \geq \hat{\theta}$, where $\hat{\theta} + r\eta(\hat{\theta}) - r(1-\lambda) = c_{med}$. As a result, the benefit of investment to each player $V(\theta) \equiv u(\theta, l(\theta))$ is as depicted by the black solid line in Figure 1.

We now show that when fundamental uncertainty σ vanishes, it is not incentive compatible for every player whose cost of investment $c > c_{med}$ to join the organization. Consider such a player with cost $c_{med} + \varepsilon$, as marked with the blue horizontal line. Since σ is close to 0, every player behaves as if he knows θ perfectly. Given the equilibrium strategy of the organization, his ideal strategy would be to invest if and only if $V(\theta) \geq c$, or equivalently $\theta \geq \hat{x}(c_{med} + \varepsilon)$ as shown in the figure. However, if he joins the organization, he is obliged to follow the median voter's strategy (that is, to invest if and only if $V(\theta) \geq c_{med}$) and thus loses the freedom not to invest when $\theta \in [V^{-1}(c_{med}), V^{-1}(c))$.[8] Anticipating this, ex ante he is better off not joining the organization. Therefore, this organization is not sustainable.

Although it suffices to analyze players whose $c > c_{med}$ in order to disprove the sustainability of organizations, to facilitate later discussion, we complete our analysis by discussing the choice of the other players, in particular those whose cost of investment $c' \in (c_{med} - r\lambda, c_{med}]$. We show that their ideal strategy perfectly coincides with that of the organization, that is,

[8]It is easy to see from Figure 1 that the inverse of V is well-defined in the relevant range.

to invest if and only if $\theta \geq \hat{\theta}$. Thus, because they have to rely on noisy signals if staying unorganized, they strictly prefer to join the organization. To see this, consider such a player, whose cost of investment is $c_{med} - \varepsilon$ as shown in Figure 1 with the green horizontal line, where $0 < \varepsilon < r\lambda$. When the organization invests, that is, when $V(\theta) \geq V\left(\hat{\theta}\right) = c_{med}$, the benefit of investment for him is $V(\theta) \geq c_{med} > c_{med} - \varepsilon$, so he also prefers to invest; When the organization does not invest, that is, when $V(\theta) < V\left(\hat{\theta}\right) = c_{med}$, the benefit of investment for him is $\theta - r\lambda < c_{med} - \varepsilon$, so he prefers not to invest as well.

Note that the organization of mass λ effectively introduces a jump of magnitude $r\lambda$ at $\theta = \hat{\theta}$ to the benefit of investment $u(\theta, l)$ that is otherwise continuous. This jump creates an asymmetry of incentive to join the organization between the continuum of players whose c_i are right below c_{med} (who want to join, marked with green line) and right above c_{med} (who want to quit, marked with blue line).

3.3 Exogenous Organizational Equilibrium

Given that no organization is sustainable when $r < \bar{c} - \underline{c}$, for the rest of the paper we will focus on the case of $r \geq \bar{c} - \underline{c}$. That is, when strategic complementarity dominates preference heterogeneity. We claim first that if $r \geq \bar{c} - \underline{c}$, in a voluntary organizational equilibrium, the organization must be downward exhaustive — a continuous interval containing $\underline{c}$ and including all players of each type c it covers, as illustrated in Figure 2. So investment costs of all unorganized players are higher than any player's in the organization. Therefore, it is without loss of generality to only focus on such organizations for the rest of the paper.

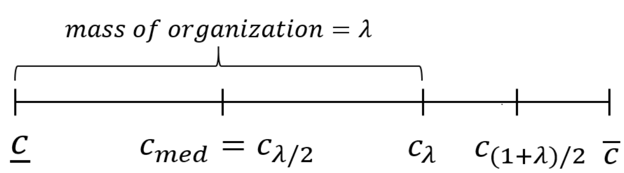

Figure 2: downward exhaustive organization

Proposition 3.2 *In any voluntary organizational equilibrium,* $\mu(c) = 1_{c \leq c_\lambda}$ *for some* $c_\lambda \in [\underline{c}, \bar{c}]$.

The proof is straightforward and thus omitted. To see the intuition, recall from Lemma 3.1 that an equilibrium organization must be a continuous interval, say $[\underline{c}_1, \bar{c}_1]$. Following the same logic as the last two paragraphs in the previous subsection, the ideal strategy of a player with $c > c_{med} - r\lambda$ coincides perfectly with that of the median voter c_{med}. Since such a player has to rely on his noisy signal (the noise approaches but is not exactly equal to zero.) if staying unorganized, he strictly prefers to join the organization. Note that $r\lambda = r\frac{1}{\bar{c}-\underline{c}}\frac{c_{med}-\underline{c}_1}{1/2} > c_{med} - \underline{c}_1$, as $r \geq \bar{c} - \underline{c}$. This implies that a positive mass of players right below the lower boundary $\underline{c}_1$ of the organization (if any) would strictly prefer to join it rather than staying outside as they are. This violates participation constraint c) of Definition 3.2 unless such players do not exist, i.e., $\underline{c}_1 = \underline{c}$.

Given this simplification, we now study how social welfare is affected by the size λ of the downward exhaustive organization (that include all players of types in $[\underline{c}, c_\lambda]$ where $c_\lambda = \underline{c} + \lambda(\bar{c} - \underline{c})$), *ignoring the sustainability issue* (which we will discuss later). To do this, taking λ as given, we first characterize optimal limiting strategies $\hat{\theta}$ and $\hat{x}$ for the organization and the unorganized players respectively. Then we calculate the resulting social welfare as a function of λ. We use the concept of *exogenous organizational equilibria* defined as follows to capture the desired optimal strategies for an *exogenously given* organization.

Definition 3.3 *An exogenous organizational equilibrium consists of*

1) an organization defined by measure μ*;*

2) a symmetric strategy $a : (x, c) \longmapsto \{0, 1\}$ *for players outside the organization; and*

3) a strategy for the organization $b : \theta \mapsto \{0, 1\}$*,*

that satisfy requirements a) and b) of Definition 3.2.

That is, an exogenous organizational equilibrium satisfies all but the two participation constraints c) and d) of a voluntary organizational equilibrium. As for voluntary organizational equilibria, we focus on symmetric equilibria in which all players are playing switching strategies. That is, in an equilibrium with fundamental uncertainty σ, $b^\sigma(\theta) = 1_{\theta \geq \hat{\theta}^\sigma}$ and $a^\sigma(x, c) = 1_{x \geq \hat{x}^\sigma(c)}$ for some switching points $\hat{\theta}^\sigma$ for the organization and $\hat{x}^\sigma(c)$ for the unorganized player with cost c. We use $\hat{\theta} \equiv \lim_{\sigma\to 0} \hat{\theta}^\sigma$ and $\hat{x}(\cdot) \equiv \lim_{\sigma \to 0} \hat{x}^\sigma(\cdot)$ to denote the limiting equilibrium switching points for the organization and the unorganized players respectively of a sequence of equilibria $\left\{\mu, \hat{\theta}^\sigma, \hat{x}^\sigma(\cdot)\right\}$.[9]

To facilitate discussion, $\forall \tau \in [0, 1]$, define

$$c_\tau \equiv \underline{c} + \tau(\bar{c} - \underline{c}). \quad (4)$$

[9] Again, it is shown in the appendix that it is without loss of generality to focus on equilibria with switching strategies when fundamental uncertainty vanishes, and that $\hat{\theta}$ and $\hat{x}$ exist.

That is, the mass of players with cost less than c_τ is τ. The following proposition characterizes the limiting exogenous organizational equilibrium with the downward exhaustive organization consisting of types in $[\underline{c}, c_\lambda]$, as illustrated by Figure 2.

Proposition 3.3 $\forall\lambda \in [0,1]$, *in the limiting exogenous organizational equilibrium in which* $\mu(c) = 1_{\{c \leq c_\lambda\}}$, *where* c_λ *is defined by (4),*

if $\lambda \leq 1 - \frac{\bar{c}-\underline{c}}{r}$,

$$\hat{\theta} = \hat{x}(c) = \frac{r}{2}\left(1-\lambda^2\right) + \mathbf{E}c,\ \forall c > c_\lambda; \tag{5}$$

if $\lambda > 1 - \frac{\bar{c}-\underline{c}}{r}$,

$$\hat{\theta} = c_{\lambda/2} + r(1-\lambda) < \hat{x}(c) = c_{(1+\lambda)/2} + \frac{r(1-\lambda)}{2},\ \forall c > c_\lambda, \tag{6}$$

where $c_{\lambda/2}$ *and* $c_{(1+\lambda)/2}$ *are defined by (4), and* $\mathbf{E}c = \frac{\bar{c}+\underline{c}}{2}$ *is the expectation of* c.

To understand Proposition 3.3, first, an argument to Proposition 2.2 shows that if and only if $r \geq \bar{c} - \underline{c}$, there is strategic uncertainty among unorganized players, that is, when the limiting switching point $\hat{x}(c)$ when fundamental uncertainty σ vanishes is constant for all unorganized players regardless of their investment cost c. The fact that investment costs of all unorganized players are higher than any player's in the organization implies that $\hat{\theta} \leq \hat{x}$.

If $\hat{\theta} < \hat{x}$, then there is strategic uncertainty only among the unorganized players, not between them and the organization. We characterize the cutoffs in this case. When $\theta = \hat{\theta}$, the median voter of the organization, whose cost of investment is $c_{\lambda/2}$, believes that the mass of non-investors is exactly $1-\lambda$, the mass of unorganized players, since $\hat{x} > \hat{\theta}$. His indifference condition there is $\hat{\theta} - r(1-\lambda) - c_{\lambda/2} = 0$, which yields the expression for $\hat{\theta}$ in (6). When $x = \hat{x}$, the unorganized players believe that $\theta = \hat{x}$ almost surely as $\sigma \approx 0$, that the organization must be investing for sure as $\hat{\theta} < \hat{x}$, and that exactly half of themselves (with mass $(1-\lambda)/2$) are investing due to strategic uncertainty among them, as $\Pr[x_{-i} < \hat{x} | x_i = \hat{x}] = 1/2$. Their average cost of investment is $c_{(1+\lambda)/2}$. Thus, the average indifference condition for them is $\hat{x} - r(1-\lambda)/2 = c_{(1+\lambda)/2}$, which yields the expression for $\hat{x}$ in (6). The assumption $\hat{\theta} < \hat{x}$ requires $\lambda > 1 - \frac{\bar{c}-\underline{c}}{r}$. Or equivalently, we must have $\hat{\theta} = \hat{x}$ if $\lambda \leq 1 - \frac{\bar{c}-\underline{c}}{r}$.

If $\hat{\theta} = \hat{x}$ and $\lambda < 1 - \frac{\bar{c}-\underline{c}}{r}$, there is also strategic uncertainty between the organization and the unorganized players. When the median voter of the organization is indifferent, he believes that the unorganized players invest with some probability, and vise versa. For each possible mass $\lambda < 1 - \frac{\bar{c}-\underline{c}}{r}$ there is a pair of such probabilities supporting the equilibrium, and the common switching point turns out to be the one in (5).

In the knife-edge case of $\lambda = 1-\frac{\bar{c}-\underline{c}}{r}$, while we have $\hat{\theta} = \hat{x}$, there is no strategic uncertainty between the organization and the unorganized players, and $\lim_{\sigma\to 0}\theta^{\sigma}$ and $\lim_{\sigma\to 0}\hat{x}^{\sigma}(c)$ in (6) both coincide with the value $\frac{r}{2}\left(1-\lambda^2\right) + \mathbf{E}c$ in (5).

Now that we have characterized the corresponding limiting exogenous organizational equilibrium for each organization size λ, we can calculate the corresponding social welfare. To accomodate the setup with organizations, we modify Definition 2.2 of social welfare as follows.

Definition 3.4 *Social welfare in an organized game (OG) with incomplete information when the organization uses strategy $b : \theta \mapsto \{0,1\}$ and other players use symmetric strategy $a : (x,c) \longmapsto \{0,1\}$ is*

$$W^{OG}(a,b) = \int_{\underline{\theta}}^{\bar{\theta}} \left\{ \begin{array}{c} \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a\left(x,c\right)|\theta\right] \cdot \left[u(\theta, l\left(\theta\right)) - c\right] \left(\frac{dc}{\bar{c}-\underline{c}} - d\mu(c)\right) \\ + \int_{\underline{c}}^{\bar{c}} b\left(\theta\right)\left[u(\theta, l\left(\theta\right)) - c\right]\mu\left(dc\right) \end{array} \right\} h\left(\theta\right) d\theta \ ,$$

where

$$l\left(\theta\right) = \frac{1}{\bar{c}-\underline{c}} \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a\left(x,c\right)|\theta\right] \left(\frac{dc}{\bar{c}-\underline{c}} - \mu(dc)\right) + \lambda \cdot b\left(\theta\right) \ .$$

Informational frictions causes strategic uncertainty, which in turn results in welfare loss. An organization obligates its members to take collective action and mechanically resolves strategic uncertainty within it. A larger organization mitigates strategic uncertainty better and results in higher social welfare. Proposition 3.4 confirms this point.

Proposition 3.4 *In an exogenous organizational equilibrium of fundamental uncertainty $\sigma > 0$ in which $\mu\left(c\right) = 1_{\{c\leq c_{\lambda}\}}$, where c_{λ} is defined by (4), let $W^{\sigma}\left(\lambda\right)$ denote the corresponding social welfare defined by Definition 3.4. Then $\lim_{\sigma\to 0} W^{\sigma}\left(\lambda\right)$ strictly increases in λ.*

Figure 3 illustrates the relation between social welfare and the size of the organization λ in exogenous organizational equilibria. Social welfare increases with λ. Note that there is a kink at $\lambda_{\max} = 1 - \frac{\bar{c}-\underline{c}}{r}$, beyond which the organization and the unorganized players start to choose different strategies asymptotically as shown in Proposition 3.3. However, it will be shown in the next subsection that organizations with size beyond $\lambda_{\max}$ are not sustainable, because larger organizations have greater internal conflict of interest as well. And thus organizations improve social welfare by no more than that achieved by the one with size $\lambda_{\max}$.

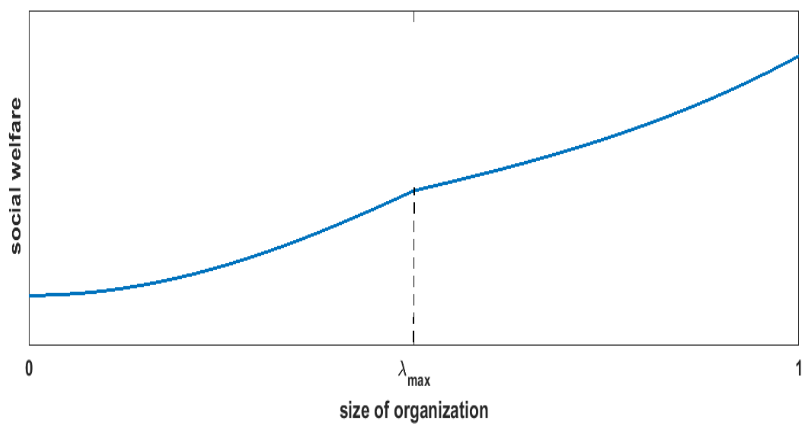

Figure 3: welfare and size of organization, $h(\theta) = \frac{1}{\theta - \underline{\theta}}$

3.4 Size Limit of Sustainable Organizations

In the previous subsection, we have characterized the corresponding exogenous organizational equilibrium and social welfare for each *exogenously given* size of organizations. Whether an organization is sustainable, as captured by participation constraints c) and d) in Definition 3.2 of a voluntary organizational equilibrium, are ignored there. In this subsection we discuss this issue. In Proposition 3.3, we find a cutoff for the size of organizations λ, namely $1 - \frac{\bar{c} - \underline{c}}{r}$, beyond which the organization behaves asymptotically differently from the unorganized players. We show in this subsection that this is also an upper bound for the size of sustainable organizations, with an argument similar to the one in Subsection 3.2.

By Proposition 3.3, if $\lambda > 1 - \frac{\bar{c} - \underline{c}}{r}$, we have $\hat{\theta} < \hat{x}$. The benefit of investment in this subcase is depicted in Figure 4. Proposition 3.3 shows that $\hat{\theta} - r(1 - \lambda) = c_{med}$. Then, an argument analogous to Subsection 3.2 shows that when fundamental uncertainty σ vanishes, it is not incentive compatible for every player whose cost of investment $\hat{\theta} < c < \hat{x}$ to join the organization, and hence the organization is not sustainable. To see this, consider such a player with cost $c_{med} + \varepsilon$ marked with the blue horizontal line in Figure 4. Given cutoffs $\hat{\theta}$ and $\hat{x}$, as $\sigma \to 0$, his ideal strategy would be to invest if and only if $\theta \geq \hat{x}'(c_{med} + \varepsilon)$ as shown in the figure, where $\hat{x}'(c_{med} + \varepsilon)$ is given by $\hat{x}'(c_{med} + \varepsilon) - r(1 - \lambda) = c_{med} + \varepsilon$.

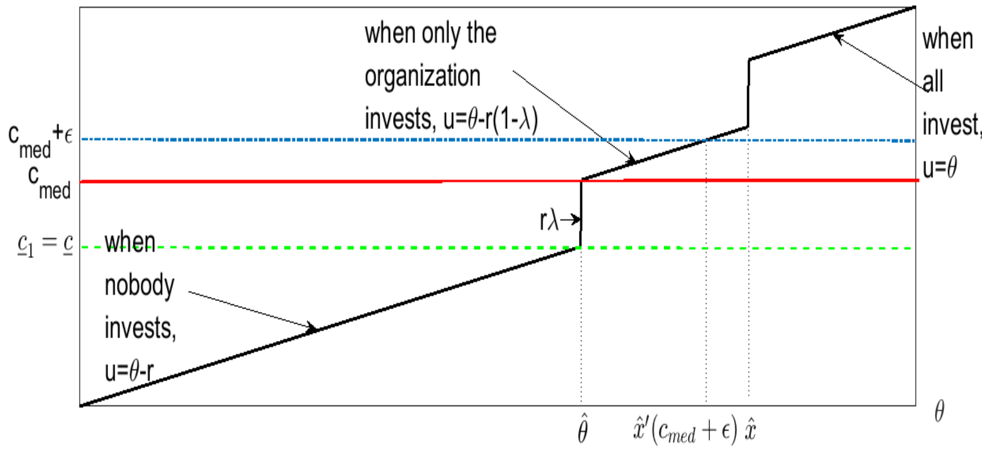

Figure 4: $\hat{\theta} < \hat{x}$

However, if he joins the organization, he is obliged to follow the median voter's strategy (that is, to invest if and only if $\theta \geq \hat{\theta}$) and thus loses the freedom not to invest when $\theta \in [\hat{\theta}, \hat{x}'(c_{med} + \varepsilon))$. Anticipating this, ex ante he is better off not joining the organization.

Analogous to Subsection 3.2, the organization of mass λ effectively introduces a jump of magnitude $r\lambda$ at $\theta = \hat{\theta}$ to the benefit of investment $u(\theta, l)$. This jump creates an asymmetry of incentive to join the organization between the continuum of players whose c_i are right below c_{med} (who want to join) and right above c_{med} (who want to quit).

For any downward-exhaustive organization with size $\lambda < 1 - \frac{\bar{c}-\underline{c}}{r}$, Proposition 3.3 shows that the limiting cutoffs for the organization and the unorganized players coincide, that is, $\hat{\theta} = \hat{x}$. This means that as fundamental uncertainty σ vanishes, a player knows ex ante that whether or not he joins the organization, he will take an asymptotically identical strategy in the investment stage, and thus becomes asymptotically indifferent between joining that organization or not. This verifies participation constraints for all players, and thus the sustainability of the organization.

Recall from Subsection 3.2 that no organization is sustainable if $r < \bar{c} - \underline{c}$. The following three propositions summarize our results concerning sustainable organizations.

Proposition 3.5 *For any voluntary organizational equilibrium with fundamental uncertainty $\sigma > 0$, let λ^σ denote the size of the organization. Then $\lambda^\sigma \leq \max\left\{0, 1 - \frac{\bar{c}-\underline{c}}{r}\right\}$ When $\sigma > 0$ is sufficiently small,*

i) if $r < \bar{c} - \underline{c}$, then $\lambda^\sigma = 0$;

ii) if $r \geq \bar{c} - \underline{c}$, then $\lambda^{\sigma} \leq \lambda_{\max} = 1 - \frac{\bar{c}-\underline{c}}{r}$, and all players, whether or not in the organization, asymptotically take the same action.

4 Welfare Implications of Organizations

Last section studies the sustainability of organizations in coordination games with incomplete information. This section complements it by exploring welfare implications of organizations. We consider four benchmarks: 1) a first-best benchmark (to be defined in the next paragraph, marked with superscript FB); 2) the welfare-maximizing equilibrium in the complete information benchmark in Subsection 2.1 (marked with superscript CI); 3) the limiting equilibrium in the unorganized game with incomplete information in Subsection 2.2 (marked with superscript UG); and 4) the welfare-maximizing limiting voluntary organizational equilibrium in Section 3 (marked with OG), which is the one with the largest sustainable organization given parameter values (Proposition 3.4). Welfare is defined as expected total utility of all players across states, as in Definition 2.2 and 3.4. We fix the strength of strategic complementarity r and the average cost of investment $\mathbf{E}c = \frac{\bar{c}+\underline{c}}{2}$, and increase preference heterogeneity $(\bar{c} - \underline{c})$ from 0.

In the first-best benchmark, a social planner assigns an action $a^{FB} : (\theta, c) \longmapsto \{0, 1\}$ at each state θ for each player c to maximize total social welfare. It is straightforward to see that $a^{FB}(\theta, c) = 1 \Rightarrow a^{FB}(\theta, c') = 1\ \forall c' < c$, and that $a^{FB}(\theta, c) = 0 \Rightarrow a^{FB}(\theta, c') = 0$ $\forall c' > c$. Thus the first-best benchmark must be characterized by the mass of investors $\alpha^{FB} : \theta \longmapsto [0, 1]$, such that $a^{FB}(\theta, c) = 1_{\left\{c \leq c_{\alpha^{FB}(\theta)}\right\}}$, where $c_{\alpha^{FB}(\theta)}$ is defined by (4).

Proposition 4.1 *Let $\alpha^{FB}(\theta)$ be such that a player with cost c invests in the first-best benchmark if and only if $c \leq c_{\alpha^{FB}(\theta)}$, where $c_{\alpha^{FB}(\theta)}$ is defined by (4).*

If $\bar{c} - \underline{c} \leq 2r$, then $\alpha^{FB}(\theta) = 1_{\{\theta \geq \mathbf{E}c\}}$;

If $\bar{c} - \underline{c} > 2r$, then
$$\alpha^{FB}(\theta) = \begin{cases} 0, \text{ if } \theta < r + \underline{c} \\ \frac{\theta - r - \underline{c}}{\bar{c} - \underline{c} - 2r}, \text{ if } \theta \in [r + \underline{c}, \bar{c} - r] \\ 1, \text{ if } \theta > \bar{c} - r \end{cases}.$$

In each of the other three benchmarks, since the benefit of investment is the same for all investors, if a player with cost c invests in equilibrium, so are all players with cost less than c. Hence, there is an analogous mass of investors $\alpha(\theta)$ in each of the rest three benchmarks, such that $a(\theta, c) = 1_{\left\{c \leq c_{\alpha(\theta)}\right\}}$. Proposition 4.1 characterizes $\alpha(\theta)$ for each of them, which is obtained from straightforward calculation from Propositions 2.1, 2.2 and Proposition 3.3, respectively.

Proposition 4.2 *Let $\alpha^{j}(\theta)$ be such that a player with cost c invests in the respective benchmark if and only if $c \leq c_{\alpha^{j}(\theta)}$, where $j \in \{CI, UG, OG\}$ and $c_{\alpha^{j}(\theta)}$ is defined by (4).*

If $\bar{c} - \underline{c} \leq r$, then $\alpha^{CI}(\theta) = 1_{\{\theta \geq \bar{c}\}}$, $\alpha^{UG}(\theta) = 1_{\left\{\theta \geq \frac{r}{2} + \mathbf{E}c\right\}}$, and $\alpha^{OG}(\theta) = 1_{\left\{\theta \geq \frac{r}{2}(1-\lambda_{\max}^{2}) + \mathbf{E}c\right\}}$, where $\lambda_{\max} = 1 - \frac{\bar{c} - \underline{c}}{r}$;

$$\text{If } \bar{c} - \underline{c} > r, \text{ then } \alpha^{CI}(\theta) = \alpha^{UG}(\theta) = \alpha^{OG}(\theta) = \begin{cases} 0, \text{ if } \theta < r + \underline{c} \\ \frac{\theta - r - \underline{c}}{\bar{c} - \underline{c} - r}, \text{ if } \theta \in [r + \underline{c}, \bar{c}] \\ 1, \text{ if } \theta > \bar{c} \end{cases}.$$

Figures 6, 7 and 8 illustrate them, and Figure 5 shows the respective levels of social welfare as functions of preference heterogeneity $\bar{c} - \underline{c}$.

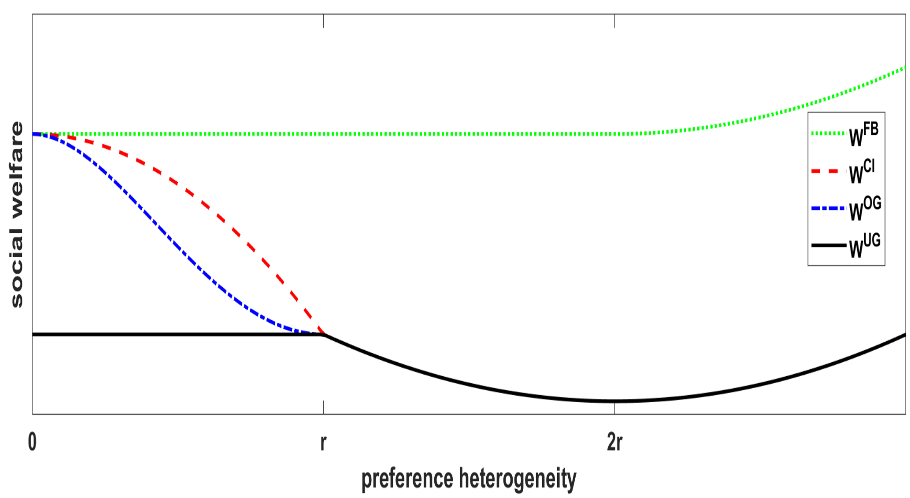

Figure 5: social welfare and preference heterogeneity $(\bar{c} - \underline{c})$, $h(\theta) = \frac{1}{\bar{\theta} - \underline{\theta}}$

There are potentially two sources of welfare loss in the game, best illustrated with the case of $\bar{c} - \underline{c} \in [0, r]$ in Figures 5 and 6. The first one is the conflict between social welfare and individual interest. This can be seen from the difference between the first-best benchmark marked with green dotted lines and the complete information benchmark (of the welfare-maximizing equilibrium) marked with red dashed lines, as there is informational frictions in neither of them. When $r \geq \bar{c} - \underline{c}$, incentive of players to coordinate their actions overweighs the heterogeneity of their preference, so the mass of investors is either 1 or 0 in all four benchmarks as shown in Figure 6. When all players invest, the benefit to every player is

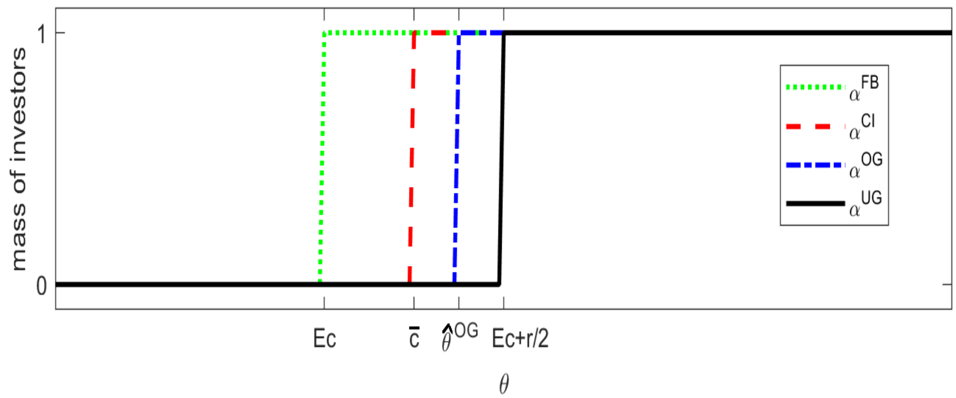

Figure 6: mass of investors and θ, $\bar{c} - \underline{c} \leq r$

θ. In the complete information benchmark, by definition all players invest if and only if the benefit θ overweighs their costs, in particular that of type $\bar{c}$. Therefore, they invest if and only if $\theta \geq \bar{c}$. However, from the perspective of the social planner, the total cost is $\mathbf{E}c$ if all players invest. So he will assign players to invest if and only if $\theta \geq \mathbf{E}c$. The conflict of interest emerges when $\theta \in [Ec, \bar{c})$. Investors with cost higher than θ are not willing to invest. But if they are forced to do so, the loss they suffer is less than the gain enjoyed by investors with cost lower than θ due to positive externality of investment. This makes it attractive for the social planner do so to increase social welfare by the difference between the green dotted line and the red dashed line in Figures 5. It is preference heterogeneity that drives this source of welfare loss, and the first-best and the complete information benchmarks coincide when $\bar{c} - \underline{c} = 0$.

The second source of welfare loss is informational frictions, i.e., miscoordination due to strategic uncertainty, as indicated by the difference in switching points $\bar{c}$ and $\mathbf{E}c + r/2$ in terms of strategies, and by the difference between the red dashed line and the black solid line in Figure 5 in terms of welfare. Organizations improve welfare from the black solid line to the blue dash-dot line in Figure 5 by mitigating strategic uncertainty and thus changing the common switching point from $\mathbf{E}c + r/2$ down to $\hat{\theta}^{OG} = \frac{r}{2}\left(1 - \lambda_{\max}^2\right) + \mathbf{E}c$ in Figure 6, as by construction there is no strategic uncertainty within any organization. Proposition 3.4 further shows that such welfare improvement strictly increases with the size of organizations. However, there is an upper limit for the size of sustainable organization (as established in Proposition 3.5) and thus for such welfare improvement as well. Because larger organizations also feature greater preference heterogeneity inside, and are thus more difficult to be sustained. — This is exactly the key tradeoff highlighted in this paper.

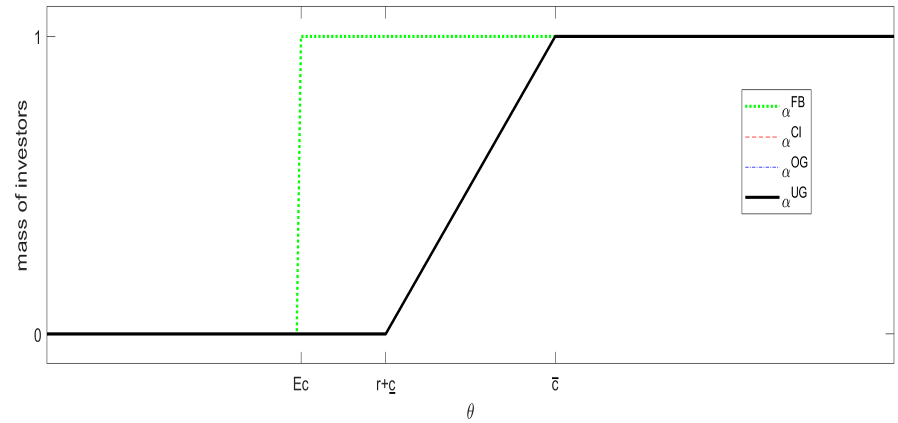

Figure 7: mass of investors and θ, $r < \bar{c} - \underline{c} \leq 2r$

In the case of $\bar{c}-\underline{c} > r$ shown in Figures 5, 7 and 8, incomplete information does not result in strategic uncertainty (Proposition 2.2), and thus equilibrium outcomes of the unorganized game with incomplete information coincide with the welfare-maximizing equilibrium with complete information. Conflict of interest is the only source of welfare loss there. Morever, in this case, no organization is sustainable (Proposition 3.5). And thus equilibrium outcomes of the organized game also coincide with the unorganized game with incomplete information.

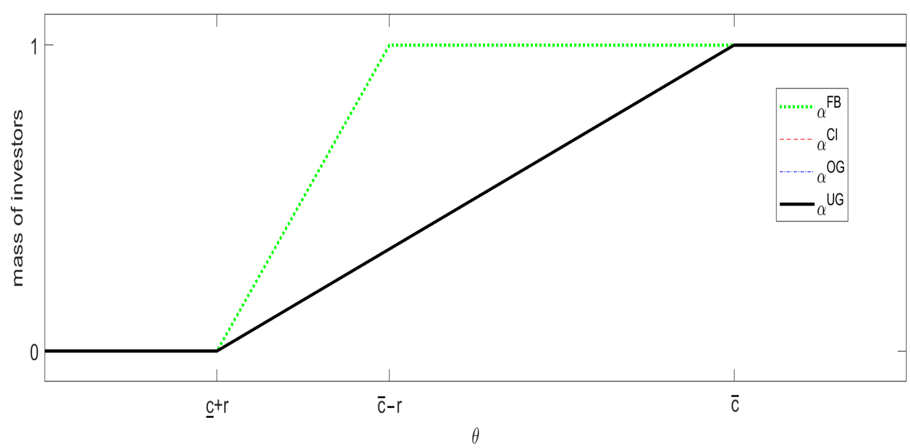

Figure 8: mass of investors and θ, $\bar{c} - \underline{c} > 2r$

Organizations can mitigate strategic uncertainty as in the case of $\bar{c}-\underline{c} \in [0, r]$. They can sometimes resolve welfare loss due to conflict of interest as well, if we ignore sustainability issue, or equivalently, allow organizations to be formed by external forces. Indeed, in the case of $\bar{c}-\underline{c} \in [0, 2r]$, the exogenous organizational equilibrium with the organization consisting of all players perfectly replicates the outcomes of the first-best benchmark (see Proposition 3.3). However, organizations cannot always resolve welfare loss even if we ignore sustainability issue. In the case of $\bar{c}-\underline{c} > 2r$ shown in Figure 8, preference heterogeneity is so large that even the social planner does not want to assign collective actions. This can be seen as the dotted green line there is no longer a step function. In this case, organizations that mandate collective actions by construction cannot resolve welfare loss.

5 Effects of Shocks to the Distribution of Preferences

As explained in Section 2, the investment cost c_i is a preference parameter that captures local variables peculiar to individual i, and the distribution from which it is drawn characterizes the preference heterogeneity of the population. Here, the largest sustainable organization, which consists of the largest possible sub-population obedient to collective decision making, can be interpreted as the scope of social order. In this sense, our model characterizes the scope of social order as a function of the distribution of individual preferences and the strength of strategic complementarity. Consequently, it can be used to study how shocks to the distribution of individual preferences affects the scope of existing social order, as captured by the shift of the boundary of the largest sustainable organization. In reality, such shocks can be generated by migration or polarization. This section tackles this issue. The main results are: 1) Inflow/outflow of population of different types with the same magnitude can have non-monotonic impact on existing social order; and 2) So can increase/decrease of social inequality.

Formally, we consider the situation where the original uniform distribution of preference is perturbed by a different distribution with density f and support $[\underline{c}, \bar{c}]$, such that the new distribution has density $\tilde{f}_\Delta(c) = \frac{1-\Delta}{\bar{c}-\underline{c}} + \Delta \cdot f(c)$, where $\Delta > 0$ is small, and that the total mass of players is still 1. Let $F(c) = \int_{\underline{c}}^{c} f(c)\, dc$ and $\tilde{F}_\Delta(c) = \int_{\underline{c}}^{c} \tilde{f}_\Delta(c)\, dc$ be the corresponding cumulative density functions. As long as Δ is small enough, it is straightforward to see that the qualitative conclusions in Section 3 still hold: No organization is sustainable if $r < \bar{c}-\underline{c}$, and sustainable organizations must be downward exhaustive if $r \geq \bar{c}-\underline{c}$. As before, we focus on the case $r \geq \bar{c}-\underline{c}$. We are interested in how such perturbation shifts the upper boundary of the largest sustainable organization as characterized by its type κ'. Recall from Section 3.4 that the boundary type before perturbation is $\kappa \equiv \underline{c} + \lambda_{\max}(\bar{c}-\underline{c})$, where $\lambda_{\max} = 1 - \frac{\bar{c}-\underline{c}}{r}$. The following proposition provides a first-order approximation for the shift of the upper boundary $(\kappa' - \kappa)$ for *any* perturbation of this form.

Proposition 5.1 *If $r \geq \bar{c} - \underline{c}$, then*

$$\kappa' - \kappa = \frac{2\left(\bar{c}-\underline{c}\right)\Delta}{r}\left\{\begin{array}{c} \frac{1-F(\kappa)}{1-\lambda_{\max}}\left(\frac{\kappa+\bar{c}}{2} - E^F\left[c|c>\kappa\right]\right) \\ +\left(\bar{c}-\underline{c}\right)\left(\frac{F(\kappa)}{2} - F\left(\frac{\underline{c}+\kappa}{2}\right)\right) \\ +\frac{r}{2}\left(\lambda_{\max} - F\left(\kappa\right)\right) \end{array}\right\} + o\left(\Delta\right), \tag{7}$$

where $\lambda_{\max} = 1 - \frac{\bar{c}-\underline{c}}{r}$ and $\kappa \equiv \underline{c} + \lambda_{\max}\left(\bar{c}-\underline{c}\right)$.

Note that $\frac{\kappa+\bar{c}}{2}$ is the average cost of the originally unorganized players, and that $\frac{\underline{c}+\kappa}{2}$ is the cost of the median voter of the original organization. The three terms in the braces of equation (7) capture the effects of perturbation due to its impact outside, inside and across the original upper boundary of the largest sustainable organization respectively.

The first term, $\frac{1-F(\kappa)}{1-\lambda_{\max}}\left(\frac{\kappa+\bar{c}}{2} - E^F\left[c|c>\kappa\right]\right)$, captures the effect of the perturbation due to its impact on the players originally outside the organization (whose $c > \kappa$), and thus we this effect of perturbation *the outside-boundary effect*. It shifts the boundary of the organization by changing the average cost of the unorganized players, which is originally $\frac{\kappa+\bar{c}}{2}$. If $E^F\left[c|c>\kappa\right] > \frac{\kappa+\bar{c}}{2}$, the average cost rises, making them more conservative and raising their common limiting switching point $\hat{x}$ so that $\hat{x} > \hat{\theta}$. Recall from Section 3.4 that the organization is not sustainable if $\hat{x} > \hat{\theta}$. To offset this, the boundary of the organization has to shrink to lower the average cost of the unorganized players. Hence, this term is negative then. The opposite story happens if $E^F\left[c|c>\kappa\right] < \frac{\kappa+\bar{c}}{2}$.

The second term, $\left(\bar{c}-\underline{c}\right)\left(\frac{F(\kappa)}{2} - F\left(\frac{\underline{c}+\kappa}{2}\right)\right)$, captures the effect of the perturbation due to its impact on the players originally inside the organization (whose $c \leq \kappa$), and thus we call this effect of perturbation *the inside-boundary effect*. It shifts the boundary of the organization by changing the median voter of the organization. If $F\left(\frac{\underline{c}+\kappa}{2}\right) > \frac{F(\kappa)}{2}$, we have $\tilde{F}_\Delta\left(\frac{\underline{c}+\kappa}{2}\right) > \frac{\tilde{F}_\Delta(\kappa)}{2}$. This means that the perturbation places more than half of the mass within the original boundary of the organization to the left of the original median voter $\frac{\underline{c}+\kappa}{2}$. This implies that the cost of the new median voter must be lower than before, increasing the conflict of interest between him and the original boundary type, who is originally just indifferent between joining the organization and not. Therefore, the boundary type would quit the organization, making this effect negative. The opposite story happens if $F\left(\frac{\underline{c}+\kappa}{2}\right) < \frac{F(\kappa)}{2}$.

The third term, $\frac{r}{2}(\lambda_{\max} - F(\kappa))$, captures the effect of the perturbation due to its impact on shifting mass across the original upper boundary of the organization, and thus we call this effect of perturbation *the across-boundary effect*. If $F(\kappa) > \lambda_{\max}$, we have $\tilde{F}_{\Delta(\kappa)}(\kappa) > \lambda_{\max}$. This means that the perturbation moves mass into the boundary of the original organization. Recall from the explanation for Proposition 3.3 that originally when $\lambda = \lambda_{\max}$, while $\hat{\theta} = \hat{x}$, there is no strategic uncertainty between the organization and the unorganized players. As explained there, at their respective switching points, the organization believes that the non-investors consist of all the unorganized players, while an unorganized player believes that they consist of only half of the unorganized players due to strategic uncertainty among themselves. Suppose mass Δ is moved into the organization. At their respective new switching points, the organization believes that the mass of non-investors drops by Δ, which is twice as much as an unorganized player believes. So $\hat{\theta}$ drops by $r\Delta$, twice as much as the decrease of $\hat{x}$. This makes $\hat{\theta} < \hat{x}$, and the boundary of the organization has to shrink to restore $\hat{\theta} = \hat{x}$ and its sustainability. The opposite story happens if mass Δ is moved out of the organization. The greater the strength of strategic complementarity, the stronger the across-boundary effect. Note also that this effect is independent of how the mass $F(\kappa)$ is distributed inside the original boundary of the organization.

5.1 Effects of Migration on the Scope of Social Order

In this subsection, we exemplify the intuition of Proposition 5.1 by the most elementary form of perturbation: $f(c) = \delta(c - c')$, where $c' \in [\underline{c}, \bar{c}]$ and $\delta(\cdot)$ is the Dirac function. That is, this perturbation uniformly scales down the density of the original uniform distribution by Δ and puts all the reduced mass Δ onto a single point c'. Any other form of perturbation can be approximated by combining such elementary ones. Note that this perturbation directly represents an inflow of players with type c', and thus the interpretation here directly illustrates the effects of migration on the scope of social order. In the next subsection, we will use a combination of such perturbations to study the impact of a change in social inequality.

Under this perturbation, equation (7) becomes

$$
\begin{aligned}
\kappa' - \kappa &= \frac{2\left(\bar{c}-\underline{c}\right)}{r} A\left(c'\right) \cdot \Delta + o\left(\Delta\right), & (8)\\
\text{where } A\left(c'\right) &= 1_{c'>\kappa} \frac{r}{\bar{c}-\underline{c}} \left(\frac{\kappa+\bar{c}}{2} - c'\right) \\
&\quad + \left(\bar{c}-\underline{c}\right)\left(\frac{1_{c'\le\kappa}}{2} - \frac{1_{c'=\frac{\underline{c}+\kappa}{2}}}{2} - 1_{c'<\frac{\underline{c}+\kappa}{2}}\right) + \frac{r}{2}\left(\lambda_{\max} - 1_{c'\le\kappa}\right) \\
&= \begin{cases} -\left(\bar{c}-\underline{c}\right), \text{ if } c' < \frac{\underline{c}+\kappa}{2} \\ -\left(\bar{c}-\underline{c}\right)/2, \text{ if } c' = \frac{\underline{c}+\kappa}{2} \\ 0, \text{ if } \frac{\underline{c}+\kappa}{2} < c' \le \kappa \\ \frac{r}{2} - \left(\bar{c}-\underline{c}\right) + r\frac{\bar{c}-c'}{\bar{c}-\underline{c}}, \text{ if } c' > \kappa \end{cases}. & (9)
\end{aligned}
$$

If c' is below the original median voter ($c' < \frac{\underline{c}+\kappa}{2}$), the outside-boundary effect $1_{c'>\kappa}\frac{r}{\bar{c}-\underline{c}}\left(\frac{\kappa+\bar{c}}{2} - c'\right) = 0$. The inside-boundary effect, $\left(\bar{c}-\underline{c}\right)\left(\frac{1_{c'\le\kappa}}{2} - 1_{c'<\frac{\underline{c}+\kappa}{2}}\right) = -\frac{\bar{c}-\underline{c}}{2} < 0$ as the cost of the median voter falls. The across-boundary effect $\frac{r}{2}\left(\lambda_{\max} - 1_{c'\le\kappa}\right) = -\frac{\bar{c}-\underline{c}}{2} < 0$ as mass is moved inside the boundary. So the overall effect is constantly $-\left(\bar{c}-\underline{c}\right) < 0$ in this range;

If c' is exactly at the original median voter ($c' = \frac{\underline{c}+\kappa}{2}$), the outside-boundary effect and the across-boundary effect are the same as the previous case, but the inside-boundary effect is zero since type $\frac{\underline{c}+\kappa}{2}$ is still the median voter. So the overall effect is $-\left(\bar{c}-\underline{c}\right)/2$.

If c' is between the original median voter and the upper boundary of the organization ($\frac{\underline{c}+\kappa}{2} < c' \le \kappa$), the outside-boundary effect is zero, the inside-boundary effect equals $\frac{\bar{c}-\underline{c}}{2} > 0$ as the cost of the median voter rises. The across-boundary effect still equals $-\frac{\bar{c}-\underline{c}}{2}$. So the overall effect is constantly 0 in this range;

If c' is outside the original boundary of the organization ($c' > \kappa$), the inside-boundary effect is zero. The outside-boundary effect, $\frac{r}{\bar{c}-\underline{c}}\left(\frac{\kappa+\bar{c}}{2} - c'\right)$, decreases linearly in c', and is positive (negative) when c' is less (greater) than $\frac{\kappa+\bar{c}}{2}$, the average cost of the original unorganized players. The across-boundary effect, $\frac{r}{2}\left(\lambda_{\max} - 1_{c'\le\kappa}\right) = \frac{r}{2}\lambda_{\max} = \frac{r}{2} - \frac{\bar{c}-\underline{c}}{2} > 0$, as mass is moved out of the original boundary. As explained before, this effect is independent of c', and is magnified by strategic complementarity r. The overall effect in this case, $\frac{r}{2} - \left(\bar{c}-\underline{c}\right) + r\frac{\bar{c}-c'}{\bar{c}-\underline{c}}$, equals $r/2 > 0$ as $c' \to c_{\lambda_{\max}}$, decreases linearly in c', and equals $\frac{r}{2} - \left(\bar{c}-\underline{c}\right)$ when $c' = \bar{c}$, which could be positive or negative, depending on the strength of strategic complementarity r. This generates the non-monotonicity result elaborated below.

Figures 9 and 10 plot $A\left(c'\right)$ against c', indicating how the scope of social order is affected

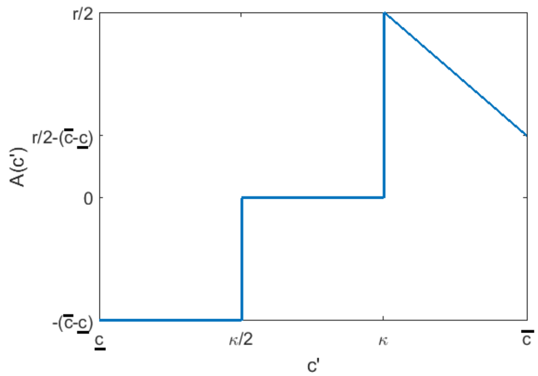

Figure 9: $A(c')$ with large r

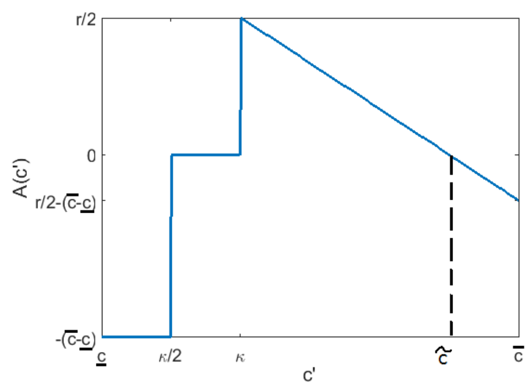

Figure 10: $A(c')$ with small r

by the type c' of the migrants. Figure 9 presents the case of $r > 2(\bar{c} - \underline{c})$. If the type c' of the migrants is below the median voter $\kappa/2$ of the original organization, the inflow of them shrinks the scope of social order, and the marginal magnitude is a constant $-(\bar{c} - \underline{c})$; if c' is above the median voter $\kappa/2$ but is still within the original boundary κ, the inflow of them has no impact of the existing social order; if c' is outside the original boundary κ, the inflow of them expands the scope of social order, but the marginal magnitude decreases linearly with c'. Hence, low cost migrants shrink the scope of social order, median cost ones do not affect it, while high cost ones expand it.

Figure 10 presents the more interesting case of $\bar{c} - \underline{c} < r < 2(\bar{c} - \underline{c})$. Now if c' is outside the original boundary κ, there is a $\tilde{c} \in (\kappa, \bar{c})$ such that $A(c') < 0$ if $c' > \tilde{c}$. This makes the effect of migrants non-monotonic in their types: Both low ($c' \leq \kappa/2$) and extremely high cost ($c' > \tilde{c}$) migrants can shrink the scope of social order, while the rest types in between either having no impact ($\kappa/2 < c' \leq \kappa$) or expanding the scope ($\kappa < c' < \tilde{c}$).

5.2 Effects of Social Inequality on the Scope of Social Order

With the intuition developed in the previous subsection, this subsection shows that an increase/decrease of social inequality can also have a non-monotonic impact on the scope of social order. We illustrate this point with the symmetric mean-preserving spread in Figures 11, 12 and 13. Mass 2Δ is equally spread from c to c' and c'', where c lies in between the median and the 75 percentile of the original organization, and $c'' - c = c - c' = d > 0$. This can be decomposed into two inflows of type c' and c" of mass Δ respectively, and an outflow of type c of mass 2Δ. Assume that $r > 2(\bar{c} - \underline{c})$ as in Figure 9. Further assume that $r < 4(\bar{c} - \underline{c})$.

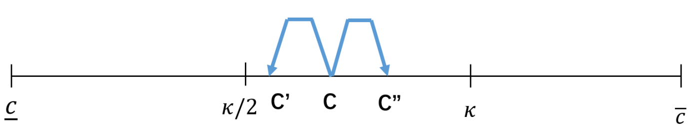

Figure 11: small mean-preserving spread

When d is small as in Figure 11, all inflows and outflows happen within the range of $(\kappa/2, \kappa)$. From Figure 9 we know that the impact of each flow and thus the whole mean-preserving spread on the organization boundary is zero.

When d is medium as in Figure 12 so that only c' falls outside $(\kappa/2, \kappa)$, from Figure 9

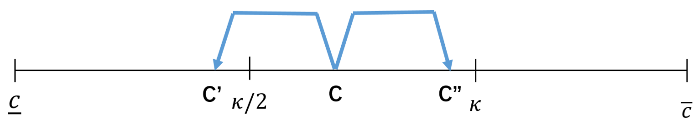

Figure 12: median mean-preserving spread

we know that the impact of the inflow of type c' is $-(\bar{c}-\underline{c})$, and that of the other flows are zero. So the scope of social order is smaller than its original level.

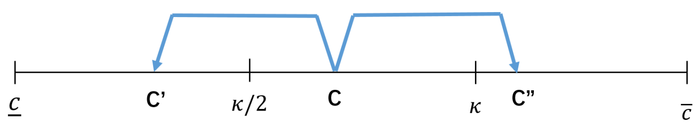

Figure 13: large mean-preserving spread

When d increases further so that c'' just falls outside $(\kappa/2, \kappa)$, from Figure 9 we know that the impact of the inflow of type c' is still $-(\bar{c}-\underline{c})$, that of the inflow of type c'' is slightly below $r/2$, and that of the outflows are zero. By our assumption that $r > 2(\bar{c}-\underline{c})$, the total impact of the mean-preserving spread is positive, and thus the scope of social order is larger than its original level. And as d increases further so that c'' moves further away from the original boundary κ, the impact of the inflow there, $\frac{r}{2}-(\bar{c}-\underline{c})+r\frac{\bar{c}-c'}{\bar{c}-\underline{c}}$, eventually falls below $\bar{c}-\underline{c}$, the absolute value of the negative impact of inflow of type c', due to the assumption that $r < 4(\bar{c}-\underline{c})$. This makes the impact of the whole mean-preserving spread negative again. Therefore, as d increases from zero, the scope of social order first stays put, then shrinks, then expands, and shrinks again at last.

6 Discussion

6.1 Multiple Organizations

To hightlight the main tension of the model, in Section 3 we focus on equilibria with at most one organization. What if we allow for multiple organizations? This subsection tackles this issue by discussing equilibria with two organizations. Three main results are presented: First, in any voluntary organizational equilibrium, the two organizations look as if it is a single one as fundamental uncertainty σ vanishes: both organizations choose the same strategy, and their union includes all players in a continuous interval with lower bound $\underline{c}$.

Second, there is an upper limit of the total mass of the two organizations as fundamental uncertainty σ vanishes. Third, every player assymptotically takes the same action as fundamental uncertainty σ vanishes. In this sense, this subsection turns out to be a robustness check of Section 3.4.

We first need to modify the Definition 3.2 of a voluntary organizational equilibrium to accommodate multiple organizations. Now there are three alternatives for each player: to join organization 1, to join organization 2, or to join neither. The modified equilibrium definition requires the choice among them to be incentive compatible.

Definition 6.1 *A voluntary multi-organizational equilibrium consists of*

1) two organizations defined by measures $\{\mu_n\}$*,* $n = 1, 2$*;*

2) a symmetric strategy $a : (x, c) \longmapsto \{0, 1\}$ *for players not in the organization; and*

3) a strategy b_i *for each organization* μ_i*, where* $b_i : \theta \mapsto \{0, 1\}$*,*

such that:

a) given $\{\mu_i\}$ *and* $\{b_i\}$*, for all* $x \in \mathbb{R}$ *and* $c \in [\underline{c}, \bar{c}]$*,*

$$a(x,c) \in \arg\max_{a'} a' \cdot \left\{ \mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right) |x\right] - c \right\} ,$$

where

$$l_{1-\lambda}(\theta) = \int_{\underline{c}}^{\bar{c}} \mathbf{E}\left[a(x,c)|\theta\right] \left(\frac{dc}{\bar{c} - \underline{c}} - \sum_i \mu_i(dc) \right)$$

and

$$\lambda_i = \int_{\underline{c}}^{\bar{c}} \mu_i(dc) > 0 \ ;$$

b) given a *and* μ*,* $\forall i = 1, 2$*, for all* $\theta \in \left[\underline{\theta}, \bar{\theta}\right]$*,*

$$b_i(\theta) \in \arg\max_{a' \in \{0,1\}} a' \cdot \left[u\left(\theta, \lambda_i + \lambda_{-i}(\theta) + l_{1-\lambda}(\theta)\right) - c_{med,i}(\mu_i)\right] ,$$

where $c_{med}(\mu_i)$ *is such that*

$$\frac{\int_{c_{med}(\mu_i)}^{\bar{c}} \mu_i(dc)}{\lambda_i} = \frac{1}{2} \ ;$$

c) for players outside the organization, $\forall i = 1, 2$*,*

$$\begin{aligned} &\mathbf{E}\left\{a(x,c)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \\ \geq\ &\mathbf{E}\left\{b_i(\theta)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \end{aligned}$$

and,

d) for players in organization i*,*

$$\begin{aligned} &\mathbf{E}\left\{b_i(\theta)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \\ \geq\ &\mathbf{E}\left\{a(x,c)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \end{aligned}$$

and

$$\begin{aligned} &\mathbf{E}\left\{b_i(\theta)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \\ \geq\ &\mathbf{E}\left\{b_{-i}(\theta)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \end{aligned}$$

In this subsection, we restrict our attention to the case $r \geq \bar{c} - \underline{c}$, as we have already established that no organization is sustainable (the same reasoning in Proposition 3.5 holds) otherwise. Also, as in Sections 2.2 and 3, we focus on symmetric equilibria of switching strategies, which is shown to be without loss of generality when fundamental uncertainty vanishes.

The first result of this subsection is that, to an outsider, regardless of the composition of each organization, both of them always take the same action *as if* they belong to the same organization, and the union of them is a continuous interval with lower bound $\underline{c}$, as illustrated by Figure 14.

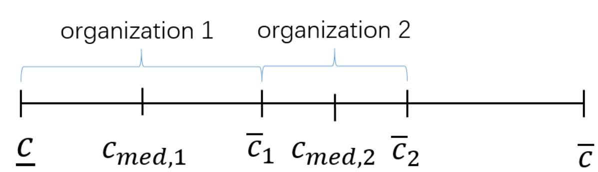

Figure 14: voluntary multi-organizational equilibrium

Proposition 6.1 *If* $r \geq \bar{c} - \underline{c}$*, in any voluntary multi-organizational equilibrium, the two organizations consist of all players in* $[\underline{c}, \bar{c}_1)$ *and* $[\bar{c}_1, \bar{c}_2]$ *respectively for some* $\bar{c}_2 > \bar{c}_1$*, and* $b_2 - b_1 \to 0$ *as* $\sigma \to 0$.

Note that type $\bar{c}_1$ is indifferent between joining either organization and type $\bar{c}_2$ may be indifferent between joining an organization and staying unorganized. Since these two types are both of mass zero, for notational convenience we assign both types to the high-cost organization.

To make sense of this, first, an argument similar to Lemma 3.1 establishes that if $c_1 < c_2$ are both in the same organization i, as $\sigma \to 0$, $\forall c \in (c_1, c_2)$ cannot stay unorganized. Second, observe that organizations choosing different strategies never overlap. To see this, suppose that two organizations are choosing strategies $\hat{\theta}_2 > \hat{\theta}_1$ respectively, and that type c is indifferent between the two. Then type $c' > c$ must strictly prefer the more conservative strategy $\hat{\theta}_2$, and the other way round for $c' < c$.

Given this, denote these organizations by $[\underline{c}_i, \bar{c}_i]$, $i = 1, 2$, with $\underline{c}_2 \geq \bar{c}_1$.[10] Following the notation used in Section 3, denote their respective median voters by $c_{med,i} \equiv \underline{c}_i + \frac{1}{2}(\bar{c}_i - \underline{c}_i)$, their respective mass by λ_i, and their respective strategy cutoffs by $\hat{\theta}_i$. $c_{med,2} > c_{med,1}$ implies that $\hat{\theta}_2 \geq \hat{\theta}_1$.

Following the same logic in Proposition 3.2, the ideal strategy of a player with $c > c_{med,i} - r\lambda_i$ coincides perfectly with that of organization $[\underline{c}_i, \bar{c}_i]$. Note that $r\lambda_i = r\frac{1}{\bar{c}-\underline{c}}\frac{c_{med,i}-\underline{c}_i}{1/2} > 2(c_{med,i} - \underline{c}_i)$, as $r > \bar{c} - \underline{c}$. This implies that players with $\underline{c}_i - (c_{med,i} - \underline{c}_i) < c < c_i$ right below the lower boundary $\underline{c}_i$ of organization $[\underline{c}_i, \bar{c}_i]$ would strictly prefer to join it, if they are outside any organization or is in an organization choosing a different strategy. Iteration of this argument on $\underline{c}_i$ yields three conclusions. First, $[\underline{c}_1, \bar{c}_1]$ must extend down to type $\underline{c}$. Second, there cannot be any player in the range $[\bar{c}_1, \underline{c}_2]$ that are unorganized, as they would be absorbed by organization $[\underline{c}_2, \bar{c}_2]$. So $\bar{c}_1 = \underline{c}_2$, that is, organizations are connected back to back. Third, we must have $\hat{\theta}_2 = \hat{\theta}_1$, otherwise organization $[\underline{c}_1, \bar{c}_1]$ would lose all its members to $[\underline{c}_2, \bar{c}_2]$. This proves the proposition.

Given that the two organizations the two organizations look as if it is a single one as fundamental uncertainty σ vanishes, it is natural to expect that, as in Proposition 3.5, there is an upper limit for the total mass of the two organizations as fundamental uncertainty

[10]Recall from Section 3.4 that although the end point types may be indifferent between joining an organization or not, and between joining two different organizations, since their total mass is zero, their membership does not affect our analysis. And thus for notational convenience, we use closed intervals to denote organizations.

vanishes, and all players assymptotically take the same action. The intuition is analogous to that of Proposition 3.5.

Proposition 6.2 *Suppose $r \geq \bar{c} - \underline{c}$. For any voluntary multi-organizational equilibrium with fundamental uncertainty $\sigma > 0$, let λ^σ denote the total mass of players in both organizations. Then $\lambda^\sigma \leq \frac{4r}{(\bar{c}-\underline{c})^2}\left[r - (\bar{c} - \underline{c})\right]$ when $\sigma > 0$ is sufficiently small. Moreover, in any voluntary multi-organizational equilibrium, all players, whether or not in an organization, assymptotically take the same action as $\sigma \to 0$.*

6.2 The Regime-Change Game

So far we have been using the investment game as the basis for discussion. In addition to its popularity in the literature per se, this game helps highlight the roles of strategic complementarity (parameterized by r) and preference heterogeneity (as parameterized by $\bar{c} - \underline{c}$) in coordination games with incomplete information. This subsection shows how the main insight of the investment game can help us understand another popular coordination game, the regime change game.

In the regime change game, the benefit of investment, $u(\theta, l) = 1_{\{l \geq 1-\theta\}}$. that is, players care about whether "the regime is changed" (i.e. the event $l \geq 1 - \theta$) in a discontinuous manner. As in the investment game, we assume that the support of players' prior distribution h is a strict superset of $[0, 1]$ to make sure it covers both dominance regions.[11] Regardless of θ, players with $c < 0$ or $c > 1$ have dominant strategy $a = 1$ and $a = 0$ respectively. So we assume in addition that $[\underline{c}, \bar{c}] \subset [0, 1]$ to rule out trivial cases. Since $\frac{\partial u}{\partial l}$ is the Dirac delta function centering at $1 - \theta$, the regime change game is similar to a coordination game with infinite strategic complementarity. The rest of this subsection confirms that all the results of the regime change game indeed resemble those of the investment game with infinite strategic complementarity r.

First, this is case for the complete information benchmark, as stated in Proposition 6.3. As the investment game with large r, the regime-change game has multiple Pareto-ordered Nash equilibria only for intermediate values of θ. Otherwise, the game has a unique Nash equilibrium, which is efficient in the sense of replicating first-best benchmark.

Proposition 6.3 *Of the regime-change game with complete information,*

[11] If $\theta > 1$, the regime changes even if no player invests, making investing a dominant strategy for every player; If $\theta < 0$, the regime does not change even if all players invest, making not investing a dominant strategy for every player.

i) If $\theta \geq 1$*, the unique equilibrium is* $a(\theta, c) = 1$ $\forall c \in [\underline{c}, \bar{c}]$*, which coincides with the first-best benchmark;*

ii) if $\theta \in [0, 1)$*, there are two equilibria: 1)* $a(\theta, c) = 1$ $\forall c \in [\underline{c}, \bar{c}]$*, and 2)* $a(\theta, c) = 0$ $\forall c \in [\underline{c}, \bar{c}]$*. 1) coincides with the first-best benchmark;*

iii) If $\theta < 0$*, the unique equilibrium is* $a(\theta, c) = 0$ $\forall c \in [\underline{c}, \bar{c}]$*, which coincides with the first-best benchmark.*

Second, this is also the case for the unorganized game with incomplete information, as formally stated in Proposition 6.4 Players with different costs choose the same cutoff $\mathbf{E}c$ assymptotically when fundamental uncertainty vanishes, which is inefficient.

Proposition 6.4 *Let* $\widehat{x}^{\sigma} : [\underline{c}, \bar{c}] \to \mathbb{R}$ *characterize an equilibrium of the regime change game given fundamental uncertainty* σ*. For any* $\delta > 0$*, there exists a* $\overline{\sigma} > 0$ *such that for all* $\sigma \in (0, \overline{\sigma})$*,*

$$\widehat{x}^{\sigma}(c) \in (\mathbf{E}c - \delta, \mathbf{E}c + \delta)$$

for all $c \in [\underline{c}, \bar{c}]$*, where* $\mathbf{E}c = \frac{\bar{c}+\underline{c}}{2}$*.*

Lastly, Proposition 6.5 states that in the regime-change game, the organization consistimg of all players is uniquely sustainable, and it induces socially efficient outcomes. This resembles the corresponding result of the investment game with $r = \infty$: in Proposition 3.5, the mass limit of a organization, $\lim_{\sigma \to 0} \lambda^{\sigma} = \frac{r-(\bar{c}-\underline{c})}{r} \to 1$ as $r \to \infty$.

Proposition 6.5 *There exists a unique voluntary organizational equilibrium in which all players choose to form an organization. In particular, the organization's equilibrium strategy is* $a_{org} = 1_{\theta \geq 0}$*. This equilibrium coincides with the first-best benchmark.*

Note that, contrary to the investment game with $r \leq \bar{c} - \underline{c}$, there is no asymmetry of incentive to join the organization between the continuum of players whose c_i are below c_{med} and above c_{med}. The primitive that drives this result is the strong strategic complementarity in this game, in the sense that the benefit of investment $u(\theta, l)$ does not change at all except at the kink $l = 1 - \theta$. Essentially, regardless of his cost of investment, everyone only cares about whether the regime changes.

7 Conclusion

Situations in which diverse individuals with incomplete information must coordinate their actions are ubiquitous. Organizations obligate individuals to take collective actions. On one hand they mitigate strategic uncertainty and facilitate coordination. On the other hand they may compel an individual to take the collective action not in his favor. In light of this tradeoff, this paper finds out that when the strategic complementarity among individuals are weak relative to the heterogeity of their preferences, organizations could be desirable, but are not sustainable; Otherwise, organizations are desirable, but there is still an upper limit for the size of sustainable organizations.

References

Acemoglu, D., G. Egorov, and K. Sonin (2008). Coalition formation in non-democracies. *The Review of Economic Studies 75*(4), 987–1009.

Acemoglu, D., G. Egorov, and K. Sonin (2012, June). Dynamics and stability of constitutions, coalitions, and clubs. *American Economic Review 102*(4), 1446–76.

Alesina, A. and E. Spolaore (1997). On the number and size of nations. *The Quarterly Journal of Economics 112*(4), 1027–1056.

Alonso, R., W. Dessein, and N. Matouschek (2008, March). When does coordination require centralization? *American Economic Review 98*(1), 145–79.

Angeletos, G.-M. and A. Pavan (2004, May). Transparency of information and coordination in economies with investment complementarities. *American Economic Review 94*(2), 91–98.

Brunnermeier, M. K., P. Bolton, and L. Veldkamp (2013). Leadership, coordination and corporate culture. *Review of Economic Studies 80*(2), 512–537.

Carlsson, H. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica 61*(5), 989–1018.

Corsetti, G., A. Dasgupta, S. Morris, and H. S. Shin (2004). Does one soros make a difference? a theory of currency crises with large and small traders. *The Review of Economic Studies 71*(1), 87–113.

Dessein, W., A. Galeotti, and T. Santos (2016, June). Rational inattention and organizational focus. *American Economic Review 106*(6), 1522–36.

Dessein, W. and T. Santos (2006). Adaptive organizations. *Journal of Political Economy 114*(5), 956–995.

Drozd, L. and R. Serrano-Padial (2017). Credit enforcement cycles. Working Papers 17-27, Federal Reserve Bank of Philadelphia.

Feddersen, T. and W. Pesendorfer (1997). Voting behavior and information aggregation in elections with private information. *Econometrica 65*(5), 1029–1058.

Frankel, D. M., S. Morris, and A. Pauzner (2003). Equilibrium selection in global games with strategic complementarities. *Journal of Economic Theory 108*(1), 1 – 44.

Gibbons, R. (2005). Four forma(lizable) theories of the firm? *Journal of Economic Behavior and Organization 58*(2), 200–245.

Gibbons, R. and J. Roberts (2012). *The Handbook of Organizational Economics*. Princeton University Press.

Grossman, S. J. and O. D. Hart (1986). The costs and benefits of ownership: A theory of vertical and lateral integration. *Journal of Political Economy 94*(4), 691–719.

Guimaraes, B. and S. Morris (2007). Risk and wealth in a model of self-fulfilling currency attacks. *Journal of Monetary Economics 54*(8), 2205 – 2230.

Hart, O. and J. Moore (1990). Property rights and the nature of the firm. *Journal of Political Economy 98*(6), 1119–1158.

Holmström, B. and P. Milgrom (1994). The firm as an incentive system. *American Economic Review 84*(4), 972–91.

Holmström, B. and J. Tirole (1991). Transfer pricing and organizational form. *Journal of Law, Economics and Organization 7*(2), 201–28.

Leister, M., Y. Zenou, and J. Zhou (2017). Coordination on networks. CEPR Discussion Papers 12392, C.E.P.R. Discussion Papers.

Meltzer, A. H. and S. F. Richard (1981). A rational theory of the size of government. *Journal of Political Economy 89*(5), 914–927.

Morris, S. and H. Shin (2003). Global games: Theory and applications. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Economet-*

rics: Proceedins of the Eight World Congress of the Econometric Society, pp. 56–114. Cambridge: Cambridge University Press.

Myerson, R. B. (1998). Population uncertainty and poisson games. *International Journal of Game Theory 27*(3), 375–392.

Rantakari, H. (2008). Governing adaptation1. *The Review of Economic Studies 75*(4), 1257–1285.

Ray, D. and R. Vohra (2015). Coalition formation. In *Handbook of Game Theory with Economic Applications*, Volume 4, Chapter 5, pp. 239–326. Elsevier.

Romer, T. (1975). Individual welfare, majority voting, and the properties of a linear income tax. *Journal of Public Economics 4*(2), 163 – 185.

Scotchmer, S. (2008). clubs. In S. N. Durlauf and L. E. Blume (Eds.), *The New Palgrave Dictionary of Economics*. Basingstoke: Palgrave Macmillan.

Sákovics, J. and J. Steiner (2012, December). Who matters in coordination problems? *American Economic Review 102*(7), 3439–61.

van Zandt, T. and X. Vives (2007). Monotone equilibria in bayesian games of strategic complementarities. *Journal of Economic Theory 134*(1), 339–360.

8 Appendix

8.1 Proof for Propositions in Section 2.1

As noted in the text, in any equilibrium, because the benefit of investment $u(\theta, l)$ is independent of an individual player's action, if a player with investment cost c is willing to invest, a player with investment cost $c' \leq c$ must also be willing to invest. Hence an equilibrium can be characterized by a cutoff function $\hat{c}(\theta)$ such that $a(\theta, c) = 1$ if $c \leq \hat{c}(\theta)$, and $a(\theta, c) = 0$ otherwise. This implies that $l = \frac{c(\theta)-\underline{c}}{\bar{c}-\underline{c}}$. So the benefit of investment $u(\theta, l) = u\left(\theta, \frac{c(\theta)-\underline{c}}{\bar{c}-\underline{c}}\right)$. Lemmas 8.1 and 8.2 characterize $\hat{c}(\theta)$ when $r \geq \bar{c} - \underline{c}$ and $r < \bar{c} - \underline{c}$ respectively. Proposition 2.1 is then straightforward from the two lemmas.

Lemma 8.1 *Of the investment game with complete information, if* $r \geq \bar{c} - \underline{c}$*,*

i) if $\theta < \theta_1$*, then the unique equilibrium is* $a(\theta, c) = 0$ $\forall c$*;*

ii) if $\theta \in [\theta_1, \theta_0]$*, then there are three symmetric equilibria: 1)* $a(\theta, c) = 1$ $\forall c$ *; 2)* $a(\theta, c) = 0$ $\forall c$*; and 3)* $a(\theta, c) = 1$ *if and only if* $c \leq \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r}$*; in addition, 1) yields higher* *social welfare than 2) and 3).*

iii) if $\theta > \theta_0$, then the unique equilibrium is $a(\theta, c) = 1\ \forall c$.

Proof. Given that $r \geq \bar{c} - \underline{c}$,

$$\theta_1 = \bar{c} < \underline{c} + r = \theta_0$$

and

$$u\left(\theta, \frac{c - \underline{c}}{\bar{c} - \underline{c}}\right) - c = \theta - r\frac{\bar{c}}{\bar{c} - \underline{c}} + \left(\frac{r}{\bar{c} - \underline{c}} - 1\right) c$$

is now strictly increasing in c..

If $\theta < \theta_1 = \bar{c}$, it is easy to see that $u\left(\theta, \frac{\underline{c}-\underline{c}}{\bar{c}-\underline{c}}\right) < c\ \forall c$. First, this holds in particular for $c = \underline{c}$, which means that given nobody investing, even player with the lowest cost of investment prefers not to invest. Nor does everyone else, which confirms that $l = \frac{\underline{c}-\underline{c}}{\bar{c}-\underline{c}} = 0$. This verifies the equilibrium. Second, the fact that $u\left(\theta, \frac{c-\underline{c}}{\bar{c}-\underline{c}}\right) < c\ \forall c$ implies that for any cutoff $c > \underline{c}$, a positive mass of players whose costs of investment are below c and above $u\left(\theta, \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}\right) < c$ strictly prefer not to invest. This verifies the uniqueness of the equilibrium. A symmetric argument applies to the case $\theta > \theta_0$.

If $\theta \in [\theta_1, \theta_0]$, we have $u(\theta, 0) < \underline{c}$, which verifies the equilibrium characterized by cutoff $\hat{c} = \underline{c}$. Similarly, we have $u(\theta, 1) > \bar{c}$, which verifies the equilibrium characterized by cutoff $\hat{c} = \bar{c}$. When $\hat{c} = \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r}$, we have $u\left(\theta, \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}\right) = \hat{c}$. Given that players follow that cutoff, the benefit of investment

$$u(\theta, l) = u\left(\theta, \frac{\hat{c}(\theta) - \underline{c}}{\bar{c} - \underline{c}}\right) = \hat{c}(\theta) \geq c$$

if and only if $c \leq \hat{c}(\theta)$, so it is indeed optimal for each player to follow the cutoff $\hat{c}(\theta)$. And this confirms that $l = \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}$. So the equilibrium characterized by cutoff $\hat{c} = \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r}$ is thus verified.

Conditional on θ, social welfare of 1) is $\theta - \mathbf{E}c > \theta - \bar{c} = \theta - \theta_1 \geq 0$, that of 2) is 0, and that of 3) is

$$\int_{\underline{c}}^{\hat{c}(\theta)} [\theta - r(1 - l) - c] \frac{dc}{\bar{c} - \underline{c}}$$
$$\leq \int_{\underline{c}}^{\bar{c}} [\theta - c] \frac{dc}{\bar{c} - \underline{c}} = \theta - \mathbf{E}c,$$

where the inequality is due to $\theta - c \geq \theta - \bar{c} = \theta - \theta_1 \geq 0 \; \forall c \in [\hat{c}(\theta), \bar{c}]$.

Note that $u\left(\theta, \frac{c-\underline{c}}{\bar{c}-\underline{c}}\right) - c$ is strictly increasing in c and is negative (positive) if and only if $c < \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r}$ $(c > \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r})$. This implies that for any cutoff $c < \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r}$ $(c > \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r})$, a positive mass of players whose costs of investment are below (above) c and above (below) $u\left(\theta, \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}\right) < c$ $(u\left(\theta, \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}\right) > c)$ strictly prefer not to invest (to invest). This verifies the results concerning equilibrium uniqueness and efficiency when $\theta \in [\theta_1, \theta_0]$. ■

Lemma 8.2 *If $0 < r < \bar{c} - \underline{c}$, then there is a unique symmetric equilibrium: $a(\theta, c) = 1$ if $c \leq \hat{c}(\theta)$, and $a(\theta, c) = 0$ otherwise, where*

$$\hat{c}(\theta) = \begin{cases} \bar{c}, \text{ if } \theta \geq \theta_1 \\ \frac{(\bar{c}-\underline{c})\theta - r\bar{c}}{\bar{c}-\underline{c}-r}, \text{ if } \theta_0 < \theta < \theta_1 \\ \underline{c}, \text{ if } \theta \leq \theta_0 \end{cases} . \tag{10}$$

Proof. Given that $r < \bar{c} - \underline{c}$,

$$\theta_1 = \bar{c} > \underline{c} + r = \theta_0$$

and

$$u\left(\theta, \frac{c-\underline{c}}{\bar{c}-\underline{c}}\right) - c = \theta - r\frac{\bar{c}}{\bar{c}-\underline{c}} + \left(\frac{r}{\bar{c}-\underline{c}} - 1\right)c \tag{11}$$

is strictly decreasing in c. If $\theta \geq \theta_1 = \bar{c}$, it is easy to see that $u\left(\theta, \frac{c-\underline{c}}{\bar{c}-\underline{c}}\right) \geq c \; \forall c$. First, this holds in particular for $c = \bar{c}$, which means that given everyone investing, even a player with the highest cost of investment prefers to invest. So does everyone else, which confirms that $l = \frac{c-\underline{c}}{\bar{c}-\underline{c}} = 1$. This verifies the equilibrium. Second, the fact that $u\left(\theta, \frac{c-\underline{c}}{\bar{c}-\underline{c}}\right) > c \; \forall c < \bar{c}$. implies that for any cutoff $c < \bar{c}$, a positive mass of players whose costs of investment are above c and below $u\left(\theta, \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}\right) > c$ strictly prefer to invest. This verifies the uniqueness of the equilibrium. A symmetric argument applies to the case $\theta < \theta_0$.

If $\theta \in [\theta_0, \theta_1)$, the candidate $\hat{c}(\theta)$ defined in (10) is such that $u\left(\theta, \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}\right) = \hat{c}$. Given that players follow that cutoff, the benefit of investment

$$u(\theta, l) = u\left(\theta, \frac{\hat{c}(\theta) - \underline{c}}{\bar{c}-\underline{c}}\right) = \hat{c}(\theta) \geq c$$

if and only if $c \leq \hat{c}(\theta)$, so it is indeed optimal for each player to follow the cutoff $\hat{c}(\theta)$. And this confirms that $l = \frac{\hat{c}-\underline{c}}{\bar{c}-\underline{c}}$. So the equilibrium is verified.

Since $u\left(\theta, \frac{c-\underline{c}}{\bar{c}-\underline{c}}\right) - c$ strictly decreases in c, if players follow a different cutoff $\hat{c}' < \hat{c}(\theta)$, the benefit of investment $u\left(\theta, \frac{\hat{c}'-\underline{c}}{\bar{c}-\underline{c}}\right) > \hat{c}'$, a positive mass of players whose costs of investment are above $\hat{c}'$ and below $\hat{c}" = u\left(\theta, \frac{\hat{c}'-\underline{c}}{\bar{c}-\underline{c}}\right)$ strictly prefer to invest. A symmetric argument applies to the case $\hat{c}' > \hat{c}(\theta)$, where players whose costs in $(\hat{c}", \hat{c}')$ would not and should not invest. These together verify the uniqueness of the equilibrium. ■

Proof. Proof of Proposition 2.1 is straightforward from Lemmas 8.1 and 8.2. ■

8.2 Proofs of Propositions in Section 2.2

Lemma 8.3 *There exists an greatest and a least equilibria of the unoganized game of any level of fundamental uncertainty $\sigma > 0$, both in switching strategies, with switching point weakly increasing in c in each of them.*

Proof. It is straightforward to see that the unorganized game in Section 2.2 is a supermodular game. Hence, the result follows from an iterated deletion of dominated strategy as in (van Zandt and Vives 2007). ■

Lemma 8.4 *For any sequence of switching strategies $\{\hat{x}^\sigma : [\underline{c}, \bar{c}] \to \mathbb{R}\}$, where $\hat{x}^\sigma(c)$ is the switching point of type c in an equilibrium with fundamental uncertainty σ. Then $\hat{x}^0(\cdot) \equiv \lim \hat{x}^\sigma(\cdot)$ exists.*

Define an equivalence relation $\sim$ on $[\underline{c}, \bar{c}]$: $c \sim c'$ iff $\hat{x}^0(c) = \hat{x}^0(c')$. Then $\forall \varepsilon > 0$, $\exists \bar{\sigma} > 0$ and $\exists$ some bounded $L(\varepsilon) \geq 0$ such that $\forall \sigma < \bar{\sigma}$:

If $\{c\} \in [\underline{c}, \bar{c}] / \sim$, then $\left|\hat{x}^0(c) - \hat{x}(c)\right| \leq L\varepsilon$, where $\hat{x}(c) = c + r\frac{\bar{c}-c}{\bar{c}-\underline{c}}$;

If $[c_1, c_2] \in [\underline{c}, \bar{c}] / \sim$, $\forall c \in [c_1, c_2]$, $\left|\hat{x}^0(c) - \hat{x}(c)\right| \leq L\varepsilon$, where $\hat{x}(c) = \left(\frac{c_1+c_2}{2} + r\frac{\bar{c}-\frac{c_1+c_2}{2}}{\bar{c}-\underline{c}}\right)$.

Proof. The assumption that the support of the prior distribution $h(\cdot)$ covers both dominance regions guarantees that the sequence $\{\hat{x}^\sigma\}$ is bounded. Hence, for any of its subsequence, there is a convergent subsequence. We will show that the limit of any convergent subsequence of $\{\hat{x}^\sigma\}$ must be the one in the statement of Lemma 8.5. This proves the existence of the limit $\hat{x}^0(\cdot)$.

Given that $[c_1, c_2] \in [\underline{c}, \bar{c}] / \sim$ and $c \in [c_1, c_2] \cap \Omega_0$, we characterize $\hat{x}^0(c_1)$. $\{c\} \in [\underline{c}, \bar{c}] / \sim$ is its special case in which $c_1 = c_2 = c$. For notational simplicity, let constant $x^0 = \hat{x}^0(c_1)$.

When fundamental uncertainty is σ, the effective strategy of a player with investment cost c, defined as his probability of investment as a function of fundamental is θ, is

$$m_c^\sigma(\theta) = \Pr\left[x_i \geq \hat{x}^\sigma(c)\,|\theta\right] = 1 - G\left(\frac{\hat{x}^\sigma(c) - \theta}{\sigma}\right).$$

For any subsequence of $\{\hat{x}^\sigma\}$ that converges to some $\hat{x}^0$, as $\sigma \to 0$, $m_c^\sigma(\theta) \to 1_{\hat{x}^0(c)}$. Since $[\underline{c}, \bar{c}]$ is compact, the convergence is uniform.

For notational convenience, let $P_- = \Pr(c < c_1) = \frac{c_1 - \underline{c}}{\bar{c} - \underline{c}}$, $P_+ = \Pr(c > c_2) = \frac{\bar{c} - c_2}{\bar{c} - \underline{c}}$, $P_= = \Pr(c \in [c_1, c_2]) = \frac{c_2 - c_1}{\bar{c} - \underline{c}}$, $m_-^\sigma(\theta) = E\left[m_c^\sigma(\theta)\,|c < c_1\right]$, $m_+^\sigma(\theta) = E\left[m_c^\sigma(\theta)\,|c > c_2\right]$, and $m_=^\sigma(\theta) = E\left[m_c^\sigma(\theta)\,|c \in [c_1, c_2]\right]$. Given that other players follow their respective equilibrium strategies, consider a player with cost $c' \in [c_1, c_2]$, if fundamental is θ, his payoff from investing is

$$\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta) - P_= m_=^\sigma(\theta)\right] - c'.$$

If he follows effective strategy $m_c^\sigma(\cdot)$, the ex ante expected payoff he gets is

$$\int_{-\infty}^{+\infty} \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta) - P_= m_=^\sigma(\theta)\right] - c'\right\} m_{c'}^\sigma(\theta)\, h(\theta)\, d\theta,$$

where h is the density of a player's prior distribution of fundamental θ.

The optimality of $m_{c_1}^\sigma$ requires that

$$\int_{-\infty}^{+\infty} \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta) - P_= m_=^\sigma(\theta)\right] - c'\right\} m_{c'}^{\sigma\prime}(\theta)\, h(\theta)\, d\theta = 0,$$

or equivalently,

$$\int_{-\infty}^{+\infty} \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta) - P_= m_=^\sigma(\theta)\right]\right\} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta = c' \int_{-\infty}^{+\infty} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta. \tag{12}$$

Because $\forall c' \in [c_1, c_2]$, $m_{c'}^\sigma(\theta) \to 1_{\{\theta \geq x^0\}}$ as $\sigma \to 0$, we have $\forall \varepsilon > 0$, $\exists \delta_1 > 0$, $\exists \sigma_1 > 0$ such that $\forall \sigma < \sigma_1$, $m_{c'}^\sigma\left(x^0 - \delta_1\right) < \varepsilon$ and $m_{c'}^\sigma\left(x^0 + \delta_1\right) > 1 - \varepsilon$. Therefore, RHS of equation (12)

$$c' \int_{-\infty}^{+\infty} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta = c'\left(\int_{-\infty}^{x^0 - \delta_1} + \int_{x^0 - \delta_1}^{x^0 + \delta_1} + \int_{x^0 + \delta_1}^{+\infty}\right) h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta.$$

$$0 \leq \int_{-\infty}^{x^0-\delta_1} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta \leq \sup_{\theta \leq x^0-\delta_1} h(\theta) \cdot \int_{-\infty}^{x^0-\delta_1} m_{c'}^{\sigma\prime}(\theta)\, d\theta < \varepsilon \sup_{\theta} h(\theta);$$

$$0 \leq \int_{x^0+\delta_1}^{+\infty} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta \leq \sup_{\theta \geq x^0+\delta_1} h(\theta) \cdot \int_{x^0+\delta_1}^{+\infty} m_{c'}^{\sigma\prime}(\theta)\, d\theta < \varepsilon \sup_{\theta} h(\theta);$$

$\sup_\theta h(\theta)$ is finite as h is bounded. Moreover, $\exists \tilde{\theta} \in \left[x^0 - \delta_1, x^0 + \delta_1\right]$ such that

$$\begin{aligned}\int_{x^0-\delta_1}^{x^0+\delta_1} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta &= h\left(\tilde{\theta}\right)\left[m_{c'}^{\sigma}\left(x^0+\delta_1\right) - m_{c'}^{\sigma}\left(x^0-\delta_1\right)\right] \\ &= h\left(x^0\right) + L_1 \varepsilon\end{aligned}$$

for some bounded $L_1(\varepsilon)$ due to the continuity of $h(\cdot)$.

Hence, $\forall \varepsilon > 0$, $\exists \delta_1 > 0$, $\exists \sigma_1 > 0$ and a bounded $L_2(\varepsilon)$ such that $\forall \sigma < \sigma_1$, $c' \int_{-\infty}^{+\infty} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta = c' h\left(x^0\right) + L_2 \varepsilon$ $\forall c' \in [c_1, c_2]$. Therefore,

$$\begin{aligned}&\mathbf{E}\left[RHS | c' \in [c_1, c_2]\right] \\ =\ &\mathbf{E}\left[c' \int_{-\infty}^{+\infty} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta | c' \in [c_1, c_2]\right] \\ =\ &\mathbf{E}\left[c | c \in [c_1, c_2]\right] h\left(x^0\right) + L_2 \varepsilon \\ =\ &\frac{c_1 + c_2}{2} h\left(x^0\right) + L_2 \varepsilon.\end{aligned}$$

Now we analyze the left hand side of equation (12). First, since $\left[1 - P_- m_-^\sigma - P_+ m_+^\sigma - P_= m_=^\sigma(\theta)\right] \in [0,1]$, $\int_{-\infty}^{+\infty} \theta h(\theta)\, d\theta$ is finite, $m_{c'}^{\sigma}\left(x^0 - \delta_1\right) < \varepsilon$ and $m_{c'}^{\sigma}\left(x^0 + \delta_1\right) > 1 - \varepsilon$, $\exists$ a bounded $L_3(\varepsilon)$ such that

$$\left(\int_{-\infty}^{x^0-\delta_1} + \int_{x^0+\delta_1}^{+\infty}\right) \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta) - P_= m_=^\sigma(\theta)\right]\right\} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta = L_3 \varepsilon.$$

Concerning $\int_{x^0-\delta_1}^{x^0+\delta_1} \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta) - P_= m_=^\sigma(\theta)\right]\right\} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta$, since $\hat{y}^0(c) < \hat{y}^0(c_1)$, $\exists \sigma_2 > 0$, such that $\forall \sigma < \sigma_2$, $\forall \theta \in \left(x^0 - \delta_1, x^0 + \delta_1\right)$, we have $m_c^\sigma(\theta) > 1 - \varepsilon$

$\forall c < c_1 - \varepsilon$, and $m_c^\sigma(\theta) < \varepsilon \ \forall c > c_2 + \varepsilon$. Therefore, we have

$$\begin{aligned} m_-^\sigma\left(\tilde{\theta}'\right) &= E\left[m_c^\sigma(\theta)\,|c < c_1\right] \\ &> (1-\varepsilon)\Pr\left[c < c_1 - \varepsilon | c < c_1\right] + 0 \times \Pr\left[c > c_1 - \varepsilon | c < c_1\right] \\ &= \left(1 - \frac{\varepsilon}{(\bar{c} - \underline{c})\, P_-}\right)(1-\varepsilon) = 1 - L_4\varepsilon \text{ for some bounded } L_4(\varepsilon), \end{aligned}$$

and

$$\begin{aligned} m_+^\sigma(\theta) &= E\left[m_c^\sigma(\theta)\,|c > c_2\right] \\ &< \varepsilon \Pr\left[c > c_2 + \varepsilon | c > c_2\right] + 1 \times \Pr\left[c < c_2 + \varepsilon | c > c_2\right] \\ &= \left(1 - \frac{\varepsilon}{(\bar{c} - \underline{c})\, P_+}\right)\varepsilon + \frac{\varepsilon}{(\bar{c} - \underline{c})\, P_+} = L_5\varepsilon \text{ for some bounded } L_5(\varepsilon). \end{aligned}$$

Intermediate value theorem implies that $\exists \tilde{\theta}' \in \left[x^0 - \delta_1, x^0 + \delta_1\right]$ such that

$$\begin{aligned} & \int_{x^0-\delta_1}^{x^0+\delta_1} \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta)\right]\right\} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta \\ = & \left[m_{c'}^\sigma\left(x^0+\delta_1\right) - m_{c'}^\sigma\left(x^0-\delta_1\right)\right] \times \\ & \left\{\tilde{\theta}' - r\left[1 - P_- m_-^\sigma\left(\tilde{\theta}'\right) - P_+ m_+^\sigma\left(\tilde{\theta}'\right)\right]\right\} h\left(\tilde{\theta}'\right) \\ = & \left\{x^0 - r\left[1 - \frac{c_1 - \underline{c}}{\bar{c} - \underline{c}}\right]\right\} h\left(x^0\right) + L_6\varepsilon \end{aligned}$$

for some bounded $L_4(\varepsilon)$. The last equality is due to the fact that $m_{c'}^\sigma\left(x^0 - \delta_1\right) \in [0, \varepsilon)$, $m_{c'}^\sigma\left(x^0 + \delta_1\right) \in (1 - \varepsilon, 1]$, $P_- = \frac{c_1 - \underline{c}}{\bar{c} - \underline{c}}$, $m_-^\sigma\left(\tilde{\theta}'\right) = 1 - L_4\varepsilon$, $P_+ = \frac{c - c_2}{\bar{c} - \underline{c}}$ and $m_+^\sigma(\theta) = L_5\varepsilon$.

$\mathbf{E}\left\{\int_{x^0-\delta_1}^{x^0+\delta_1} rP_= m_=^\sigma(\theta)\, h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta | c' \in [c_1, c_2]\right\} = \int_{m_=^\sigma(x^0-\delta_1)}^{m_=^\sigma(x^0+\delta_1)} rP_= m_=^\sigma(\theta)\, h(\theta)\, dm_=^\sigma(\theta) = \frac{1}{2} r \frac{c_2 - c_1}{\bar{c} - \underline{c}} h\left(x^0\right) + L_7(\varepsilon)$ for some bounded $L_7(\varepsilon)$.

Therefore, concerning the left hand side of equation (12), we have

$$\begin{aligned} & E\left[LHS | c' \in [c_1, c_2]\right] \\ = & \mathbf{E}\left\{\int_{x^0-\delta_1}^{x^0+\delta_1} \left\{\theta - r\left[1 - P_- m_-^\sigma(\theta) - P_+ m_+^\sigma(\theta)\right]\right\} h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta | c' \in [c_1, c_2]\right\} \\ & - \mathbf{E}\left\{\int_{x^0-\delta_1}^{x^0+\delta_1} rP_= m_=^\sigma(\theta)\, h(\theta)\, m_{c'}^{\sigma\prime}(\theta)\, d\theta | c' \in [c_1, c_2]\right\} \\ = & h\left(x^0\right)\left\{x^0 - r\left[1 - \frac{c_1 - \underline{c}}{\bar{c} - \underline{c}} - \frac{1}{2}\frac{c_2 - c_1}{\bar{c} - \underline{c}}\right]\right\} + L_8\varepsilon \end{aligned}$$

for some bounded $L_8(\varepsilon)$.

Previously we have already established that the right hand side of of equation (12) satisfies

$$\mathbf{E}\left[RHS|c' \in [c_1, c_2]\right] = \frac{c_1 + c_2}{2} h\left(x^0\right) + L_2\varepsilon$$

for some bounded $L_2(\varepsilon)$. Therefore,

$$h\left(x^0\right)\left\{x^0 - r\left[1 - \frac{c_1 - \underline{c}}{\bar{c} - \underline{c}} - \frac{1}{2}\frac{c_2 - c_1}{\bar{c} - \underline{c}}\right]\right\} + L_8\varepsilon = \frac{c_1 + c_2}{2} h\left(x^0\right) + L_2\varepsilon$$

Divide both sides by $h\left(x^0\right)$, then we have

$$\begin{aligned} x^0 &= \frac{c_1 + c_2}{2} + r\left[1 - \frac{c_1 - \underline{c}}{\bar{c} - \underline{c}} - \frac{1}{2}\frac{c_2 - c_1}{\bar{c} - \underline{c}}\right] + \frac{(L_2 - L_8)}{h\left(x^0\right)}\varepsilon \\ &= \frac{c_1 + c_2}{2} + r\frac{\bar{c} - \frac{c_1 + c_2}{2}}{\bar{c} - \underline{c}} + \frac{(L_2 - L_8)}{h\left(x^0\right)}\varepsilon. \end{aligned}$$

This concludes the proof. ■

While Lemma 8.4 characterizes the limiting switching points $\hat{x}(\cdot)$, it does not show how the equivalence relation $\sim$ partitions $[\underline{c}, \bar{c}]$. Lemma 8.5 fills this gap.

Lemma 8.5 *$\forall \underline{c} \leq c < c' \leq \bar{c}$, $\hat{x}(c) < \hat{x}(c')$ if and only if $r < \bar{c} - \underline{c}$.*

Proof. $c < c'$ implies $\hat{x}(c) \leq \hat{x}(c')$. If $r < \bar{c} - \underline{c}$, suppose $\hat{x}(c) = \hat{x}(c') = x$, let $[c_1, c_2] = \{c'' \in [\underline{c}, \bar{c}] : \hat{x}(c'') = x\}$. Then we have:

First, $\lim_{\sigma \to 0} \mathbf{E}[\theta | x_i = \hat{x}^\sigma(c_1)] = \lim_{\sigma \to 0} \mathbf{E}[\theta | x_i = \hat{x}^\sigma(c_2)] = x$. Second, $\hat{x}(c'') < \hat{x}^0(c_1)$ if and only if $c'' < c_1$. Also, $|x_i - x_{-i}| \to 0$ as $\sigma \to 0$. Thus, $\lim_{\sigma \to 0} \Pr[x_{-i} < \hat{x}^\sigma(c_{-i}) | x_i = \hat{x}^\sigma(c_1)] = \Pr\left[\hat{x}^0(c_1) < \hat{x}^0(c_{-i})\right] = \Pr[c_{-i} > c_1] = \frac{\bar{c} - c_1}{\bar{c} - \underline{c}}$. Similarly, $\lim_{\sigma \to 0} \Pr[x_{-i} < \hat{x}^\sigma(c_{-i}) | x_i = \hat{x}^\sigma(c_2)] = \frac{\bar{c} - c_2}{\bar{c} - \underline{c}}$. Third, by definition of $\hat{x}^\sigma$, we have

$$\begin{aligned} & x - r\frac{\bar{c} - c_1}{\bar{c} - \underline{c}} - c_1 \\ =\ & \lim_{\sigma \to 0} \mathbf{E}[\theta | x_i = \hat{x}^\sigma(c_1)] - \lim_{\sigma \to 0} r \Pr[x_{-i} < \hat{x}^\sigma(c_{-i}) | x_i = \hat{x}^\sigma(c_1)] - c_1 \\ =\ & 0 \\ =\ & \lim_{\sigma \to 0} \mathbf{E}[\theta | x_i = \hat{x}^\sigma(c_2)] - \lim_{\sigma \to 0} \Pr[x_{-i} < \hat{x}^\sigma(c_{-i}) | x_i = \hat{x}^\sigma(c_2)] - c_2 \\ =\ & x - r\frac{\bar{c} - c_2}{\bar{c} - \underline{c}} - c_2 \end{aligned}$$

The second and the third equality is due to the indifference conditions of type $(\hat{x}^\sigma(c_1), c_1)$ and $(\hat{x}^\sigma(c_2), c_2)$, respectively. Since $c_1 < c_2$, we have $\frac{r}{\bar{c} - \underline{c}} = 1$, contradicting the assumption that $r < \bar{c} - \underline{c}$. This proves the "if" claim.

To prove the "only if" claim, assume $\hat{x}(c') > \hat{x}(c)$. Let $[c_1, c_2] = \{c'' \in [\underline{c}, \bar{c}] : \hat{x}(c'') = \hat{x}(c)\}$ and $[c_1', c_2'] = \{c'' \in [\underline{c}, \bar{c}] : \hat{x}(c'') = \hat{x}(c')\}$. We must have $c_2 < c_1'$. Then by Lemma 8.4, $\hat{x}(c) = \frac{c_1+c_2}{2} + r\frac{\bar{c} - \frac{c_1+c_2}{2}}{\bar{c}-\underline{c}}$, and $\hat{x}(c') = \frac{c_1'+c_2'}{2} + r\frac{\bar{c} - \frac{c_1'+c_2'}{2}}{\bar{c}-\underline{c}}$. Then $\hat{x}(c') - \hat{x}(c) = \frac{1}{2}\left(1 - \frac{r}{\bar{c}-\underline{c}}\right)(c_1' + c_2' - c_1 - c_2) \leq 0$, a contradiction. ■

Proof of Proposition 2.2

Proof. The proof is straightforward from Lemmas 8.4 and 8.5. ■

8.3 Proof for Propositions in Section 3

Proof of Lemma 3.1

Proof. By Law of Iterated Expectation, the condition (3) of incentive compatibility for player c to join an organization is equivalent to

$$DU(c) \equiv \mathbf{E}\left[(a(x,c) - b(\theta))(u(\theta, l(\theta) - c))\right] \leq 0.$$

By assumption, $DU(c_1) \leq 0$ for some $c_1 < c$ and $DU(c_2) \leq 0$ for some $c_2 > c$. Since we focus on equilibria with switching strategies, let $a(x,c) = 1_{\{x \geq \hat{x}(c)\}}$ and $b(\theta) = 1_{\{\theta \geq \hat{\theta}\}}$.

If $\mathbf{E}\left[1_{\{x \geq \hat{x}(c)\}} - 1_{\{\theta \geq \hat{\theta}\}}\right] \leq 0$, we show that $DU(c_2) > DU(c)$. This, together with $DU(c_2) \leq 0$, implies that $DU(c) \leq 0$ as desired.

$$\begin{aligned}
& DU(c_2) - DU(c) \\
= \; & \mathbf{E}\left[(a(x,c_2) - b(\theta))(u(\theta, l(\theta) - c_2))\right] - \mathbf{E}\left[(a(x,c) - b(\theta))(u(\theta, l(\theta)) - c)\right] \\
= \; & \mathbf{E}\left[(a(x,c_2) - a(x,c))(u(\theta, l(\theta) - c_2))\right] - (c_2 - c)\,\mathbf{E}\left[a(x,c) - b(\theta)\right] \\
= \; & \mathbf{E}\left[\left(1_{\{x \geq \hat{x}(c_2)\}} - 1_{\{x \geq \hat{x}(c)\}}\right)(u(\theta, l(\theta)) - c_2)\right] - (c_2 - c)\,\mathbf{E}\left[1_{\{x \geq \hat{x}(c)\}} - 1_{\{\theta \geq \hat{\theta}\}}\right] \\
= \; & -\mathbf{E}\left[1_{\{\hat{x}(c) \leq x < \hat{x}(c_2)\}}(u(\theta, l(\theta)) - c_2)\,|x\right] - (c_2 - c)\,\mathbf{E}\left[1_{\{x \geq \hat{x}(c)\}} - 1_{\{\theta \geq \hat{\theta}\}}\right].
\end{aligned}$$

The last line is due to Law of Iterated Expectation.

$\mathbf{E}\left[1_{\{x \geq \hat{x}(c)\}} - 1_{\{\theta \geq \hat{\theta}\}}\right] \leq 0$ and $c_2 > c$ imply that the second term is positive. To see that the first term is also positive, first note that $\hat{x}(c) < \hat{x}(c_2)$ due to the monotonicity of $\hat{x}(\cdot)$. In addition, $\mathbf{E}\left[u(\theta, l(\theta)) - c_2 | x\right]$ increases in x. This is because 1) $u(\cdot,\cdot)$ is increasing in both arguments, 2) $l(\cdot)$ is also increasing in θ, as every player uses monotonically increasing strategies, and 3) SMLRP is assumed. Moreover, the optimality of the threshold

$\hat{x}(c_2)$ requires that $\mathbf{E}[u(\theta, l(\theta) - c_2)|x = \hat{x}(c_2)] = 0$. This yields $\mathbf{E}[u(\theta, l(\theta)) - c_2|x] < 0$ $\forall x \in [\hat{x}(c), \hat{x}(c_2))$, and thus $\mathbf{E}\left[1_{\{\hat{x}(c) \leq x < \hat{x}(c_2)\}}(u(\theta, l(\theta)) - c_2)|x\right] < 0$. Therefore, we have $DU(c_2) - DU(c) > 0$ as desired.

If $\mathbf{E}\left[1_{\{x \geq \hat{x}(c)\}} - 1_{\{\theta \geq \hat{\theta}\}}\right] > 0$, a symmetric argument shows that $0 \geq DU(c_1) > DU(c)$. This concludes the proof. ■

Lemma 8.6 *Given any organization μ, there exists an greatest and a least exogenous organizational equilibria of the unoganized game of any level of fundamental uncertainty $\sigma > 0$, both in switching strategies, with switching points of the unorganized players weakly increasing in c in each of them.*

Proof. It is straightforward to see that given any organization μ, the organized game in Section 3 is also a supermodular game. Hence, the result again follows from an iterated deletion of dominated strategy as in (van Zandt and Vives 2007). ■

Proof of Proposition 3.1

Proof. An argument analogous to Lemma 8.5 shows that in the limiting equilibrium, the cutoff for unorganized types $\hat{x}(\cdot)$ must be strictly increasing in c. Thus, the mass of unorganized investors $\eta(\theta) = \lim_{\sigma \to 0} \int_{\{c: c \text{ is unorganized}\}} \Pr[x > \hat{x}^\sigma(c)|\theta] \frac{dc}{\bar{c} - \underline{c}} = \int_{\{c: c \text{ is unorganized}\}} 1_{\theta > \hat{x}^\sigma(c)} \frac{dc}{\bar{c} - \underline{c}}$ is strictly increasing in θ. The rest of the proof is already given in the text of Section 3.2.

■

Proof of Proposition 3.3

Proof. A rigorous proof is similar to that of Lemma 8.4. We provide an intuitive sketch here. Given that $r \geq \bar{c} - \underline{c}$, for any exogenous oganizational equilibrium sequence $\left(\mu, \hat{\theta}^\sigma, \hat{x}^\sigma(\cdot)\right)$ with μ being a downward-exhaustive organization that converges to some $\left(\mu, \hat{\theta}, \hat{x}(\cdot)\right)$ as $\sigma \to 0$, first, by Lemma 8.5, $\hat{x}$ must be a constant over $[c_\lambda, \bar{c}]$. Second, monotonicity of strategies in c implies that we must have $\hat{\theta} \leq \hat{x}$.

Similar to the proof of Lemma 8.5, optimality of $\hat{x}^\sigma$ for $c \in [c_\lambda, \bar{c}]$ implies that

$$\int_{-\infty}^{+\infty} \left\{\theta - r\left[1 - \lambda 1_{\theta \geq \hat{\theta}^\sigma} - (1-\lambda)\hat{m}^\sigma(\theta)\right] - c\right\} m_c^{\sigma\prime}(\theta)\, h(\theta)\, d\theta = 0,$$

where λ is the mass of the organization, $m_c^\sigma(\theta) = \Pr[x \geq \hat{x}^\sigma(c)|\theta]$ is the effective strategy of player $c \in [c_\lambda, \bar{c}]$, and $\hat{m}^\sigma(\theta) = E[m_c(\theta)|c \in [c_\lambda, \bar{c}]]$. Taking conditional expectation over

$c \in [c_\lambda, \bar{c}]$ and letting $\sigma \to 0$, we have

$$\int_0^1 \left\{\theta - r\left[1 - \lambda 1_{\theta \geq \hat{\theta}} - (1-\lambda)\hat{m}(\theta)\right]\right\} d\hat{m}(\theta) = c_{\frac{1+\lambda}{2}} + O(\sigma). \tag{13}$$

If $\hat{\theta} < \hat{x}$, (13) becomes

$$\int_0^1 \{\theta - r[1 - \lambda - (1-\lambda)\hat{m}(\theta)]\} d\hat{m}(\theta) = c_{\frac{1+\lambda}{2}} + O(\sigma).$$

This yields

$$\hat{x} = c_{\frac{1+\lambda}{2}} + \frac{r(1-\lambda)}{2}.$$

On the other hand, since $\hat{\theta} < \hat{x}$, optimality of $\hat{\theta}$ implies that

$$\hat{\theta} - r(1-\lambda) = c_{\lambda/2}.$$

Thus,.

$$\hat{\theta} = r(1-\lambda) + c_{\lambda/2}, \text{ and}$$

$$\begin{aligned} \hat{x} - \hat{\theta} &= c_{\frac{1+\lambda}{2}} - c_{\lambda/2} - \frac{r(1-\lambda)}{2} \\ &= \frac{\bar{c} - \underline{c}}{2} - \frac{r(1-\lambda)}{2} > 0 \\ \Leftrightarrow & \ \lambda > 1 - \frac{\bar{c} - \underline{c}}{r}. \end{aligned}$$

This means if $\lambda \leq 1 - \frac{\bar{c} - \underline{c}}{r}$, we must have $\hat{\theta} = \hat{x}$.

If $\hat{\theta} = \hat{x}$, (13) becomes

$$\hat{x} - r(1 - \lambda t) + \frac{r(1-\lambda)}{2} = c_{\frac{1+\lambda}{2}},$$

where $t = \int_0^1 1_{\theta \geq \hat{\theta}} d\hat{m}(\theta) = \hat{m}(\infty) - \hat{m}\left(\hat{\theta}\right) = 1 - \hat{m}\left(\hat{\theta}\right) \in [0,1]$. Optimality of $\hat{\theta}$ requires that

$$\hat{\theta} - r\left(1 - \lambda - (1-\lambda)\hat{m}\left(\hat{\theta}\right)\right) = c_{\lambda/2}.$$

Therefore, we have

$$\begin{aligned}\hat{\theta} &= r(1-\lambda)t + c_{\lambda/2}, \text{ and} \\ \hat{x} &= r(1-\lambda t) - \frac{r(1-\lambda)}{2} + c_{\frac{1+\lambda}{2}}.\end{aligned}$$

Elimination of t yields

$$\hat{x} = \hat{\theta} = \frac{r}{2}\left(1-\lambda^2\right) + \mathbf{E}c,$$

and in turn $t = 1 - \frac{r(1-\lambda)-(\bar{c}-\underline{c})}{2r} \geq 1/2 > 0$. $t \leq 1$ implies that $\lambda \leq 1 - \frac{\bar{c}-\underline{c}}{r}$. That is, if $\lambda > 1 - \frac{\bar{c}-\underline{c}}{r}$, we must have $\hat{\theta} < \hat{x}$. This concludes the proof. ■

Proof of Proposition 3.4

Proof. Consider an exogenous organizational equilibrium with a downward exhaustive organization of size λ. We first show that the limiting social welfare $W(\lambda) \equiv \lim_{\sigma\to 0} W^{\sigma}(\lambda)$ increases in both $[0, \lambda_{\max}]$ and $(\lambda_{\max}, 1]$, where $\lambda_{\max} = 1 - \frac{\bar{c}-\underline{c}}{r}$. Recall from equation (4) that $\forall \tau \in [0,1]$, $c_\tau \equiv \underline{c} + \tau(\bar{c} - \underline{c})$.

If $\lambda \leq \lambda_{\max}$, by Proposition 3.3, $\hat{\theta} = \hat{x} = \frac{r}{2}\left(1-\lambda^2\right) + \mathbf{E}c$, and the resulting limiting social welfare

$$W(\lambda) = \int_{\frac{r}{2}(1-\lambda^2)+\mathbf{E}c}^{\bar{\theta}} (\theta - \mathbf{E}c)\, h(\theta)\, d\theta.$$

So for $\lambda \in [0, \lambda_{\max})$, we have

$$W'(\lambda) = r\lambda\left(1-\lambda^2\right) h\left(\frac{r}{2}\left(1-\lambda^2\right) + \mathbf{E}c\right) > 0.$$

If $\lambda > \lambda_{\max}$, by Proposition 3.3, $\hat{\theta} = r(1-\lambda) + c_{\lambda/2}$ and $\hat{x} = c_{\frac{1+\lambda}{2}} + \frac{r(1-\lambda)}{2}$. The resulting limiting social welfare

$$\begin{aligned} W(\lambda) &= \int_{\hat{\theta}(\lambda)}^{\hat{x}(\lambda)} \left[\int_{\underline{c}}^{c_\lambda} (\theta - r(1-\lambda) - c) \frac{dc}{\bar{c} - \underline{c}} \right] h(\theta) \, d\theta \\ &\quad + \int_{\hat{x}(\lambda)}^{\bar{\theta}} (\theta - \mathbf{E}c) h(\theta) \, d\theta \\ &= \int_{\hat{\theta}(\lambda)}^{\hat{x}(\lambda)} \left\{ \int_{\underline{c}}^{c_\lambda} \lambda [\theta - r(1-\lambda)] - \frac{c_\lambda^2 - \underline{c}^2}{2(\bar{c} - \underline{c})} \right\} h(\theta) \, d\theta \\ &\quad + \int_{\hat{x}(\lambda)}^{\bar{\theta}} (\theta - \mathbf{E}c) h(\theta) \, d\theta \\ &= \int_{\hat{\theta}(\lambda)}^{\hat{x}(\lambda)} \lambda \left[\theta - r(1-\lambda) - c_{\lambda/2} \right] h(\theta) \, d\theta + \int_{\hat{x}(\lambda)}^{\bar{\theta}} (\theta - \mathbf{E}c) h(\theta) \, d\theta \\ &= . \int_{\hat{\theta}(\lambda)}^{\hat{x}(\lambda)} \lambda \left[\theta - \hat{\theta}(\lambda) \right] h(\theta) \, d\theta + \int_{\hat{x}(\lambda)}^{\bar{\theta}} (\theta - \mathbf{E}c) h(\theta) \, d\theta \end{aligned}$$

The third equality results from $\frac{c_\lambda^2 - \underline{c}^2}{2(\bar{c} - \underline{c})} = \frac{c_\lambda - \underline{c}}{\bar{c} - \underline{c}} \frac{c_\lambda + \underline{c}}{2} = \lambda c_{\lambda/2}$. Because $\frac{\partial \hat{x}}{\partial \lambda} = \frac{\bar{c} - \underline{c} - r}{2}$ and $\frac{\partial \hat{\theta}}{\partial \lambda} = \frac{\bar{c} - \underline{c}}{2} - r$, for $\lambda \in (\lambda_{\max}, 1]$ we have

$$\begin{aligned} W'(\lambda) &= \int_{\hat{\theta}(\lambda)}^{\hat{x}(\lambda)} \left[\theta - r(1-\lambda) - c_{\lambda/2} \right] h(\theta) \, d\theta + \lambda \int_{\hat{\theta}(\lambda)}^{\hat{x}(\lambda)} \left(r - \frac{\bar{c} - \underline{c}}{2} \right) h(\theta) \, d\theta \\ &\quad + \left[\hat{x}(\lambda) - r(1-\lambda) - c_{\lambda/2} \right] h(\hat{x}(\lambda)) \frac{\bar{c} - \underline{c} - r}{2} \\ &\quad - \left[\hat{\theta}(\lambda) - r(1-\lambda) - c_{\lambda/2} \right] h\left(\hat{\theta}(\lambda) \right) \left(\frac{\bar{c} - \underline{c}}{2} - r \right) \\ &\quad - (\hat{x}(\lambda) - \mathbf{E}c) h(\hat{x}(\lambda)) \frac{\bar{c} - \underline{c} - r}{2}. \end{aligned}$$

Because $\hat{x} > \hat{\theta} = r(1-\lambda) + c_{\lambda/2}$ and $r \geq \bar{c} - \underline{c} > \frac{\bar{c} - \underline{c}}{2}$, the first two terms are strictly positive and the fourth term is zero. The rest two terms sum up to

$$\begin{aligned} & h(\hat{x}(\lambda)) \frac{\bar{c} - \underline{c} - r}{2} \left(\mathbf{E}c - r(1-\lambda) - c_{\lambda/2} \right) \\ = \; & h(\hat{x}(\lambda)) \frac{\bar{c} - \underline{c} - r}{2} (1-\lambda) \left(\frac{\bar{c} - \underline{c}}{2} - r \right) > 0. \end{aligned}$$

Therefore, $W'(\lambda) > 0$ in $(\lambda_{\max}, 1]$ as well. Because both $\hat{x}$ and $\hat{\theta}$ are continuous in λ, so is W. This concludes the proof. ■

Proof of Proposition 3.5

Proof. The proof concerning the size limit of the organization is given in the text and thus omitted here. And Proposition 3.3 shows that if $r \geq \bar{c} - \underline{c}$, $\hat{x} = \hat{\theta}$ in any voluntary organizational equilibrium with the size of the organization below the limit. ■

8.4 Proofs of Propositions in Section 4

Proof of Proposition 4.1

Proof. We characterize α^{FB}. It is straightforward to see that $\forall\theta$, if the player with cost c is assigned to invest (not to invest) in the first-best benchmark, so (neither) is any player with cost $c' < (>)\, c$. Therefore, it suffices to characterize the mass of investors α^{FB} as a function of θ, where all players whose costs are below $c_{\alpha^{FB}} = \underline{c} + \alpha^{FB}(\bar{c} - \underline{c})$ are the investors when fundamental is θ.

Conditional on θ, social welfare as a function of mass of investors α is

$$\begin{aligned} W(\alpha;\theta) &= \int_{\underline{c}}^{c_\alpha} [\theta - r(1-\alpha) - c] \frac{dc}{\bar{c} - \underline{c}} \\ &= [\theta - r(1-\alpha)] \frac{c_\alpha - \underline{c}}{\bar{c} - \underline{c}} - \frac{c_\alpha^2 - \underline{c}^2}{2(\bar{c} - \underline{c})} \\ &= \left(r - \frac{\bar{c} - \underline{c}}{2}\right)\alpha^2 + (\theta - r - \underline{c})\,\alpha, \\ \text{where } c_\alpha &= \underline{c} + \alpha(\bar{c} - \underline{c}). \end{aligned}$$

If $\bar{c} - \underline{c} < 2r$, then $W(\cdot;\theta)$ is a parabola opening upward, which is maximized at either $\alpha = 0$ or $\alpha = 1$. $W(0;\theta) = 0$, $W(1;\theta) = \theta - \mathbf{E}c$. Hence, $\alpha^{FB}(\theta) = 1_{\theta \geq \mathbf{E}c}$.

If $\bar{c} - \underline{c} > 2r$, then $W(\cdot;\theta)$ is a parabola opening downward centered at $\frac{\theta - r - \underline{c}}{\bar{c} - \underline{c} - 2r}$. If $\frac{\theta - r - \underline{c}}{\bar{c} - \underline{c} - 2r} \in [0,1]$, i.e., if $\theta \in [r + \underline{c}, \bar{c} - r]$, then $\alpha^{FB}(\theta) = \frac{\theta - r - \underline{c}}{\bar{c} - \underline{c} - 2r}$; If $\theta < r + \underline{c}$, then $W(\cdot;\theta)$ decreases in $[0,1]$, and thus $\alpha^{FB}(\theta) = 0$; Similarly, if $\theta > \bar{c} - r$, then $\alpha^{FB}(\theta) = 1$.

If $\bar{c} - \underline{c} = 2r$, then $W(\alpha;\theta) = (\theta - \mathbf{E}c)\,\alpha$. So $\alpha^{FB}(\theta) = 1_{\theta \geq \mathbf{E}c}$. ■

Proof of Proposition 4.2

Proof. The proof follows from straightforward calculation from Propositions 2.1**,** 2.2 and Proposition 3.3 and is thus omitted here. ■

8.5 Proofs of Propositions in Section 5

Proof of Proposition 5.1

Proof. Let λ_0 be such that $c_{\lambda_0} = \underline{c} + \lambda_0(\bar{c} - \underline{c})$ is the upper boundary type of the organization, and

$$\begin{aligned}\lambda &= \int_{\underline{c}}^{c_{\lambda_0}} k(c)\,dc = \int_{\underline{c}}^{c_{\lambda_0}} \left[\frac{1-\Delta}{\bar{c}-\underline{c}} + \Delta \cdot f(c)\right] dc \\ &= (1-\Delta)\,\lambda_0 + \Delta \cdot F(c_{\lambda_0}) \end{aligned} \tag{14}$$

be the mass of the organization. Under the assumption that $r > \bar{c} - \underline{c}$, the unorganized players must have the same limiting switching point $\hat{x}$. As before, denote the limiting switching point of the organization by $\hat{\theta}$. Recall that we must have $\hat{\theta} \leq \hat{x}$, and from Section 3.4 that an organization is not sustainable if $\hat{\theta} < \hat{x}$. We characterize $\hat{\theta}$ and $\hat{x}$ given $\hat{\theta} < \hat{x}$, and rule out the λ_0's that makes $\hat{\theta} < \hat{x}$. As in Proposition 3.3, there is a $\lambda'_{\max}$ such that $\hat{\theta} < \hat{x}$ iff $\lambda_0 > \lambda'_{\max}$. And $\kappa' = \underline{c} + \lambda'_{\max}(\bar{c} - \underline{c})$ and $\kappa = \underline{c} + \lambda_{\max}(\bar{c} - \underline{c})$ are what we want in the statement of the proposition.

Analogous to Proposition 3.3, given that $\hat{\theta} < \hat{x}$, we have $\hat{x} = \mathbf{E}^K[c|c > c_{\lambda_0}] + \frac{r(1-\lambda)}{2}$ and $\hat{\theta} - r(1-\lambda) = c_{med}$, where $\mathbf{E}^K$ refers to expectation under measure K, and c_{med} refers to the median voter of the organization. We now approximate $\mathbf{E}^K[c|c > c_{\lambda_0}]$ and c_{med} in terms of λ_0, and then obtain $\lambda'_{\max}$ from the condition $\hat{\theta} < \hat{x}$.

To obtain an approximation in terms of λ_0 for $\mathbf{E}^K[c|c > c_{\lambda_0}]$, by (14),

$$\frac{1}{1-\lambda} = \frac{1}{1-\lambda_0 + (\lambda_0 - F(c_{\lambda_0}))\Delta} = \frac{1}{1-\lambda_0} + \frac{F(c_{\lambda_0}) - \lambda_0}{(1-\lambda_0)^2}\Delta + o(\Delta),$$

and thus,

$$\begin{aligned} &\mathbf{E}^K[c|c > c_{\lambda_0}] \\ = &\int_{c_{\lambda_0}}^{\bar{c}} \left[\frac{1-\Delta}{\bar{c}-\underline{c}} + \Delta \cdot f(c)\right]\frac{dc}{1-\lambda} \\ = &(1-\Delta)\frac{1-\lambda_0}{1-\lambda}\frac{1}{1-\lambda_0}\int_{c_{\lambda_0}}^{\bar{c}} \frac{dc}{\bar{c}-\underline{c}} + \Delta \cdot \frac{1-F(c_{\lambda_0})}{1-\lambda}\int_{c_{\lambda_0}}^{\bar{c}} \frac{f(c)\,dc}{1-F(c_{\lambda_0})} \\ = &\frac{1}{1-\lambda}\left\{(1-\Delta)(1-\lambda_0)\,c_{\frac{1+\lambda_0}{2}} + \Delta \cdot (1-F(c_{\lambda_0}))\,\mathbf{E}^F[c|c > c_{\lambda_0}]\right\} \\ = &c_{\frac{1+\lambda_0}{2}} + \Delta \cdot \frac{1-F(c_{\lambda_0})}{1-\lambda_0}\left(\mathbf{E}^F[c|c > c_{\lambda_0}] - c_{\frac{1+\lambda_0}{2}}\right) + o(\Delta). \end{aligned} \tag{15}$$

where the third equality is due to

$$\frac{1}{1-\lambda_0}\int_{c_{\lambda_0}}^{\bar{c}} \frac{dc}{\bar{c}-\underline{c}} = E^U[c|c > c_{\lambda_0}] = c_{\frac{1+\lambda_0}{2}},$$

with U referring to the original uniform distribution on $[\underline{c}, \bar{c}]$.

To obtain an approximation in terms of λ_0 for c_{med}, the definition of c_{med} implies that

$$\begin{aligned}\frac{\lambda}{2} &= \int_{\underline{c}}^{c_{med}} k(c)\,dc = \frac{1-\Delta}{\bar{c}-\underline{c}}\left(c_{med}-\underline{c}\right) + \Delta\cdot F\left(c_{med}\right) \\ &= \frac{1-\Delta}{\bar{c}-\underline{c}}\left(c_{med}-\underline{c}\right) + \Delta\cdot\left[F\left(c_{\lambda_0/2}\right) + f\left(c_{\lambda_0/2}\right)\left(c_{med}-c_{\lambda_0/2}\right) + O\left(\Delta\right)\right] \\ &= \frac{1-\Delta}{\bar{c}-\underline{c}}\left(c_{med}-\underline{c}\right) + \Delta\cdot\left[F\left(c_{\lambda_0/2}\right) + f\left(c_{\lambda_0/2}\right)\left(c_{med}-c_{\lambda_0/2}\right)\right] + o\left(\Delta\right).\end{aligned}$$

On the other hand, (14) yields

$$\frac{\lambda}{2} = \left(1-\Delta\right)\lambda_0/2 + \Delta\cdot F\left(c_{\lambda_0}\right)/2 = \frac{1-\Delta}{\bar{c}-\underline{c}}\left(c_{\lambda_0/2}-\underline{c}\right) + \Delta\cdot F\left(c_{\lambda_0}\right)/2.$$

These two equations together yield

$$\begin{aligned}c_{med} &= c_{\lambda_0/2} + \Delta\cdot\frac{F\left(c_{\lambda_0/2}\right) - F\left(c_{\lambda_0}\right)/2}{\frac{1-\Delta}{\bar{c}-\underline{c}} + \Delta\cdot f\left(c_{\lambda_0/2}\right)} + o\left(\Delta\right) \\ &= c_{\lambda_0/2} + \Delta\cdot\left(\bar{c}-\underline{c}\right)\left[F\left(c_{\lambda_0/2}\right) - F\left(c_{\lambda_0}\right)/2\right] + o\left(\Delta\right). \qquad (16)\end{aligned}$$

Equations (14), (15) and (16) imply that

$$\hat{\theta} < \hat{x} \Leftrightarrow \lambda_0 > \lambda_{\max} + \frac{2\Delta}{r}\left\{\begin{array}{c}\frac{1-F\left(c_{\lambda_0}\right)}{1-\lambda_0}\left(c_{\frac{1+\lambda_0}{2}} - \mathbf{E}^F\left[c|c > c_{\lambda_0}\right]\right) \\ +\left(\bar{c}-\underline{c}\right)\left[F\left(c_{\lambda_0}\right)/2 - F\left(c_{\lambda_0/2}\right)\right] \\ +\frac{r}{2}\left(\bar{c}-\underline{c}\right)\left[\lambda_0 - F\left(c_{\lambda_0}\right)\right]\end{array}\right\} + o\left(\Delta\right).$$

To rule out the possibility of $\hat{\theta} < \hat{x}$, λ_0 must be no greater than the right hand side. As the greatest among those λ_0's, $\lambda'_{\max}$ exact equals the right hand side. Due to the facts that $\lambda'_{\max} = \lambda_{\max} + O(\Delta)$ and that $\kappa' - \kappa = (\bar{c}-\underline{c})\left(\lambda'_{\max} - \lambda_{\max}\right)$, we get Equation (7) as desired. ■

8.6 Proofs of Propositions in Section 6

Proof of Proposition 6.1

Proof. First, the same argument in the proof of Lemma 3.1 establishes that if $c_1 < c_2$ are both in the same organization i, as $\sigma \to 0$, $\forall c \in (c_1, c_2)$ cannot stay unorganized. Second, We establish formally that organizations choosing different strategies never overlap. Suppose that two organizations are choosing strategies $\hat{\theta}_1 < \hat{\theta}_2$ respectively, and that type

c is indifferent between the two. Then

$$\begin{aligned}
& \mathbf{E}\left\{b_1(\theta)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \\
= & \mathbf{E}\left\{b_2(\theta)\left(\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right] - c\right)\right\} \\
\Leftrightarrow & \mathbf{E}\left\{b_1(\theta)\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right]\right\} \\
& -\mathbf{E}\left\{b_2(\theta)\mathbf{E}\left[u\left(\theta, \sum_i \lambda_i b_i(\theta) + l_{1-\lambda}(\theta)\right)|x\right]\right\} \\
= & c\mathbf{E}\left\{b_1(\theta) - b_2(\theta)\right\}
\end{aligned}$$

Note that LHS is independent of c and RHS is linear in c. If $\hat{\theta}_1 < \hat{\theta}_2$, then LHS and $\mathbf{E}\{b_1(\theta) - b_2(\theta)\}$ are both strictly positive. This implies that type $c' > c$ must strictly prefer the more conservative strategy $\hat{\theta}_2$, and the other way round for $c' < c$. This establishes the claim.

The rest of the proof is already given in the text. ■

Proof of Proposition 6.2

Proof. By Proposition 6.1, we know that the two organizations must consist of all players in $[\underline{c}, \bar{c}_1]$ and $[\bar{c}_1, \bar{c}_2]$ respectively for some $\bar{c}_1 < \bar{c}_2$, and have the same cutoff $\hat{\theta}^0$. Let $c_{med,1} = \frac{1}{2}[\underline{c} + \bar{c}_1]$ and $c_{med,2} = \frac{1}{2}[\bar{c}_1 + \bar{c}_2]$ be the investment cost of their respective median voter, $\lambda_1 = \frac{\bar{c}_1 - \underline{c}}{\bar{c} - \underline{c}}$ and $\lambda_1 = \frac{\bar{c}_2 - \bar{c}_1}{\bar{c} - \underline{c}}$ be their respective mass, and $\lambda = \lambda_1 + \lambda_2$.

We use a strategy analogous to the proof of Proposition 3.5. We first assume $\hat{\theta} < \hat{x}(\bar{c})$ and characterize $\hat{\theta}$ and $\hat{x}(\bar{c})$, then show if $\lambda > \frac{4r}{(\bar{c} - \underline{c})^2}[r - (\bar{c} - \underline{c})]$, indeed $\hat{\theta} < \hat{x}(\bar{c})$. Then an analogous argument to the last one in the proof of Proposition 3.5 would show that organizations with total mass greater than $\frac{4r}{(\bar{c} - \underline{c})^2}[r - (\bar{c} - \underline{c})]$ cannot survive in equilibrium.

Given that $\hat{\theta} < \hat{x}(\bar{c})$, as fundamental uncertainty vanishes, the mass of unorganized investors $\lim_{\sigma \to 0} \eta^{\sigma}(\hat{\theta}) = 0$. Lemma8.5 yields

$$\hat{x}(\bar{c}) = \frac{\bar{c} + \underline{c} + r}{2} + \frac{\bar{c} - \underline{c} - r}{2}\lambda,$$

which is independent of $\bar{c}_1$ given λ (or equivalently $\bar{c}_2$).

In order for both median voters to switch exactly at $\hat{\theta}$, we must have

$$\hat{\theta} - r(1 - \lambda) \geq c_{med,i} \geq \hat{\theta} - r(1 - \lambda_i) \ \ \forall i = 1, 2$$

which is equivalent to

$$c_{med,i} + r(1-\lambda) \leq \hat{\theta} \leq c_{med,i} + r(1-\lambda_i) \equiv A_i \ \forall i = 1,2$$

Because $c_{med,1} < c_{med,2}$, the first inequality holds for $i = 1$ if it holds for $i = 2$. Note that $A_i > c_{med,i} + r(1-\lambda)\ \forall i$, so if the second inequality holds $\forall i$ and binds for some i, then the corresponding $\hat{\theta}$, call it $\hat{\theta}_{\max}$, is its greatest possible value in equilibrium.

For fixed λ (or equivalently, $\bar{c}_2$), we look for $\max_{\bar{c}_1}[\min\{A_1, A_2\}]$, which gives the greatest equilibrium $\hat{\theta}$. This makes $\hat{\theta} < \hat{x}(\bar{c})$ least likely to hold, and thus generates the desired contradiction with the upper bound for λ. Note that tedious algebra yields

$$\begin{aligned} A_2 &= \bar{c}_1 + r - \left[r - \frac{\bar{c} - \underline{c}}{2}\right] \frac{\bar{c}_2 - \bar{c}_1}{\bar{c} - \underline{c}} \\ A_1 &= \underline{c} + r - \left[r - \frac{\bar{c} - \underline{c}}{2}\right] \frac{\bar{c}_1 - \underline{c}}{\bar{c} - \underline{c}}. \end{aligned}$$

For fixed $\bar{c}_2$, since $r \geq \bar{c} - \underline{c} > \frac{\bar{c}-\underline{c}}{2}$, A_2 is increasing in $\bar{c}_1$ and A_1 is decreasing in $\bar{c}_1$ if $\bar{c}_1 \in (\underline{c}, \bar{c}_2)$. In addition, $A_2 > A_1$ when $\bar{c}_1 = \bar{c}_2$, and $A_2 < A_1$ when $\bar{c}_1 = \underline{c}$. Thus, for fixed λ (or equivalently, $\bar{c}_2$), $\min\{A_1, A_2\}$ is maximized when $A_1 = A_2$, i.e., when $\frac{\lambda_1}{\lambda_2} = \frac{r-(\bar{c}-\underline{c})/2}{r+(\bar{c}-\underline{c})/2}$, and the corresponding $\hat{\theta}_{\max} = \underline{c} + r - \frac{[r-(\bar{c}-\underline{c})/2]^2}{2r}\lambda$.

Now, $\hat{x}(\bar{c}) > \hat{\theta}_{\max} \Leftrightarrow \lambda > \frac{4r}{(\bar{c}-\underline{c})^2}[r - (\bar{c} - \underline{c})]$. An argument analogous to the last one in the proof of Proposition 3.5 shows that this cannot be an equilibrium.

An argument analogous to that of the counterpart in Proposition 3.5 shows the all players asymptotically take the same action in equilibrium. ■

Proof of Proposition 6.3

Proof. If $\theta \geq 1$, regime changes even if nobody invests, so every player would invest. This makes $l = 1$ and confirms the change of regime. So the unique equilibrium is characterized by $\hat{c}(\theta) = \bar{c}$. This corresponds to $a(\theta, c) = 1\ \forall c$.

Similarly, if $\theta < 0$, regime does not change even if everyone invests. So no player would invest. This makes $l = 0$ and confirms the survival of regime. So the unique equilibrium is characterized by $\hat{c}(\theta) = \underline{c}$. This corresponds to $a(\theta, c) = 0\ \forall c$.

If $\theta \in [0, 1)$, in any equilibrium either the regime changes or it does not. If the regime changes, then it is optimal for every player to invest. This makes $l = 1$ and indeed changes the regime. This verifies the equilibrium characterized by $\hat{c}(\theta) = \bar{c}$. This corresponds to $a(\theta, c) = 1\ \forall c$. A similar argument applies to verifies the other equilibrium characterized by $\hat{c}(\theta) = \underline{c}$. This corresponds to $a(\theta, c) = 0\ \forall c$.

From the perspective of the social planner, since $0 \leq \underline{c} \leq \bar{c} \leq 1$, it is straightforward that he would set $l = 1_{\theta \geq 0}$. This verifies the efficiency statements. ■

Proof of Proposition 6.4

Proof. A rigorous proof of Proposition 3.3 is similar to that of Lemma 8.4. We provide an intuitive sketch here. The regime change is a supermodular game. Again, an argument analogous to the investment game shows that the limiting equilibrium is essentially unique and in switching strategies. To characterize it, again let $m_c^\sigma(\theta) = \Pr[x \geq \hat{x}^\sigma(c)|\theta]$ be the effective strategy of player c, and $\hat{m}^\sigma(\theta) = E[m_c(\theta)|c \in [\underline{c}, \bar{c}]]$. First, the benefit of investment is $1_{\theta \geq 1 - l(\theta)} > \bar{c}$, so every player only cares about whether $\theta \geq 1 - l(\theta)$, and thus must have the same switching point $\hat{x}$ in the limiting equilibrium as $\sigma \to 0$. So

$$\hat{x} = 1 - l(\hat{x}) = 1 - m(\hat{x}). \tag{17}$$

The optimality of m_c^σ requires

$$\int_{\underline{\theta}}^{\bar{\theta}} \left(1_{\theta \geq 1 - l(\theta)} - c\right) m_c^{\sigma\prime}(\theta) h(\theta) d\theta = 0, \text{ where } \theta^\sigma = 1 - m^\sigma(\theta^\sigma).$$

Taking expectation over $[\underline{c}, \bar{c}]$ and letting $\sigma \to 0$, since $\theta \geq 1 - l(\theta)$ iff $\theta \geq \hat{x}$, and $\hat{m}^\sigma(\theta) \to 1_{\theta \geq \hat{x},}$ we have

$$\begin{aligned} \mathbf{E}c + O(\sigma) &= \int_{\underline{\theta}}^{\bar{\theta}} 1_{\theta \geq \hat{x}} dm(\theta) \\ &= m(\bar{\theta}) - m(\hat{x}) = 1 - m(\hat{x}), \end{aligned}$$

which yields

$$1 - m(\hat{x}) = \mathbf{E}c + O(\sigma).$$

Together with (17), this yields $\hat{x} = \mathbf{E}c + O(\sigma)$ as desired. ■

Proof of Proposition 6.5

Proof. First, the equilibrium strategy of any organization must be weakly increasing in θ, and the effective strategy $\mathbf{E}[a(x, c)|\theta]$ of any player with c and outside any organization must be strictly increasing in θ. Therefore, $\exists$ a unique θ^* such that $\theta \geq 1 - l$ if and only if $\theta \geq \theta^*$. Thus, every player's payoff is maximized if he invests if and only if $\theta \geq \theta^*$. This is the equilibrium strategy of any organization. And this is feasible to any player only if he joins such an organization, because otherwise his action can only be contingent on his own noisy signal x. Therefore, in equilibrium, all players join an organization, and every such organization invests if and only if $\theta \geq \theta^*$, which is equivalent to the case where a single organization includes all such players. Given this, the equilibrium strategy of the organization is $a_{org} = 1_{\theta \geq 0}$. This coincides with the first-best benchmark characterized in Proposition 6.3. ■