Inaccurate Statistical Discrimination: An Identification Problem

Aislinn Bohren, Kareem Haggag, Alex Imas & Devin G. Pope

Discrimination in economics

Literature defines two potential sources of discrimination.

- **Taste- or preference-based:** due to animus against members of a group (Becker 1957).
- **Statistical:** uncertainty about productivity; group identity informative – believe group discriminated against has:
  - Lower avg. productivity (Phelps 1972; Arrow 1973);
  - Higher/lower variance in productivity (Aigner Cain 1977; Bartoš Bauer Chytilová Matějka 2016);
  - Less/more informative or biased signal of productivity.

Methods to distinguish between these sources often rely on assumption that beliefs are accurate.

Inaccurate beliefs

- Evidence that people have systematic biases in beliefs.
  - Inaccurate stereotypes based on representativeness heuristic (Bordalo et al. (2016)).
  - Enormous literature in psychology discussing how stereotypes can be inaccurate (see Fiske (2018) for review).
- Inaccurate beliefs may arise from a lack of information.
  - Inexperience (e.g. aliens arrive from Mars).
  - Subjective information: unsure of others' beliefs + preferences (e.g. interpreting a recommendation letter).
  - Dynamics: accurate priors about population averages but unsure how selection impacts current population (e.g. hiring & promotion criteria).

Inaccurate beliefs affect discrimination

- Evaluators use beliefs to learn about trait of candidate (e.g. productivity).
- Inaccurate beliefs lead to inaccurate inference about the candidate.
  - May persist even in the face of a lot of information about candidate (learning with model misspecification literature).
  - May cause inefficient exit from market.
  - Bohren Imas Rosenberg (2019): dynamics of discrimination with inaccurate beliefs.

Why do inaccurate beliefs matter?

Methodological

- Erroneously assuming accurate beliefs leads to misidentification of source in outcome-based tests — common method used to distinguish source.
- Relaxing assumption of accurate beliefs creates an identification problem.

Policy

- Effective interventions when beliefs are inaccurate differ from those for accurate statistical or taste-based sources.
- Welfare implications also differ.

Literature survey

Most economics papers on discrimination ignore the possibility of inaccurate beliefs.

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Measure beliefs	7	6.7%

Scope: empirical papers published between 1990-2018 in AEJ: Applied, AEJ: Policy, AER, EMA, JEEA, JLE, JPE, ReStud, ReStat, QJE.

A model of discrimination with inaccurate beliefs

Worker:

- Group identity $g \in \{M, F\}$.
- Productivity $a \sim N(\mu_g, 1/\tau_g)$.
- Signal $s = a + \epsilon$, $\epsilon \sim N(0, 1/\eta_g)$.
- Focus on discrimination against group F.

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Evaluator:

- Decides whether to hire worker, $v \in \{0, 1\}$.
- Holds subjective beliefs $(\hat{\mu}_g, \hat{\tau}_g, \hat{\eta}_g)$.
  - Misspecified model when subjective parameters $\neq$ true parameters.
- Observes g and s; uses Bayes rule to updates belief about a.
- Hires worker if subjective posterior $\hat{E}_\theta[a|s, g]$ above group-specific hiring threshold $u_g \in \mathbb{R}$ (reduced form for preferences).
- $\theta = \{u_g, \hat{\mu}_g, \hat{\tau}_g, \hat{\eta}_g\}_{g \in \{M,F\}}$ denotes evaluator's type.

Partiality

Categorize different forms of preferences and beliefs.

- Use partiality to refer to properties of model primitives.
- To distinguish from discrimination – property of behavior and consequence of primitives.

Definition (Preference Partiality)

An evaluator has *preference partiality* if $u_F \neq u_M$.

Definition (Belief Partiality)

An evaluator has *belief partiality* if $(\hat{\mu}_F, \hat{\tau}_F, \hat{\eta}_F) \neq (\hat{\mu}_M, \hat{\tau}_M, \hat{\eta}_M)$. Beliefs are accurate if $(\hat{\mu}_g, \hat{\tau}_g, \hat{\eta}_g) = (\mu_g, \tau_g, \eta_g)$ for $g \in \{M, F\}$ and otherwise inaccurate.

Discrimination

- Difference between hiring decisions

$$D(s,\theta) \equiv v(s,M,\theta) - v(s,F,\theta)$$

where $v(s,g,\theta) \equiv \mathbb{1}\{\hat{E}_\theta[a|s,g] \geq u_g\}$ denotes optimal hiring decision.

- Discrimination occurs against group F if $\exists s$ s.t. $D(s,\theta) > 0$.
- Interested in when different sets of beliefs and preferences give rise to same discriminatory behavior.

Definition

Two evaluators of types θ and θ' exhibit equivalent discrimination if $D(s,\theta) = D(s,\theta')$ for all $s \in \mathbb{R}$.

Equivalent discrimination

Posterior mean productivity monotonic wrt s

- Represent optimal hiring decision as cut-off rule wrt signal: hire if $s \geq \bar{s}(\theta, g)$.
- Two types have same signal cut-offs $\Rightarrow$ equivalent discrimination.

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Proposition (Set of types that exhibit equivalent discrimination)

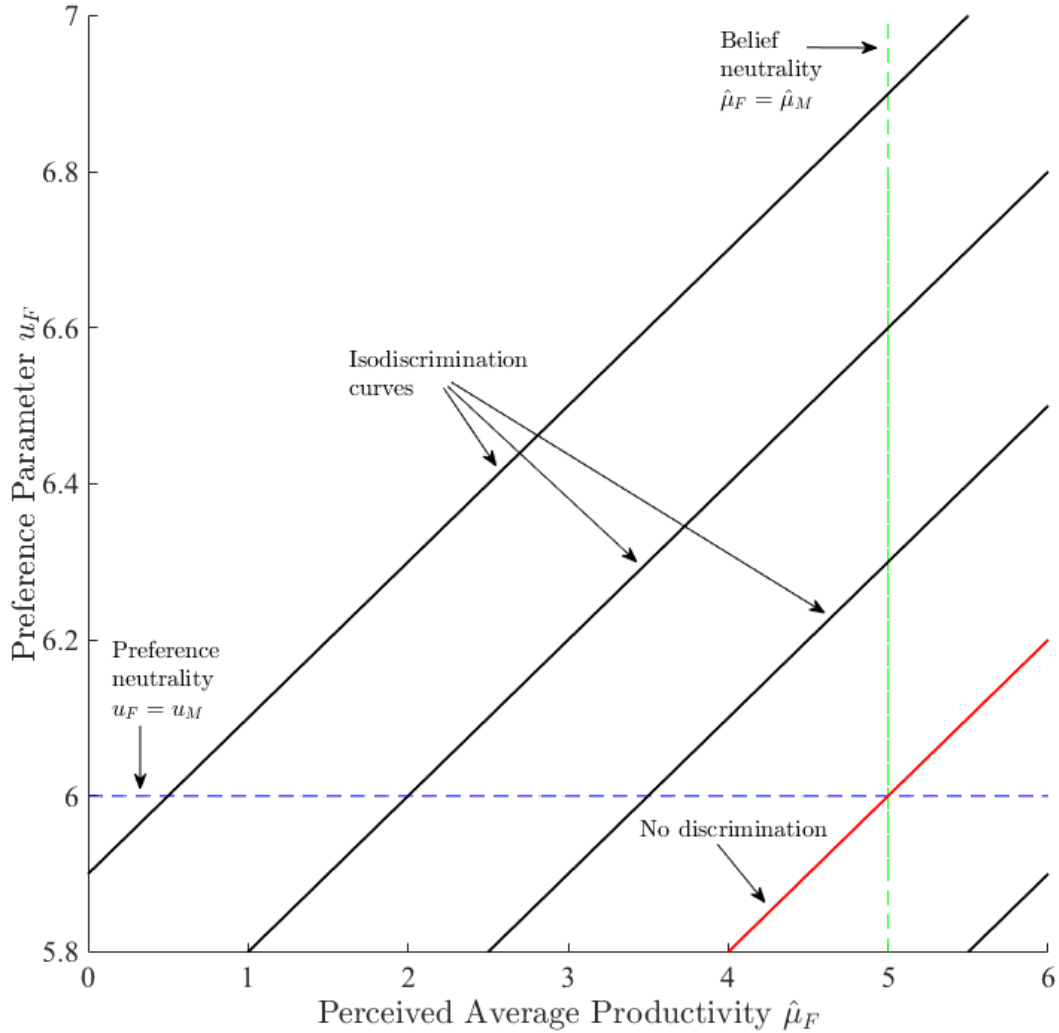
For any constants $(s_M, s_F) \in \mathbb{R}^2$ with $s_M \neq s_F$, equivalent discrimination occurs for the set of types $\{\theta = (u_g, \hat{\mu}_g, \hat{\tau}_g, \hat{\eta}_g)\}_{g \in \{M,F\}}$ s.t.

$$\frac{\hat{\tau}_M + \hat{\eta}_M}{\hat{\eta}_M} u_M - \frac{\hat{\tau}_M}{\hat{\eta}_M} \hat{\mu}_M = s_M \tag{1}$$

$$\frac{\hat{\tau}_F + \hat{\eta}_F}{\hat{\eta}_F} u_F - \frac{\hat{\tau}_F}{\hat{\eta}_F} \hat{\mu}_F = s_F. \tag{2}$$

No discrimination occurs for types that satisfy (1), (2) when $s_M = s_F$.

Isodiscrimination curves



$(u_M, \hat{\mu}_M, \hat{\tau}_M, \hat{\eta}_M) = (6, 5, .5, 2)$, $(\hat{\tau}_F, \hat{\eta}_F) = (.5, 2)$

Identifying discrimination

Can property of interest can be backed out from available data?

- When evaluators may have inaccurate beliefs, what can we say about preferences and beliefs in common empirical designs used to study discrimination?

Existence of discrimination

Show there exists an s such that $D(s, \theta) > 0$.

- Assume researcher observes group g and hiring decision v for each worker.
- If researcher also observes s, this is straightforward (direct method).
- Otherwise, can use correspondence or audit study method: create fictitious workers with randomly assigned g and s.
- Provided observe hiring decisions for sufficiently rich set of signals, can also identify isodiscrimination curve.

Source of discrimination

Proceed with assuming researcher can identify isodiscrimination curve (i.e. using correspondence study).

- What form of partiality generates observed discriminatory behavior?
- Observing isodiscrimination curve does not allow researcher to distinguish between preference, accurate belief and inaccurate belief partiality.
- Moreover, not possible to rule out any potential source.

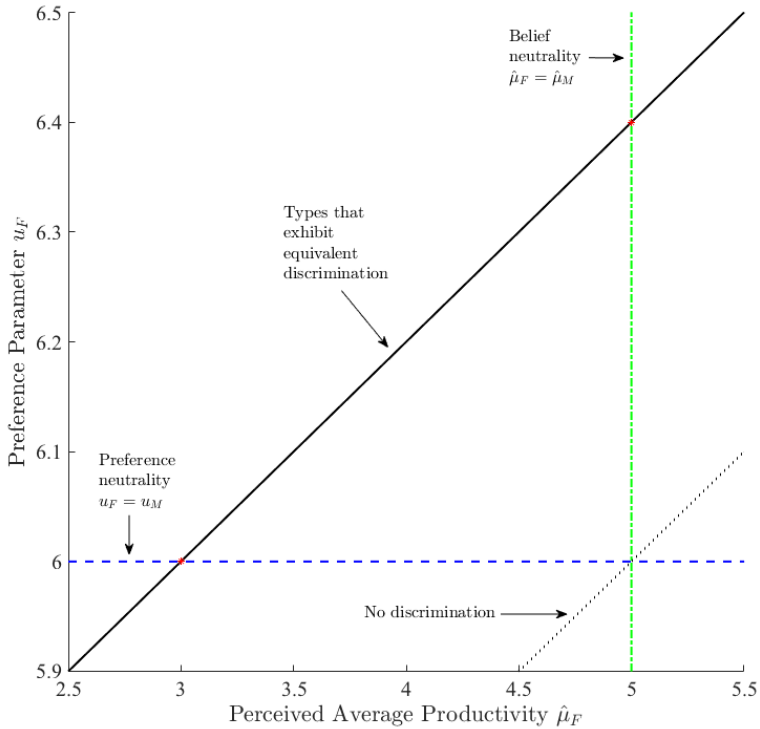
Each form of partiality in isolation can generate given pattern of discrimination

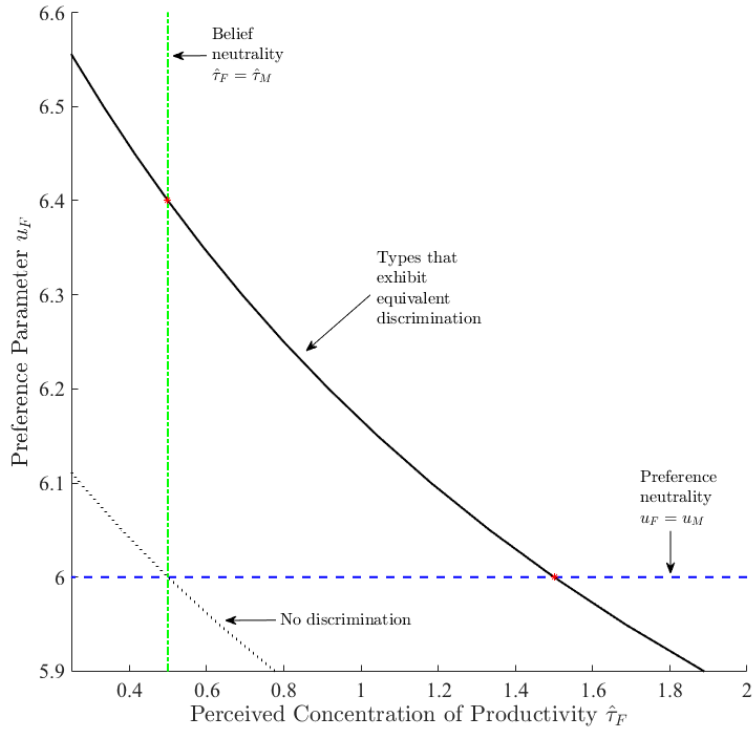
Proposition

For any type θ that discriminates against group F, there are a continuum of types that exhibit equivalent discrimination, including:

1. *A type θ' with preference partiality against group F and belief neutrality, $u'_F > u'_M$;*
2. *A type θ' with preference neutrality and belief partiality due to lower expected productivity, $\hat{\mu}'_F < \hat{\mu}'_M$;*
3. *A type θ' that believes the market is cherry-picking (lemon-dropping) and has preference neutrality and belief partiality due to higher (lower) concentration of productivity, $\hat{\tau}'_F > \hat{\tau}'_M$;*
4. *A type θ' that believes the market is cherry-picking (lemon-dropping) and has preference neutrality and belief partiality due to lower (higher) signal precision, $\hat{\eta}'_F < \hat{\eta}'_M$ ($\hat{\eta}'_F > \hat{\eta}'_M$).*

Each form of partiality in isolation can generate given pattern of discrimination





Isodiscrimination curve $(s_M, s_F) = (6.25, 6.75)$; $(u_M, \hat{\mu}_M, \hat{\tau}_M, \hat{\eta}_M) = (6, 5, .5, 2)$

Left figure: $(\hat{\tau}_F, \hat{\eta}_F) = (.5, 2)$; Right figure: $(\hat{\mu}_F, \hat{\eta}_F) = (5, 2)$

Source of discrimination

Corollary

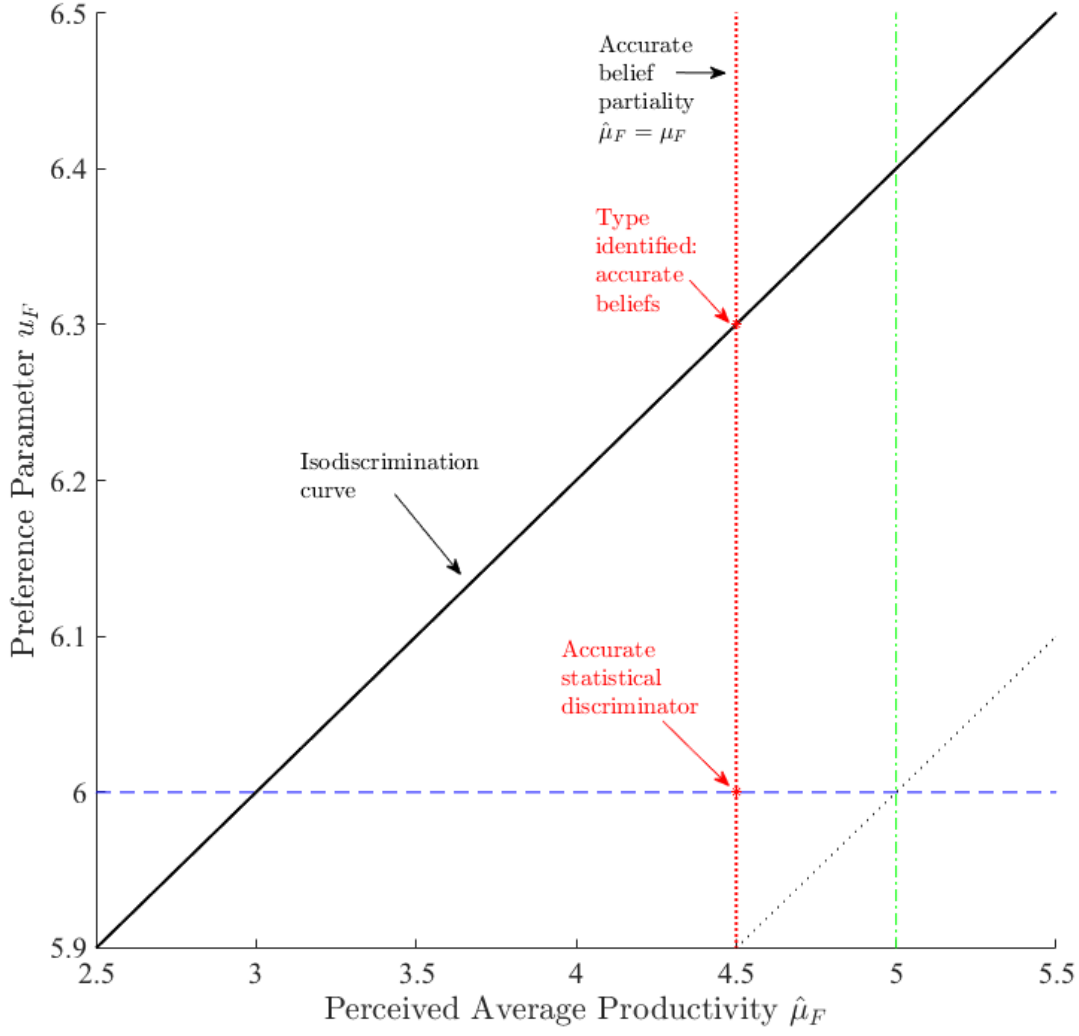
Observing the isodiscrimination curve does not rule out any category of partiality.

Outcomes-based test

Suppose it is also possible to observe the true productivity and signal distributions for each group.

- Common method to identify source of discrimination.
- Under assumption of accurate beliefs, this identifies evaluator's type.

Outcomes-based test: accurate beliefs



Isodiscrimination curve $(s_M, s_F) = (6.25, 6.75)$; true distributions $(\mu_M, \tau_M, \eta_M) = (5, .5, 2)$, $(\mu_F, \tau_F, \eta_F) = (4.5, .5, 2)$; $(u_M, \hat{\mu}_M, \hat{\tau}_M, \hat{\eta}_M) = (6, 5, .5, 2)$.

Identification crucially depends on accurate belief assumption

- Type no longer identified when inaccurate beliefs are possible.
- Erroneously assuming accurate beliefs leads researcher to misattribute discrimination from inaccurate beliefs to preferences.
- Inaccurate beliefs + preference partiality can be mistaken for accurate beliefs.

Proposition (Misidentified Preferences)

Suppose a researcher incorrectly assumes an evaluator has accurate beliefs and uses the outcomes-based method to identify the type.

1. *For a generic set of types and true distributions, the researcher misidentifies preferences.*
2. *If inaccurate beliefs increase (decrease) discrimination against group F, then the researcher overestimates (underestimates) the evaluator's preference partiality against group F.*

Dynamics of inaccurate beliefs

Suppose inaccurate beliefs + preference partiality generate equivalent discrimination to accurate statistical discrimination.

- May be tempted to conclude inaccurate beliefs innocuous.
- But inaccurate beliefs may negatively affect worker in future performance evaluations and promotions.
- Example:

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- Example:
  - Inaccurate beliefs exaggerate true difference in avg. productivity between groups.
  - Preferences somewhat favor disadvantaged group through lower hiring threshold ($u_F < u_M$) for entry-level position.
  - Evaluator only feels compelled to favor disadvantaged group for entry-level hiring.
  - $\Rightarrow$ Inaccurate beliefs lead to persistently lower rates of promotion and advancement.

Sometimes possible to rule out accurate statistical discrimination

- Is pattern of discrimination consistent with accurate belief partiality + preference neutrality?
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Proposition (Rejecting accurate statistical discrimination)

Suppose a researcher observes the true productivity and signal distributions for each group and can identify the isodiscrimination curve with thresholds (s_M, s_F)*. If*

$$\frac{\tau_M \mu_M + \eta_M s_M}{\tau_M + \eta_M} \neq \frac{\tau_F \mu_F + \eta_F s_F}{\tau_F + \eta_F}, \tag{3}$$

the evaluator is not an accurate statistical discriminator.

Source of discrimination

So how can we identify the source of discrimination when beliefs may be inaccurate?

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- Elicit subjective beliefs.
  - Analogous technique to outcomes-method.
  - Identifies preferences.
  - Does not identify whether beliefs are accurate – to do so, must also observe true distributions.

Source of discrimination

So how can we identify the source of discrimination when beliefs may be inaccurate?

- Elicit subjective beliefs.
  - Analogous technique to outcomes-method.
  - Identifies preferences.
  - Does not identify whether beliefs are accurate – to do so, must also observe true distributions.
- Manipulate information.
  - Vary number of signal draws evaluator observes.
  - Important caveat: crucial that multiple signals drawn from same distribution.
  - Online reviews vs. SAT score + GPA.

Manipulating information

Proposition (Set of types that exhibit equivalent discrimination)

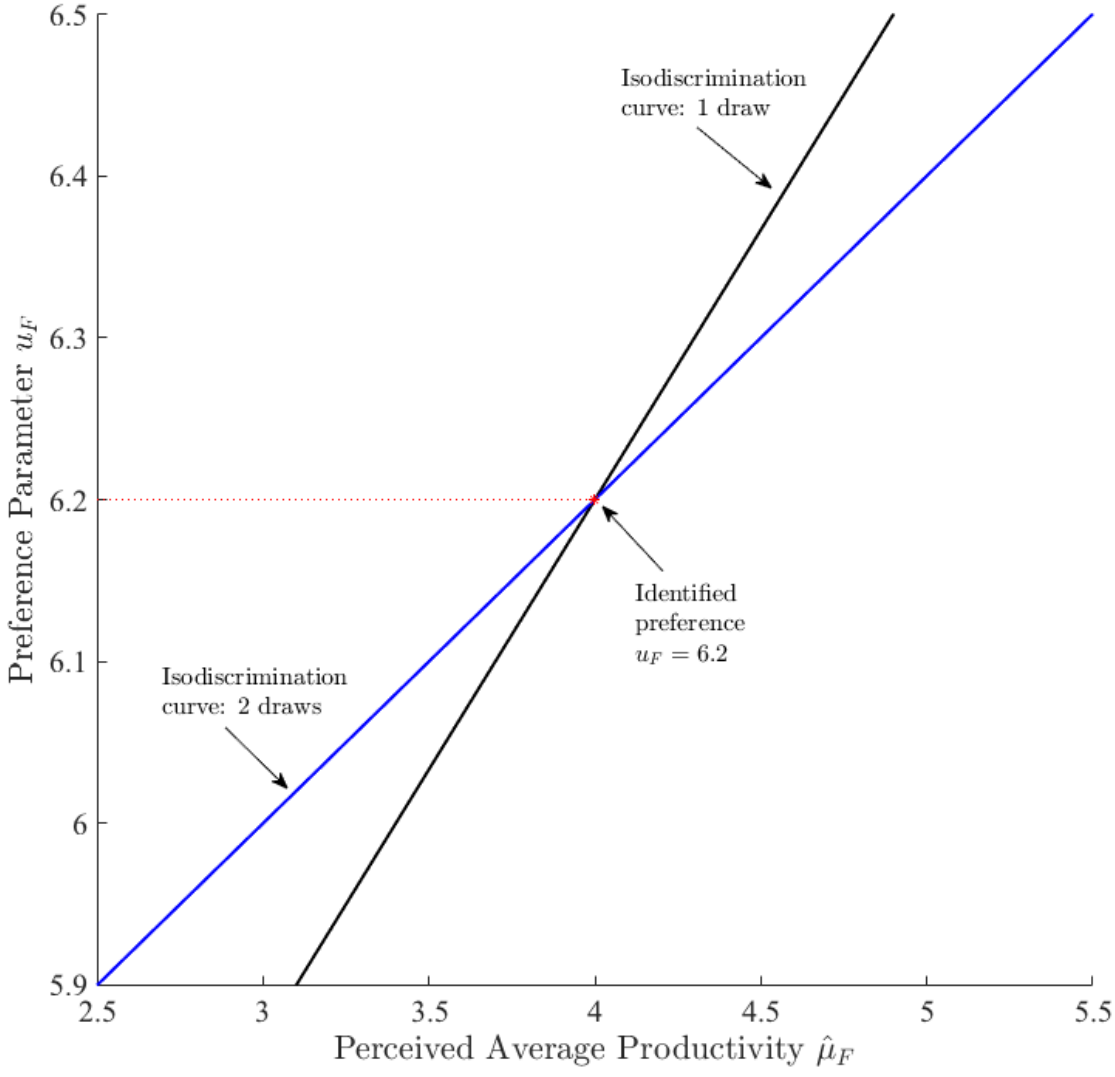
Suppose an evaluator of type $\theta = (u_g, \hat{\mu}_g, \hat{\tau}_g, \hat{\eta}_g)_{g \in \{M,F\}}$ *observes either* x_1 *or* $x_2 \neq x_1$ *signal draws. Then the set of types*

$$\left\{ \theta' = (u_g, \hat{\mu}'_g, \hat{\tau}'_g, \hat{\eta}'_g)_{g \in \{M,F\}} \middle| \hat{\mu}'_g = u_g - \frac{\hat{\tau}_g / \hat{\eta}_g}{\hat{\tau}'_g / \hat{\eta}'_g}(u_g - \hat{\mu}_g) \right\}$$

exhibit equivalent discrimination to θ *across both informational treatments. This set of types also exhibit equivalent discrimination for any number* $x \geq 1$ *of signal draws.*

- Unique level of preference partiality yields equivalent discrimination across informational treatments.
- Identifies preferences but not beliefs.

Information manipulation



Isodiscrimination curves for $(u_F, \hat{\mu}_F, \hat{\tau}_F, \hat{\eta}_F) = (6.2, 4, .5, 1)$

Experiment

Illustrative example

- Demonstrate importance of inaccurate beliefs in how we study discrimination.
- Suggest possible intervention to correct inaccurate beliefs.

Experiment

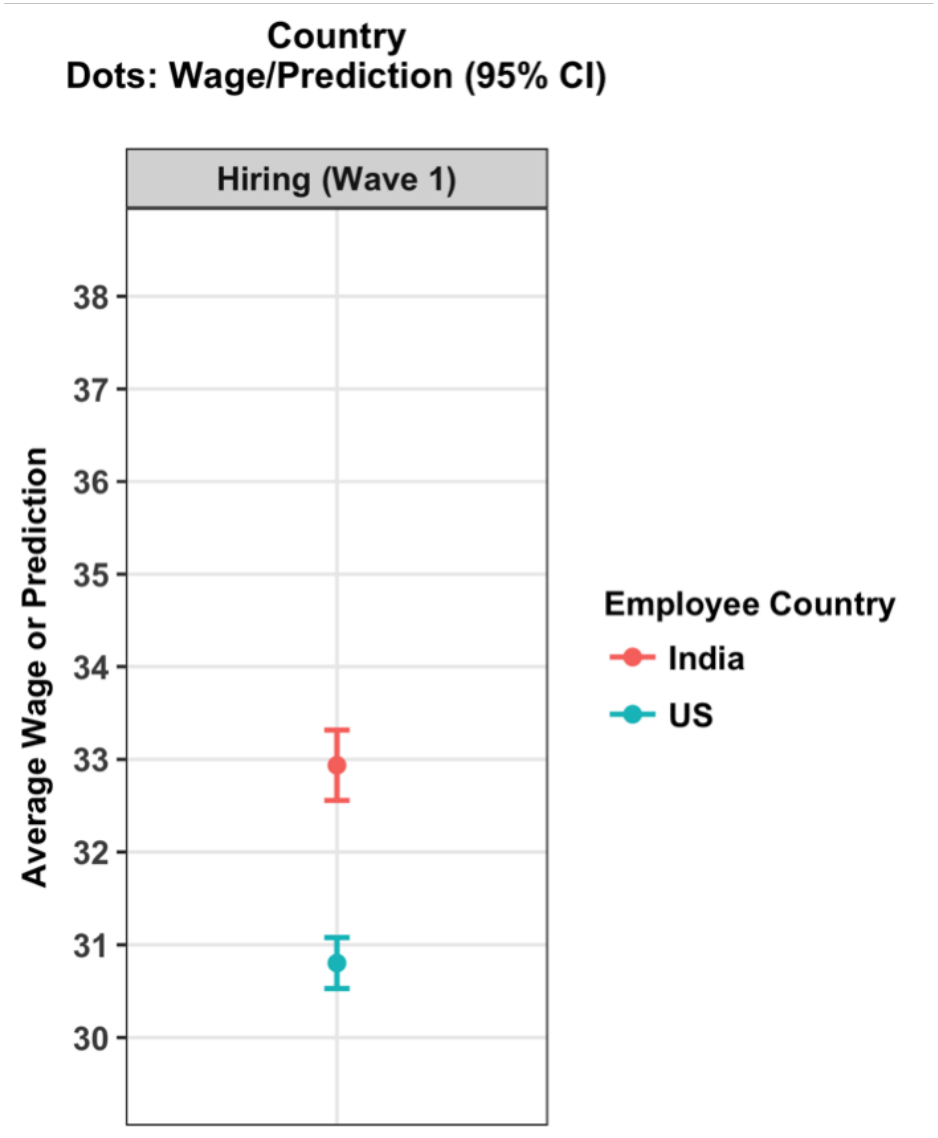
- **Part 1:** 600 Mturkers (400 from US; 200 from India) recruited to be "employees."
  - Completed 50 question math test.
  - Answered 8 questions to develop personal profile.
- **Part 2:** 600 different Mturkers (400 from US; 200 from India) recruited to be "employers."
  - Observed profiles of 20 different employees.
  - Decided maximum wage (wtp) to hire each (0-50 cents).
- **Payment**
  - Drawn random number x between 0 and 50 for each potential employee; if wtp $\geq x$, employer hires employee and pays wage x.
  - Employers earn 1 cent for each question that hired employee answered correctly.
  - Employers told avg # correct answers was 37.
  - Comprehension questions to check understanding.

Step 1: check for differential treatment

Three sets of group identities in experiment:

1. **Indian vs. American**
2. Male vs. Female
3. Old vs. Young

Indians employees receive higher wages than Americans



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- Indians favored relative to Americans.

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- Compare to true performance distributions.

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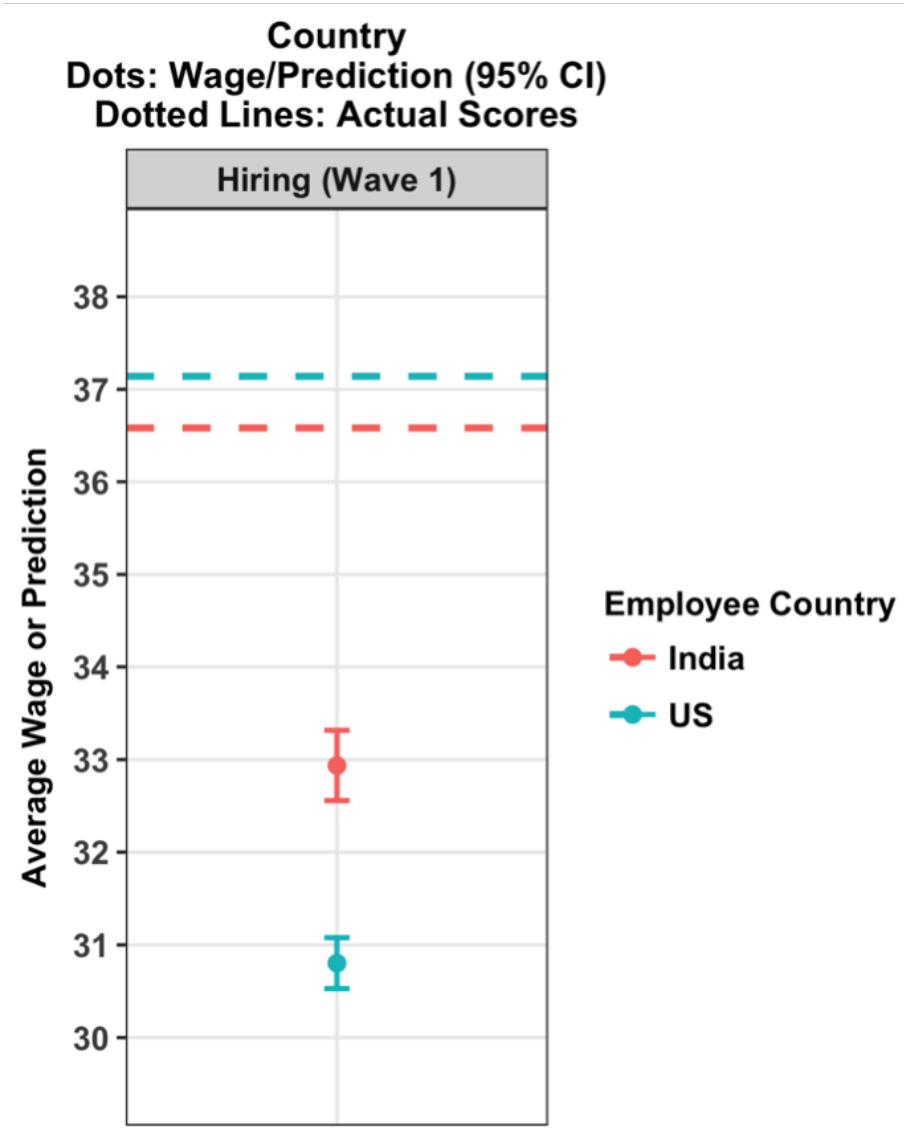


Summary

Part 1: Document Differential Treatment

- Indians favored relative to Americans.

Part 2: Compare to True Distributions (e.g. standard technique)

- Taste-based discrimination against Americans.

But what if beliefs are wrong?

Part 3: Elicit Average Beliefs

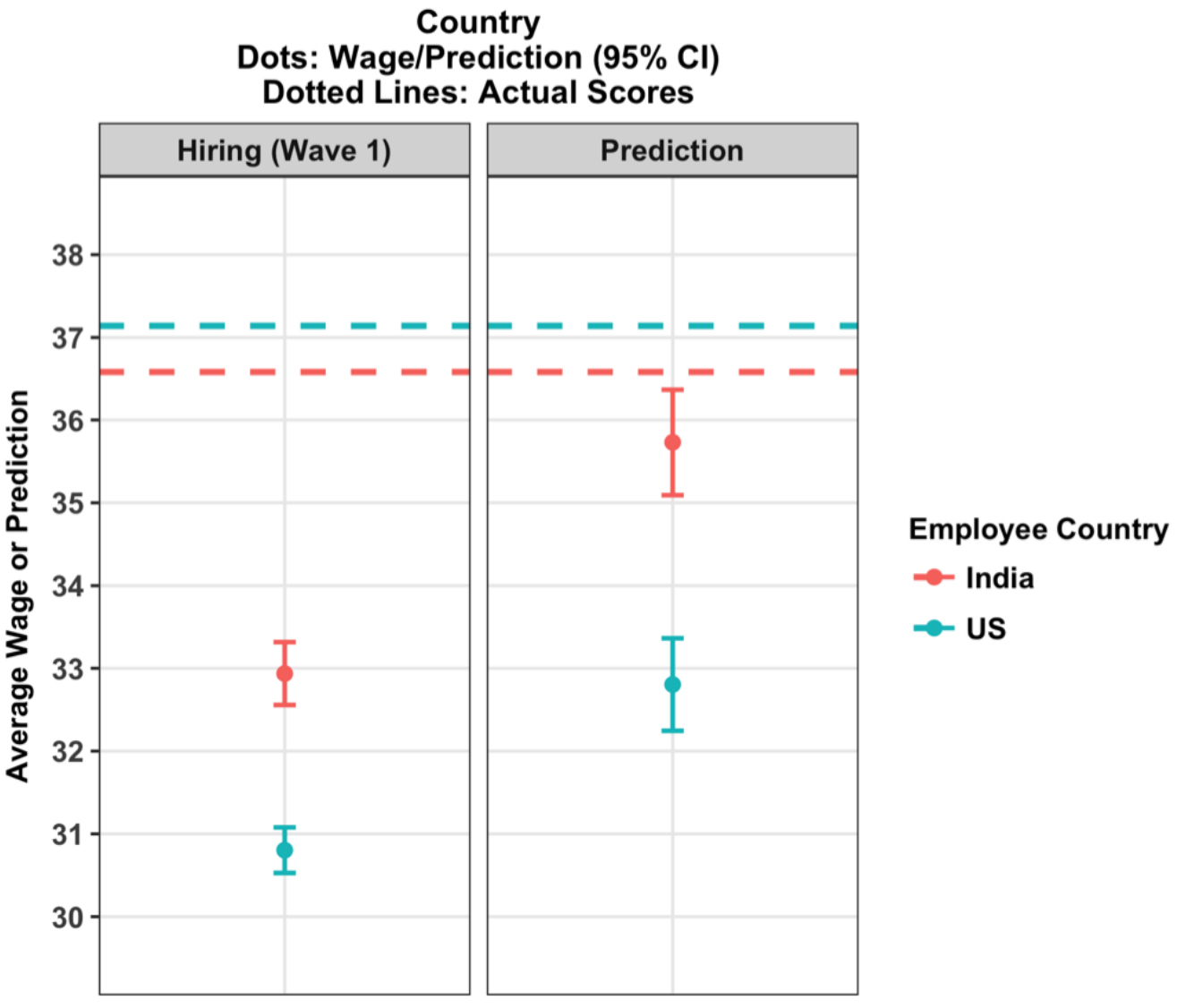
Employers asked following question:

"On average, how many questions do you think g answered correctly?"

where $g \in \{Americans, Indians\}$.

- Provided incentive compatible bonuses for performance on questions.
- Randomized between small and large incentives – no differences between two groups.

Inaccurate statistical discrimination against Americans.



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Part 3: Compare True and Perceived Distributions

- Inaccurate statistical discrimination against Americans.

Intervention: is it possible to correct beliefs?

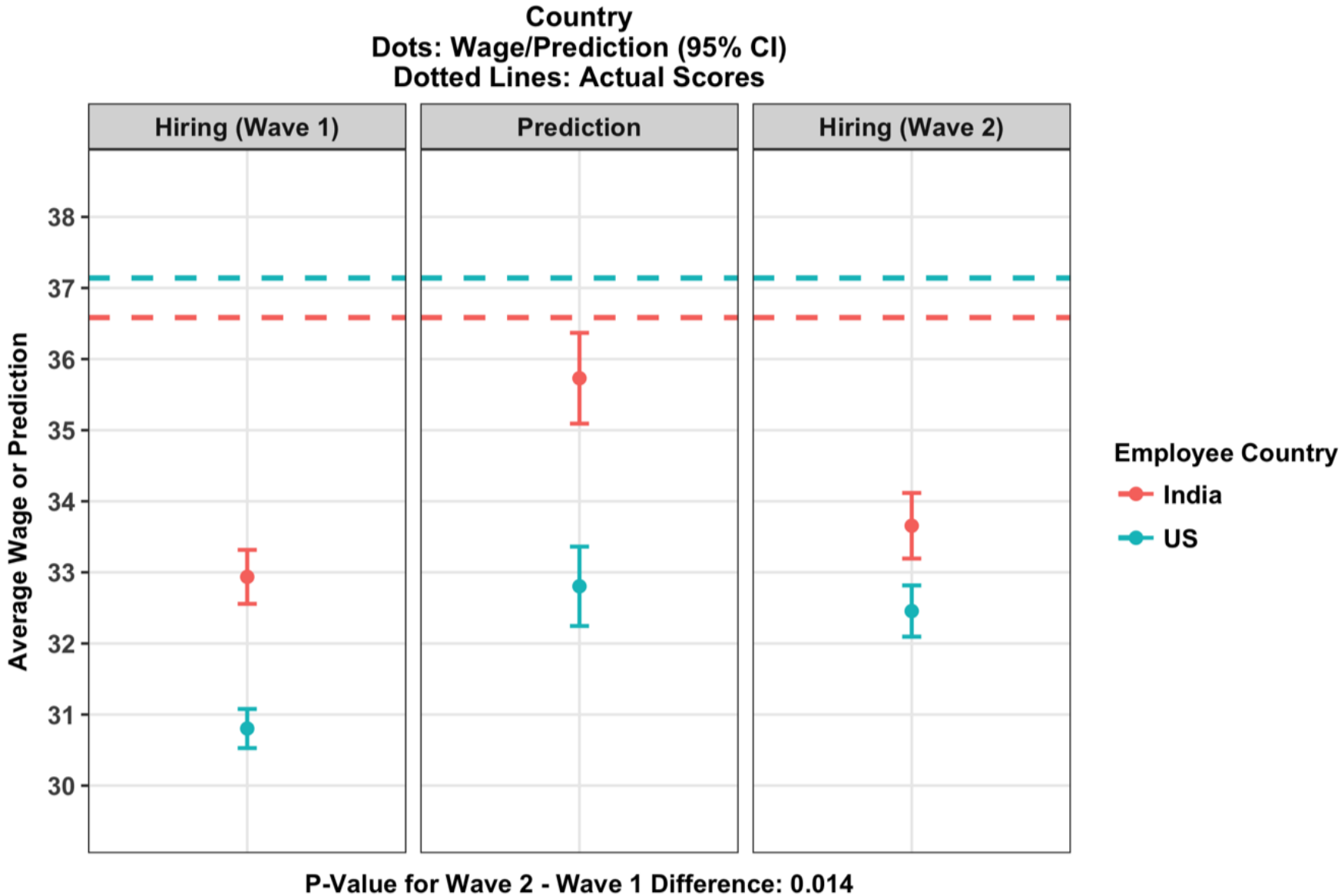
Employers were told:

"Here are the correct answers for the belief questions that you answered."

- Americans answered 37.14 questions correctly on average.
- Indians answered 36.58 questions correctly on average.

Asked to make wage offers based on 10 additional employee profiles (same hiring/payment scheme).

Either taste-based or still some inaccurate statistical discrimination against Americans.



Conclusions

Inaccurate beliefs may be prevalent in many domains currently being studied by discrimination researchers.

- Most tests to separate statistical and taste-based sources rely on strong assumption: people hold accurate beliefs.
- Incorrectly imposing this assumption can lead to mistaken attribution of source, and therefore, misguided policy/welfare analysis.
- Relaxing this assumption generates an identification problem.
- Illustrative example using results from an online experiment demonstrates the importance of inaccurate beliefs in how we study discrimination.

Conclusions: Two Key Forms of Inaccurate Beliefs

Inaccurate priors / beliefs about population.

- Persistent discrimination due to selection effects / exit
- May not "wash out" in the long-run.
- Discrimination reversals can be sign of selection (Bohren Imas Rosenberg (2019)).

Inaccurate beliefs about signals.

- Persistent discrimination due to incorrect updating.
- Could be about exogenous signals (e.g. how predictive is a test) or endogenous signals (e.g. interpreting reference letter).