

# Dynamics of Some Iterated Games of Cooperation

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**Abstract.** The dynamics of iterated games have been widely studied by game theorists to examine strategic cooperation. In this paper, we examine the dynamics of iterated games of Prisoner's Dilemma, Stag-Hunt, and some other games that might be useful in modeling social contracts. We use computer simulations to investigate the relative success of various strategies in each case.

**Keywords:** Iterated Games, Iterated Prisoner's Dilemma, Social Dynamics

## EXTENDED ABSTRACT

## 1 Introduction

A standard question that game theorists have studied is how individuals who interact with each other repeatedly over a period of time modify and improve their strategies. In this regard, the techniques of evolutionary game theory have been a useful tool to study the dynamics of populations which over time adopt effective strategies and reduce the usage of ineffective ones to adapt to complex situations.[1]

We first examine the classical game of repeated prisoner's dilemma. We suggest some likely strategies for a population of  $M$  players, use computer simulations to identify which strategies work well after  $N$ -rounds, and report associated bounds on ratios of payoffs.

Similarly, we take a look at the case of an iterated stag-hunt game and derive conclusions on the efficacy of various strategies and relative payoffs. Finally, we simulate a two-player game that models the strategies in an election game and, once again, observe the performance of various strategies against each other.

## 2 Models

### 2.1 Prisoner's dilemma

The iterated Prison's dilemma problem has been widely studied by game theorists. Consider the following two-person game shown below. [2] [4][5] Clearly, this game models a prisoner's dilemma game. Here C and D represent cooperate and defect respectively.

		Player 2	
		C	D
Player 1	C	(3, 3)	(0, 5)
	D	(5, 0)	(1, 1)

Consider a population with  $M$  players who can pick one of the following strategies:

- The player can choose to cooperate always
- The player can choose to defect always.
- The player randomly chooses to defect or cooperate.
- The player prefers to cooperate. However, if the other player chooses to defect, she will also defect.
- The player prefers to defect. However, if the other player choose to cooperate, she will also choose cooperate.
- The player prefers to defect. However, the move that he picks is the opposite of his opponent's most recent move.
- The player wants to choose cooperate. However, he picks a move that is the opposite of his opponent's most recent move.

We wrote a program in Maple to simulate a tournament in which a population of  $M$  players choose between these strategies and compete with each other  $N$  times to see which strategies are the best and by how much the payoffs of the best strategies outperform the worst strategies. For small values of  $M$ , with all strategies being equally likely to be chosen initially, we found that if the players competed against each other for 10000 rounds, defecting always was the best strategy while cooperating always was the worst and the payoff from the best strategy was at least three times as much the payoff from the worst. We repeat the experiment for different values of  $M$ , while varying the proportion of the population using each of the seven strategies.

### 2.2 Stag-Hunt

The Stag-Hunt game is one of the classic cooperation games in game theory.[3] Consider the following two-person game shown below; clearly, this models a Stag-Hunt game.

		Player 2	
		C	D
Player 1	C	(5, 5)	(0, 3)
	D	(3, 0)	(3, 3)

We repeated the simulations from the previous section on Prisoner's Dilemma with the same set of strategies. For small values of M with all strategies being equally likely to be chosen initially, we found that if the players competed against each other for 10000 rounds, defecting always was the best strategy while confessing always was the worst and the payoff from the best strategy was at least 1.5 times as much the payoff from the worst. We also repeat the experiment for different values of M, while varying the proportion of the population using each of the seven strategies. However, when we replaced the payoff in the first row and first column of the matrix with (10,10), the best and worst strategies changes as did the payoffs. We intend to repeat our simulations with different values in the payoff matrix, to draw more more meaningful conclusions.

### 2.3 Election Game

Suppose two high school students, Alice and Bob, are both competing against each other in a very close race to be the president of their school's Math Club. Their classmate, Charlie, who is looking to use the elections for his person gain approaches Alice and makes the following proposition. In exchange for 100 dollars, he offers to fabricate evidence of Bob bad-mouthing the Math Club. To increase his odds of a payout, he approaches Bob with a similar proposition. Alice and Bob suspect that Charlie has reported their discussions to his best friend, Danielle.

There are now three possible outcomes for Alice (and for Bob). She can choose to reject Charlie's offer and hope that she wins based on the issues that she has campaigned on. Alternatively, she can accept Charlie's offer. This, in turn, can go two ways. One outcome is that the fabricated evidence dooms Bob's chances in the election and Alice gets the result that she wants. However, if Danielle reveals Alice's plot to the voters, not only will Alice lose the race, but she might also have to face disciplinary action. The payoffs can be modeled by the following matrix. Here R refers to the situation when Charlie's offer is rejected, AS depicts the situation when the offer is accepted and Danielle stays silent about the plot, and AC depicts the situation when the offer is accepted and Danielle reveals the plot to the voters.

After talking to former presidents of the club who had also faced a similar predicament, Alice and Bob individually come up with the following strategies:

- Always reject Charlie's offer.
- Always accept Charlie's offer.
- They random choose whether to accept or reject the offer.

		Player 2		
		<i>R</i>	<i>AS</i>	<i>AC</i>
Player 1	<i>R</i>	(0, 0)	(-2, 2)	(0, -3)
	<i>AS</i>	(2, -2)	(-1, -1)	(3, -3)
	<i>AC</i>	(-3, 0)	(-3, 3)	(-3, -3)

- Initially, they reject the offer but if they find out that their opponent has accepted the offer, they also accept the offer.
- Initially, they accept the offer but if they find out that their opponent has accepted the offer, they reject the offer and prefer to expose their opponent's scheme.
- Initially, they accept the offer but if they find out that their opponent has rejected the offer, they also reject the offer.
- Initially, they reject the offer but if they find out that their opponent has rejected the offer, they accept the offer.

Once again, use our computer model to simulate repeated interaction of Alice and Bob and to see how the strategies perform over generations of class presidents for a population of M players after N rounds.

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