Competing Mechanisms: Communication and Equilibrium Robustness

Andrea Attar* Eloisa Campioni† Gwenaël Piaser ‡

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Abstract

We study competing mechanism games with principals simultaneously designing contracts to deal with agents. Following Epstein and Peters (1999), the traditional approach to characterize equilibrium mechanisms focuses on increasing the complexity of the agents' message spaces to incorporate all the relevant market information generated by the competing principals. Principals, instead, could not send any signal to agents. Along the lines of Myerson (1982), we extend the traditional approach to allow for principals' communication. We focus on complete information settings, and show by means of three examples that the restriction to one-sided communication mechanisms involves a loss of generality.

*Toulouse School of Economics and University of Rome Tor Vergata, e-mail: andrea.attar@tse-fr.eu
†University of Rome Tor Vergata, e-mail: eloisa.campioni@uniroma2.it
‡Ipag Business School Paris, e-mail: piaser@gmail.com

1 Introduction

We study competing mechanism games: several principals simultaneously design contracts to deal with multiple agents. Such a strategic scenario has become a reference framework to model competition in a large number of market settings.[1]

As originally pointed out in the seminal works of McAfee (1993) and Peck (1997), the equilibrium allocations derived in these contexts crucially depend on the set of mechanisms that are made available to principals. For example, letting agents communicate to principals additional information on top of their "exogenous" type typically generates additional equilibrium outcomes.[2] This raises the issue of identifying a class of mechanisms that allow players to communicate all relevant market information. In an important contribution, Epstein and Peters (1999) have constructed a general communication device that makes it possible to incorporate any sophisticated mechanism. The corresponding mechanisms require each agent to send messages from a *universal*, potentially very large, type space. Epstein and Peters (1999) show that there is no loss of generality in restricting principals to such mechanisms.[3] A prominent feature of this approach is that communication is one-sided, that is, principals are do not send any (public or private) signal to agents. This restriction has been postulated in the subsequent competing mechanism literature, both in complete and incomplete information settings, and is at the centre stage of economic applications.

In general terms, the assumption that a single principal cannot recommend agents the behavior they should take vis-a-vis his opponents appears to be debatable, being in contrast with the traditional insights of single-principal mechanism design theory (Myerson (1982)). The present paper investigates this issue in greater detail. That is, we focus on *complete information* competing mechanism games, in which agents' actions are observable and can be contracted upon. In such a scenario, which has been extensively analyzed in economic applications, we show that the restriction to one-sided communication mechanisms involves a loss of generality. We develop our arguments by means of three examples.

Example 1 shows that equilibria supported by pay-for-effort contracts do not survive if principals can deviate to two-sided communication mechanisms that involve private signals to agents. By sending private signals, a single principal can make agents differently informed about his decisions. This transforms the agents' effort game into an incomplete information game and allows to sustain continuation equilibria that cannot be supported by means of one-sided communication mechanisms. The result suggests that there is little rationale to

[1] See Peters (2014) for a recent survey.

[2] This result, which has been documented in single agent contexts by Martimort and Stole (2002) and Peters (2001), is often acknowledged as a "failure" of the revelation principle in games with multiple principals.

[3] See Theorem 3.1 in Epstein and Peters (1999).

limit principals to simple pay-for-effort contracts, as it has been done in most applications.[4] On a more theoretical ground, the example shows that the robustness result of Han (2007) does not hold if two-sided communication is allowed.

Example 2 further investigates the folk-theorem results exhibited by Yamashita (2010).[5] Specifically, we first show, along the lines of Yamashita (2010), how a multiplicity of equilibrium allocations may be supported in a complete information scenario by letting principals use recommendation mechanisms. Following a principal's unilateral deviation, the recommendation mechanisms posted by his competitors are sufficiently flexible to allow agents to coordinate on recommending them to issue contracts which implement the largest available punishment. This in turn supports multiplicity: as in Theorem 1 of Yamashita (2010), any incentive compatible allocation can be supported in a pure strategy equilibrium. Yet, in the context of our example, none of these allocations survives against a deviation to a well chosen two-sided communication mechanism. The intuition is similar to that of Example 1 and exploits the power of a deviating principals to correlate agents' actions by privately communicating with them.

Since two-sided communication cannot in general be prevented, the results above suggest that pure strategy equilibria supported by one-sided communication may not survive the introduction of private communication by principals. This calls for the characterization of a "safe" class of mechanisms supporting robust equilibria. To be relevant for applications, such mechanisms must be sufficiently simple and tractable. In this respect, a natural candidate is the class of direct mechanisms introduced by Myerson (1982) for generalized principal-agent problems. In complete information competing mechanism settings, they correspond to two-sided mechanisms in which the set of signals available to each principal coincides with the set of agents' actions, and agents send no relevant message to principals.

Example 3 therefore focuses on the game in which two principals are restricted to use such direct mechanisms, and characterizes a pure strategy equilibrium. However, we show that the corresponding allocation does not survive when principals are allowed to use arbitrary sets of signals. In the example, using a larger signals' space allows the deviating principal to induce agents to play different correlated equilibria for different signals they may receive from the other principal.

Overall, our results suggest that further work is needed to assess the role of two-sided communication in competing mechanism settings. This appears to be a relevant issue for most economic applications, and a fundamental task to provide a general characterization of equilibrium mechanisms.

[4] See Prat and Rustichini (2003) among others.

[5] See also Szentes (2009), and Xiong (2013).

2 The Model

We study competing mechanism games of complete information in which the agents' actions are observable.

There are $J \geq 2$ principals dealing with $I \geq 2$ agents. Each agent $i = 1, 2, \cdots, I$ takes an effort e^i from a finite set E^i, with $e = \left(e^1, e^2, \ldots, e^I\right) \in E = \underset{i=1}{\overset{I}{\times}} E^i$. Let Y_j be the set of actions available to principal j and $y_j \in Y_j$ a generic element of that set, and let $Y = \underset{j=1}{\overset{J}{\times}} Y_j$. The functions $u_i : E \times Y \to \mathbb{R}$ and $v_j : E \times Y \to \mathbb{R}$ denote the payoff to agent i and to principal j, respectively.

Agents' efforts are observable, so each principal j can choose an action y_j contingent on the array of efforts e. Final allocations are determined by the public mechanisms that principals simultaneously commit to.

To model communication, we refer to the canonical protocol introduced by Myerson (1982) for the analysis of generalized principal-agent models. That is, we here allow for *two-sided* communication between principals and agents. Each agent i sends a private message $m_j^i \in M_j^i$ to each principal j. Given the messages received from the agents, $m_j \in M_j = \underset{i=1}{\overset{I}{\times}} M_j^i$, each principal j sends a private signal $r_j^i \in R_j^i$ to each agent i. A mechanism for principal j is the mapping $\gamma_j : E \times M_j \to \Delta\left(Y_j \times R_j\right)$, where $R_j = \underset{i=1}{\overset{I}{\times}} R_j^i$ and $\Delta\left(Y_j \times R_j\right)$ is the set of probability distributions over $Y_j \times R_j$. Let Γ_j be the set of mechanisms available to principal j, and denote $\Gamma = \underset{j=1}{\overset{J}{\times}} \Gamma_j$. Throughout the paper, we will refer to $\gamma_j \in \Gamma_j$ as to a mechanism with signals, and to $\delta_j \in \Delta(\Gamma_j)$ as a probability distribution over such mechanisms.

As in Peters (2001) and Han (2007), we assume that, whenever a measurable structure is needed, the corresponding Borel sigma-algebra is used.

Mechanisms are publicly observed, but the message from agent i to principal j and the signal from principal j to agent i are only observed by i and j. Since signals are private, a principal can induce a correlated equilibrium in the continuation game in which agents choose efforts. There are two stages at which agent i moves in the game. First, having observed the mechanisms $\gamma = (\gamma_1, \ldots, \gamma_J)$, she sends an array of messages $m^i = (m_1^i, \ldots, m_J^i)$ to the principals. The efforts e and vector m_j of messages received by principal j determine the lottery $\gamma_j(m_j, e)$ over actions and signals, and we denote $\gamma(m, e) = \left(\gamma_1(m_1, e), \gamma_2(m_2, e), \cdots, \gamma_J(m_J, e)\right)$. Second, having observed her private signals $r^i = (r_1^i, \ldots, r_J^i)$, agent i then chooses an effort $e^i \in E^i$.

Given Γ, we take $\mu^i : \Gamma \to \Delta\left(M^i\right)$ to be the message strategy of agent i, and $\eta^i : \Gamma \times M^i \times R^i \to \Delta\left(E^i\right)$ to be her strategy in the effort game, with $M^i = \underset{j=1}{\overset{J}{\times}} M_j^i$ and

$R^i = \underset{j=1}{\overset{J}{\times}} R^i_j$. We let $\beta^i = (\mu^i, \eta^i)$ be agent i's overall continuation strategy, and we denote $\beta = (\beta^1, \ldots, \beta^I)$. The mechanisms posted by principals, γ, and the agents' strategies β, induce a probability distribution $z_\beta(.;\gamma)$ over the space $Y \times E$. Let the expected utilities of the players be denoted as $U^i(\gamma, \beta)$ for agent i and $V_j(\gamma, \beta)$ for principal j. We denote G^Γ the game in which agents send messages to principals through the sets $(M^1, M^2, ..., M^I)$ and principals post mechanisms $\gamma \in \Gamma$, and send signals to agents through the sets $(R_1, R_2, ..., R_J)$. As usual, we choose Perfect Bayesian Equilibrium (PBE) as the relevant solution concept.[6]

One should observe that if principals are not allowed to send any signal to agents, and communication is embedded in the (virtually large) message spaces $(M^1, M^2, ..., M^I)$, the model above reduces to the standard competing mechanism framework first analyzed by Epstein and Peters (1999). We denote the corresponding game, in which each set of signals is a singleton, G^{Γ^M}.

Along the lines of Peters (2001), we formalize the notion of a robust equilibrium in a game G^Γ as follows. For any pair $\left(\Gamma, \Gamma'\right)$, we say that $\Gamma' \succeq \Gamma$, that is Γ' is "larger" than Γ, if there exists an embedding $\alpha : \Gamma \to \Gamma'$. Let Γ be the set of available mechanisms in the game G^Γ, and consider a continuation equilibrium $\beta = (\mu, \eta)$. With reference to the game $G^{\Gamma'}$ with $\Gamma' \succeq \Gamma$, we say that the agents' continuation equilibrium $\beta' = (\mu', \eta')$ extends β if there is an embedding α such that, for every $\gamma \in \Gamma$, we have $z_\beta(.;\gamma) = z_{\beta'}(.;\alpha(\gamma))$. Thus, an equilibrium (β, γ) of G^Γ is *robust* if, for every $\Gamma' \succeq \Gamma$ there exists an extension β' such that $(\beta', \alpha(\gamma))$ constitutes a PBE for the game $G^{\Gamma'}$. That is, the original equilibrium survives to a unilateral deviation of a principal toward a more sophisticated mechanism if there exists *at least one* continuation equilibrium of the enlarged game which makes the deviation unprofitable.

This paper studies the robustness of pure strategy equilibria relative to different classes of competing mechanism games when principals' enlarged set of mechanisms involve two-sided communication.

[6] To simplify exposition, we did not explicitly consider the case in which principals play mixed strategies, by randomizing over mechanisms. This can accomplished by appropriately redefining the relevant expected payoffs.

3 Robust Equilibria and pay-for-effort contracts

This section considers the class of complete information competing mechanism games G^{Γ^M}, in which communication is one-sided. That is, a mechanism for principal j is a mapping $\tilde{\gamma}_j : M_j \times E \longrightarrow \Delta(Y_j)$ associating any array of agents' messages and efforts to a probability distribution over his decisions.

This framework is at the center stage of several economic applications, as documented by Prat and Rustichini (2003) among others. In particular, it can be seen as an extension of the traditional models of lobbying of Dixit et al. (1997) and Bernheim and Whinston (1986) to multiple agency contexts. As pointed out by Han (2007), all these economic applications share the restriction to a simple class of mechanisms for principals. That is, principals are only allowed to post pay-for-effort contracts, that specify a (possibly stochastic) allocation for every array of observed efforts. Competition between principals is therefore framed in a communication-less game, which we refer to as G^0. It is therefore a relevant question from the viewpoint of economic applications to understand whether the equilibria of the game G^0 survive when principals deviate to more complex mechanisms involving some communication.

Theorem 1 of Han (2007) provides a positive answer to this question, identifying a set of equilibria that are robust against unilateral deviations to mechanisms involving one-sided communication. These are the *pure strategy strongly robust* equilibria of G^0, that is, the PBE in which no principal has a profitable deviation to any $\tilde{\gamma}_j$, *regardless of the continuation equilibrium selected by agents.* The result does not naturally extend to mixed strategy equilibria as shown in Example 1 of Han (2007).

We revisit this issue by checking whether these equilibria further survive when principals deviate to the richer class of mechanisms with signals, identified in Section 2. This is a rather natural requirement, to the extent that preventing principals from sending private signals to agents appears hard to justify.

The following example exhibits a strongly robust equilibrium in the communication-less game G^0 which does not survive against deviations to mechanisms involving two-sided communications.

Example 1. Consider a complete information setting with four players: two principals, $P1$ and $P2$, and two agents, $A1$ and $A2$, who take contractible actions in the sets $E^1 = E^2 = \{a, b\}$. Let $P1$'s decision set be $Y_1 = \{x_1, x_2\}$, and $P2$'s one be $Y_2 = \{y_1, y_2\}$.

The payoffs of this game are represented in the matrix below, where the first two numbers in each cell denote the payoff to P1 and P2, respectively, and the remaining two numbers denote the payoffs to $A1$ and $A2$.

		y_1			y_2	
		a	b		a	b
x_1	a	$(1,1,1,1)$	$(-1,-1,-1,-1)$	a	$(1,2,0,0)$	$(-1,-1,0,0)$
	b	$(-1,-1,-1,1)$	$(1,1,-1,-1)$	b	$(1+\varepsilon,-1,-2,\zeta)$	$(1,0,1,-2)$
		a	b		a	b
x_2	a	$(2,1,0,0)$	$(-1,-1,0,-1)$	a	$(2,2,-1,-1)$	$(-1,-1,-2,0)$
	b	$(-1,-1,-1,0)$	$(0,1,-1,-1)$	b	$(-1,-1,-2,-2)$	$(1,0,1,-2)$

in which $\varepsilon > 0$ and $\zeta > 0$.

We first consider the game G^0, in which there is no communication, and a strategy for principal j is the pay-for-effort contract $\alpha_j : E^1 \times E^2 \longrightarrow \Delta(Y_j)$.

Claim 1 *The principals' strategies $\alpha_1(e^1, e^2) = x_1$ and $\alpha_2(e^1, e^2) = y_2$ for every effort array $(e^1, e^2) \in E^1 \times E^2$, and the agents' strategies that prescribe to choose $e^1 = e^2 = a$ upon observing (x_1, y_2) support a strongly-robust equilibrium in G^0, with corresponding outcome $(1, 2, 0, 0)$.*

Proof. To show this, observe that the strategies (α_1, α_2) induce the following continuation game over efforts:

	a	b
a	$(0,0)$	$(0,0)$
b	$(-2,\zeta)$	$(1,-2)$

in which (a, a) is the only Nash equilibrium. We show that no principal has any profitable deviation. Indeed, we only have to consider P1's deviations, since P2 achieves his highest payoff of 2. A deviation of P1 can be represented by an array of lotteries contingent on agents' efforts. Let these be $(\delta_{11}, \delta_{12}, \delta_{13}, \delta_{14})$ with $\delta_{11} = prob(x_1|a, a)$, $\delta_{12} = prob(x_1|a, b)$, $\delta_{13} = prob(x_1|b, a)$ and $\delta_{14} = prob(x_1|b, b)$, with $\delta_{1i} \in [0, 1]$ for every $i = 1, \dots, 4$. Given α_2, the agents' continuation game induced by P1's deviation is:

	a	b
a	$-(1-\delta_{11}), -(1-\delta_{11})$	$-2(1-\delta_{12}), 0$
b	$-2, \zeta\delta_{13} - 2(1-\delta_{13})$	$1, -2$

For any such deviation to be profitable, the agents' continuation equilibrium must put a positive probability upon playing (a, a) or (b, a). The following cases should therefore be considered:

(i) $\delta_{11} = 1$, $\delta_{13} \in (0, 1]$. In this case, (a, a) is the unique continuation equilibrium. However, P1 chooses x_1 deterministically, which does not constitute a profitable deviation.

(ii) $\delta_{11} \in [0, 1)$ $\delta_{13} = 0$. In this case, (b, b) is the unique equilibrium, yielding P1 his equilibrium payoff of 1.

(iii) $\delta_{11} = 1$, $\delta_{13} = 0$. In this case, we have to consider mixed strategy equilibria. Observe that for A2 a is a weakly dominated, hence it is not in the support of any mixed strategy equilibrium. Anticipating that A2 will play b deterministically, A1's best reply is to choose b. Hence, this deviation is not profitable either.

To summarize: if principals play (α_1, α_2) agents play (a, a) in the unique continuation equilibrium; furthermore, if a principal deviates towards any alternative pay-for-effort contract the unique continuation equilibrium in the agents' effort game is such that the deviation is not profitable. Taken together, these features imply that the pure strategy equilibrium we characterize is strongly robust.[7]

One can therefore apply Theorem 1 in Han (2007) to conclude that this equilibrium survives in any one-sided communication game G^{Γ^M}. That is, for a given message space M_j, principal j cannot profitably deviate to any mechanism $\tilde{\gamma}_j : M_j \times E \longrightarrow \Delta(Y_j)$.

We now show that P1 can profitably deviate using a two-sided communication mechanism.

Claim 2 *If P2 keeps playing α_2, then P1 can profitably deviate by sending private signals to agents.*

Proof. We establish the result considering the simple case in which $M_1^1 = M_1^2 = \{m\}$ and $R_1^1 = R_1^2 = \{a, b\}$. The deviating mechanism $\gamma_1 : E \longrightarrow \Delta(Y \times E)$ for P1 is such that he plays (x_1, b, a) with probability $\pi \in (0, 3/5)$, and (x_2, b, b) with the complementary probability $(1 - \pi)$ *for every (e^1, e^2) pair.*

Following the deviation to γ_1, $A1$ remains uninformed about P1's decision, since she always receives the same signal, while A2 gets perfectly informed of P1's choice once she

[7] Whenever each relevant sub-game features only one continuation equilibrium the notion of strong robustness collapses to the general one we defined in Section 2, typically denoted as weak robustness, see Peters (2001).

gets her signal. We prove that the unique continuation equilibrium in the induced effort game corresponds to each agent playing according to P1's signal. This in turn supports a profitable deviation for P1.

Consider the agents' continuation game. We first show that it admits a pure strategy equilibrium in which A1 plays b, and A2 plays $a(b)$ when she observes the signal $a(b)$.

When receiving the signal a, A2 knows that P1 chooses x_1 with probability one. In the cell (x_1, y_2), a is a weakly dominant strategy for A2. When receiving the signal b, A2 knows that P1 chooses x_2 with probability one. In the cell (x_2, y_2), b is a weakly dominant strategy for A2. We therefore only need to show that it is optimal for A1 to choose b when A2 plays according to the received signals. By playing b, A1 gets $-2\pi + (1 - \pi) = 1 - 3\pi$, while she gets $-2(1 - \pi)$ when playing a. Playing b is therefore a best reply since $\pi < 3/5$.

We now show that this continuation equilibrium is unique. First, it is immediate to verify that this is the only equilibrium in which A1 plays b. Indeed, if A2 plays a when receiving the signal b, then A1 would rather prefer to play a. Next, we remark that there is no equilibrium in which A1 plays a. Suppose this were the case, that is, A1 plays a at equilibrium. In this case, A2 strictly prefers playing b upon receiving the signal b. Given such a behavior, the unique optimal choice for A1 is to choose b regardless of the effort chosen by A2 when she gets the signal a. This constitutes a contradiction and establishes the result. ■

4 Folk Theorems and two-sided communication

Economic applications of competing mechanism games typically exhibit a multiplicity of Pareto-ranked equilibrium allocations.[8] The recent work of Yamashita (2010) provides a theoretical foundation for these results, identifying a class of *one-sided* communication mechanisms that support a folk theorem-like result.

Specifically, Yamashita (2010) refers to a G^{Γ^M} game in which every agent uses the entire space of pay-for-effort contracts to communicate with every principal. Hence, letting $\mathcal{A}_j$ be the set of pay-for-effort contracts available to principal j, we have $M_j^i = \mathcal{A}_j$ for every $j \in J$ and $i \in I$. In this game, Yamashita (2010) focuses on recommendation mechanisms, that is, mappings $\tilde{\gamma}_j^R : (\mathcal{A}_j)^I \times E \longrightarrow \Delta Y_j$ such that given the reports $m_j^i \in \mathcal{A}_j$ of every agent $i \in I$

[8]Under complete information, this feature is shared by single agent settings (Bernheim and Whinston (1986)) and Dixit et al. (1997) and multiple agent ones (Prat and Rustichini (2003)).

to principal j,

$$\tilde{\gamma}_j(m_j^i, \ldots, m_j^I) = \begin{cases} \alpha_j & \text{if } |\left\{i : m_j^i = \alpha_j\right\}| \geq I - 1 \\ \text{any } \bar{\alpha}_j \in \mathcal{A}_j & \text{otherwise.} \end{cases}$$

That is, in a recommendation mechanism, agents' messages to any principal j determine the incentive scheme offered by that principal, provided that coordination is high enough. Theorem 1 of Yamashita (2010) shows that every incentive compatible allocation that yields each principal a payoff above a given threshold can be supported at equilibrium. Under complete information, the intuition for the result can be resumed as follows: if a principal unilaterally deviates towards a simple pay-for-effort contract, agents coordinate on recommending to the non deviating principals contracts that implement the necessary punishment.[9] A relevant question from the viewpoint of economic applications is whether such a folk-theorem result survives the introduction of two-sided communication mechanisms.

To answer this question, we construct an example of a G^{Γ^M} game in which recommendation mechanisms support every incentive compatible allocation at a pure strategy equilibrium. We then show that none of them survives against deviations to a well chosen two-sided communication mechanism.

Example 2. Consider a complete information setting with five players: two principals, $P1$ and $P2$, and three agents, $A1$, $A2$ and $A3$, who take contractible actions in the sets $E^1 = E^2 = E^3 = \{a, b\}$. Let $P1$'s decision set be $Y_1 = \{x_1, x_2\}$, and $P2$'s one be $Y_2 = \{y_1, y_2\}$.

We assume that A3 strictly prefers to choose a, irrespective of the principals' decisions and of the other agents' actions. The payoffs corresponding to A3 choosing a are represented in the matrix below, where the first two numbers in each cell denote the payoff to P1 and P2, respectively, and the remaining three numbers denote the payoffs to A1, A2 and A3.

		y_1			y_2	
		a	b		a	b
x_1	a	$(2, 10, 10, 5, 1)$	$(2, \zeta, 3/2, 8, 1)$	a	$(2, 0, 0, 0, 1)$	$(2, \zeta, 0, 8, 1)$
	b	$(2, 0, 0, 0, 1)$	$(2, \zeta, 0, 10, 1)$	b	$(2, 5, 5, 5, 1)$	$(2, \zeta, 1, -1, 1)$
		a	b		a	b
x_2	a	$(2, 10, 10, 5, 1)$	$(2, \zeta, 3/2, 8, 1)$	a	$(2, 0, -1, 4, 1)$	$(2, \zeta, -1, 8, 1)$
	b	$(2, 3, 5, 5, 1)$	$(2, \zeta, -1, 4, 1)$	b	$(2, 5, 0, 0, 1)$	$(2, \zeta, 0, -1, 1)$

[9] This logic extends to multiple agency the role played by menus and latent contracts under common agency (Martimort and Stole (2002)).

In the matrix, in which the payoffs to $P1$ and $A3$ are constantly equal to 2 and to 1 respectively, we let $\zeta < 0$. Following Yamashita (2010), we restrict attention to *incentive compatible* allocations. In our complete information setting, an allocation $(\alpha_1(e), \alpha_2(e), e^1, e^2, e^3)$ is incentive compatible if (e^1, e^2) is a pure strategy equilibrium in the continuation game induced by the incentive scheme $(\alpha_1(.), \alpha_2(.))$.

Claim 3 *There is no incentive compatible allocation yielding P2 a payoff strictly greater than* 5.

Proof. Consider an arbitrary profile of pay-for-effort contracts, $\alpha_j : E^1 \times E^2 \to \Delta(Y_j)$ with $j = 1, 2$. Specifically, denote $\delta_1 = prob(x_1|a, a)$ the probability with which P1 chooses x_1 given the effort pair (a, a), and, similarly, denote $\delta_2 = prob(x_1|a, b)$, $\delta_3 = prob(x_1|b, a)$, and $\delta_4 = prob(x_1|b, b)$. As for P2, denote $\sigma_1 = prob(y_1|a, a)$, $\sigma_2 = prob(y_1|a, b)$, $\sigma_3 = prob(y_1|b, a)$ and $\sigma_4 = prob(y_1|b, b)$. The induced continuation game among A1 and A2 is given by:

	a	b
a	$10\sigma_1 - (1-\delta_1)(1-\sigma_1)$, $5\sigma_1 + 4(1-\delta_1)(1-\sigma_1)$	$\frac{3}{2}\sigma_2 - (1-\delta_2)(1-\sigma_2), 8$
b	$5(1-\sigma_3)\delta_3 + 5\sigma_3(1-\delta_3)$, $5(1-\sigma_3)\delta_3 + 5\sigma_3(1-\delta_3)$	$\delta_4(1-\sigma_4) - \sigma_4(1-\delta_4)$, $10\delta_4\sigma_4 + 4\sigma_4(1-\delta_4) - (1-\sigma_4)$

which we can rewrite as

	a	b
a	$11\sigma_1 + \delta_1(1-\sigma_1) - 1$, $\sigma_1 + 4(1-\delta_1+\delta_1\sigma_1)$	$\frac{5}{2}\sigma_2 + \delta_2(1-\sigma_2) - 1, 8$
b	$5(\sigma_3+\delta_3) - 10\sigma_3\delta_3$, $5(\sigma_3+\delta_3) - 10\sigma_3\delta_3$	$\delta_4(1-\sigma_4) - \sigma_4(1-\delta_4)$, $\sigma_4(6\delta_4+5) - 1$

Table 1: The efforts' continuation game

To establish the claim, it is enough to observe that, in order for P2 to get a payoff strictly above 5, principals' mechanisms should be designed to induce a (pure strategy) continuation equilibrium in which agents choose (a, a). Yet, observe that A2 strictly prefers to choose b rather than a whenever $\sigma_1 + 4(1 - \delta_1 + \delta_1\sigma_1) < 8$, which can be rewritten as $\sigma_1(1 + 4\delta_1) < 4(1 + \delta_1)$. It is immediate to check that the inequality is satisfied for every $\delta_1 \geq 0$, and $\sigma_1 \geq 0$. Thus, there is no profile of pay-for-effort contracts inducing a continuation equilibrium yielding P2 a payoff strictly greater than 5. ■

Remark 1 *Although P2 cannot achieve a payoff greater than 5 through pay-for-effort contracts, the set of incentive compatible allocations is nonempty. In particular, it includes the allocations* $(\alpha_1(e) = x_2, \alpha_2(e) = y_1, e^1 = a, e^2 = b)$*. One can check that, given* α_1 *and* α_2*, it is a continuation equilibrium for A1 to plays a and for A2 to play b which implements the payoff profile* $(2, \zeta, 3/2, 8, 1)$*, yielding P2 her lowest utility* ζ*. It also contains the allocation* $(\alpha_1(e) = x_1, \alpha_2(e) = y_2, e^1 = b, e^2 = a)$*. One can also check that, given* α_1 *and* α_2*, it is a continuation equilibrium for A1 to plays b and for A2 to play a, and this implements the payoff profile* $(2, 5, 5, 5, 1)$*, yielding P2 her highest incentive compatible payoff* 5.

Claim 4 *Every incentive compatible allocation yielding at least* ζ *to P2, can be sustained in a pure strategy equilibrium of the the game* G^{Γ^M} *in which* $M_j^i = \mathcal{A}_j$ *for every* $j \in J$ *and* $i \in I$*.*

Proof. Let each principal j uses the recommendation mechanism $\tilde{\gamma}_j^R$ defined above. To show that any incentive compatible allocation yielding each principal a payoff above a given threshold can be sustained at equilibrium, we first establish the following Lemma.

Lemma 1 *If P1 plays a given recommendation mechanism, for every pay-for-effort contract posted by P2 there exists a continuation equilibrium in the agents' game which yields* ζ *to P2.*

Proof of Lemma 1. Let $\tilde{\gamma}_1^R$ be the recommendation mechanism played by P1 with $(\delta_1, \delta_2, \delta_3, \delta_4)$ being the probability distributions over $\{x_1, x_2\}$ associated to every possible combination of agents' efforts. Letting $(\sigma_1, \sigma_2, \sigma_3, \sigma_4)$ be the probability distributions associated to a given pay-for-effort contract α_2, agents play the continuation game described in Table 1.

First, we show that there exists an array $(\delta_1, \delta_2, \delta_3, \delta_4)$ such that (a, b, a) is a equilibrium of that game. Specifically, consider any randomization such that $\delta_2 = 1$ and $\delta_4 \leq \sigma_4$. Then, if A1 plays a, A2 will choose b because $8 > 5\sigma_1 + 4(1-\delta_1)(1-\sigma_1)$ for every (σ_1, δ_1) pair. When A2 chooses b, A1's payoff by choosing a is $3/2\sigma_2 \in [0, 3/2]$ and her payoff from choosing b is not greater than zero since $\delta_4 \leq \sigma_4$, which guarantees that (a, b, a) is a continuation equilibrium.

Next, we construct the agents' continuation equilibrium induced by $(\tilde{\gamma}_1^R, \alpha_2)$ by letting them send messages to $P1$ that select an array $(\delta_1, \delta_2, \delta_3, \delta_4)$ such that $\delta_2 = 1$ and $\delta_4 \leq \sigma_4$. As a consequence, following any deviation of P2 to a pay-for-effort contract α_2, agents coordinate on the continuation equilibrium that yields him exactly ζ. ■

This reasoning reproduces that of Lemma 2 of Yamashita (2010) and shows that ζ is the threshold payoff for P2. Hence, as in Theorem 1 of Yamashita (2010), every incentive compatible allocation yielding P2 at least a payoff of ζ can be sustained at equilibrium. ■

It then follows from Remark 1 and Claim 3 that the game exhibits several equilibria yielding at most a payoff of 5 to P2.

We now study a strategic situation in which communication is unrestricted in the following sense: agents can still use the entire set of pay-for-effort contracts to communicate with each principal, but principals can in addition send private signals to each agent. This allows us to evaluate the robustness of the Yamashita (2010) folk theorem-like result, to principals' deviations that involve two-sided communication.

Claim 5 *Consider a general game G^{MS}, in which each agent's message space coincides with the space of pay-for-effort contracts. Suppose that P1 plays a recommendation mechanism. Then, given this mechanism, there is a mechanism for P2 which yields him a payoff strictly greater than* 5 *in every continuation equilibrium.*

Proof. As in Claim 2, we let $M_j^i = \mathcal{A}_j$ for every $j \in J$ and $i \in I$, in addition we allow only P2 to send private signals to agents, from the set $S_2^i = \{a, b\}$ for every $i \in I$.

Consider the following probability distribution over the signals sent by P2: with probability $k > 0$ he privately communicates a to both agents, with probability $(1-k)$ he privately communicates b to A1 and a to A2. All the other signals are sent with probability zero. Let P2 associate to such signals a simple decision rule, which selects y_1 when (a, a) is sent, and and y_2 when (b, a) is sent, for every combination of agents' efforts and messages.

Consider now the agents' continuation game induced by a recommendation mechanism $\hat{\alpha}_1$ ankd the mechanism above for P2. From the perspective of A1, when she receives the signal a she knows that with probability one P2 has chosen y_1. By choosing a she gets 10 if A2 plays a and 3/2 if A2 plays b. By choosing b, instead, she gets $5\,(1-\delta_3)$ if A2 plays a and $-\,(1-\delta_4)$ if A2 plays b. Playing a is hence a strictly dominant effort strategy for A1 irrespective of the message she sends to P1, i.e. for every δ_3 and δ_4. Alternatively, if she receives the signal b she knows that with probability one P2 chooses y_2. By choosing a she gets $-(1-\delta_1)$ if A2 plays a and $-\,(1-\delta_2)$ if A2 plays b. By choosing b, instead, she gets $5\delta_3$ if A2 plays a and δ_4 if A2 plays b. Playing b is a strictly dominant effort strategy for A1 for every $(\delta_1, \delta_2, \delta_3, \delta_4)$.

The above remarks allow to pin down A2's beliefs on A1's equilibrium behavior. From the perspective of A2, signals are uninformative, hence she chooses a if:

$$k[5\delta_1 + 5(1-\delta_1)] + (1-k)[5\delta_3] \geq 8k - (1-k)$$

which boils down to a condition on k, that is

$$k \leq \frac{5\delta_3 + 1}{5\delta_3 + 4}$$

in which the right-hand side is strictly greater than zero for every δ_3.

Thus, for every δ there exists a two-sided communication mechanism with $k \in \Big(0, \frac{5\delta_3+1}{5\delta_3+4}\Big)$, that induces a unique equilibrium in the agents' effort game yielding to P2 a payoff of $10k + (1-k)5 = 5 + 5k > 5$. ■

5 Competing mechanisms under two-sided communication

Our results indicate that, when principals can send private signals to agents, pure strategy equilibria supported by (possibly sophisticated) one-sided communication may not survive. This puts again at the centre-stage the issue of identifying a class of mechanisms supporting robust equilibria. To the extent that two-sided communication cannot in general be restricted, the characterization of such mechanisms appears to be a relevant task for economic applications. In this perspective, the tractability of the relevant class of mechanisms is a key requirement. A natural first step is therefore to rely on the standard "direct" mechanisms introduced by Myerson (1982) for generalized principal-agent problems. In a complete information setting, they correspond to two-sided mechanisms in which the set of signals available to each principal coincides with the set of agents' actions, i.e. $R^i_j = E^i$ for every i and j, and agents send no relevant message to principals, i.e. $M^i_j = \{m\}$ for each i and for each j. As in Myerson (1982), a direct mechanism is *incentive compatible* from the point of view of principal j if, given the mechanisms offered by the other principals, it induces a continuation equilibrium in which agents are obedient to the private signals he is sending. A direct mechanism for principal j can therefore be incentive compatible for a given array of mechanisms posted by $-j$ principals, but not not for some other arrays.

We hence focus on the game in which a mechanism available to each principal j is the mapping $\hat{\gamma}_j : E \longrightarrow \Delta\Big(Y_j \times E\Big)$. Unfortunately, this restriction does not allow to characterize robust equilibria. Indeed, we provide an instance of an equilibrium allocation sustained by the direct mechanisms above that does not survive when principals are allowed to use arbitrary sets of signals.

Example 3. Consider a complete information setting with four players: two principals, $P1$ and $P2$, and two agents, $A1$ and $A2$, who take contractible actions in the sets $E^1 = E^2 = \{a, b\}$. Let $P1$'s decision set be $Y_1 = \{x_1, x_2\}$, and $P2$'s one be $Y_2 = \{y_1, y_2\}$.

The payoffs of this game are represented in the matrix below, where the first two numbers in each cell denote the payoff to P1 and P2, respectively, and the remaining two numbers denote the payoffs to $A1$ and $A2$.

		y_1			y_2	
		a	b		a	b
x_1	a	$(-100, 2, 0, 0)$	$(0, 2, 8, 3)$	a	$(-100, 2, 0, 0)$	$(0, 2, 8, 3)$
	b	$(0, 2, 3, 8)$	$(1, 2, 6, 6)$	b	$(0, 2, 3, 8)$	$(1, 2, 7, 7)$
		a	b		a	b
x_2	a	$(4, 2, 0, 0)$	$(0, 2, 8, 3)$	a	$(4, 2, 0, 0)$	$(0, 2, 8, 3)$
	b	$(0, 2, 3, 8)$	$(-100, 2, 6, 6)$	b	$(0, 2, 3, 8)$	$(-100, 2, 7, 7)$

We let $M_1^i = M_2^i = \{m\}$. Then, we let $P2$ post the direct mechanism $\hat{\gamma}_2$ that equally randomizes over $\{y_1, y_2\} \times \{a, b\}^2$ for every pair of agents' efforts (e^1, e^2). Observe that, since P2's payoff is everywhere constant in the game, this strategy constitutes a best reply.

Claim 6 *Given $\hat{\gamma}_2$, for any direct mechanism $\hat{\gamma}_1$ posted by P1, there is a continuation equilibrium which yields P1 at most a payoff of* $8/9$.

Proof. Supppose that, given $\hat{\gamma}_2$, P1 is restricted to direct mechanisms. It is convenient to write any such mechanism as a joint probability distribution over $\mathcal{A}_1 \times \{a, b\}^2$ with $\mathcal{A}_1$ being the set of pay-for-effort contracts available to P1 and $\alpha_1 : \{a, b\}^2 \longrightarrow \Delta\{x_1, x_2\}$ an element of this set. For a given $\alpha_1 \in \mathcal{A}_1$, a mechanism $\hat{\gamma}_1$ can therefore be represented by the probability distribution $\pi = (\pi_1, \pi_2, \pi_3, \pi_4,)$ that it induces on decisions and signals, with $\pi_i \geq 0$ for $i = 1, \ldots, 4$ and $\underset{i}{\Sigma}\pi_i = 1$. Specifically, we denote $\pi_1 = prob(\alpha_1, a, a)$, $\pi_2 = prob(\alpha_1, a, b)$, $\pi_3 = prob(\alpha_1, b, a)$ and $\pi_4 = prob(\alpha_1, b, b)$.

To construct a best reply for P1 to $\hat{\gamma}_2$, one can hence focus on the optimal pay-for-effort contract $\hat{\alpha}_1$. Observe that, when looking for an optimal $\hat{\gamma}_1$, there is no loss of generality in setting $\hat{\alpha}_1$ such that P1 chooses x_1 with probability one upon observing the efforts (b, b), and x_2 with probability one for all other effort pairs. This follows from the fact that P1's payoff does not depend on agents' efforts in such instances. In addition, it follows from Myerson (1982) that, given $\hat{\gamma}_2$, an optimal mechanism for P1 can be characterized by restricting attention to those $\hat{\gamma}_1$ that induce a continuation equilibrium in which agents obey to the private signals he is sending.[10] We hence consider the continuation game induced by $(\hat{\gamma}_1, \hat{\gamma}_2)$, focusing on the continuation equilibria in which each of the agents plays according to the private signals he receives from $P1$. Consider first $A1$. If she gets the signal a by $P1$, she will choose effort a if

$$8\frac{\pi_2}{\pi_1+\pi_2} \geq 3\frac{\pi_1}{\pi_1+\pi_2} + \frac{13}{2}\frac{\pi_2}{\pi_1+\pi_2}$$

which simplifies to $\frac{3}{2}\pi_2 \geq 3\pi_1$. Similarly, if $A1$ gets the signal b she will choose the effort accordingly as long as:

$$3\frac{\pi_3}{\pi_3+\pi_4} + \frac{13}{2}\frac{\pi_4}{\pi_3+\pi_4} \geq 8\frac{\pi_4}{\pi_3+\pi_4}$$

which reduces to $3\pi_3 \geq \frac{3}{2}\pi_4$.

Consider now $A2$: if she gets the signal a by $P1$, she will follow it if $\frac{3}{2}\pi_3 \geq 3\pi_1$, while if she gets the signal b she will play accordingly if $3\pi_2 \geq \frac{3}{2}\pi_4$.

To determine the optimal probabilities from the perspective of $P1$, we must solve the following constrained maximization problem:

$$\begin{aligned} &\max_{\pi_1,\pi_2,\pi_3,\pi_4} 4\pi_1 + \pi_4 \\ s.t. \quad &\tfrac{3}{2}\pi_2 \geq 3\pi_1 \\ &3\pi_3 \geq \tfrac{3}{2}\pi_4 \\ &\tfrac{3}{2}\pi_3 \geq 3\pi_1 \\ &3\pi_2 \geq \tfrac{3}{2}\pi_4 \\ &\pi_i \geq 0 \text{ for } i = 1,\ldots,4 \\ &\Sigma_i \pi_i = 1, \pi_2 = \pi_3 \end{aligned}$$

Observe that π_2 and π_3 are irrelevant for the objective of $P1$. However, given the symmetry in the incentive compatibility constraints of the agents, if $P1$ tries to set π_2 and/or π_3 equal to zero this directly implies that also π_1 and π_4 must be equal to zero which is clearly suboptimal. Hence, any candidate probability distributions will be such that $\pi_2 > 0$ and $\pi_3 > 0$. Without loss of generality, we consider distributions with $\pi_2 = \pi_3$; the set of

[10] This is the traditional notion of incentive compatibility (obedience) for complete information settings in Myerson (1982).

constraints of the problem above thus reduces to:

$$\tfrac{3}{2}\pi_2 \geq 3\pi_1 \tag{1}$$

$$3\pi_2 \geq \tfrac{3}{2}\pi_4 \tag{2}$$

$$\pi_1 = (1 - 2\pi_2 - \pi_4) \geq 0, \tag{3}$$

$$\pi_2 = \pi_3 > 0, \pi_4 \geq 0$$

We prove that $P1$ maximizes his payoff by letting (1) and (2) bind. The corresponding distribution over private signals allows $P1$ to correlate agents' behaviors so to sustain at equilibrium a payoff equal to 8/9.

From (2) binding, we have $\pi_4 = 2\pi_2$, which substituted into (1) together with (3) yields $\frac{3}{2}\pi_2 \geq 3(1 - 4\pi_2)$, that is $\pi_2 \geq \frac{2}{9}$. The array of probabilities $(\pi_1, \pi_2, \pi_3, \pi_4) = (\frac{1}{9}, \frac{2}{9}, \frac{2}{9}, \frac{4}{9})$ yields to $P1$ an expected payoff of $4\frac{1}{9} + \frac{4}{9}1 = \frac{8}{9}$.

It is easy to show that $P1$ cannot achieve a higher payoff than $\frac{8}{9}$, we do so by examining the set of constraints (1)-(3) in the Cartesian space (π_1, π_4) upon which $P1$'s payoff depends. From equation (3), $\pi_2 = \frac{1}{2}(1 - \pi_1 - \pi_4)$, which substituted into (1) and (2), gives us the system of linear inequalities:

$$\frac{15}{4}\pi_1 \leq \frac{3}{4}(1 - \pi_4) \tag{4}$$

$$\pi_1 + 2\pi_4 \leq 1 \tag{5}$$

The set of incentive compatible mechanisms for $P1$, which maximizes his payoff, corresponds to the dashed area at the intersection of (4)-(5). Clearly, our candidate $(\pi_1, \pi_4) = \left(\frac{1}{9}, \frac{4}{9}\right)$ lies at the intersection of the two lines in Figure 1 (point B). Given that the iso-profit curves of P1 exhibit a $\frac{\pi_4}{\pi_1}$ ratio equal to $-4 \in [-5, -1/2]$ no corner solution is preferred to point B, which identifies P1's preferred distribution. Hence, $P1$'s maximized expected profits reach the value of $\frac{8}{9}$. ■

Claim 2: Given $\hat{\gamma}_2$, $P1$ can do better by means of a sophisticated system of signals, reaching a payoff of $\frac{5}{4} > \frac{8}{9}$.

Proof. Consider the game G^{MS} in which $P1$'s the message spaces are singletons and the signal spaces are $R_1^1 = \{r_1, r_2, r_3, r_4\}$ for $A1$ and $R_1^2 = \{s_1, s_2, s_3, s_4\}$ for $A2$. Each private signal $r_i \in R_1^1$ prescribes the following behavior to $A1$:

r_1: play a regardless of the signal sent by P2.

r_2: play a if P2 sends signal a and b if P2 sends signal b

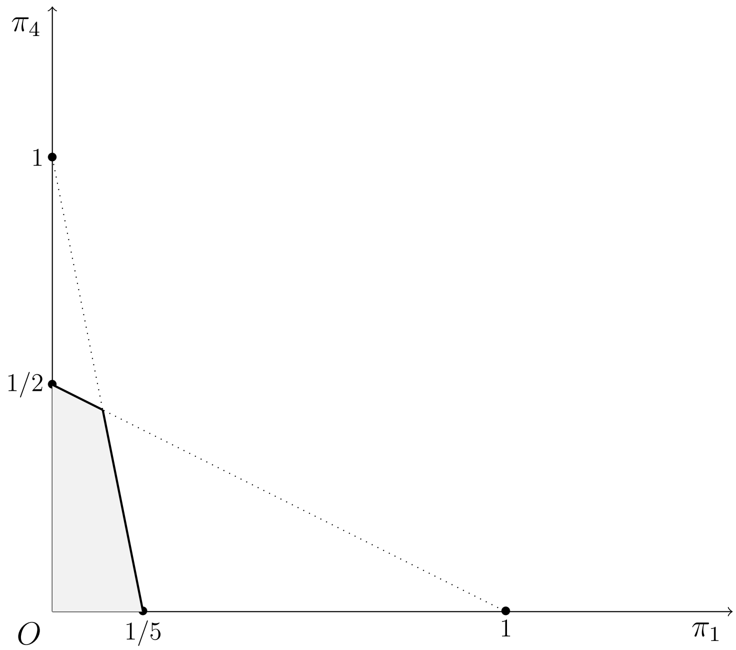

Figure 1: Set of incentive-feasible probability distributions for $P1$.

r_3: play b if P2 sends signal a and a if P2 sends signal b
r_4: play b regardless of the signal sent by P2.

Each recommendation $s_i \in R_1^2$ sent to $A2$ can be interpreted in the same way. Let π_{ij} be the (joint) probability with which $P1$ recommends r_i to $A1$ and s_j to $A2$ respectively, i.e. $\pi_{ij} = prob(r_i, s_j)$ with $i = j = 1, \ldots, 4$. Let $P1$'s mechanism at the deviation be given by γ_1' which is constituted by a probability distribution $\pi' = (\pi_{ij})_{i,j=1,\ldots,4}$ on arrays of signals and the incentive contract $\alpha_1' (e^1, e^2) = \alpha_1 (e^1, e^2)$.

Given $P2$ keeps playing the simple mechanism $\hat{\gamma}_2$ described before, we construct the set of incentive compatibility constraints for $P1$. To do so, define $\tilde{\pi}_i = \sum_{j=1}^{4} \pi_{ij}$ the marginal probability with which $P1$ sends the signal r_i to $A1$, and $\bar{\pi}_j = \sum_{i=1}^{4} \pi_{ij}$ the marginal probability with which he sends the signal s_j to $A2$. So that, for instance, $\tilde{\pi}_1 = \pi_{11} + \pi_{12} + \pi_{13} + \pi_{14}$ is the marginal probability over r_1, and $\bar{\pi}_1 = \pi_{11} + \pi_{21} + \pi_{31} + \pi_{41}$ is the corresponding marginal probability over s_1. Consider $A1$, if she gets the signal r_1, she will follow it if:

$$\frac{8}{2} \frac{(\pi_{12} + \pi_{13})}{\tilde{\pi}_1} + 8 \frac{\pi_{14}}{\tilde{\pi}_1} \geq 3 \frac{\pi_{11}}{\tilde{\pi}_1} + \left(\frac{3}{2} + \frac{13}{4} \right) \frac{(\pi_{12} + \pi_{13})}{\tilde{\pi}_1} + \frac{13}{2} \frac{\pi_{14}}{\tilde{\pi}_1},$$

which simplifies to,

$$3\pi_{11} + \frac{3}{4}\Big(\pi_{12} + \pi_{13}\Big) \leq \frac{3}{2}\pi_{14}. \tag{6}$$

provided that $\tilde{\pi}_1 \neq 0$.[11] If $A1$ gets the signal r_2, she will follow it if:

$$\frac{3}{2}\frac{\pi_{21}}{\tilde{\pi}_2} + \frac{35}{8}\frac{(\pi_{22} + \pi_{23})}{\tilde{\pi}_2} + \frac{29}{4}\frac{\pi_{24}}{\tilde{\pi}_2} \geq \frac{3}{2}\frac{\pi_{21}}{\tilde{\pi}_2} + \frac{35}{8}\frac{(\pi_{22} + \pi_{23})}{\tilde{\pi}_2} + \frac{29}{4}\frac{\pi_{24}}{\tilde{\pi}_2},$$

which implies that this incentive constraint must hold with equality. The same occurs if $A1$ gets the signal r_3. Eventually, if she gets the signal r_4, she will follow it if:

$$3\frac{\pi_{41}}{\tilde{\pi}_4} + \frac{19}{4}\frac{(\pi_{42} + \pi_{43})}{\tilde{\pi}_4} + \frac{13}{2}\frac{\pi_{44}}{\tilde{\pi}_4} \geq 0\frac{\pi_{41}}{\tilde{\pi}_4} + \frac{8}{2}\frac{(\pi_{42} + \pi_{43})}{\tilde{\pi}_4} + 8\frac{\pi_{44}}{\tilde{\pi}_4},$$

which reduces to

$$3\pi_{41} + \frac{3}{4}\Big(\pi_{42} + \pi_{43}\Big) \geq \frac{3}{2}\pi_{44}. \tag{7}$$

The relevant incentive constraints for $P1$, when dealing with $A1$, are given by the inequalities:

$$3\pi_{11} + \frac{3}{4}\Big(\pi_{12} + \pi_{13}\Big) \leq \frac{3}{2}\pi_{14} \qquad (6)$$
$$3\pi_{41} + \frac{3}{4}\Big(\pi_{42} + \pi_{43}\Big) \geq \frac{3}{2}\pi_{44} \qquad (7)$$

By applying the same procedure we can identify the incentive compatibility conditions for $A2$, which result into:

$$3\pi_{11} + \frac{3}{4}\Big(\pi_{21} + \pi_{31}\Big) \leq \frac{3}{2}\pi_{41} \tag{8}$$
$$3\pi_{14} + \frac{3}{4}\Big(\pi_{24} + \pi_{34}\Big) \geq \frac{3}{2}\pi_{44} \tag{9}$$

The probability distribution induced by the agents' joint signals can be visualised in matrix form:

	s_1	s_2	s_3	s_4
r_1	π_{11}	π_{12}	π_{13}	π_{14}
r_2	π_{21}	π_{22}	π_{23}	π_{24}
r_3	π_{31}	π_{32}	π_{33}	π_{34}
r_4	π_{41}	π_{42}	π_{43}	π_{44}

[11] In the entire construction, we will consider marginal probabilities different from zero for both agents, i.e. $\tilde{\pi}_i \neq 0$ and $\bar{\pi}_j \neq 0$ for all $i, j = 1, \ldots, 4$.

Table 2: Probability distribution over sophisticated signals

We show that there exists one such distribution that allows $P1$ to achieve a payoff of $\frac{5}{4} > \frac{8}{9}$. Consider a symmetric matrix, so that $\pi_{ij} = \pi_{ji}$ for every $i = j = 1, \ldots, 4$ with $i \neq j$. Any such distribution would imply that equation (6) is identical to (8), and (7) to (9). Hence, we are left with the following system:

$$3\pi_{11} + \frac{3}{4}\Big(\pi_{21} + \pi_{31}\Big) \leq \frac{3}{2}\pi_{41} \qquad (6bis)$$

$$3\pi_{41} + \frac{3}{4}\Big(\pi_{42} + \pi_{43}\Big) \geq \frac{3}{2}\pi_{44} \qquad (7bis)$$

First, let equation $(6bis)$ bind and fix $\pi_{11} = 0$, then every $\pi_{21} = \pi_{31} = \pi_{41}$ satisfies $(6bis)$. Now, from equations $(7bis)$ and $\sum_{j=1}^{4}\sum_{i=1}^{4}\pi_{ij} = 1$, look for $\pi_{44} > 0$ and $\pi_{14} > 0$ that solve the system having fixed all remaining π_{ij} equal to zero. Substituting $\pi_{41} = \frac{1}{6}(1 - \pi_{44})$ into $(7bis)$, we get $\pi_{44} \leq \frac{1}{4}$. Take $\pi_{44} = \frac{1}{4}$, then $\pi_{41} = \pi_{14} = \pi_{12} = \pi_{13} = \pi_{21} = \pi_{31} = \frac{1}{8}$.

	s_1	s_2	s_3	s_4
r_1	0	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
r_2	$\frac{1}{8}$	0	0	0
r_3	$\frac{1}{8}$	0	0	0
r_4	$\frac{1}{8}$	0	0	$\frac{1}{4}$

Table 2: Probability distribution over sophisticated signals

The matrix of the resulting probability distribution is represented in Table 2. The expected payoff that $P1$ can achieve by means of such sophisticated signals amounts to $\pi_{41}8 + \pi_{44} = \frac{5}{4} > \frac{8}{9}$ that gives us the result. ■

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