Information Processing: Contracts versus Communication

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The Puzzle – Broad Outline

- A principal expects to receive private information.
- The principal has to rely on an agent acting on that information.
- The principal has limited power to commit to state-contingent instructions.
- The principal can always engage in *ad hoc* (cheap-talk) communication.

Question: When to commit (to instructions) and when to communicate?

The Puzzle – More Detail

- Prior to receiving information the principal writes a contract.
- Contract is not fully detailed complete – due to writing costs.
- May choose a contract that is not obligationally complete.
- Principal receives information after writing, before execution.
- The agent acts, either in response to a contract instruction or to (cheap talk) communication.
- (Coarsely) state-contingent contract actions are strictly enforced.

Question: What information will be governed by the contract and what by ad hoc *(cheap talk) communication? How do contract and cheap talk interact?*

Timing

- Stage 1: Principal writes contract.

  The contract codifies language that makes conditions (sets of states) and actions (instructions) verifiable to third parties.

  No contingent payments. Non-contingent payments are largely ignored.
- Stage 2: Principal observes the state.
- Stage 3: Principal sends instruction (contract) or a cheap-talk message (gap).

  Ad hoc communication (cheap talk) is informal, private, and non-verifiable.
- Stage 4: Agent takes an action by following an instruction or responding to a message.
- Stage 5: Contracted states and actions become verifiable.

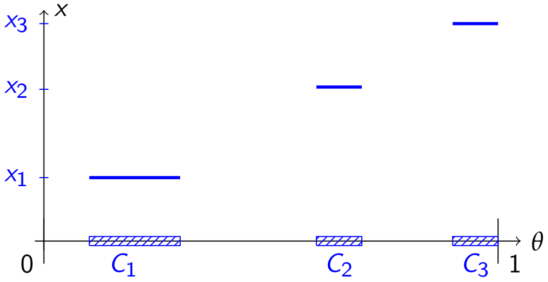
Environment

- Players:
  - Sender (Principal)
  - Receiver (Agent)
- State $\theta \in [0, 1]$ with $\theta \sim F, f(\theta) > 0$
- Receiver's action $y \in \mathbb{R}$
- Payoffs: concave loss functions satisfying positive mixed-partial condition
  - Sender $U^S(y, \theta, b)$
  - Receiver $U^R(y, \theta)$
  - positive bias b – for any θ the sender's ideal point exceeds the receiver's ideal point

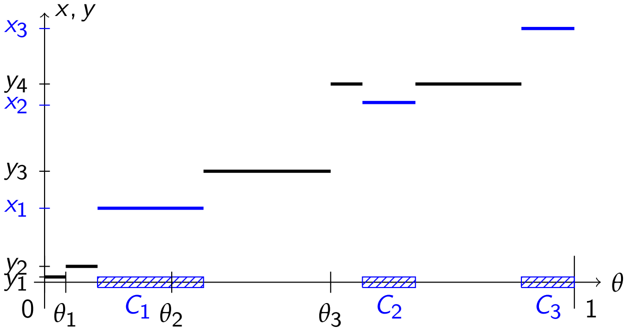
Timing of the contract writing game G

1. Sender writes a contract – $\mathcal{C}$
   - simple – not fully detailed complete
   - potentially obligationally incomplete – giving rise to a *gap*
2. Sender observes the state
   - contract induces action
   - gap induces communication
3. Communication subgame – $\Gamma^{\mathcal{C}}$ on the gap
   - sender sends message
   - receiver takes action

Goal: characterize sender-optimal SPEa



x
x_3
x_2
x_1
θ
0
C_1
C_2
C_3
1



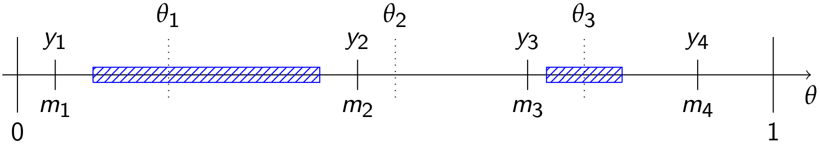
x, y
x3
y4
x2
y3
x1
y2
y1
0
θ1
C1
θ2
θ3
C2
C3
1
θ

Contract writing game $G(\widehat{K}, b)$

- Sender writes **contract** $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{K}$
- Clauses (C_k, x_k), $k = 1, \ldots, K$
- Conditions $C_k \subseteq [0, 1]$ are intervals in the state space.
- Instructions $x_k \in \mathbb{R}$
- $K \leq \widehat{K}$
- **Commitment**:
  If $\theta \in C_k$, action x_k implemented

Communication subgame $\Gamma^{\mathcal{C}}$

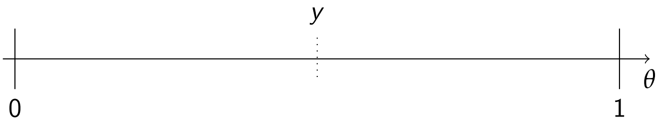
- **Gap** in the contract: $\mathcal{L}(\mathcal{C}) := [0,1] \setminus \bigcup_{k=1}^{K} C_k$
- Sender strategy (messages) $\sigma : \mathcal{L}(\mathcal{C}) \to \Delta(M)$
- Receiver strategy (actions) $\rho : M \to \mathbb{R}$
- Γ^0 induced by $\mathcal{C}^0$ is a CS game
- No commitment
- Partitional equilibria
  - Critical types θ_i
  - "steps" = induced actions y_i



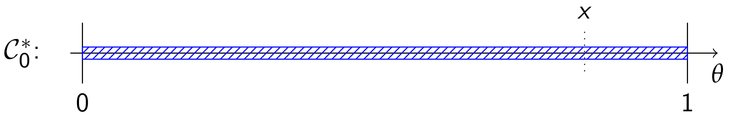
Example: $\widehat{K} = 1$ and $b = \frac{1}{3}$

- Uniform distribution and quadratic payoffs
  - Sender: $U^S(y, \theta, b) = -(\theta + b - y)^2$
  - Receiver: $U^R(y, \theta) = -(\theta - y)^2$
- There cannot be an equilibrium with more than two steps
- No contract, 0-step, 1-step, and 2-step contracts: find the optimum in each class and compare.

No contract = CS communication (babbling):

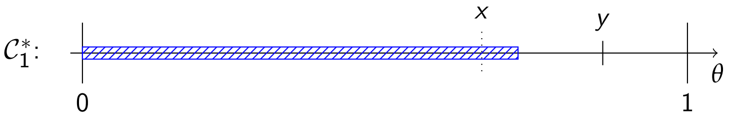


Every state covered by contract = no communication (0-step):

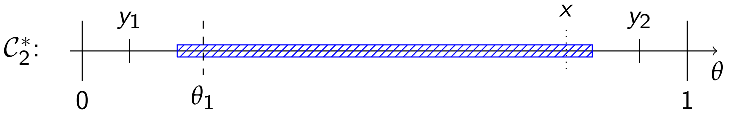


Example: 1-step and 2-step

Allowing for 1-step communication:



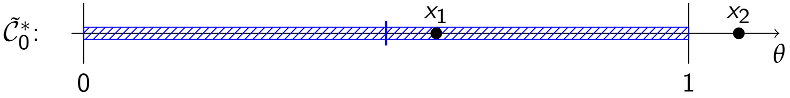
Allowing for 2-step communication:



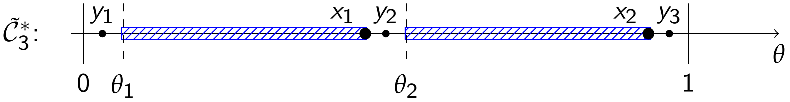
The sender's payoffs are ordered:
no contract $\prec$ 0-step contract $\prec$ 1-step comm. $\prec$ 2-step comm.

Example with different parameters

Increase number of clauses to $\widehat{K} = 2$, keeping $b = \frac{1}{3}$:



Increase number of clauses to $\widehat{K} = 2$ and decrease bias to $b = \frac{1}{5}$ (recall that w/o a contract maximally two actions can be induced in equilibrium):



Example: take-aways

- More clauses improve payoff
- More clauses can drive out communication
- Communication can replace contracting for smaller bias
- More communication actions with contract compared to CS:

  Contract relaxes incentive constraints in communication

General results

The sender optimally uses as many clauses as possible:

Proposition 1

If $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{K}$ *is an optimal contract in* $G(\widehat{K}, b)$, *then* $K = \widehat{K}$.

- Intuition:
  - Replace communication interval: sender imposes her bias
  - Split existing clause: actions more precise

General results

If the maximal number of clauses grows large, contracting drives out communication:

Proposition 2

For any sequence of $\{\mathcal{L}_{\widehat{K}}\}_{\widehat{K}=1}^{\infty}$ *of gaps arising in sender-optimal equilibria* $e(\widehat{K}, b)$ *of contract-writing games* $G(\widehat{K}, b)$, $\widehat{K} = 1, 2, \ldots,$

$$\lim_{\widehat{K}\to\infty} \mathrm{Prob}(\mathcal{L}_{\widehat{K}}) = 0.$$

General results

If the bias goes to zero, communication drives out contracting:

Proposition 3

Suppose that the continuity property holds for the games $\Gamma^0(b_i)$. For any sequence $\{\mathcal{L}_i\}_{i=1}^{\infty}$ of gaps in sender-optimal equilibria $e(b_i)$ of games $G(\widehat{K}, b_i)$ with $\lim_{i\to\infty} b_i = 0$,

$$\lim_{i\to\infty} \mathrm{Prob}(\mathcal{L}_i) = 1.$$

Results for uniform-quadratic environment

(Not necessarily optimal) contracts can increase the number of steps in communication:

Proposition 4

For any b, there exist a $\widehat{K}$ and a contract $\mathcal{C}$ such that there is an equilibrium of the communication subgame $\Gamma^{\mathcal{C}}$ with n induced actions if and only if $n < 1 + \frac{1}{2b}$.

- Comparison to CS for $b < \frac{1}{2}$:

$$\frac{1}{2} + \sqrt{\frac{1}{4} + \frac{1}{2b}} < 1 + \frac{1}{2b}$$

- Example $b = \frac{1}{10}$, $\hat{K}$ arbitrary: $N_{CS} = 2$ and $\hat{N} = 5$

Results for uniform-quadratic environment

Sufficiently many clauses – relative to the bias – result in no communication:

Proposition 5

If $\widehat{K} > \frac{1}{2b}$, then any optimal contract is obligationally complete.

Results for uniform-quadratic environment

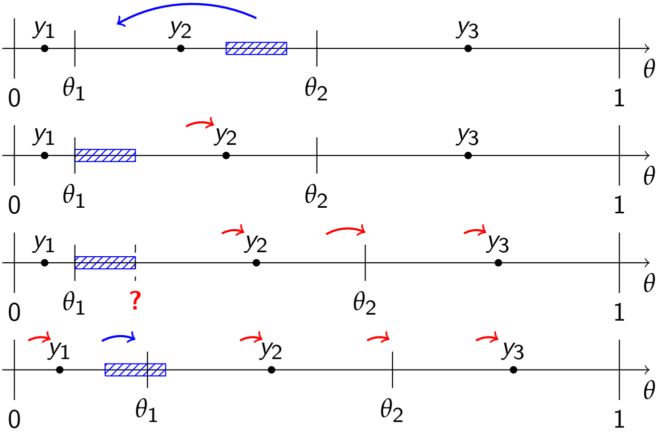
An optimal contract relaxes incentive constraints:

- Every "condition cluster" contains a critical type (equivalently, no condition cluster belongs to the interior of a communication interval)
- With influential communication, there is a condition cluster containing a critical type that is not 0 or 1

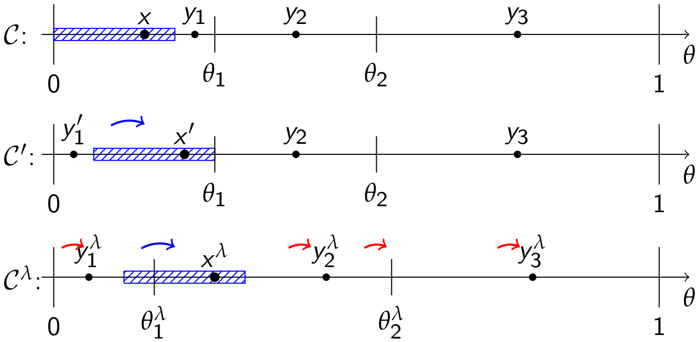
Proposition 6

Suppose that the contract $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{\widehat{K}}$ *is optimal in the contract-writing game* G*, and the equilibrium* $e^{\mathcal{C}}$ *is sender-optimal in the communication subgame* $\Gamma^{\mathcal{C}}$*. Then, for every condition cluster* $\boldsymbol{C}$*, there is a critical type* θ *with* $\boldsymbol{C} \cap \{\theta\} \neq \emptyset$*. If, in addition, the equilibrium* $e^{\mathcal{C}}$ *induces at least two communication actions, then there is a condition cluster* $\boldsymbol{C}$ *and a critical type* $\theta \neq 0, 1$ *with* $\boldsymbol{C} \cap \{\theta\} \neq \emptyset$.

Intuition for: Condition clusters contain critical types

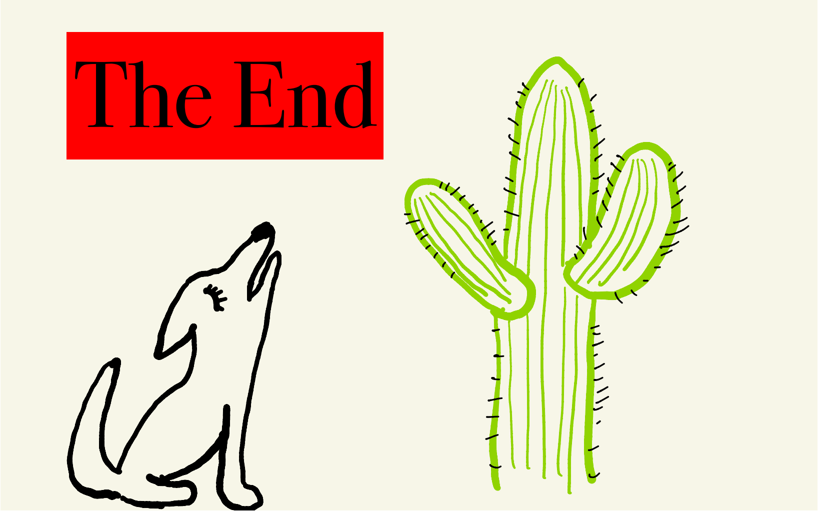


Condition with critical type that is not 0 or 1



Concluding remarks

- Model of interaction between contracts and communication
- Tradeoff: Ex ante commitment versus ex post discretion
- At the extremes:
  - Small disagreement: communication dominates
  - Many clauses: contracting dominates
- Insight: two benefits from contracts
  - direct: shift control to principal
  - indirect: relaxation of incentive constraints $\rightarrow$ potential for more actions induced by communication and for "more even communication"


The End

If there is time or there are questions:

Structure of optimal contracts

- Equilibrium is partitional and monotonic

Corollary 7

Suppose that the contract $\mathcal{C} = \{(C_k, x_k)\}_{k=1}^{\widehat{K}}$ *is part of a sender-optimal equilibrium* e^G *in the contract-writing game* G *and induces a sender-optimal n-step equilibrium* $e^{\mathcal{C}}$ *in the communication subgame* $\Gamma^{\mathcal{C}}$. *Then, the equilibrium* e^G *is*

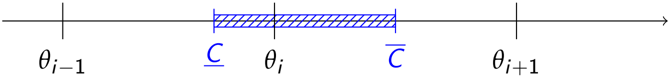
1. *partitional – there is a partition* $\mathcal{P} = \{P_1, P_2, \ldots, P_{\widehat{K}+n}\}$ *of the type space* $[0, 1]$ *into intervals such that each* $P \in \mathcal{P}$ *is either a condition of* $\mathcal{C}$ *or a communication interval in* $e^{\mathcal{C}}$*; and,*
2. *monotonic – for any two* $P, P' \in \mathcal{P}$, $P \neq P'$, *with* $\inf(P') \geq \sup(P)$, *the actions* $a(P')$ *and* $a(P)$ *taken for states in* P' *and* P *satisfy* $a(P') > a(P)$.

Equalizing communication intervals

- Contracts relax incentive constraints
- Lengths of communication intervals can be equalized

Corollary 8

Suppose $\widehat{K} = 1$, the contract $\mathcal{C}$ with condition $[\underline{C}, \overline{C}]$ is optimal, and $\mathcal{C}$ induces at least two communication actions in the sender-optimal equilibrium $e^{\mathcal{C}}$ of the communication subgame $\Gamma^{\mathcal{C}}$. If θ_{i-1}, θ_i, and θ_{i+1} are critical types in the equilibrium $e^{\mathcal{C}}$ with $\theta_i \in \left[\underline{C}, \overline{C}\right]$, then $\left|\theta_{i+1} - \overline{C}\right| < \left|\underline{C} - \theta_{i-1}\right| + 4b$; and, if $\theta_i \in \left(\underline{C}, \overline{C}\right)$, then $\left|\theta_{i+1} - \overline{C}\right| \leq \left|\underline{C} - \theta_{i-1}\right|$.



Variations of our introductory example

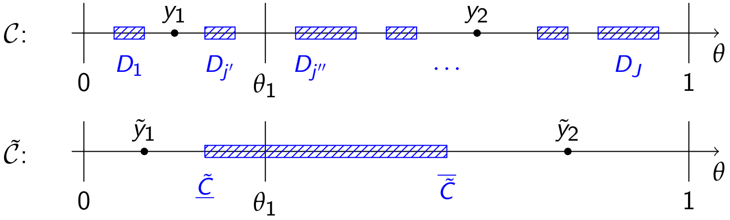
Recall that there $b = \frac{1}{3}$ and $\widehat{K} = 1$.

Finite unions of disjoint closed intervals as conditions

- It is never optimal to split the condition into finitely many disjoint intervals.

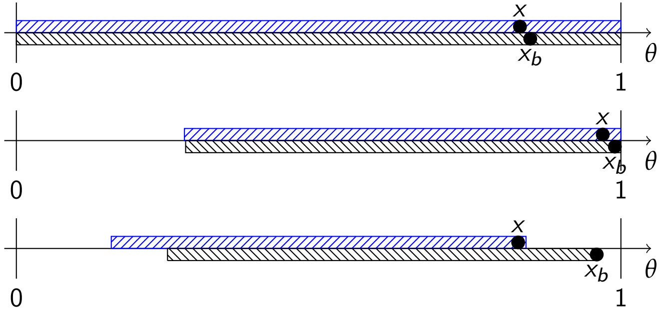
Proposition 9

Suppose that we allow contracts with conditions C that are finite unions of disjoint closed intervals. Then, for $b > \frac{1}{4}$ and $\widehat{K} = 1$, any optimal contract is nonempty and the condition in that contract is a single interval.



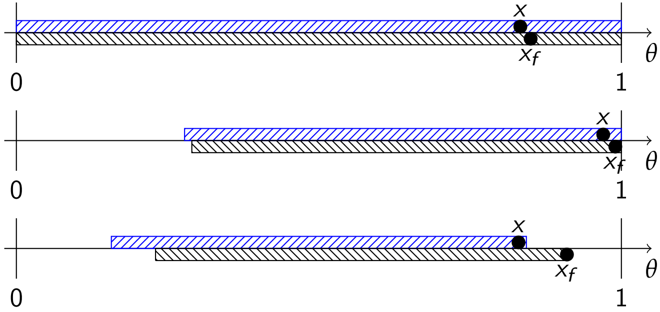
Example: nonconstant bias

- Assume $b(\theta) = \frac{1}{3} + \frac{1}{30}\theta$
- Optimal contract covers states with relatively higher bias



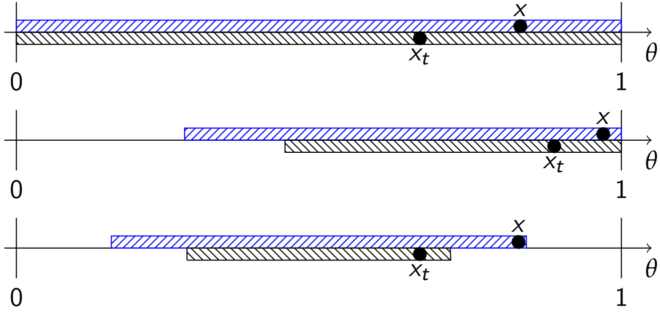
Example: nonuniform distribution

- Assume $f(\theta) = \frac{9}{10} + \frac{2}{10}\theta$
- Optimal contract covers more likely states



Example: transfers

- Sender: $U^S(y, \theta, b, w) = -(\theta + b - y)^2 - w$
- Receiver: $U^R(y, \theta, w) = -\alpha(\theta - y)^2 + (1 - \alpha)w$
- Sender maximizes:
  $\mathbb{E}U^S(y, \theta, b, w) \quad \text{s.t.} \quad \mathbb{E}U^R(y, \theta, w) = \overline{u}^R$
- Optimal contract covers fewer states



Extensions: take-aways

- Some motivation for assuming intervals
- Some robustness with respect to: bias, distribution, transfers
- Contract covers states with higher conflict
- Contract covers states that are more likely
- Transfers reduce the set of states in the contract

Continuity assumption

Nothing unexpected happens for $b \to 0$:

Continuity Property. *For any sequence of biases* $\{b_i\}_{i=1}^{\infty}$ *with* $b_i \to 0$ *and any sequence* $\{e(b_i)\}_{i=1}^{\infty}$ *of sender-optimal equilibria in the games* $\{\Gamma^0(b_i)\}_{i=1}^{\infty}$, *the sender's payoffs in those equilibria converge to* $\int_{[0,1]} U^S(y^S(\theta), \theta, 0) dF(\theta)$.

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