

Managing Strategic Buyers: Should a Seller Ban Resale?

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April 16, 2017

Abstract

We study the seller's pricing strategy of one good (finite inventory) that can be sold in two bargaining periods (before a deadline) when she faces two strategic buyers with private valuations. In particular, we are interested in comparing the outcomes of this game in two environments: allowing versus forbidding a resale option. Without resale, the seller charges prices high in the first bargaining period to motivate high valuation consumers to buy, but prices are reduced if no buyer expresses their willingness to buy. Compared with this benchmark case, introducing the resale option generates two effects: there is an increase in consumers willingness to buy in the first period, motivating an increase in the price of the first period, but there is an increase in demand price-elasticity of the first period, motivating a decrease in the price of the first period. We show that the second effect dominates for a bunch of reasonable parameters, motivating a reduction in first period price and generating an increase in profits, aggregate consumer surplus, and, thus, in welfare.

Keywords: resale, bargaining, price discrimination, strategic buyers.

JEL Classification: L11, D4.

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1 Introduction

Suppose a monopolist with a finite number of units of a good to sell before a deadline. The good can be consumed only at that deadline. That is, before the deadline the good can be bought but it cannot be consumed and after it the non-traded units are valueless. For instance, consider sports' or concerts' tickets, airline tickets, hotels reservations, etc... Thus, through time, the monopolist proposes (potentially) different take-it-or-leave-it price offers to a finite number of strategic buyers with private information about their willingness to pay. The buyers decide whether to accept or not one of those prices, anticipating that waiting may imply that no good is available in the future. In this revenue management scenario, we are interested in studying the impact of a resale option in prices, profits, consumer surplus, and welfare. We show that under some reasonable conditions, introducing the resale option induces the monopolist to reduce optimal prices, increasing her profits and the aggregate consumer surplus. Thus, allowing for resale may increase total welfare.

The revenue management problem without the resale option has already been analyzed in many previous articles (we discuss about them below). In particular, Hörner & Samuelson (2011) is the first one in characterizing the optimal sequence of prices offered by a monopolist without commitment who faces strategic buyers. We use this model as our benchmark case and we study the important and realistic contractual option of resale.

In particular, we consider the simplified problem of a seller with one unit of an indivisible and costless good that can be traded in one of two periods. The seller faces two strategic buyers with private and heterogeneous valuations, and unit demand. In every period the monopolist offers a price and each buyer faces the following trade-off: he can accept the current price or he can wait for a future lower price with the risk that, if the rival buyer has accepted the current offer, he will end up empty-handed. If only one buyer accepts the current price, he receives the unit and pays the price announced. If both buyers are willing to buy, a random tie-breaking rule allocates the unit to one of them. Otherwise, the game moves to the next stage with the same logic. Since we allow for resale, when a buyer succeed in getting the good in the first period, he can make a take-it-or-leave-it price offer to the remaining buyer.

As Hörner & Samuelson (2011) shows, in the benchmark case the seller chooses a decreasing price sequence that balances a trade-off between price

discriminating consumers and charging a high reserve price. When resale is allowed consumers' buying decisions is affected by the following two effects. First, at a given price, their willingness to buy sooner increases since they can get some extra surplus out of the resale option. Everything else equal, this effects induces the monopolist to ask for a larger price. Second, the buyer in the model with resale is more sensitive to changes in prices. In the benchmark case, an increment in the price gives incentive to buyers to wait for a lower price. This effect is also present in the model with resale. However, in this case, an increment in the price also reduces the probability of reselling. In other words, the new demand has greater price-elasticity. Everything else equal, this second effect induces the monopolist to reduce the price.¹

Our first goal is to define under which conditions the effect of greater price-elasticity of demand dominates the effect of upward shift of demand, motivating the seller to lower the price sequence. We show that if the distribution of buyer's valuation implies a concave demand function, the impact of resale option on willingness to pay is higher in consumers with valuations approaching the price from below; consequently, the seller has incentives to reduce the price, increasing her profits by raising the probability of selling sooner.

Next, we study the effect on welfare of introducing reselling. We show that welfare may increase. In this sense, while the seller increases his profits, the buyers face a lower price. The increasing in the probability of selling also affects positively the welfare. Finally, notice that the reduction in the price can make that consumers with lower valuation end up with the product. However, there is a high probability that these buyers resale the unit when there is a consumer with high valuation. The net effect is undetermined, but we show that it is likely that the aggregate welfare increases. Notice that there is a distribution of surplus among buyers with positive net effect on welfare.

The organization of the paper is as follows. After reviewing the literature, we use section 2 to present our main results by solving a simple discrete valuation model with and without resale; they are compared in section 2.3. Section 3 considers the continuous model, with particular emphasis on the

¹Notice that if the seller had more units, the increment in the number of offers (seller and resale-buyers) would increase competition in the following stages, motivating a reduction in price.

uniform distribution case. Section 4 concludes.

1.1 Literature review

This paper relates with several strands of literature. First, it belongs to the revenue management literature. As we have stated above, the closest article is Hörner & Samuelson (2011), that help us to define a benchmark.

On the other hand, resale is a realistic feature that has been studied under different models but not in the revenue management literature. For instance, Möller & Watanabe (2010) consider a dynamic model in which buyers are uncertain about their valuations. In particular, they study a two-period framework where a one ticket seller faces heterogeneous buyers that learn their valuations after the first period has finished. When considering resale, they assume that the seller commits to a sequence of prices. They find that resale motivates an increase in first period prices. We relax this commitment of future prices assumption and we find the new interesting result that resale may motivate a reduction in first period prices.

Also Courty (2003*b*) studies the ticket resale when heterogeneous buyers learn at different speed about their valuation through time and the seller commits to a sequence of prices.² They consider the role of brokers, allowing for competition in the resale market. They find no relevant impact of resale.

Calzolari & Pavan (2006) consider the possibility of reselling a durable good in two scenarios: when the buyers resell to new consumers (that the seller cannot approach directly) and when the buyers resell to the same buyers than the seller can approach (as in our setting). Allowing for resale increases revenues in the first case but decreases revenue in the second case. In contrast, we find positive impact of resale on seller's profits. The main differences with our setting are that they study a durable good and that their approach does not consider a repeated interaction between the monopolist and the buyers.

To the best of our knowledge, this is the first attempt to study the effect of resale without commitment in a revenue management framework.

²See also Courty (2003*a*) and Courty & Hao (2000).

2 A simple model

A seller has a unique good to sell in a two-period model, $t = 1, 2$ (from now on, we refer to $t = 1$ as *Today* and $t = 2$ as *Tomorrow*). The day after Tomorrow the good is valueless. The production and opportunity costs of the good are normalized to zero. In the demand side there are two buyers with private valuations. Valuations are $v \in \{a, b, 1\}$ satisfying $0 < a < b < 1$ with probabilities $q := (q_a, q_b, 1 - q_a - q_b)$.

The timing is the following: first nature chooses both buyers' valuations, which are private information. In the second stage the seller sets a price for Today. In the third stage each buyer announces whether he is willing to buy the product. If only one buyer expresses his willingness to buy, he receives the good and pays the price. If both buyers express their willingness to buy, a random tie-breaking rule decides who gets the good (and pays the price).³ If none of them express the willingness to buy, the seller keeps the good until the next day. In the fourth stage we move to Tomorrow where there are two scenarios: in one scenario the seller still has the good and she announces a price, following the same procedure described for the second and third stages. In the second scenario one buyer has the good: with no-resale the buyer makes no offer and keeps the good; with resale the buyer announces a resell price, that is accepted or rejected by the other buyer. At the end of Tomorrow payoffs deliver.

2.1 Equilibrium with no resale

In this simple model the value of the parameters a, b, q_a, q_b define different cases. For a non-trivial case, we restrict to those cases where the seller wants to sell the good with probability one. With no resale and restricting to symmetric strategies, then there are two options: (A) types $v \in \{b, 1\}$ buy Today and type $v = a$ buys Tomorrow; and (B) type $v = 1$ buys Today and types $v \in \{a, b\}$ buy Tomorrow.⁴

- **Case (A):** To motivate types $v \in \{b, 1\}$ to buy Today, the seller solves

$$\max_{p_1} p_1(1 - q_a^2) + p_2 q_a^2,$$

³For this simple example it is irrelevant whether each buyer knows the other buyer decision. For an extended version, this information may affect results.

⁴We ignore those cases where the seller charges a high price that only type $v = 1$ or types $v \in \{b, 1\}$ finally buy the good.

subject to Incentive Compatibility (IC) constraint for the buyer types $v \in \{b, 1\}$ (buy Today instead of waiting), and seller's pricing policy for Tomorrow p_2 . This implies that type $v = a$ remains until Tomorrow and, thus, the seller's pricing strategy Tomorrow is $p_2 = a$. Buyers' beliefs must be right in equilibrium, anticipating that if the other buyer valuation is $v = a$ he waits (otherwise he buys Today) and that Tomorrow price is $p_2 = a$.⁵ Buyers' utility to wait until Tomorrow and buy Today are:

$$E[U(\text{wait}|v = b)] = \frac{q_a}{2}(b - a),$$

$$E[U(\text{buy}|v = b)] = q_a(b - p_1) + \frac{1 - q_a}{2}(b - p_1) = \frac{1 + q_a}{2}(b - p_1).$$

Notice that if type $v = b$ prefers to buy Today than waiting until Tomorrow, then it is also true that type $v = 1$ will prefer to buy Today as well. Then, the seller incorporates only the type- b IC binding. The IC constraint faced by the seller is $p_1 \leq \frac{b + a q_a}{1 + q_a}$. Then, the prices

$$\left(\hat{p}_1, \hat{p}_2 \right) = \left(\frac{b + a q_a}{1 + q_a}, a \right), \quad (1)$$

characterize the solution to the seller's problem. Formally, a symmetric subgame perfect Nash equilibrium is a sequence of prices (\hat{p}_1, \hat{p}_2) that solve seller's problem, and each buyer's price acceptance rule for each type $v \in \{a, b, 1\}$. In this case, prices are characterized in (1) and buyer's acceptance rule is: type $v \in \{b, 1\}$ buy Today if the price is lower or equal than \hat{p}_1 and wait otherwise; and $v = a$ buys Tomorrow if $\hat{p} > a$ and buys Today otherwise. In equilibrium seller's profit is $\Pi_A = b - (b - a)q_a$.

- **Case (B):** another possible case is to motivate type $v = 1$ to buy Today and types $v \in \{a, b\}$ wait until Tomorrow. To implement this case, the seller solves

$$\max_{p_1} p_1(1 - (q_b + q_a)^2) + p_2(q_b + q_a)^2,$$

⁵Recall that the seller has no credible commitment Today to set prices for Tomorrow. Like in the Coase conjecture, it may be optimal for the firm to commit to $p_2 = b$ but this threat is not self-fulfilled.

subject to Incentive Compatibility constraints of buyers and seller's pricing policy for Tomorrow p_2 . If only $v = 1$ buys Today, the seller may charge a price $p_2 \in \{a, b\}$. Our assumption of trade with probability one requires that $p_2 = a$ or, equivalently, that b is not too high (i.e., $b < \frac{a}{1-q_a}$). Consumers utility to wait and buy are:

$$\begin{aligned} E[U(\text{wait}|v = 1)] &= \frac{q_a + q_b}{2}(1 - a), \\ E[U(\text{buy}|v = 1)] &= (q_a + q_b)(1 - p_1) + \frac{1 - q_a - q_b}{2}(1 - p_1) \\ &= \frac{1 + q_a + q_b}{2}(1 - p_1). \end{aligned}$$

In this case the (IC) constraint is $p_1 \leq \frac{1+a(q_a+q_b)}{1+q_a+q_b}$. Then, prices are

$$\left(\hat{p}_1, \hat{p}_2 \right) = \left(\frac{1 + a(q_a + q_b)}{1 + q_a + q_b}, a \right). \quad (2)$$

A symmetric subgame perfect Nash equilibrium is a sequence of prices characterized in (2) and the following buyers' acceptance rule: type $v = 1$ buys Today if the price is lower or equal than \hat{p}_1 and wait otherwise; and types $v \in \{a, b\}$ wait until Tomorrow if $\hat{p}_1 > a$ and buys Today otherwise. In equilibrium buyer's type $v = 1$ buys Today and $v \in \{a, b\}$ buy Tomorrow. The seller's profit is $\Pi_B = 1 - (1-a)(q_a + q_b)$.

The symmetric subgame perfect Nash equilibrium is the one that maximizes seller's profits comparing cases A and B. Then, the equilibrium is the one in case B (where only the highest type $v = 1$ buys Today) if $\Pi_B > \Pi_A$, or $1 - (1-a)(q_a + q_b) \geq b - (b-a)q_a$. Consequently, if

$$b < 1 - \frac{(1-a)q_b}{1-q_a}, \quad (3)$$

2.2 Equilibrium With resale

As before, there two possible cases with resale: (C) Two types of buyers buy Today and one buys Tomorrow, and (D) One type of buyer buys Today and two buy Tomorrow. However, if one buyer buys Today, he has the option to resale the product Tomorrow, consequently he may introduce a resale price.

- **Case (C):** If two types of buyers buy Today, those are $v\{b, 1\}$. The seller solves

$$\max_{p_1} p_1(1 - q_a^2) + p_2 q_a^2.$$

As before, buyers anticipate that $p_2 = a$ (if the good is not sold, both buyers are type $v = a$). Buyer's utility to wait and buy are:

$$\begin{aligned} E[U(\text{wait}|v = b)] &= \frac{q_a}{2}(b - a), \\ E[U(\text{buy}|v = b)] &= q_a(b - p_1) + \frac{q_b}{2}(b - p_1) + \frac{1 - q_a - q_b}{2}(r - p_1), \\ &= \frac{1 + q_a}{2}(b - p_1) + \frac{1 - q_a - q_b}{2}(r - b). \end{aligned}$$

Notice that $r = 1$ is the optimal resale price. Resale is accepted by a buyer of type $v = 1$ and rejected otherwise. Also, if type $v = b$ prefers to buy Today, then type $v = 1$ also prefers to buy Today. Then, the constraint is $p_1 \leq \frac{a q_a + b(q_a + q_b) + 1 - q_a - q_b}{1 + q_a}$. Then, seller's prices and resale prices (in case of resale) are:

$$(p_1^*, p_2^*, r^*) = \left(\frac{a q_a + b(q_a + q_b) + 1 - q_a - q_b}{1 + q_a}, a, 1 \right). \quad (4)$$

A symmetric subgame perfect Nash equilibrium is now characterized by a sequence of prices (p_1, p_2, r) and buyer's acceptance in each stage. In this case, prices are characterized by (4). Buyer types $v \in \{b, 1\}$ buy Today and charge a resale price $r^* = 1$, buyer type $v = a$ wait until Tomorrow and buy if $p_2^* \leq a$.

The seller's profit is

$$\begin{aligned} \Pi_C &= [b(q_a + q_b) + 1 - q_a - q_b](1 - q_a) + a q_a, \\ \Pi_C &= 1 - (1 - a)q_a - (q_a + q_b)(1 - q_a)(1 - b). \end{aligned}$$

The second expression is to compared profit with one buyer Today (with no resale) and two types of buyers buying today (with resale).

- **Case (D):** If one buyer buys Today, then only $v = 1$ buys Today. As he can only resale to another $v = 1$ type of buyer, resale does not generate any additional value compare to the case with no resale

(there is no gain from resale). Consequently, equilibrium is the same as the one described in case (B), and seller's profits are $\Pi_D = \Pi_B = 1 - (1 - a)(q_a + q_b)$.

The symmetric subgame perfect Nash equilibrium is the one that maximizes seller's profits. Then, the equilibrium is the one where only the highest type $v = 1$ buys Today if $\Pi_D > \Pi_C$, or

$$\begin{aligned} 1 - (1 - a)(q_a + q_b) &> 1 - (1 - a)q_a - (q_a + q_b)(1 - q_a)(1 - b), \\ b &< \frac{(q_a + q_b)(1 - q_a) - (1 - a)q_b}{(q_a + q_b)(1 - q_a)}. \end{aligned} \quad (5)$$

2.3 Analyzing the effect of resale

The impact of resale depends on parameter values. Profits will never go down but prices may go up or down. The first results is quite trivial.

Proposition 1. *If $b < 1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)}$, resale has no impact on profits, prices or sales.*

If the valuation of the intermediate type $v = b$ is low enough, the seller focus on attracting type $v = 1$ buyers to buy Today, independently of whether resale is allowed or not. Then, resale has no impact either on demand or on willingness to pay.

Now, we move to the interesting cases.

Proposition 2. *If $1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)} < b$, resale motivates the seller to increase the price Today.*

Proposition 2 says that if the valuation of the intermediate type is high enough, the resale option motivates a higher price Today. Two cases explain this result: (i) if $b \geq 1 - \frac{(1-a)q_b}{1-q_a}$ the seller prefers that types $v = \{b, 1\}$ buy Today with or without resale; thereby, resale affects a redistribution of surplus among buyers, increasing the willingness to pay buyers for the product of type $v = b$ that ultimately the seller appropriates with a higher price Today. There is no effect on demand.

(ii) if $1 - \frac{(1-a)q_b}{1-q_a} > b > 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}$ the good is sold to type $v = 1$ buyer Today without resale, but with resale the seller does attract types $v = b$ and $v = 1$ to buy Today. This increment in demand is motivated by the increase in type $v = b$ willingness to pay due to the resale option. This

increase in willingness to pay is high enough that they are willing to buy the product Today at price p_1 without resale. This motivates the seller to increase the price. In this case there are both effects on demand and on the willingness to pay of type $v = b$ buyers.

Proposition 3. *If $1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)} \leq b < 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}$, resale motivates the seller to decrease the price Today.*

Proposition 3 shows that under some reasonable parameters the seller may reduce the price Today to attract more buyer types to purchase Today. In particular, the seller attracts only type $v = 1$ to buy Today without resale, but she attracts types $v \in \{b, 1\}$ to buy Today with resale. The resale option increases the willingness to pay of type $v = b$ for buying Today, but this effect is not enough to buy Today at no-resale prices; as a result the seller prefers to reduce the price in order to increase demand. A reduction in the price, increases more than proportionally the probability of selling Today, and total expected profits of the seller.

Before moving to the welfare analysis, we present two numerical examples to represent our results that Today's price may increase or decrease when allowing for resale.

*Example 1: Suppose valuations are $v := \{0.8, 0.95, 1\}$ with probabilities $q := (0.2, 0.15, 0.65)$. In this case, prices change from $(p_1, p_2) = (0.948, 0.8)$ with no resale to $(p_1, p_2) = (0.952, 0.8)$ with resale, **with an increase in p_1** . Profits increase from $\Pi = 0.93$ to $\Pi = 0.946$.*

Given $(a, q_a, q_b) = (0.8, 0.2, 0.15)$, if $b > 0.9365$ then introducing the resale option motivates an increase in price. For lower values of b , if $b \in [0.8928, 0.9365]$, it motivates a reduction in price.

*Example 2: Suppose valuations are $v := \{0.8, 0.95, 1\}$ with probabilities $q := (0.35, 0.15, 0.5)$. In this case, prices change from $(p_1, p_2) = (0.9333, 0.8)$ with no resale to $(p_1, p_2) = (0.9296, 0.8)$ with resale, **with a decrease in p_1** . Profits increase from $\Pi = 0.9$ to $\Pi = 0.91375$.*

Given $(a, q_a, q_b) = (0.8, 0.35, 0.15)$, if $b > 0.96$ then introducing the resale option motivates an increase in price. For $b \in [0.9077, 0.96]$, it motivates a reduction in price.

2.4 Welfare

In this 3-valuation model the welfare comparison is quite simple because: 1) we assume that there is always trade (at minimum price $p = a$), 2) the value of the welfare is determined by the type of the buyer who finally receives the good.

We first calculate the welfare for each of the four cases (A), (B), (C), and (D) described above, and then we make a welfare analysis.

- Case (A)

$$\begin{aligned}
 W(A) &= \underbrace{(1 - q_a - q_b)^2 + 2q_a(1 - q_a - q_b) + q_b(1 - q_a - q_b)}_{\text{type } v=1}, \\
 &\quad + b \underbrace{(q_b(1 - q_a - q_b) + q_b^2 + 2q_a q_b)}_{\text{type } v=b} + a \underbrace{(q_a^2)}_{\text{type } v=a}, \\
 &= (1 - q_a - q_b)(1 + q_a) + q_b(1 + q_a)b + q_a^2 a.
 \end{aligned}$$

- Cases (B) and (D)

$$\begin{aligned}
 W(B) = W(D) &= \underbrace{(1 - q_a - q_b)^2 + 2(q_a + q_b)(1 - q_a - q_b)}_{\text{type } v=1}, \\
 &\quad + b \underbrace{\left(q_b^2 + \frac{2(q_a q_b)}{2}\right)}_{\text{type } v=b} + a \underbrace{\left(q_a^2 + \frac{2(q_a q_b)}{2}\right)}_{\text{type } v=a}, \\
 &= 1 - (q_a + q_b)^2 + (a q_a + b q_b)(q_a + q_b).
 \end{aligned}$$

- Case (C)

$$\begin{aligned}
 W(C) &= \underbrace{(1 - q_a - q_b)^2 + 2(q_a + q_b)(1 - q_a - q_b)}_{\text{type } v=1 \text{ direct/resale}}, \\
 &\quad + b \underbrace{(q_b^2 + 2(q_a q_b))}_{\text{type } v=b} + a \underbrace{(q_a^2)}_{\text{type } v=a}, \\
 &= 1 - (q_a + q_b)^2 + (a q_a + b q_b)q_a + b q_b(q_a + q_b).
 \end{aligned}$$

In this simple model, allowing for resale never decreases welfare. To see this, notice that if allowing for resale does not change the buyer's type

who buys in each period, welfare will never decrease.⁶ When comparing cases (C) with (A), the difference in welfare is $\Delta W = W(C) - W(A) = q_b(1 - q_b - q_a + b(1 + q_b + q_a))$, this is positive when case (A) is optimal. Notice that $W(D) = W(B)$, then $\Delta W = 0$. When comparing cases (C) with (B) the result remains because the difference in welfare is $\Delta W = W(C) - W(B) = q_a q_b(b - a)$.

The difference in profits are: (i) $\Pi(C) - \Pi(B) = \Delta\Pi = q_b(b - a) - (1 - b)q_a[1 - (q_a + q_b)]$; (ii) $\Pi(C) - \Pi(A) = \Delta\Pi = (1 - q_b - q_a)(1 - b)(1 - q_a)$; and (iii) $\Pi(D) - \Pi(B) = \Delta\Pi = 0$.

As welfare and profits never decrease, the remaining question is whether aggregate consumer surplus increases or not. Calculating the change in consumer surplus as $\Delta CS = \Delta W - \Delta\Pi$, the following results appear:

- Comparing (D) with (B), $CS(D) - CS(B) = 0$.

- Comparing (C) with (B), $\Delta CS = CS(C) - CS(B)$

$$\Delta CS = q_a(1 - q_a - q_b) + a q_b(1 - q_a) - b[q_a(1 - q_a - q_b) + q_b(1 - q_a)],$$

which is positive if

$$b \leq 1 - \frac{(1 - a)q_b(1 - q_a)}{q_a(1 - q_a - q_b) + q_b(1 - q_a)} = 1 - \frac{(1 - a)q_b(1 - q_a)}{(q_a + q_b)(1 - q_a) - q_a q_b},$$

- Comparing (C) with (A),

$$\Delta CS = CS(C) - CS(A) = -(1 - b)[1 - 2q_a + (q_a + q_b)^2] + 2q_b$$

which is positive if

$$b \geq 1 - \frac{2q_b}{1 - 2q_a + (q_a + q_b)^2}.$$

but it seems unlikely to hold.

Comparing this cutoffs we can state that there are always parameters where consumer surplus increases given the reduction in price.

⁶Two cases fit this description: (i) $v = 1$ buys Today and $v \in \{a, b\}$ Tomorrow with and without resale (comparing (D) with (B)); and (ii) a) $v \in \{1, b\}$ buy Today and $v = a$ Tomorrow with and without resale (comparing (C) with (A)).

Proposition 4. *If $1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)} \leq b < 1 - \frac{(1-a)q_b(1-q_a)}{(q_a+q_b)(1-q_a)-q_a q_b}$, then allowing for resale reduces Today's price and increase aggregated consumer surplus, profits, and, thus, welfare.*

Proposition 4 presents our key welfare results. Under certain conditions Today's price decrease so much that consumers' surplus increase on average by two different sources. First, a reduction in Today's price increases type $v = 1$ net surplus. Second, the allocation is more efficient; in particular, in our simple discrete example, there is an optimal allocation of the good: the highest valuation consumer receives the product with probability one.⁷ However, type b buyers now must pay a higher price for the product. Proposition 4 shows that there are cases where we have good news and aggregate consumer surplus increases as price decreases.

However, there are also cases where the reduction in price is not high, and consumer surplus decreases. This is proved in the following proposition.

Proposition 5. *If $1 - \frac{(1-a)q_b(1-q_a)}{(q_a+q_b)(1-q_a)-q_a q_b} \leq b < 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}$, then allowing for resale reduces Today's price but aggregated consumer surplus decreases.*

3 Continuous valuation model

We now extend our results to the continuous valuation case.

Suppose a two period game in which there is one seller who has one unit of an indivisible good (one ticket) to sell and $n = 2$ buyers with private information about their valuation for it. The good can be purchased in any of both periods but can only be consumed at the end of the second period. A non-traded unit is valueless after it. We assume that there is no discounting of time (i.e., the discount factor is equal to one).

Buyer's private valuation v is independently and identically distributed according to F . We normalize its support to $[v_0, v_1] = [0, 1]$. The distribution F is continuous and differentiable with its density f also continuous. We assume that f is log-concave. Players (buyers and seller) are risk neutral and maximize their expected surpluses. The production and opportunity costs are normalized to zero.

⁷this optimality is not a general result for the continuous case but, there is an efficiency gain in this direction.

The timing is as follows: at $t = 1$ (*Today*) the seller (who has no commitment to set prices in advance) announces a price p_1 . If only one buyer accepts the offer, he pays the price and receives the good. If two buyers accept the offer, a random tie-breaking rule allocates the good to one of them. If no buyer accept the offer, the game moves to the next period. At $t = 2$ (*Tomorrow*) there are two scenarios: (A) the seller still has the good or (B) one buyer has bought it. In the scenario (A) the seller has the good because no buyer has accepted the offer, in which case he announces a new price p_2 taking into account buyers' rejection, and the allocation rule is the same as the one described for period $t = 1$. In the scenario (B) one of the buyers has bought the good at $t = 1$. Without resale, the game ends. With resale, the buyer announces a resale price r that is accepted or not by the remaining buyer. After this the game ends.

3.1 Benchmark: resale is not allowed

If resale is not allowed we are in the environment of Hörner & Samuelson (2011). The seller profit at period $t = 1$ is

$$\max_{p_1} p_1 (1 - F(\bar{v})^2) + F(\bar{v})^2 p_2 \left(1 - \frac{F(p_2)^2}{F(\bar{v})^2}\right).$$

This profit is given by the price p_1 times the probability of acceptance (i.e., the probability that at least one of the buyers have $v \geq \bar{v}$) plus the continuation value $p_2 \left(1 - \frac{F(p_2)^2}{F(\bar{v})^2}\right)$ times the probability of rejection.

The value of \bar{v} is determined by the buyer type who is indifferent between buying Today and waiting until Tomorrow taking into account the other buyer and the future seller behavior.

When a buyer with valuation v accepts the price, he achieves

$$(v - p_1) \left(\frac{1 - F(\bar{v})}{2} + F(\bar{v}) \right),$$

and, when he rejects and waits for a lower price he expect to get

$$(v - p_2) \left(\frac{F(\bar{v}) - F(p_2)}{2} + F(p_2) \right).$$

Notice, that the buyer v takes into account the probability with which the remaining buyer also buys (in the first case) and the one with which his rival also waits until Tomorrow to buy the good.

Thus, the indifferent buyer \bar{v} is the one that satisfies

$$(\bar{v} - p_1) \left(\frac{1 - F(\bar{v})}{2} + F(\bar{v}) \right) = (\bar{v} - p_2) \left(\frac{F(\bar{v}) - F(p_2)}{2} + F(p_2) \right).$$

We look for symmetric strategies, then the problem of the seller at $t = 1$ is

$$\max_{p_1} p_1 (1 - F(\bar{v})^2) + F(\bar{v})^2 p_2 \left(1 - \frac{F(p_2)^2}{F(\bar{v})^2} \right), \quad \text{s.t.}, \quad (SP1)$$

$$(\bar{v} - p_1) \frac{1 + F(\bar{v})}{2} = (\bar{v} - p_2) \left(\frac{F(\bar{v}) + F(p_2)}{2} \right), \quad (IC)$$

$$p_2(\bar{v}) := \arg \max_{p_2} p_2 \left(1 - \frac{F(p_2)^2}{F(\bar{v})^2} \right). \quad (SP2)$$

We illustrate the case for the uniform distribution $[0, 1]$.

Example 6. *Benchmark Case.* Suppose v uniformly distributed on $[0, 1]$. Solving the benchmark problem, the seller makes in equilibrium $\pi^* = 0.400$, and posts prices $p_1^* = 0.579$ and $p_2^* = 0.479$. The indifference valuation is $\bar{v}^* = 0.829$.

3.2 Resale is allowed

The resale option slightly changes the problem of the seller.⁸ In particular, a buyer at $t = 1$ has the option to resell the product purchased in the next stage. Then, the expected utility of buying at $t = 1$ becomes:

$$(v - p_1) \left(\frac{F(r) + F(\bar{v})}{2} \right) + (r - p_1) \frac{1 - F(r)}{2},$$

⁸Adding units and seller complicates the model, as the seller and buyers that resell units compete eventually to attract new buyers. Of course, the competition is asymmetric as the opportunity and production costs differ, adding a new dimension to the problem

where r depends on buyer's valuation v , $r(v) := \arg \max_r [1 - F(r)](r - p_1) + [F(r) - F(\bar{v})](v - p_1)$. Then, the seller's problem becomes

$$\max_{p_1} p_1 (1 - F(\bar{v})^2) + F(\bar{v})^2 p_2 \left(1 - \frac{F(p_2)^2}{F(\bar{v})^2} \right), \quad \text{s.t.}, \quad (SPR1)$$

$$\frac{1 - F(r)}{2}(r - \bar{v}) + \frac{1 + F(\bar{v})}{2}(\bar{v} - p_1) = (\bar{v} - p_2) \left(\frac{F(\bar{v}) + F(p_2)}{2} \right), \quad (ICR)$$

$$p_2^* \equiv \operatorname{argmax}_{p_2} \left(1 - \frac{F(p_2)^2}{F(\bar{v})^2} \right) p_2, \quad (SPR2)$$

$$r^* \equiv \operatorname{argmax}_r [1 - F(r)](r - p_1) + [F(r) - F(\bar{v})](\bar{v} - p_1). \quad (RP)$$

Again, we illustrate using the uniform distribution $[0, 1]$.

Example 7. Extending for Resale Markets. Suppose we allow for a resale market. Now, the buyer who gets the good in $t = 1$ can resale it in $t = 2$. Hence, in $t = 2$ there are two possibilities: *i*) in case of not selling in $t = 1$, the seller solves in $t = 2$ the same problem than in the benchmark case. This is, the seller solves (SPR2). *ii*) in case the seller success in selling in period $t = 1$, the buyer who acquired the good can resale it, solving (RP). Now, the seller makes in equilibrium $\pi^* = 0.402$, and posts prices $p_1^* = 0.57$ and $p_2^* = 0.464$. The indifference valuation is $\bar{v}^* = 0.803$ and the reselling price of a consumer with valuation v is $r^*(v) = \frac{1+v}{2}$.

Comparing both examples we see that the seller makes larger profits under reselling, that the initial demand increases (the valuation that makes the buyer indifferent decreases), and that prices posted by the seller are lower in both periods.

In the next subsection, we study the robustness of the results illustrated in the examples.

3.3 Comparing the resale option effects

Notice first that the demand for a given price p_1 is always larger in the model under resale than in the benchmark case. The intuition is the following. In the benchmark case the accepting buyer makes surplus by consuming the good. However, in the model with resale, the accepting buyer can get a larger surplus by reselling the good than by consuming it. Thus, for every p_1 , the valuation of the indifferent buyer under reselling (for comparison purposes

denoted from now on as \bar{v}_R) must be at most the one without reselling. We collect this result in the following lemma.

Lemma 8. *For a given price p_1 , the indifferent buyer in the reselling case has a lower or equal valuation than in the benchmark case, i.e., $\bar{v}_R \leq \bar{v}$.*

Proof. Given a p_1 , the surplus that a consumer gets by rejecting it is the same for both cases, i.e., right hand side (RHS) in equation (IC) is the same than in equation (ICR). On the other hand, the consumer gets at least the same surplus by accepting the price in the resale case than in the benchmark one. Suppose the contrary. In this case, the buyer can always accept the price without putting the good on resale. Since buyer's surplus is increasing in his valuation, $\bar{v}_R \leq \bar{v}$ for a given price p_1 . \square

In the benchmark case, an increment of p_1 gives incentive to a buyer with valuation v to wait for a lower price. This effect is also present in the model with resale. However, in this case, an increment in p_1 also reduces the probability of reselling. As consequence, that buyer is now more sensitive to changes in prices.

Lemma 9. *Suppose a buyer with valuation v . This buyer is more sensitive to a change in prices in the reselling model than in the benchmark case; i.e., $\partial v / \partial p_1 |_{Resale} \geq \partial v / \partial p_1 |_{Benchmark}$.*

When resale is allowed there are two effects. First, there is an increase in demand in $t = 1$ since including resale increases the willingness to pay of the buyer (Lemma 8). Everything else equal, this effects induces the monopolist to ask for a larger price. Second, the buyer in the model with resale is more sensitive to changes in prices. Everything else equal, this second effect induces the monopolist to reduce the price in $t = 1$. These two effects impact price p_1 in opposite directions.

In our examples with the continuous distribution function, the latter effect dominates the former one and the optimal prices with resale are lower than in the benchmark case. Consequently, a reduction in price is motivated by a first order effect of increasing the price-elasticity of Today's probability of selling (or Today's demand). However, as we have seen in the discrete case, it is possible to find distribution functions under which this does not happen. Thus, the change in price at $t = 1$ is undetermined. So far we can establish the following result.

Proposition 10. *The presence of resale is profitable for the monopolist. Moreover, if $1 - F(v(p_1))^2$ is concave in p_1 , the resale option motivates a reduction in the optimal price p_1^* .*

When demand is concave in prices, then the effect on price-elasticity of Today's demand dominates an upward shift of Today's demand, motivating a reduction in price.

The welfare analysis is also undetermined. An increase in the probability of selling Today is driven by the fact that buyers with lower valuation buy Today. If this buyers buy the product, do not resale, and the other buyer has higher valuation, there is an inefficiency. A lower valuation buyer ends up with the product. Given the concavity of demand the impact on welfare is positive. For a given \bar{v} there is a probability of reallocating the good to a buyer with higher valuation that dominates negative effects.

4 Conclusions

We have presented a simple version to illustrate the impact of introducing a resale option into the revenue management problem of strategic buyers when there is a sequence of bargaining stages. We have shown that under reasonable parameters the first order effect in the seller's pricing strategy motivates a reduction in prices in the first stages of the game.

This results opens the door for further research. For instance, how this model is affected by the fact that the seller has several units, several periods, and/or a combination of both is not obvious. Allowing for resale may generate competition between the current seller and the reselling buyer/s.

A Proofs

Proposition 1 . Since $1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)} < 1 - \frac{(1-a)q_b}{(1-q_a)}$, if $b < 1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)}$ then $b < 1 - \frac{(1-a)q_b}{(1-q_a)}$; consequently, selling to $v = 1$ Today is optimal with and without resale. A buyer with $v = 1$ has no incentives to resale, implying no change in Today's willingness to pay. \square

Proposition 2 . If $1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)} < b$, there are two cases: (i) if $b \geq 1 - \frac{(1-a)q_b}{1-q_a}$ the seller prefers to sell Today to types $v = b$ and $v = 1$ with or without resale; thereby, resale affects a redistribution of surplus among

buyers, increasing their willingness to pay for the product that ultimately the seller appropriates with a higher prices. There is no effect on demand and only an effect on increasing the willingness to pay of type $v = b$ buyers. The price always goes up from $\frac{b+a q_a}{1+q_a}$ to $\frac{a q_a+b(q_a+q_b)+1-q_a-q_b}{1+q_a}$.

(ii) if $1 - \frac{(1-a)q_b}{1-q_a} > b > 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}$ the seller attracts type $v = 1$ to buy Today without resale, but she attracts types $v = b$ and $v = 1$ to buy Today with resale. This increment in demand is motivated by the increase in type $v = b$ willingness to pay due to the resale option. This increment is high enough to motivate the seller to increase the price. There are both effects on demand and on the willingness to pay of type $v = b$ buyers.

The price increases from $\frac{1+a(q_a+q_b)}{1+q_a+q_b}$ to $\frac{a q_a+b(q_a+q_b)+1-q_a-q_b}{1+q_a}$ if

$$\begin{aligned} \frac{a q_a + b(q_a + q_b) + 1 - q_a - q_b}{1 + q_a} &\geq \frac{1 + a(q_a + q_b)}{1 + q_a + q_b}, \\ b &\geq \frac{(q_a + q_b)^2 + (q_a + q_b) - (1 - a)q_b}{(q_a + q_b)^2 + (q_a + q_b)}, \\ b &\geq 1 - \frac{(1 - a)q_b}{(q_a + q_b)^2 + (q_a + q_b)}. \end{aligned}$$

□

Proposition 3. If $1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)} < b < 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}$, the seller attracts type $v = 1$ to buy Today without resale, but she attracts types $v = b$ and $v = 1$ to buy Today with resale. This increment in demand is motivated by the increase in type $v = b$ willingness to pay due to the resale option. This increment is not high, so the seller is motivated to reduce the price in such a way that type $v = b$ buys Today. This increase in the number of buyers Today motivate an increase in expected profits. The price decreases from $\frac{1+a(q_a+q_b)}{1+q_a+q_b}$ to $\frac{a q_a+b(q_a+q_b)+1-q_a-q_b}{1+q_a}$ if

$$\begin{aligned} \frac{a q_a + b(q_a + q_b) + 1 - q_a - q_b}{1 + q_a} &< \frac{1 + a(q_a + q_b)}{1 + q_a + q_b}, \\ b &< \frac{(q_a + q_b)^2 + (q_a + q_b) - (1 - a)q_b}{(q_a + q_b)^2 + (q_a + q_b)}, \\ b &< 1 - \frac{(1 - a)q_b}{(q_a + q_b)^2 + (q_a + q_b)}. \end{aligned}$$

□

Proposition 4. First notice that the set $1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)} \leq b < 1 - \frac{(1-a)q_b(1-q_a)}{(q_a+q_b)(1-q_a) - q_a q_b}$ is not empty:

$$\begin{aligned} 1 - \frac{(1-a)q_b}{(1-q_a)(q_a+q_b)} &< 1 - \frac{(1-a)q_b(1-q_a)}{(q_a+q_b)(1-q_a) - q_a q_b}, \text{ implies} \\ (1-q_a)^2(q_a+q_b) &< (q_a+q_b)(1-q_a) - q_a q_b, \\ 0 &< q_a(1-q_a-q_b). \end{aligned}$$

Then, we use the comparison in the consumer surplus variation and the result of Proposition 3 to have our result. \square

Proposition 5. First notice that the set $1 - \frac{(1-a)q_b(1-q_a)}{(q_a+q_b)(1-q_a) - q_a q_b} \leq b < 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}$ is not empty:

$$\begin{aligned} 1 - \frac{(1-a)q_b(1-q_a)}{(q_a+q_b)(1-q_a) - q_a q_b} &< 1 - \frac{(1-a)q_b}{(1+q_a+q_b)(q_a+q_b)}, \text{ implies} \\ (q_a+q_b)(1-q_a) - q_a q_b &< (1-q_a)(q_a+q_b)(1+q_a+q_b), \\ -q_a q_b &< (1-q_a)(q_a+q_b)(q_a+q_b). \end{aligned}$$

Then, we use the comparison in the consumer surplus variation and the result of Proposition 3 to have our result. \square

Lemma 8. Given a p_1 , the surplus that a consumer gets by rejecting it is the same for both cases, i.e. RHS in equation (IC) is the same than in equation (ICR). On the other hand, the consumer gets at least the same surplus by accepting the price in the resale case than in the benchmark one. Suppose the contrary. In this case, the buyer can always accept the price without putting the good on resale. Since buyer's surplus is increasing in his valuation, $\bar{v}_R \leq \bar{v}$ for a given price p_1 . \square

Lemma 9. From equation (IC) we get the following expression for p_1 ,

$$p_1(v) = \frac{v[1 - F(p_2)] + p_2[F(v) + F(p_2)]}{1 + F(v)}. \quad (6)$$

Similarly, from equations (ICR), (SPR2) and (RP), we get for the resale case

$$p_{1,R}(v) = p_1(v) + \frac{[1 - F(r)][r - v]}{1 + F(v)}. \quad (7)$$

Taking $\partial p_{1,R}/\partial v$,

$$\frac{\partial p_{1,R}}{\partial v} = \frac{\partial p_1}{\partial v} + \frac{(1 + F(v)) \left[\frac{\partial(1-F(r))}{\partial v} [r - v] + (1 - F(r)) \frac{\partial[r-v]}{\partial v} \right] - f(v)(1 - F(r))[r - v]}{(1 + F(v))^2}.$$

From (RP),

$$r^* = \frac{1 - F(r^*)}{f(r^*)} + v.$$

Notice that, since we have assume log-concavity of f , the first term is non-increasing in r^* (see Bagnoli & Bergstrom (2005)). Hence $\partial r^*/\partial v \in [0, 1]$ and $\partial[r - v]/\partial v \leq 0$.⁹ Additionally, $\partial[1 - F(r)]/\partial v \leq 0$. Therefore, $\partial p_{1,R}/\partial v \leq \partial p_1/\partial v$, or equivalently, $\partial v/\partial p_{1,R} \geq \partial v/\partial p_1$. \square

Proposition 10. To prove the first part, suppose $\bar{v}_R = \bar{v}$. Therefore, $1 - F(\bar{v}_R)^2 = 1 - F(\bar{v})^2$ and the monopolist's problem in the last period is the same in both environments. Hence, $p_{2,R} = p_2$ and, from Lemma 8, $p_{1,R} \geq p_1$. Thus, $\pi_R \geq \pi$ for any $\bar{v} \in [0, 1]$.

For the second part, we proceed in several steps.

- Suppose $t = 1$ and let's denote the monopolist's revenues for period as

$$R_1(v) = (1 - F(v)^2)p_1(v), \quad R_2(v) = (F(v)^2 - F(p_2(v))^2)p_2(v).$$

\square

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$\frac{\partial r^*}{\partial v} = \frac{\partial \frac{1-F(r^*)}{f(r^*)}}{\partial r^*} \frac{\partial r^*}{\partial v} + 1$, with $\frac{\partial \frac{1-F(r^*)}{f(r^*)}}{\partial r^*} \leq 0$ by log-concavity.

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