Gambling over Public Opinion*

Deepal Basak
Indian School of Business

Joyee Deb
Yale University

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Abstract

We consider bargaining environments where two agents make demands, following which public opinion forms. Agents then bargain again, and suffer costs of compromise if they scale back their initial demands. If public favors one agent's position, it is more costly for her to compromise. In a simple model with symmetric uncertainty about public opinion, we show that there is a unique equilibrium in which agents never make compatible demands in the first stage, rather take a gamble over public opinion. This implies inevitable welfare loss with at least one party making a costly compromise. We analyze how the extent of gambling varies with the distribution of public opinion.

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1 Introduction

In many bargaining situations, public opinion is used as a tool for credible commitment. Bargainers who have strong public support can more easily convince their opponents that they will not back down from their position. For instance, domestic public opinion can lend credibility to a politician's tough stance in international negotiations.[1] This is because it is understood that if a politician has the public mandate, but ends up softening his position, he would face significant domestic political costs (Fearon (1994)).[2] In 2012, just weeks before the impending fiscal cliff, many Republican politicians held firm on their vote in favor of spending cuts. Their stance was credible because it was integral to their promise to their own constituents and, backing down would involve reputational costs and had the potential to adversely impact their re-election probabilities.

However, in many situations, public opinion is not really known with certainty at the time bargaining starts, but rather evolves over time. Yet bargaining parties stick to their positions until it becomes finally clear that the public favors one side or the other, and this determines who finally gives in. Consider the following examples: In case of the fiscal cliff in 2012, there was initially no clear understanding of what the public opinion was, and both Republicans and Democrats held firm in their respective positions for the longest time.[3] In December 2012, a Gallup poll found that public opinion had moved: *"Sixty-two percent of Americans would like to see federal government leaders compromise on an agreement... more than twice the 25% who want leaders to stick to their principles..."*(Jones (2012)). It became increasingly clear that the public wanted the Republicans to compromise. Finally, shortly before the deadline, enough Republicans did back down to avert the fiscal cliff. Similarly, in April 2011, the Indian government and a prominent activist Anna Hazare were deadlocked over the drafting of an anti-corruption bill. Hazare went on a very public hunger strike and made demands that the Government refused to accept. Such anti-corruption activism was the first of its kind in India and a priori, it was not clear how the public would respond. By late July, Hazare had attracted enormous popular support, and it became

[1] Schelling (1960) writes, in the context of international negotiations: *"national representatives . . . seem often to create a bargaining position by public statements, statements calculated to arouse a public opinion that permits no concessions to be made. If a binding public opinion can be cultivated and made evident to the other side, the initial position can thereby be made visibly "final.""*

[2] Hans Morgenthau (1948) famously wrote that "No man who has taken such a stand before the attentive eyes and ears of the world can in full public view agree to a compromise without looking like a fool. . . "

[3] CNN reported in November 2012: *"Yet while the American people have chosen Barack Obama to navigate these shoals over the next four years, they remain deeply divided over what to do about these challenges. Obama has a mandate to govern, but his mandate on specific issues is far from clear..."*

clear that he had the public mandate, making backing down and letting down millions of supporters now impossible. In August, the Government finally yielded to all demands.

Such bargaining impasses followed by compromise by one or both sides is not efficient. Even ignoring costs of delay, such compromises imply that one or both parties will have to back down from their initial positions, and will suffer reputational costs. The objective of this paper is to examine such bargaining environments with uncertain public opinion, and ask what outcomes emerge, and why inefficiency arises.

Formally, we study a two-stage bilateral bargaining process. In the first stage both parties simultaneously and publicly make their demands. If the demands are compatible, then an agreement is reached right away. If the demands are not compatible, then the bargainers are deadlocked and move into the second stage. At the start of the next stage, they observe the public opinion on the issue, and then engage in bargaining again. We assume that the agents alternately make offers as is standard after Rubinstein (1982). We also assume that an agent can make a counter offer almost immediately after an offer is made. If a party ends up backing down from its initial demand, it suffers a cost. We can interpret this as the cost of compromise or a reputational cost. Note that at the time of making their initial demands, the bargainers do not know the public opinion, and hence the reputational cost they may have to incur from backing down later.

Motivated by the two leading examples above, our environment has three key features. First, public opinion is unidimensional: higher support for one party implies lower support for the other. We interpret this as both agents being accountable to the same public. Second, if any party compromises from her initial bargaining position she suffers a cost that depends on the fraction of the public that ends up favoring her. This captures the idea that if the public strongly supports one party's position it is hard for that party to compromise and let them down. Finally, if a bargainer backs down from her initial position, the cost she faces depends on the extent of the compromise. This is to capture the idea that the public doesn't uniformly punish all compromises. For instance, if a labor union leader were trying to negotiate a wage of \$17 per hour up from the current \$15, and settled at \$16.75, her stakeholders are less likely to lose faith in her than if she were to give in completely.

In the main result of the paper, we show that under symmetric uncertainty about the public opinion, inefficiency is inevitable. Formally, we show that there is a unique symmetric equilibrium, in which players make incompatible demands in the first stage. This immediately implies that any outcome from bargaining is inefficient, because once bargaining goes into the second stage, one or both players have to compromise and bear the associated costs. It turns out that each party prefers to gamble over public opinion at the first stage and back down later if public opinion is unfavorable, thereby incurring a cost of compromise.

It is worth highlighting that in the existing literature on bargaining, when delay or inefficiency arises in equilibrium, there is either some source of asymmetric information, or the existence of agent types that are exogenously committed to sticking to demands, or both. Here, inefficiency arises under symmetric uncertainty, with no possibility of exogenous commitment.

To see the intuition behind our result, consider a situation in which both players demand exactly half of the surplus at the first stage and settle immediately. Given a symmetric environment this may be a reasonable outcome to expect. However, this cannot be optimal. If an agent increases her demand of $\frac{1}{2}$ by a very small ε, then she takes a gamble: with probability $\frac{1}{2}$ the public opinion will lean towards her, forcing her opponent to back down eventually and thus giving her additional ε, and with probability $\frac{1}{2}$ the public opinion will lean towards her opponent, forcing her to back down and thus costing her. However, she needs to bear this cost of compromise exactly when compromise is not too costly for her: We show[4] that the expected loss is $\varepsilon \cdot (2ln(2) - 1)$. Since $(1 - ln(2)) > 0$, the marginal benefit always outweighs the marginal cost. The key insight is when uncertainty is resolved and one party finds out that the public opinion favors her opponent, it is no longer that costly to back down on her initial demand. This induces agents to gamble rather than settle right away.

But, to what extent do agents gamble? We provide an explicit characterization of the initial demands made by agents in the unique equilibrium, and show that, with uniformly distributed public opinion, while agents each demand more than half of the surplus, they do not ever demand the full surplus. This is consistent with empirical evidence. The intuition behind this is that though agents prefer to take a gamble with public opinion, they also know that they will need to back down with positive probability and the cost of compromise they face depends also on the extent of compromise. This pushes them to demand less and reduce the expected cost of compromise. Uniqueness of equilibrium allows us to derive comparative statics on the extent of gambling. We show that the extent of gambling (and inefficiency) increases with the uncertainty about public opinion.

[4] For most of the paper, we restrict attention to uniformly distributed public opinion and a specific functional form for the cost of compromise. These assumptions are made mainly for tractability. At the end of the paper, we consider general symmetric distributions of public opinion and general cost functions, and provide sufficient conditions under which our main results continue to hold.

The reader may wonder if our results are driven by the symmetry of agents. For instance, suppose that one agent starts out with a disadvantage ex ante, in the sense that it is commonly known that public opinion is extremely likely to favor her opponent, which means that she is more likely to have to compromise in the second stage. In this case, we might guess that she may settle right away rather than take a gamble. It turns out that inefficiency will still arise (and there is no equilibrium in which players make compatible demands in the first stage). The intuition is that if public opinion is likely to favor one agent, her incentive to make high initial demands is dampened. But at the same time, her opponent now has an even greater incentive to overstate his demand, because now he knows that with high likelihood he will dominate and get her share once public opinion is revealed.

A key force that pushes each agent to gamble over public opinion is the knowledge that if she needs to compromise it will be exactly when compromise is not that costly. Indeed, because public opinion is unidimensional, the costs of the two agents are perfectly negatively correlated. However, in many applications, this may not be a reasonable assumption, because the two parties may not be accountable to the same public. For instance, a labor union leader may be accountable to the labor union, and management may be answerable to the stakeholders of the business, and while their interests are opposed, the opinions of the two stakeholder groups may not be perfectly negatively correlated. We show that the qualitative result of inevitable inefficiency in equilibrium is robust even when there is some imperfect negative correlation.

In our setting, we assume that the public opinion is independent of the demands made by the two agents. However, in the current times, with political negotiations being made increasingly public by media and social media, it is easy to see how the bargaining positions taken by parties might influence public opinion. For instance, the public may be tired of political gridlock and therefore strongly prefer a moderate position to one that is extreme. To explore this idea, we study an extension in which the initial demands made by the two agents can affect the distribution of public opinion, and show that our results continue to hold in this setting.

1.1 Related Literature

This paper is related to several strands of the literature on bargaining. Schelling (1956) was the first paper to propose the idea that costly commitment to a bargaining position can provide leverage in bargaining. This idea was formalized in Crawford (1982). Fearon (1994) was among the first to discuss "audience costs" in bargaining, or the costs that political leaders have to face in terms of lost reputation or poor electoral outcomes when they back down in international negotiations.

There is a growing literature on bargaining with commitment types, where agents can get forever committed to their demands. Inefficient delay can arise in these settings. See for example, Abreu and Gul (2000) who studies bargaining between agents when agents privately know if they are commitment types. Kambe (1999) considers a somewhat related model with commitment types, but in which agents do not know if they will become committed at the start of the game. Ellingsen and Miettinen (2008) show that if there is a cost to getting committed, but the probability of getting committed is less than 1, then conflict will arise with positive probability. The key difference between our paper and this literature is that we do not assume the existence of commitment types. In our setting, there is no possibility of exogenous commitment; nor is there any private information about it.[5]

Our framework is similar to an early paper by Muthoo (1992) who augments a Rubinstein bargaining game with a first stage, but does not consider any uncertainty. More closely related is Dutta (2011) who extends Crawford (1982) and, like us, considers a model of symmetric information where revoking costs become public after incompatible demands. Dutta (2011) shows that if the revoking cost distribution functions are identical and perfectly positively correlated then there are multiple equilibria, but under a global game perturbation, disagreement cannot be sustained in equilibrium. The baseline model in our paper assumes negatively interdependent revoking costs, and we show that disagreement and inefficiency is indeed inevitable.[6]

There is also a connection with the literature on reference dependent preferences in bargaining. In our setting, the initial demands serve as a reference point for computing the costs in the event of compromise. In contrast, Li (2007) and Compte and Jehiel (2003) consider the references points as the earlier offers made by the opponent.

Our paper is also related to Ortner (2017) who studies how electoral incentives influence political bargaining between two parties. The focus of the paper is completely different from ours in that it analyzes the dynamics of bargaining to predict when gridlocks are most

[5]In a related paper, Li (2011) modifies the model in Ellingsen and Miettinen (2008) by allowing players to make demands and commitment decisions sequentially, and show that both compatible and incompatible demands can arise in equilibrium.

[6]The case of perfectly positively correlated costs can be easily incorporated in our framework. In that case, we would get a unique equilibrium with agreement (efficient).

likely to arise, when popularity of parties changes over time. The paper finds that periods of gridlock arise close to an election, and when parties have similar levels of popularity.

The rest of the paper is organized as follows: Section 2 describes the model. In Section 3, we characterize the unique equilibrium and show inefficiency must arise with agents making incompatible demands at the first stage. In Section 4, we show that the results are robust to asymmetry in public opinion.In Section 7, we consider general symmetric distributions of public opinion and general cost functions and present a sufficient condition under which the inefficiency result continues to hold. Finally, in Section 6, we discuss settings in which the costs of the agents are not perfectly negatively correlated. Some of the proofs are in the Appendix.

2 Model

Suppose that there are two agents i and j that are negotiating. Each party wants to maximize her share of surplus which is normalized to 1. At the start of the game, the parties simultaneously and publicly announce their demands: Let $z_j \in [0,1]$ denote the demand of party j, which means that she offers her opponent the remainder $1 - z_j$. We let $\mathbf{z}$ denote the vector of initial demands (z_i, z_j). We say the demands are compatible if $z_i + z_j \leq 1$. In this case, agreement is reached. If $z_i + z_j < 1$, then one party's offer is randomly chosen, and the other party is given the remainder. If demands are incompatible, i.e., $z_i > 1 - z_j$, then conflict occurs.

During conflict, a mass 1 size public decide whether to stand behind a party i's demand. Let k_i be the proportion of public that support party i. We refer to k_i as public opinion. Both party are facing the same public and hence the public support for party j is $k_j = 1 - k_i$.

When parties make their initial demands they are uncertain of the public opinion, and commonly believe that k_i is drawn from an exogenous[7] distribution F_i. Note the two public opinions k_i and k_j are perfectly negatively interdependent.[8] We assume that F_i has a continuous density f_i and it is symmetric around $1/2$.

Once public opinion is realized, it is observed and determines the parties' cost of subsequent compromise. After observing the public opinion, the parties negotiate again. This second stage proceeds as an alternating offer bargaining game à la Rubinstein (1982), with

[7]In Section 5 we consider the case in which initial demand can influence the distribution of the public opinion.

[8]In Section 6 we consider more general interdependence.

frequent offers: One of the parties is picked randomly to make the first demand. If the other party accepts this demand then an agreement is reached and the game ends, otherwise it continues to the next round. In the next round, the other party makes the demand and the same process continues until an agreement is reached. Thus, potentially the game can go on forever. We assume that the time interval between two rounds $\Delta \to 0$.

If a party i demanded z_i at the start of the game, but settles for $x < z_i$ at the end of bargaining at the second stage, then she bears a cost of compromise $C_i(z_i - x, k_i)$, that depends on the extent of compromise and the public opinion. Formally, we assume that

$$C_i(z_i - x, k_i) = c_i(k_i) \cdot (z_i - x),$$

where $c_i(k_i)$ is a strictly positive function that is increasing in k_i. We can think of $c_i(k_i)$ as representing a "constant marginal cost per unit of compromise", where this marginal cost depends on the fraction of public support.

The specification of the cost function captures two natural features of our setting. First, when a party ends up with strong support of the public, it is very difficult for her to scale back her initial demand. Put differently, the party with the greater public support has a higher cost of compromise and greater leverage. To go back to the motivating examples, with regard to the fiscal cliff, when polls showed that most of the public wanted their federal leaders to compromise to avoid the fiscal cliff, it became easier for Republican leaders to change their position and reach a settlement. They realized that not sticking to their stated positions would not have adverse electoral implications. Similarly, when the public opinion in India was overwhelmingly in support of activist Hazare, it became close to impossible for Hazare to back down and let down his supporters. At the same time, it became easier for the Government of India to back down and settle.

Second, the specification captures the natural idea that public opinion does not punish all compromises uniformly, and that the cost of compromise depends on the extent of the compromise, i.e., the difference between the initial demand and the final settlement.[9]

If party i gets share x at time t with initial incompatible demand z_i and realized public opinion k_i, then her payoff is given by $e^{-r_i t} u_i(x_i, z_i, k_i)$ where :

$$u_i(x_i, z_i, k_i) := x - c_i(k_i) \cdot (z_i - x)\mathbf{1}_{(z_i > x)},$$

[9]It is worth highlighting that the multiplicative form that we assume makes the solution of the second stage bargaining problem more tractable. Relaxing this would make it impossible to get a closed-form analytical solution to the second stage problem.

where $\mathbf{1}$ is an indicator function. In words, $u_i(x_i, z_i, k_i)$ is the utility to agent i who initially demands z_i, but after observing the public opinion k_i settles for x_i immediately.

For simplicity, we assume a symmetric environment : the two agents have the same discount rate ($r_i = r_j = r$), the same cost functions ($c_i : [0, 1] \to R_+$ is same for both i) and they face the same distribution of public opinion ($F_i : [0, 1] \to [0, 1]$ is same for both i). For most part of the paper, we assume that F_i is uniform and

$$c_i(k_i) = \frac{k_i}{1 - k_i}.$$

However, we generalize this in Section 7. In the next section, we characterize the (symmetric) equilibrium of this game, and show that inefficiency must arise in equilibrium.

3 Gambling is inevitable

We first consider the second stage of bargaining, after the public opinion has been formed.

3.1 Second stage of bargaining

The second stage is a standard alternating offer bargaining game. Lemma 1 below characterizes the outcome in the second stage.

For any player i, we define two thresholds of public opinion:

$$\begin{aligned} \underline{k}_i(\mathbf{z}) &:= \frac{(1 - z_j)}{z_i + z_j} \\ \overline{k}_i(\mathbf{z}) &:= 1 - \frac{(1 - z_i)}{z_i + z_j}. \end{aligned}$$

These thresholds define three regions $D_i(\mathbf{z}), D_j(\mathbf{z}), C(\mathbf{z})$ based on the realized public opinion, which help us fully characterize behavior in the second stage. For any i

$$D_i(\mathbf{z}) = \{k_i | k_i \geq \overline{k}_i(\mathbf{z})\}.$$

Since the thresholds for the two parties are such that $\underline{k}_i = 1 - \overline{k}_j$, we can equivalently write $D_i(\mathbf{z}) = \{k_j | k_j \leq \underline{k}_j(\mathbf{z})\}$.

$$C(\mathbf{z}) = \{k_i | k_i \in [\underline{k}_i(\mathbf{z}), \overline{k}_i(\mathbf{z})]\} = \{k_j | k_j \in [\underline{k}_j(\mathbf{z}), \overline{k}_j(\mathbf{z})]\}.$$

Lemma 1 *Given initial demands* $\mathbf{z} = (z_i, z_j)$*, for any agent* i *and realized public opinion* $\mathbf{k} = (k_i, k_j)$ *(where* $k_j = 1 - k_i$*),*

1. *If* $k_i \in D_i(\mathbf{z})$*, then agent* i *dominates and agent* j *surrenders and accepts agent* i*'s demand, i.e., agent* i *gets her demand* z_i *and* j *gets* $1 - z_i$*.*

2. *If* $k_i \in C(\mathbf{z})$*, then they both compromise and agent* i *gets* $v_i^c(\mathbf{z}, \mathbf{k}) := z_i k_i + \frac{1}{2}(1 - z_i k_i - z_j k_j)$ *and analogously agent* j *gets* $v_j^c(\mathbf{z}, \mathbf{k}) := z_j k_j + \frac{1}{2}(1 - z_i k_i - z_j k_j)$*.*

The details of the proof are in the Appendix. An agent i takes nothing below $z_i k_i$ because the reputation cost will be so high that the actual payoff is negative. In equilibrium, when they both compromise, each of them takes this minimum acceptable share and equally divide the rest $(1 - z_i k_i - z_j k_j)$. Note that if the public opinion favors any agent i (high k_i), then agent i needs to compromise less. If $k_i \geq \overline{k}_i$, then she does not need to compromise at all. Accordingly, we call $D_i(\mathbf{z})$ the dominance region for any player i. Analogously, $C(\mathbf{z})$ is termed the compromise region, where both agents need to compromise from their initial demands.

3.2 Initial demands

We can use the characterization of behavior in the second stage to now determine the demands made in the first stage.

Definition 1 *A pure strategy Nash equilibrium is said to be a* ***settle equilibrium*** *if the initial demands* (z_i^*, z_j^*) *are compatible and a* ***gamble equilibrium*** *if the initial demands are incompatible.*

Consider the expected utility of an agent j from making an initial demand z_j. Let $\mathbf{z} = (z_i, z_j)$ denote the pair of initial demands made by agents i and j in the first stage of bargaining. Given our characterization of the second stage from Lemma 1, we have

$$\begin{aligned} U_j(\mathbf{z}) &:= \int_0^{\underline{k}_j} u_j(1 - z_i, z_j, k_j) dF_j(k_j) + \int_{\underline{k}_j}^{\overline{k}_j} u_j(v_j^c(\mathbf{z}, \mathbf{k}), z_j, k_j) dF_j(k_j) + \int_{\overline{k}_j}^1 z_j dF_j(k_j), \\ &= F_j(\underline{k}_j) U_j^{D_i} + (F_j(\overline{k}_j) - F_j(\underline{k}_j)) U_j^C + (1 - F_j(\overline{k}_j)) U_j^{D_j}, \end{aligned}$$

where U_j^X denotes agent j's expected utility when $k_i \in X$. We establish a lemma that will be used later.

Lemma 2 $\frac{\partial U_j}{\partial \overline{k}_j} = \frac{\partial U_j}{\partial \underline{k}_j} = 0.$

Proof.

$$\begin{aligned}\frac{\partial U_j}{\partial \overline{k}_j} =& - z_j f_j(\overline{k}_j) + u_j(v_j^c(\mathbf{z}, 1 - \overline{k}_j, \overline{k}_j), z_j, \overline{k}_j) f_j(\overline{k}_j) \text{ (by Leibnitz Rule)} \\ =& 0 \text{ (since } v_j^c(\mathbf{z}, \mathbf{k}) = z_j \text{ when } k_j = \overline{k}_j).\end{aligned}$$

Similarly, $\frac{\partial U_j}{\partial \underline{k}_j} = 0$. ■

3.3 No Settle Equilibrium

Suppose that the public opinion is uniformly distributed in $[0, 1]$. The main result of this paper shows that two parties can never come to an immediate agreement.

Proposition 1 *There is no settle equilibrium.*

Proof. Suppose that there exists a settle equilibrium $\mathbf{z} = (z_i, z_j)$ such that $z_i + z_j = 1$. (Clearly, $z_i + z_j < 1$ cannot be an equilibrium.) It is easy to check that if $z_i + z_j = 1$, then

$$\underline{k}_j = \overline{k}_j = z_j.$$

Without loss of generality, suppose that j makes the weakly lower demand, i.e., $z_j \leq z_i$. Let us consider a deviation by agent j to a slightly higher demand $(z_j + \varepsilon)$, and analyze the effect on j's expected payoff. Differentiating $U_j(\mathbf{z})$ w.r.t z_j we get

$$\frac{dU_j}{dz_j} = \frac{\partial U_j}{\partial z_j} + \frac{\partial U_j}{\partial \overline{k}_j} \frac{d\overline{k}_j}{dz_j} + \frac{\partial U_j}{\partial \underline{k}_j} \frac{d\underline{k}_j}{dz_j}.$$

We call the first term the direct effect of altering her demand and the last two terms the indirect effect. By Lemma 2, we know the indirect effect is 0, and we need only consider the direct effect of increasing z_j. When the two parties have made almost compatible demands, then in the second stage, if the public opinion $k_j \leq \underline{k}_j$ then j surrenders, otherwise i surrenders. The probability that both agents will compromise is almost 0.

The benefit to agent j of making a slightly higher initial demand is that if the public opinion leans towards her ($k_j \geq \overline{k}_j$), then i will surrender and she will get a higher payoff. In this case, an increase in z_j increases her payoff by exactly the same amount. This benefit is weighted by the probability that her opponent surrenders - i.e., the marginal benefit of increasing z_j is given by

$$\frac{\partial(1-F_j(\overline{k}_j))U_j^{D_j}}{\partial z_j} = 1 - F_j(\overline{k}_j).$$

The cost of making a marginally higher initial demand for party j is that if the public opinion leans towards her opponent ($k_j \leq \underline{k}_j$), then she has to surrender and this requires larger compromise. From Equation 1, we can see that the marginal cost of increasing z_j is given by

$$-\frac{\partial F_j(\underline{k}_j)U_j^{D_i}}{\partial z_j} = \int_0^{\underline{k}_j} c_j dk_j$$

Agent j will deviates from the compatible demand and marginally increase her initial demand, only if the marginal benefit of doing so outweighs the marginal cost, i.e., the net benefit from deviating is positive if the following holds:

$$1 - F_j(\overline{k}_j) \geq \int_0^{\underline{k}_j} c_j dk_j \tag{NB}$$

Simplifying the above, we have

$$\begin{aligned} 1-\overline{k}_j &\geq \int_0^{\underline{k}_j} \frac{k_j}{1-k_j} dk_j \\ \implies 1-\overline{k}_j &\geq -\underline{k}_j - ln(1-\underline{k}_j) \\ \implies 1+ln(1-z_j) &\geq 0. \end{aligned}$$

The last inequality follows from $\underline{k}_j = \overline{k}_j = z_j$. Thus, the net benefit from deviation is positive if $1+ln(1-z_j) > 0$. Since $z_j \leq \frac{1}{2}$, the above inequality is always satisfied. Thus, among two symmetric parties, the one making a weakly lower initial demand, will always find it in her interest to unilaterally deviate. Therefore (z_i, z_j) with $z_i + z_j = 1$ is not an equilibrium. ■

The key intuition is that when uncertainty is resolved and one agent finds out that the public opinion favors her opponent, it is no longer that costly to back down on her initial demand. More precisely, any agent j knows that the event of having to bear the cost of compromise arises exactly when the realized public opinion is in the dominance region of her opponent ($k_j \in [0, \underline{k}_j]$), and in this case compromise is not that costly.

Remark 1 *Note that the net benefit of marginally increasing one's demand from a compatible pair* $\mathbf{z} = (z, 1-z)$ *is minimized at* $z = \frac{1}{2}$. *We use this fact later to shorten some arguments by only inspecting the incentive to deviate from* $\mathbf{z} = (\frac{1}{2}, \frac{1}{2})$.

It is worth highlighting Proposition 1 does not depend on the uniform distribution or the exact functional form of c. The result for arbitrary distributions and cost functions is in Section 7, where we present the general sufficient conditions for the non-existence of a settle equilbrium.

3.4 Gamble Equilibrium

We focus on symmetric equilibrium. Below we show that when public opinion is distributed uniformly, in the unique symmetric equilibrium, both agents make incompatible offers at the first stage.

Proposition 2 ***[Unique Symmetric Gamble equilibrium:]*** *The unique symmetric equilibrium is a gamble equilibrium, i.e., the initial demands* $\mathbf{z}^* = (z^*, z^*)$ *are incompatible* ($z^* > \frac{1}{2}$). *Further,* z^* *satisfies*

$$H(z^*) := \frac{1}{z^*} + ln\left(\frac{1-z^*}{2z^*}\left(1 - \frac{1-z^*}{2z^*}\right)\right) = 0.$$

Proof. Given z_i and an incompatible initial demand $z_j > 1 - z_i$, agent j's expected utility is

$$U_j(\mathbf{z}) = \int_0^{\underline{k}_j} u_j(1-z_i, z_j, k_j)dF_j(k_j) + \int_{\underline{k}_j}^{\overline{k}_j} u_j(v_j^c(\mathbf{z}, \mathbf{k}), z_j, k_j)dF_j(k_j) + \int_{\overline{k}_j}^1 z_j dF_j(k_j).$$

In equilibrium, we must have $\frac{dU_j}{dz_j} = 0$. We know from Lemma 2 that $\frac{\partial U_j}{\partial \overline{k}_j} = \frac{\partial U_j}{\partial \underline{k}_j} = 0$, and we need focus only on the direct effect of increasing z_j, i.e., $\frac{\partial U_j}{\partial z_j}$, which is given by:

$$\begin{aligned}
\frac{\partial U_j}{\partial z_j} &= -\int_0^{\underline{k}_j} c_j dF_j(k_j) + \int_{\underline{k}_j}^{\overline{k}_j} \left((1+c_j)\frac{\partial v_j^c}{\partial z_j} - c_j\right) dF_j(k_j) + \int_{\overline{k}_j}^1 dF_j(k_j) \\
&= -\int_0^{\underline{k}_j} c_j dF_j(k_j) - \frac{1}{2} \cdot \int_{\underline{k}_j}^{\overline{k}_j} c_j dF_j(k_j) + (1 - F_j(\overline{k}_j))
\end{aligned}$$

The above simplifies to:

$$\frac{\partial U_j}{\partial z_j} = -\frac{1}{2}\left(\int_0^{\underline{k}_j} c_j dF_j(k_j) + \int_0^{\overline{k}_j} c_j dF_j(k_j)\right) + (1 - F_j(\overline{k}_j)) \tag{DE}$$

Since F_j is uniform, and $c_j(k_j) = \frac{k_j}{1-k_j}$, we have

$$\begin{aligned}\frac{\partial U_j}{\partial z_j} &= \frac{1}{2}\left(\underline{k}_j + ln(1-\underline{k}_j) + \overline{k}_j + ln(1-\overline{k}_j)\right) + (1-\overline{k}_j) \\ &= 1 - \frac{1}{2}(\overline{k}_j - \underline{k}_j) + \frac{1}{2}(\ln(1-\overline{k}_j) + \ln(1-\underline{k}_j))\end{aligned}$$

For $z_i = z_j = z$, we have

$$\underline{k}_j = 1 - \frac{1-z}{2z} \text{and } \overline{k}_j = \frac{1-z}{2z}.$$

Substituting the values of $\overline{k}_j$ and $\underline{k}_j$ in the expression above, we have,

$$\frac{1}{2}\left(\frac{1}{z} + \ln\left(\frac{1-z}{2z}\left(1 - \frac{1-z}{2z}\right)\right)\right) = 0.$$

Let us define

$$H(z) := \frac{1}{z} + ln\left(\frac{1-z}{2z}\left(1 - \frac{1-z}{2z}\right)\right).$$

At symmetric equilibrium $z_i = z_j = z^*$, we must have $H(z^*) = 0$. Note that

$$H'(z) = -\frac{1}{z^2} + \frac{1}{\frac{1-z}{2z}\left(1 - \frac{1-z}{2z}\right)} \cdot \left(-\frac{1}{z^2}\left(1 - \frac{1-z}{2z}\right) + \left(\frac{1-z}{2z}\right)\frac{1}{z^2}\right).$$

It is easy to check that when $z \geq \frac{1}{2}$, the above expression is negative. Further,

$$\begin{aligned}\lim_{z\to\frac{1}{2}} H(z) =& 2 - ln(4) > 0. \\ \lim_{z\to 1} H(z) =& -\infty.\end{aligned}$$

Therefore, there exists a unique z^* that solves $H(z^*) = 0$, and is optimal. ■

Numerically, the above expression implies that parties each demand approximately 63%

of the pie. In a symmetric equilibrium, when an agent makes an initial demand of $z > \frac{1}{2}$, we say she gambles over public opinion. When z increases, we say the extent of gambling increases. Notice that any gamble equilibrium involves inefficiency, because if bargaining enters the second stage, then one or both players will have to compromise from their initial demand, and therefore incur a cost of compromise. We show below that for any realized public opinion, the joint payoff is decreasing in the initial demands, i.e., the inefficiency increases with the extent of gambling.

Claim 1 *The joint payoff in the symmetric equilibrium is decreasing in the initial demands.*

3.4.1 Variance of public opinion

The above result shows that uncertainty about the public opinion inevitably gives rise to inefficiency in equilibrium. A natural question is to ask if efficiency can be achieved if there was less uncertainty about the public opinion. At the extreme, if it were commonly known that the public opinion is going to be exactly $\frac{1}{2}$, agents would settle immediately (dividing the surplus equally in a symmetric equilibrium). This is because if an agent deviates and demands more than $\frac{1}{2}$, she would have to surely compromise in the second stage, and such compromise is costly. So no agent will deviate from settling. But what happens with some small uncertainty?

To answer this question, suppose that the public opinion is uniformly distributed in an interval around $\frac{1}{2}$. For example, suppose that $k_i \sim \mathscr{U}[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, for some $\varepsilon \in (0, \frac{1}{2}]$. Note that here, the variance of the public opinion is increasing in ε. Will agents make compatible demands of $(\frac{1}{2}, \frac{1}{2})$ in the initial stage for ε small enough? Note that if agents make incompatible demands $z_i = z_j = z > \frac{1}{2(1-\varepsilon)}$, then $\overline{k}_i(\mathbf{z}) = 1 - \frac{1-z}{2z} > \frac{1}{2} + \varepsilon$, and both agents would have to compromise at the second stage. This cannot be an equilibrium. However, the agents may still make incompatible demands $z \in (\frac{1}{2}, \frac{1}{2(1-\varepsilon)})$.

Proposition 3 *Suppose that public opinion is distributed uniformly in $[\frac{1}{2} - \varepsilon, \frac{1}{2} + \varepsilon]$, for $\varepsilon \in (0, \frac{1}{2}]$. There is no settle equilibrium. There is a unique symmetric gamble equilibrium, in which each agent demands $z^* \in \left(\frac{1}{2}, \frac{1}{2(1-\varepsilon)}\right)$ that satisfies*

$$H^\varepsilon(z^*) := 2\left(2\varepsilon - 1 - ln(\frac{1}{2} + \varepsilon)\right) + H(z^*) = 0.$$

Further, the extent of gambling increases with ε.

The interested reader may refer to the Appendix for the formal proof. Note that $H^{\varepsilon}(z)$ is a parallel downward shift of $H(z)$. As ε increases, $H^{\varepsilon}(z)$ increases for any z and $H^{\frac{1}{2}}(z) = H(z)$. Since $\frac{dH^{\varepsilon}(z)}{dz} < 0$, the initial demand z^* that satisfies $H^{\varepsilon}(z^*) = 0$ increases with ε.

3.4.2 Polarized Public Opinion

Proposition 3 suggests that the extent of gambling increases with the variance in public opinion. A natural question is whether there are situations in which the extent of gambling is extreme and agents each demand the full surplus.[10]

We know from Proposition 2 that when public opinion is uniformly distributed on $[0, 1]$, this cannot happen in equilibrium: The unique symmetric equilibrium demand $z^* \in (\frac{1}{2}, 1)$, i.e., while agents do make incompatible demands they do not demand the entire surplus. However, extreme demands can arise for other distributions of public opinion. For instance, consider a situation in which the public has extreme yet correlated opinions in that they are likely to be either completely in favor of agent i or completely in favor of agent j. In this case, it is possible that agents not only make incompatible demands, but indeed each demand the entire surplus. The intuition is that with such polarized opinions, if an agent starts out making an incompatible demand, she knows that in the next stage only one of them will surrender. It is not possible that both will compromise, since public opinion will be extreme in one direction or the other. If an agent needs to surrender in the second stage, it will be precisely when it costs her almost nothing to do so. Therefore, she always has an incentive to increase her demand in the first stage. Indeed, in such settings, in equilibrium both agents will demand the whole surplus. To see this formally, assume that the public opinion is equally like to be ε close to 0 or 1 for some $\varepsilon < 1/2$.

$$f_i(k_i) = \begin{cases} \frac{1}{2\varepsilon} \text{ for } k_i \in [0, \varepsilon] \cup [1-\varepsilon, 1] \\ 0 \text{ otherwise} \end{cases} \tag{1}$$

Proposition 4 *With polarized public opinion as in (1), in any symmetric gamble equilibrium, the agents will demand at least* $\frac{1}{1+2\varepsilon}$.

[10] Ellingsen and Miettinen (2008) present an extreme result in which each bargainer demands the entire pie. Extreme demands also arise in the literature on final offer arbitration. (See, for instance, Chatterjee (1981), which reduces to a special case of our model with zero cost of compromise.)

4 Asymmetric public opinion

So far, we have restricted attention to a symmetric environment and shown that immediate settlement is impossible in equilibrium: The agent who makes the weakly smaller demand will have an incentive to deviate to a higher demand. However, it would be interesting to ask whether symmetry drives this result. For instance, suppose that one agent starts out with a disadvantage, in the sense that public opinion is less likely to favor her, which makes it more likely that that she would have to compromise in the second stage. In this case, might she want to settle right away rather than gamble?

Suppose that it is commonly known that the public opinion is more likely to lean towards party i. For instance, suppose that k_i is distributed uniformly on the interval $[\frac{1}{2}, 1]$; i.e., $f(k_i) = 2$ for $k_i \in (\frac{1}{2}, 1)$ and 0 otherwise. As before, $k_j = 1 - k_i$. Will agent j now want to deviate from $(1/2, 1/2)$? Surely, she will end up compromising in the second stage, hence the answer is - No. However, agent i will not be satisfied with $1/2$ when the public opinion is more likely to favor her. To incentivize agent i to settle right away, agent j must offer agent i more. Can she offer enough without violating her own incentive to settle? The following proposition shows that the answer is - No.

Proposition 5 *Suppose that k_i is distributed uniformly on the interval $[\frac{1}{2}, 1]$, and as before, $k_j = 1 - k_i$. There is no settle equilibrium.*

Proof. Suppose, for a contradiction that agents made compatible demands $\mathbf{z} = (z_i, z_j)$ such that $z_i + z_j = 1$. Agent i will not deviate from this demand if the marginal benefit from increasing her demand does not exceed the marginal cost of doing so, i.e., Following (NB), we see that she will not deviate as long as

$$1 - F_i(\overline{k}_i) \quad \leq \quad \int_{\frac{1}{2}}^{\underline{k}_i} c_i dk_i.$$

Now, at compatible demands, we know that $\underline{k}_i = \overline{k}_i = z_i$. Moreover, given that $c_i(k_i) = \frac{k_i}{1-k_i}$ and F_i is the uniform distribution on $[\frac{1}{2}, 1]$, the above condition simplifies to

$$\begin{aligned} 1 - F_i(z_i) - \int_{\frac{1}{2}}^{z_i} \frac{k_i}{1-k_i} f_i(k_i) dk_i &\leq 0 \\ \implies 1 - F_i(z_i) + (F_i(k_i) + 2ln(1-k_i))|_{k_i=1/2}^{z_i} &\leq 0 \\ \implies 1 + 2ln(2(1-z_i)) &\leq 0 \end{aligned}$$

which simplifies to $z_i \geq 1 - \frac{1}{2\sqrt{e}}$. Analogously, agent j will not deviate from compatible demand (z_i, z_j) as long as $z_j \geq 1 - \frac{1}{2\sqrt{e}}$. Therefore, $z_i + z_j \geq 2 - \frac{1}{\sqrt{e}} > 1$, which is a contradiction. Therefore there is no settle equilibrium. The simple example above shows that if z_i is sufficiently large to stop i from deviating, then it is also sufficiently small to force agent j to deviate. ■

The intuition here is that when agent i has a clear advantage, then even though agent j will have a higher propensity to make a lower demand to settle, now agent i has an even stronger incentive to make an aggressive demand (since she knows that in the second stage, her opponent is likely to bear the cost of compromise).

Finally, consider the extreme case that the public opinion k_i is known to be almost surely 1. Then we expect that agents will settle for $(z_i, z_j) = (1, 0)$ right away. The proposition below shows that this is indeed the only case in which agents will settle. As long as there is small uncertainty, there cannot exist a settle equilibrium.

Proposition 6 *If $k_j \sim \mathscr{U}[a, b]$ with $0 \leq a < b \leq 1$, there is no settle equilibrium.*

It is worth mentioning that there can be other sources of asymmetry in our setting. For instance, agents could have different discount factors. It is straightforward to show that our main result about the lack of immediate agreement extends. Another example of asymmetry could be that the cost of compromise is one-sided in the sense that only one agent suffers a reputation cost when she backs down, while the other does not. The lack of immediate agreement in equilibrium is robust to this as well. The interested reader may refer to an Online Appendix for the formal proofs of these results.

5 Shaping public opinion

So far, we have assumed that the public opinion is independent of the demands made by the two agents. However, in the current times, with diplomacy and negotiations being made increasingly public by media, it is easy to see how the bargaining positions taken by parties might influence public opinion. For instance, the public may be tired of political gridlock and therefore strongly prefer a party that makes conciliatory demands to one that makes very extreme demands. To explore this idea, we study an extension in which the initial demands made by the two agents can affect the distribution of public opinion. In particular, we introduce a parameter α that measures the public's aversion to extreme demands.

Suppose that given initial public demands $\mathbf{z} = (z_i, z_j)$, the public opinion k_j is drawn from the uniform distribution $F_j^\alpha := \mathscr{U}[a, b]$ where given $\alpha \in [0, 1]$,

$$a = \frac{\alpha z_i + (1-\alpha) z_j}{2(z_i + z_j)} \text{ and } b = \frac{1}{2} + a.$$

We can interpret the parameter α as measuring the public's aversion to extreme demands. To see why, notice that

$$\frac{da}{dz_j} = \frac{db}{dz_j} = \frac{(1-2\alpha)}{2} \cdot \frac{z_i}{(z_i + z_j)^2}.$$

So, when $\alpha \in [1/2, 1]$, then a higher initial demand by agent j moves the distribution of public opinion against her, capturing the public's aversion to an extreme demand. On the other hand, when $\alpha \in [0, 1/2]$, agent j can move the distribution of public opinion in her favor by increasing her demand z_j. The benchmark case is when $\alpha = 1/2$. This corresponds to the case of exogenous public opinion.[11]

A reasonable conjecture might be that when the public is averse to extreme demands, bargaining agents may be pushed towards settling in the first stage. However, we show below that while public aversion to extreme demands reduces the extent of inefficiency, the unique symmetric equilibrium still involves gambling over public opinion.

Proposition 7 *For any $\alpha \in [0, 1]$, there is no settle equilibrium. The unique symmetric equilibrium is a gamble equilibrium. The extent of gambling decreases with α.*

The interested reader can refer to the proof in the appendix. It is easy to see that if the public opinion likes bold demands ($\alpha \in (0, \frac{1}{2})$), then each agent will have an incentive to gamble. However, it turns out that even in the case of extreme public aversion to extreme demands ($\alpha = 1$), agents still gamble. To get the intuition, imagine a small increase in demand by agent j from compatible demands $(\frac{1}{2}, \frac{1}{2})$. In addition to the standard incentive to gamble driven by the uncertainty, now there is also an effect on the distribution of public opinion: both a and b goes down. As a changes the utility marginally decreases by $(1 - z_i)$ and as b changes the utility marginally improves by z_j. These effects cancel each other when the demands are just compatible - i.e., $z_i + z_j = 1$. Thus, the distribution effect cannot eliminate the agents' incentive to gamble at compatible demands. The extent of public aversion to extreme demands can only affect the extent of gambling in equilibrium.

[11] Note that instead of interval $[0, 1]$, we are considering $[1/4, 3/4]$.

6 Interdependence of Public Opinion

In our setting, an agent wants to bet on the public opinion favoring her because she knows that, she will need to compromise and accept a share lower than her initial demand only when it is not costly for her to do so. Since public opinion favors one party or another, the realized costs of compromise for the two parties are perfectly negatively correlated.

However, perfect negative correlation may not always be a realistic assumption. Consider, for instance, negotiations between a labor union and management over the minimum hourly wage. The two negotiating parties are not accountable to the same stakeholders: A labor union leader may be accountable to the labor union, while management may be answerable to the stakeholders of the business, and while their interests are opposed, the opinions of the two stakeholder groups may not be perfectly negatively correlated. For instance, say the management wants to maintain the current hourly wage while the union wants to increase it. If 75% of the union supports the wage increase, it does not imply that that exactly 25% of the shareholders support no wage increase. A natural question is whether the lack of immediate agreement in bargaining is robust in such settings. Below, we show that perfect negative correlation is indeed not necessary. Some negative interdependence is enough to guarantee that agents will never choose to settle at the first stage.

6.1 Two Imperfectly Correlated Public Opinions

Let $(k_i, k_j) \in [0,1] \times [0,1]$ denote the public opinion for parties i and j respectively. (The cost function is defined as before.) Given l, suppose that (k_i, k_j) is uniformly distributed in the region

$$K_l = \{(k_i, k_j) \in [0,1]^2 : 1 - k_j - l \leq k_i \leq 1 - k_j + l\}.$$

Thus, $f(k) = \frac{1}{l(2-l)}$ for any $k \in K_l$. Notice that K_l is a strip of width $2l$ symmetric about the line $k_i + k_j = 1$ inside the unit square. The value of l ranges from $l = 0$ being the case of perfect negative correlation ($k_i = 1 - k_j$), to $l = 1$ being the case of independence. (This is a tractable way of capturing correlation in values, and is related to the approach used in the literature on global games (See for instance, Tsoy (2017).)

For $\mathbf{z} = (\frac{1}{2}, \frac{1}{2})$, the dominance region $D_i(\mathbf{z}) = K_l \cap \{k_j \leq k_i\}$. As before, Equation (NB)) implies that the condition for non-existence of a settle equilibrium is given by

$$\frac{1}{2} > \iint_{D_i} c_j dF(k).$$

This condition guarantees that an agent will have an incentive to deviate from $\mathbf{z} = (, \frac{1}{2})$. From Remark 1, we know that it suffices to check that an agent has an incentive to deviate from the compatible demand pair $\mathbf{z} = (, \frac{1}{2})$. Under symmetry, for any compatible demand $\mathbf{z} = (z_i, z_j)$, an agent j has even higher incentive to deviate if $z_j < 1/2$. It turns out that for any $l \in (0, 1)$, the above condition is satisfied. Accordingly, we have the proposition below which states that as long as there is some negative correlation between the costs of compromise for two parties, agreement in the first stage is not an equilibrium. The proof is relegated to the appendix.

Proposition 8 *In the bargaining game with public opinion* $\mathbf{k}$ *uniformly distributed in* K_l *where* $l \in (0, 1)$*, there is no settle equilibrium.*

6.2 Two independent public opinions

Finally, in some settings, while both parties suffer some reputational costs of compromise, these costs could be independent of each other. For instance, consider two countries negotiating over terms of trade. Each negotiating leader is now accountable to the people of her respective country: In the bailout negotiations between Greece and the rest of the EU, Greek Prime Minister Tsipras's cost of compromise depended on public opinion in Greece while the ECB negotiators cared about their home electorates.

Below, we consider a setting in which the two agents' costs of compromise are independent, i.e., each party cares about the opinion about its respective public, i.e.,k_i and k_j are drawn i.id from distribution F, that is symmetric about $\frac{1}{2}$. We show that in this case, there exists a settle equilibrium. The intuition is that if the two public opinions (and costs of compromise) are independent, depending on the distribution, it is possible that with high probability, both parties end up with very high costs of compromise, and then one of them will have to compromise, even with high costs. It turns out that this can indeed deter agents from gambling over public opinion.

Proposition 9 *If* k_i *and* k_j *are drawn independently from the uniform distribution over* $[0, 1]$*, then agents settle at the first stage with compatible demands* $(\frac{1}{2}, \frac{1}{2})$*.*

Proof. First note that using the arguments in Lemma 1, we know that for each realized k_i, there exist thresholds $\underline{k}_i(k_j)$ and $\underline{k}_i(k_j)$ that define the respective dominance and compromise regions D_i and C. Then, the expected utility to agent j from initial demands $\mathbf{z} = (z_i, z_j)$ is given by

$$\begin{aligned} U_j(\mathbf{z}) = \; & \int_{k_i=0}^{1} \int_{0}^{\underline{k}_j(k_i)} u_j(1 - z_i, z_j, k_j) F_j(\underline{k}_j(k_i)) dF_i(k_i) \\ + \; & \int_{k_i=0}^{1} \int_{\underline{k}_j}^{\overline{k}_j(k_i)} u_j(v_j^c(z, k), z_j, k_j) dF_j(k_j) dF_i(k_i) \\ + \; & \int_{k_i=0}^{1} \int_{\overline{k}_j(k_i)}^{1} z_j dF_j(k_j) dF_i(k_i). \end{aligned}$$

As before, Lemma 2 holds, and it is easy to show that $\underline{k}_j(k_i) = \overline{k}_j(k_i) = k_i$ when $\mathbf{z} = (\frac{1}{2}, \frac{1}{2})$. Thus, the condition for agent j to deviate from the compatible demand $(\frac{1}{2}, \frac{1}{2})$ is

$$\int_{k_i=0}^{1} \int_{0}^{k_i} c_j dF_j(k_j) dF_i(k_i) < \int_{k_i=0}^{1} \int_{k_i}^{1} dF_j(k_j) dF_i(k_i)$$

Given that $F_i = F_j$ and the distribution is symmetric around $\frac{1}{2}$, we can rewrite the right hand side of the above expression (the benefit from deviation) as

$$\int_{k_i=0}^{1} \int_{k_i}^{1} dF_j(k_j) dF_i(k_i) = \int_{k_i=0}^{1} (1 - F_j(k_i)) dF_i(k_i) = \frac{1}{2}.$$

Also, we can write the left hand side (the cost of deviation) as

$$\int_{k_i=0}^{1} \int_{0}^{k_i} c_j dk_j dk_i = \int_{k_j=0}^{1} \left(\int_{k_j}^{1} dk_i \right) c_j dk_j = \int_{k_j=0}^{1} (1 - k_j) c_j dk_j = \int_{k_i=0}^{1} k_j dk_j = \frac{1}{2}.$$

It follows that agents cannot improve their payoff by deviating from $(\frac{1}{2}, \frac{1}{2})$. ■ This result makes transparent the main economic insight of the paper. The motivation to gamble is driven by the knowledge that one needs to bear a reputation cost exactly when it is not so costly to do so. The presence of two independent public opinions eliminates this effect. In fact, it is easy to find examples of other distributions, where agents have a strict incentive to settle at the initial stage. For example, suppose that the distribution F_i was such that $F_i(k_i) \to 1$ for $k_i \in (1 - \varepsilon, 1)$ and $F_i(k_i) \to \frac{1}{2}$ for $k_i \in (0, \varepsilon)$ for some small $\varepsilon > 0$. We see that the $E(c_j | k_j < k_i) > \frac{1}{8} \cdot c(1 - \varepsilon) = \frac{1-\varepsilon}{8\varepsilon}$. Thus, $\lim_{\varepsilon \to 0} E(c_j | k_j < k_i) = \infty$. Thus, agent j will not want to deviate from $(\frac{1}{2}, \frac{1}{2})$. The intuition is that now there is a significant probability $\frac{1}{4}$ that both parties end up with a very high cost of compromise, and half of the time, each agent will need to give up and incur a cost of compromise. This will deter her from gambling in the first stage.[12]

7 Generalizing the Cost and Distribution of Opinion

In this final section, we allow for arbitrary cost functions $c_i(k_i)$ and any distribution of public opinion F that is symmetric about $\frac{1}{2}$. We provide a necessary and sufficient condition for non-existence of the settle equilibrium. We also prove the existence of a unique symmetric gamble equilibrium under this condition. Intuitively, what the condition guarantees is that an agent is required to compromise in the second stage precisely when it is not costly for her to compromise.

Proposition 10 *Suppose that F is symmetric around $1/2$. Given any compatible first-stage demands $\mathbf{z} = (z_i, z_j)$ with $z_j \leq \frac{1}{2}$, agent j has an incentive to deviate if and only if*

$$\mathbb{E}\left[c_j | k_j \leq \frac{1}{2}\right] < 1.$$

Proof. For sufficiency, suppose that $\mathbb{E}[c_j|k_j \leq \frac{1}{2}] < 1$. We first examine the thresholds $\overline{k}_i$ and $\underline{k}_i$ that define the dominance and compromise regions at the second stage.

Lemma 3 *For initial compatible demands $\mathbf{z} = (z_i, z_j)$, $\underline{k}_j = \overline{k}_j$. The threshold increases with z_j and equals $1/2$ when $z_j = 1/2$.*

The proof of this lemma is in the appendix.

The net benefit for agent j from making a marginally higher demand is (see equation (NB))

$$1 - F_j(\overline{k}_j) - \left(\int_0^{\overline{k}_j} c_j dF_j(k_j)\right),$$

which is decreasing in $\overline{k}_j$. From lemma 3 we know $\overline{k}_j$ is increasing in z_j. Therefore, the minimum net benefit is at $z_j = 1/2$ and at $z_j = 1/2$, lemma 3 shows that $\overline{k}_j = 1/2$. Hence, the net benefit is at least

$$1 - F_j\left(\frac{1}{2}\right) - \int_0^{\frac{1}{2}} c_j dF_j(k_j) \geq \frac{1}{2}\left(1 - E\left[c_j | k_j \leq \frac{1}{2}\right]\right) > 0,$$

[12] It is also straightforward to characterize the equilibrium in the case of perfectly positive correlation in public opinion. In this case, settling with compatible demands of $(\frac{1}{2}, \frac{1}{2})$ would be an equilibrium.

where the last inequality follows from our assumption. Therefore, at any pair of compatible demands, the agent making the weakly lower initial demand can profitably deviate by increasing her demand.

Conversely, it is easy to check that if the condition in the proposition is violated, then $\mathbf{z} = (\frac{1}{2}, \frac{1}{2})$ is an equilibrium. So, this is a necessary and sufficient condition for no settle equilibrium. ■

We can show that, also, in this general setting, there exists a unique gamble symmetric equilibrium.

Proposition 11 *There is a unique symmetric gamble equilibrium.*

The interested reader can refer to the appendix for the proof.

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8 Appendix

8.1 Proof of Lemma 1

Proof. Suppose that the agents' initial demands are $\mathbf{z} = (z_i, z_j)$ such that $z_1 + z_2 > 1$, and the realized public opinion is $\mathbf{k} = (k_i, k_j)$ (with the resulting costs of compromise c_i and c_j respectively). Let us consider the incentives of an agent i in the alternating offers bargaining game. Consider two options for player i: accepting an offer of x now or receiving y after time Δ. Define $\xi(y, z_i)$ such that if $x = \xi(y, z_i)$, then agent i is indifferent between the two options. Then,

$$\xi(y, z_i) - c_i \cdot (z_i - \xi(y, z_i))\mathbf{1}_{\{\xi(y,z_i)<z_i\}} = e^{-r\Delta}(y - c_i \cdot (z_i - y))\mathbf{1}_{\{y<z_i\}})$$

First consider the case of both $\xi(y, z_i)$ and y being less than the initial demand z_i. Then, we have

$$\begin{aligned}
\xi(y, z_i) - \frac{k_i}{1-k_i} \cdot (z_i - \xi(y, z_i)) =& e^{-r\Delta} \left(y - \frac{k_i}{1-k_i} \cdot (z_i - y) \right) \\
\xi(y, z_i) \left(\frac{1}{1-k_i} \right) - z_i \left(\frac{k_i}{1-k_i} \right) =& e^{-r\Delta} \left(y \left(\frac{1}{1-k_i} \right) - z_i \left(\frac{k_i}{1-k_i} \right) \right) t \\
\xi(y, z_i) - z_i k_i =& e^{-r\Delta} (y - z_i k_i) \\
\Rightarrow \xi(y, z_i) =& e^{-r\Delta} y + (1 - e^{-r_i\Delta}) z_i k_i.
\end{aligned}$$

Similarly, if $\xi(y, z_i)$ and y are both greater than the initial demand z_i, then we have

$$\xi(y, z_i) = e^{-r\Delta} y.$$

It is easy to check that the remaining case of $\xi(y, z_i) < z_i < y$ is irrelevant as $\Delta \to 0$. Note that in both cases above, $\xi(y, z_i)$ is increasing in y, and therefore, following standard arguments (see, for instance, Shaked and Sutton (1984)), we know that there are threshold offers M_i and m_i, such that agent i accepts any offer higher than M_i and rejects any offer less than m_i. We have for any $i \neq j$,

$$m_i \geq \xi_i(1 - M_j, z_i), \; M_j \leq \xi_j(1 - m_i, z_j).$$

Solving these inequalities, using $M_i \geq m_i$, and taking $\Delta \to 0$, we can say that if agents accept offers in the second stage bargaining, that are less than their initial demands, then they must settle for a generalized Rubinstein solution:[13][14]

$$v_i^c(\mathbf{z}, \mathbf{k}) := z_i k_i + \frac{1}{2}(1 - z_i k_i - z_j k_j) \tag{2}$$

Note that if party i settles for less than $z_i k_i$, then she gets a negative payoff. But, given $k_i = 1 - k_j$, we cannot have $1 - z_i k_i - z_j k_j < 0$. This implies $v_i^c(\mathbf{z}, \mathbf{k})$ always gives agents a positive payoff. Further note that $v_i^c(\mathbf{z}, \mathbf{k}) \leq z_i$, i.e., it is not possible that any agent i gets more than her initial demand z_i in the second stage. To see this consider the best case scenario for agent i : it is infinitely costly for i to compromise and not at all costly for agent j to compromise. We can interpret z_i as an outside option for agent i in the second stage. As Binmore, Shaked and Sutton (1989) argue, given such an outside option, agent i cannot get more than z_i.

Thus, whenever, the realized public opinion $\mathbf{k}$ is such that $v_i^c(\mathbf{z}, \mathbf{k}) \geq z_i$, agent j surrenders and accepts agent i's offer. Given $\mathbf{z}$, we call the set of such $\mathbf{k}$ *i-dominating*, denoted by $D_i(\mathbf{z})$, where

$$D_i(\mathbf{z}) := \{k_i | v_i^c(\mathbf{z}, \mathbf{k}) \geq z_i\}.$$

Simplifying further, we have

$$D_i(\mathbf{z}) = \{k_i | k_i \geq 1 - \frac{1 - z_i}{z_i + z_j}\} \qquad \text{and} \qquad D_j(\mathbf{z}) = \{k_i | k_i \leq \frac{1 - z_j}{z_i + z_j}\}$$

Finally, if $\mathbf{k}$ is such that $v_i^c(\mathbf{z}, \mathbf{k}) < z_i$ for both agents, then they both compromise. We call such realization of public opinion *compromising*, denoted by $C(\mathbf{z})$, where

$$C(\mathbf{z}) := \{k_j | v_i^c(\mathbf{z}, \mathbf{k}) < z_i \text{ for all } i\},$$

which simplifies to

$$C(\mathbf{z}) = \{k_i | k_i \in \left[\frac{1 - z_j}{z_i + z_j}, 1 - \frac{1 - z_i}{z_i + z_j}\right]\}$$

Let us define thresholds $\underline{k}_i(\mathbf{z})$ and $\overline{k}_i(\mathbf{z})$ as follows:

[13] For details, see Basak (2016)

[14] One can also consider this to be a result of Nash bargaining where for any agent i, $k_i z_i$ serves as the disagreement point.

$$\underline{k}_i(\mathbf{z}) := \frac{1 - z_j}{z_i + z_j} \qquad \text{and} \qquad \overline{k}_i(\mathbf{z}) := 1 - \frac{1 - z_i}{z_i + z_j}. \tag{3}$$

Therefore, for given any initial demand $\mathbf{z} = (z_i, z_j)$,

1. If $k_i \in D_i(\mathbf{z})$ for some i, then agent i dominates and agent j surrenders and accept agent i's demand.

2. $k_i \in C(\mathbf{z})$, then they both compromise and agent i gets $v_i^c(\mathbf{z}, \mathbf{k})$.

■

8.2 Proof of Claim 1

Proof. Consider any k_i. W.o.l.o.g suppose that $k_i \geq 1/2$. If the initial demand is $z \in (\frac{1}{2}, \frac{1}{1+2(1-k_i)}]$, then such $k_i \geq 1 - \frac{1-z}{2z} = \overline{k}_i$ - i.e., $k_i \in D(\mathbf{z})$. And if $z \in [\frac{1}{1+2(1-k_i)}, 1]$, then such $k_i \leq \overline{k}_i$ - i.e., $k_i \in C(\mathbf{z})$.

Given any $k_i \geq 1/2$, if $z \in (\frac{1}{2}, \frac{1}{1+2(1-k_i)}]$, then agent i dominates and gets her initial demand z while agent j takes $(1 - z)$ and gets the payoff $(1 - z) - c_j(2z - 1)$. Hence, the join payoff is $1 - \frac{1-k_i}{k_i} \cdot (2z - 1)$. Clearly this is decreasing in z. One the other hand, if $z \in [\frac{1}{1+2(1-k_i)}, 1]$, then both agents compromise and any agent i takes the share v_i^c which gives her the following payoff

$$\begin{aligned} v_i^c - c_i(z - v_i^c) &= v_i^c(1 + c_i) - zc_i \\ &= (zk_i + \frac{1}{2} \cdot (1 - zk_i - zk_j))\frac{1}{1 - k_i} - z\frac{k_i}{1 - k_i} \\ &= \frac{1 - z}{2(1 - k_i)}. \end{aligned}$$

Hence, the joint payoff is $(1 - z) \cdot \frac{1}{2} \left(\frac{1}{1-k_i} + \frac{1}{k_i} \right)$. Clearly this is decreasing in z as well. Since the payoffs are continuous in z, this establishes that the joint payoff is decreasing in z for any k_i. ■

8.3 Proof of Proposition 3:

Proof. Recall that by Equation (NB), agent j will deviate from a compatible demand $\mathbf{z} = (z_i, z_j)$ with $z_j \leq z_i$ if

$$1 - F_j(\overline{k}_j) \geq \int_{\frac{1}{2}-\varepsilon}^{\underline{k}_j} c_j \frac{1}{2\varepsilon} dk_j.$$

Here, $\overline{k}_j = \underline{k}_j = z_j$. So the above reduces to

$$\frac{\frac{1}{2} + \varepsilon - z_j}{2\varepsilon} \geq -\frac{1}{2\varepsilon}\left((k_j + ln(1-k_j)) \,|_{\frac{1}{2}-\varepsilon}^{z_j} \right)$$

$$\implies \frac{1}{2} + \varepsilon - z_j \geq -z_j - ln(1-z_j) + \left(\frac{1}{2} - \varepsilon\right) + ln\left(\frac{1}{2} + \varepsilon\right)$$

$$\implies ln(1-z_j) + 2\varepsilon - ln\left(\frac{1}{2} + \varepsilon\right) \geq 0$$

Note that the left hand side of the above expression is decreasing in z_j. Therefore it suffices to show that the above holds for $z_j = \frac{1}{2}$, since we know that $z_j \leq z_i$ and $z_i + z_j = 1$. Substituting $z_j = \frac{1}{2}$, the above becomes

$$2\varepsilon - ln(1 + 2\varepsilon) \geq 0$$

which is true because $x - ln(1+x) \geq 0$ for any $x > 0$. Therefore, there is no settle equilibrium as the agent making the weakly lower demand will have an incentive to deviate.

Next, we characterize the unique symmetric gamble equilibrium. Suppose agents make initial incompatible demands $(z_i, z_j) = (z, z)$ such that $z \in (\frac{1}{2}, \frac{1}{2(1-\varepsilon)})$. From Equation (DE), we have the net benefit to agent j from deviating from (z, z) is given by:

$$\frac{\partial U_j}{\partial z_j} = -\frac{1}{2}\left(\int_{\frac{1}{2}-\varepsilon}^{\underline{k}_j} c_j dF_j(k_j) + \int_{\frac{1}{2}-\varepsilon}^{\overline{k}_j} c_j dF_j(k_j) \right) + (1 - F_j(\overline{k}_j))$$

This simplifies to

$$\frac{1}{2\varepsilon}\left[-\left(\frac{1}{2} - \varepsilon\right) - ln\left(\frac{1}{2} + \varepsilon\right) + \frac{1}{2} + \varepsilon - 1 + 1 - \frac{1}{2}(\overline{k}_j - \underline{k}_j) + \frac{1}{2}(\ln(1 - \overline{k}_j) + \ln(1 - \underline{k}_j)) \right].$$

Recall that, for $z_i = z_j = z$, $\overline{k} = 1 - \frac{1-z}{2z}$ and $\underline{k} = \frac{1-z}{2z}$, and $H(z)$ is defined as in Proposition 2. So the above expression further simplifies to

$$\frac{1}{4\varepsilon}\left[2\left(2\varepsilon-1-ln\left(\frac{1}{2}+\varepsilon\right)\right)+\underbrace{\frac{1}{z}+ln\left(\frac{1-z}{2z}\left(1-\frac{1-z}{2z}\right)\right)}_{=H(z)}\right].$$

The necessary condition for equilibrium is that $\frac{dU_j}{dz_j}=0$. By Lemma 2, we know that $\frac{\partial U_j}{\partial \bar{k}}=\frac{\partial U_j}{\partial \underline{k}}=0$. Therefore, $\frac{dU_j}{dz_j}=0$ if, and only if

$$H^{\varepsilon}(z):=H(z)+2\left(2\varepsilon-1-ln\left(\frac{1}{2}+\varepsilon\right)\right)=0.$$

For any $\varepsilon\in(0,\frac{1}{2})$, we have

$$\lim_{z\to\frac{1}{2}}H^{\varepsilon}(z)=2-2ln2+2\left(2\varepsilon-1-ln\left(\frac{1}{2}+\varepsilon\right)\right)=2\left(2\varepsilon-ln\left(1+2\varepsilon\right)\right)\geq 0.$$

Note that this limit is 0 only when $\varepsilon=0$. Also, at $z=\frac{1}{2(1-\varepsilon)}$,

$$\begin{aligned}H^{\varepsilon}(z)=&2\varepsilon+ln\left(\left(\frac{1}{2}+\varepsilon\right)\left(\frac{1}{2}-\varepsilon\right)\right)+2\left(2\varepsilon-1-ln\left(\frac{1}{2}+\varepsilon\right)\right)\\=&2\varepsilon-ln\left(\frac{1+2\varepsilon}{1-2\varepsilon}\right)<0.\end{aligned}$$

Finally, $H^{\varepsilon'}(z)=H'(z)<0$ for $z>\frac{1}{2}$. Hence, there is a unique $z^*\in(\frac{1}{2},\frac{1}{2(1-\varepsilon)})$ that satisfies $H^{\varepsilon}(z^*)=0$.

Note that

$$\frac{\partial H^{\varepsilon}(z)}{\partial\varepsilon}=2\left(2-\frac{1}{\frac{1}{2}+\varepsilon}\right)>0$$

for any $\varepsilon>0$ and $\frac{dH^{\varepsilon}(z)}{dz}<0$. Therefore,

$$\frac{dz^*}{d\varepsilon}=-\frac{\frac{\partial H^{\varepsilon}(z)}{\partial\varepsilon}}{\frac{dH^{\varepsilon}(z)}{dz}}>0.$$

This implies the extent of gambling increases with ε. ∎

8.4 Proof of Proposition 4:

Proof. Suppose, for contradiction, agents demand $z < \frac{1}{1+2\varepsilon}$. Then $\overline{k}_i < 1-\varepsilon$ and $\underline{k}_i > \varepsilon$. Therefore, the net benefit from deviation is an in equation (NB)

$$\begin{aligned} 1 - F_j(1-\varepsilon) - \int_0^{\varepsilon} c_j \frac{1}{2\varepsilon} dk_j &= \frac{1}{2} + \frac{1}{2\varepsilon}(k_j + ln(1-k_j))|_0^{\varepsilon} \\ &= 1 + \frac{ln(1-\varepsilon)}{2\varepsilon} > 0 \end{aligned}$$

The last inequality follows from $\frac{ln(1-\varepsilon)}{2\varepsilon}$ being decreasing in ε and equal to $1 - ln(2) > 0$ at $\varepsilon = 1/2$. ■

8.5 Proof of Proposition 6

Proof. Suppose, for contradiction, that there is a settle equilibrium in which the agents make compatible demand $\mathbf{z} = (z_i, z_j)$. Agent i will not deviate only if (see equation (NB))

$$1 - F_j(\overline{k}_j) \leq \int_{1-b}^{\underline{k}_j} c_j \left(\frac{1}{b-a}\right) dk_j$$

At compatible demands $\overline{k}_j = \underline{k}_j = z_j$. Note that if $z_j < a$, then a small deviation means agent j does not have to compromise in the next stage. This implies she will surely deviate. So we can restrict our attention to $z_j \geq a$. The above inequality simplifies to

$$b - z_j \leq -(k_j + ln(1-k_j))|_a^{z_j}$$

$$(b-a) - ln(1-a) \leq -ln(1-z_j).$$

$$\frac{1}{1-z_j} \geq \frac{\exp(b-a)}{1-a}$$

Simplifying this, we

$$z_j \geq 1 - (1-a)\exp(b-a).$$

Similarly, agent j will not deviate if

$$z_i \geq 1 - b\exp(b-a).$$

A settle equilibrium requires $z_i + z_j = 1$. Therefore, a settle equilibrium exists only if

$$1 - (1 - (1 + b - a)\exp(b - a)) \leq 1$$

$$1 - (1 + b - a)\exp(b - a)) \leq 0.$$

Note that $1 - (1 + x)\exp(x)$ is increasing in x for any $x \in (0, 1]$ and hence the above expression is always positive. Hence contradiction. ■

8.6 Proof of proposition 7

Proof.

$$\begin{aligned} U_j &= \int_a^{\max\{\underline{k}_j, a\}} u_j(1 - z_i, z_j, k_j)dF_j^\alpha(k_j) \\ &+ \int_{\max\{\underline{k}_j, a\}}^{\min\{\overline{k}_j, b\}} u_j(v_j^c, z_j, k_j)dF_j^\alpha(k_j) + \int_{\min\{\overline{k}_j, b\}}^b z_j dF_j^\alpha(k_j). \end{aligned}$$

We first look into a symmetric equilibrium $z_i = z_j = z$. This implies in equilibrium, the public opinion will be distributed in $[1/4, 3/4]$. For $z > 2/3$, $\underline{k}_j < 1/4$ and $\overline{k}_j > 3/4$. This means both agents have to compromise in the second stage. This cannot be an equilibrium. So, we can restrict our attention to $z \leq 2/3$. When an agent contemplates the impact of increasing her initial demand, like before she considers $(\partial U_j / \partial z_j)$ and recall from lemma 2 that $\frac{\partial U_j}{\partial \overline{k}_j} = \frac{\partial U_j}{\partial \underline{k}_j} = 0$, but in addition she also considers the impact on the change in distribution of the public opinion:

$$\frac{dU_j}{dz_j} = \frac{\partial U_j}{\partial z_j} + \frac{\partial U_j}{\partial b}\frac{db}{dz_j} + \frac{\partial U_j}{\partial a}\frac{da}{dz_j}.$$

As in equation (DE),

$$\begin{aligned} \frac{\partial U_j}{\partial z_j} &= -\frac{1}{2}\left(\int_a^{\underline{k}_j} c_j dF_j^\alpha(k_j) + \int_a^{\overline{k}_j} c_j dF_j^\alpha(k_j)\right) + (1 - F_j^\alpha(\overline{k}_j)) \\ &= -\left(\int_a^{\underline{k}_j} c_j dk_j + \int_a^{\overline{k}_j} c_j dk_j\right) + 2(b - \overline{k}_j) \\ &= 2\left[1 - \frac{1}{2}\left(\overline{k}_j - \underline{k}_j\right) + \frac{1}{2} ln(1 - \overline{k}_j) + ln(1 - \underline{k}_j) - (1 - (b - a) + ln(1 - a))\right]. \end{aligned}$$

At $z_i = z_j = z$, we have $1-\overline{k}_j = (1-z)/2z$ and $1-\underline{k}_j = 1-((1-z)/2z)$. Therefore,

$$\frac{\partial U_j}{\partial z_j} = \frac{1}{z} + ln\left(\frac{1-z}{2z}\left(1-\frac{1-z}{2z}\right)\right) - 2\left(\frac{1}{2} + ln\left(\frac{3}{4}\right)\right) \quad := \quad H^{\alpha}(z)$$

Next we look into the effect due to change in the distribution of public opinion.

$$\begin{aligned} \frac{\partial U_j}{\partial b}\frac{db}{dz_j} + \frac{\partial U_j}{\partial a}\frac{da}{dz_j} &= (-(1-z_i) - c_j(a)(z_j - (1-z_i)) + z_j) \cdot \frac{z_i}{(z_i+z_j)^2}(1-2\alpha) \\ &= (z_i + z_j - 1)\left(1 - \frac{a}{1-a}\right)\frac{z_i}{(z_i+z_j)^2}(1-2\alpha). \end{aligned} \tag{4}$$

At $z_i = z_j = z$, this simplifies to

$$J^{\alpha}(z) = \frac{(2z-1)(1-2\alpha)}{6z} = \frac{(1-2\alpha)}{6}\left(2-\frac{1}{z}\right).$$

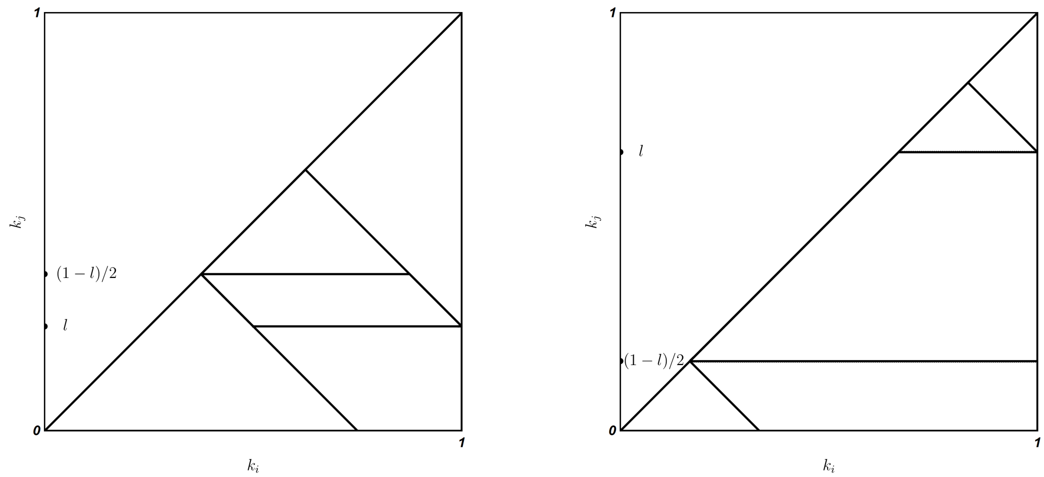
A symmetric equilibrium $\mathbf{z} = (z^*, z^*)$, it must be such that

$$H^{\alpha}(z^*) + J^{\alpha}(z^*) = 0.$$

Note that $\frac{\partial}{\partial z}(H^{\alpha} + J^{\alpha}) < 0$, $H^{\alpha}(1/2) > 0$, $J^{\alpha}(1/2) = 0$ and $H^{\alpha} + J^{\alpha} \to -\infty$ as $z \to 1$. Therefore, there is a unique $z^* > 1/2$ that solves the above equation. This implies the unique symmetric equilibrium is a gamble equilibrium.

No Settle equilibrium: At $z = 1/2$, the indirect effect $J^{\alpha}(1/2) = 0$ and $H^{\alpha}(1/2) > 0$. Therefore, an agent is better off by marginally increasing her demand. In fact, for any compatible demand $\mathbf{z} = (z_i, z_j)$, we can see (equation (4)) that the effect due to change is the distribution of public opinion is 0. By proposition 6, we know that at least one agent gets positive net marginal benefit from deviating from any compatible demand $\mathbf{z} = (z_i, z_j)$.

Effect of Public Aversion to extreme opinion (α) : First consider the becnhmark case $\alpha = 1/2$ in which public opinion is exogenous. As we can see the indirect effect $J^{\alpha}(z) = 0$. Since $H^{\alpha}(z)$ is decreasing in z with $H^{\alpha}(1/2) > 0$ and $H^{\alpha}(1) < 0$, there is a unique z^* with $H^{\alpha}(z^*) = 0$. Hence, there is a unique symmetric equilibrium in which the



(a) Case 1: $l \leq 1/3$, or $(1-l)/2 \geq l$ (b) Case 2: $l \geq 1/3$, or $l \geq (1-l)/2$

Figure 1: Two negatively correlated public opinions

negotiation cannot be resolved in the first stage. Note that $\frac{\partial}{\partial \alpha}(H^\alpha + J^\alpha) < 0$. Therefore,

$$\frac{dz^*}{d\alpha} = -\frac{\frac{\partial}{\partial z}(H^\alpha + J^\alpha)}{\frac{\partial}{\partial \alpha}(H^\alpha + J^\alpha)} < 0.$$

This implies that the more the public is averse to extreme demands, in equilibrium the less the extent of gambling. Hence, efficiency improves. Nevertheless, full efficiency cannot be reached since the agents will gamble. ■

8.7 Proof of Proposition 8

Proof. There are two cases to consider.

Case 1: $l \leq 1/3$

For $k_j \in [0, l]$, $k_i \sim \mathscr{U}[1-k_j-l, 1]$, for $k_j \in [l, (1-l)/2]$, $k_i \sim \mathscr{U}[1-k_j-l, 1-k_j+l]$, and for $k_j \in [(1-l)/2, (1+l)/2]$, $k_i \sim \mathscr{U}[k_j, 1-k_j+l]$.

Case 2: $l \geq 1/3$

For $k_j \in [0, (1-l)/2]$, $k_i \sim \mathscr{U}[1-k_j-l, 1]$, for $k_j \in [(1-l)/2, l]$, $k_i \sim \mathscr{U}[k_j, 1]$, and for $k_j \in [l, (1+l)/2]$, $k_i \sim \mathscr{U}[k_j, 1-k_j+l]$.

The following two results will be very useful.

$$\int \frac{k_j}{1-k_j} dk_j = \int \left(\frac{1}{1-k_j} - 1 \right) dk_j = -ln(1-k_j) - k_j$$
$$\int \frac{k_j^2}{1-k_j} dk_j = \int \left(\frac{1}{1-k_j} - (1+k_j) \right) dk_j = -ln(1-k_j) - k_j - \frac{k_j^2}{2}.$$

The first one we have used on multiple occasions already. Let us consider the first case $l \leq 1/3$.

$$\begin{aligned}
& \iint\limits_{D_i} c_j dF_j(k) \\
= \; & \frac{1}{l(2-l)} \left[\int_0^l \frac{(k_j+l)k_j}{1-k_j} dk_j + \int_l^{(1-l)/2} \frac{2lk_j}{1-k_j} dk_j + \int_{(1-l)/2}^{(1+l)/2} \frac{(1-2k_j+l)k_j}{1-k_j} dk_j \right] \\
= \; & \frac{l}{l(2-l)} [-ln(1-k_j) - k_j]_0^l + \frac{1}{l(2-l)} [-ln(1-k_j) - k_j - \frac{k_j^2}{2}]_0^l \\
& + \frac{2l}{l(2-l)} [-ln(1-k_j) - k_j]_l^{(1-l)/2} \\
& + \frac{(1+l)}{l(2-l)} [-ln(1-k_j) - k_j]_{(1-l)/2}^{(1+l)/2} - \frac{2}{l(2-l)} [-ln(1-k_j) - k_j - \frac{k_j^2}{2}]_{(1-l)/2}^{(1+l)/2} \\
= \; & \frac{1+l}{2-l} \left(-\frac{ln(1-l)}{l} - 1 \right) - \frac{l}{2(2-l)} \\
& + \frac{1}{(2-l)} \left(-2 \cdot ln(1+l) + 2 \cdot ln(2) - (1-l) + 2 \cdot ln(1-l) + 2l \right) \\
& + \frac{(l-1)}{(2-l)} \left(-\frac{ln(1-l)}{l} + \frac{ln(1+l)}{l} - 1 \right) + \frac{1}{(2-l)}
\end{aligned}$$

Note that as $l \to 0$, the first and the last part converges to 0 and the second expression $ln(2) - 1/2$. Recall that $ln(2) - 1/2 < 1/2$ and hence there is no settle equilibrium. For $l > 0$, we can rewrite the above expression as

$$\iint\limits_{D_i} c_j dF_j(k) = -\left(\frac{1+l}{2-l} \right) \frac{ln(1+l)}{l} + \frac{l}{2(2-l)} + \frac{2ln(2)}{2-l}.$$

Next, consider the second case $l \geq \frac{1}{3}$.

$$\begin{aligned}
& \iint\limits_{D_i} c_j dF_j(k) \\
= \ & \frac{1}{l(2-l)} \left[\int_0^{(1-l)/2} \frac{(k_j+l)k_j}{1-k_j} dk_j + \int_{(1-l)/2}^{l} \frac{(1-k_j)k_j}{1-k_j} dk_j + \int_l^{(1+l)/2} \frac{(1-2k_j+l)k_j}{1-k_j} dk_j \right] \\
= \ & \frac{l}{l(2-l)} [-ln(1-k_j) - k_j]_0^{(1-l)/2} + \frac{1}{l(2-l)} [-ln(1-k_j) - k_j - \frac{k_j^2}{2}]_0^{(1-l)/2} \\
& + \frac{1}{l(2-l)} [\frac{k_j^2}{2}]_{(1-l)/2}^{l} \\
& + \frac{(1+l)}{l(2-l)} [-ln(1-k_j) - k_j]_l^{(1+l)/2} - \frac{2}{l(2-l)} [-ln(1-k_j) - k_j - \frac{k_j^2}{2}]_l^{(1+l)/2}
\end{aligned}$$

which simplifies to

$$\iint\limits_{D_i} c_j dF_j(k) = -\left(\frac{1+l}{2-l}\right) \frac{ln(1+l)}{l} + \frac{l}{2(2-l)} + \frac{2ln(2)}{2-l}.$$

Note that it is same as in case 1. This is an increasing function in l in the range $(0, 1)$ and

$$\lim_{l \to 1} \iint\limits_{D_i} c_j dF_j(k) = \frac{1}{2}.$$

For any $l < 1$, $\iint\limits_{D_i} c_j dF_j(k) < 1/2$. ■

8.8 Proof of Lemma 3:

Proof. Agent i surrenders to agent j if

$$\begin{aligned}
v_j^c(z,k) = z_j \frac{c_j}{1+c_j} + \frac{1}{2}\left(1 - z_i \frac{c_i}{1+c_i} - z_j \frac{c_j}{1+c_j}\right) \ & \geq \ z_j \\
z_j \left(\frac{c_j}{1+c_j}\right) - z_i \left(\frac{c_i}{1+c_i}\right) \ & \geq \ 2z_j - 1.
\end{aligned}$$

Let us define $\xi(k) = \frac{c(k)}{1+c(k)}$. Since $c_j = c(k_j)$ and $c_i = c(1 - k_j)$, the above inequality simplifies to

$$z_j \xi(k_j) - z_i \xi(1 - k_j) \geq 2z_j - 1.$$

Since $\xi(k_j)$ is an increasing function in k_j, there exists $\overline{k}_j$ such that

$$z_j \xi(\overline{k}_j) - z_i \xi(1 - \overline{k}_j) = 2z_j - 1. \tag{5}$$

Similarly, for agent i

$$z_i \xi(\overline{k}_i) - z_j \xi(1 - \underline{k}_i) = 2z_i - 1.$$

Since, $\underline{k}_j = 1 - \overline{k}_i$, we have

$$z_j \xi(\underline{k}_j) - z_i \xi(1 - \underline{k}_j) = 1 - 2z_i.$$

Note that for compatible demands $2z_j - 1 = 1 - 2z_i$. Hence the above equation is the same as equation (5). Since $\xi(k_j)$ is an increasing function, this implies $\overline{k}_j = \underline{k}_j$.

Finally, differentiating both sides of equation (5), we get

$$\frac{d\overline{k}_j}{dz_j} = -\frac{\xi(\overline{k}_j) + \xi(1 - \overline{k}_j) - 2}{z_j \xi'(\overline{k}_j) + z_i \xi'(1 - \overline{k}_j)} > 0.$$

At $z_j = \frac{1}{2}$, $\xi(\overline{k}_j) = \xi(1 - \overline{k}_j)$. Since $\xi(k_j)$ in increasing this implies $\overline{k}_j = 1/2$. ■

8.9 Proof of Proposition 11:

Proof. Consider a symmetric equilibrium where the initial demand is $\mathbf{z} = (z, z)$. Let us first characterize some properties of the thresholds in a symmetric equilibrium.

Lemma 4 *In a symmetric gamble equilibrium* $\mathbf{z} = (z, z)$ *with* $z > 1/2$*, for any* j*,* $1 - \overline{k}_j = \underline{k}_j$*. The threshold* $\overline{k}_j$ *is increasing in* z *and converges to* 1 *as* $z \to 1$*.*

Proof. Substituting $z_i = z_j = z > 1/2$ in equation 5, we get

$$\xi(\overline{k}_j) - \xi(1 - \overline{k}_j) = 2 - \frac{1}{z}.$$

The same is true for agent i as well. This implies $\overline{k}_j = \overline{k}_i = 1 - \underline{k}_j$. Since $\xi(k_j)$ is increasing in k_j, $\overline{k}_j$ is increasing in z. Since $\xi(.) < 1$, when $z = 1$ it must be that $\overline{k}_j = 1$.

For the equality to hold it also required $c(k_j) \to \infty$ as $k_j \to 1$. ■

Recall from equation (DE) that the net benefit for agent j from marginally increasing the initial demand is

$$\frac{dU_j}{dz_j} = -\frac{1}{2}\left(\int_0^{\underline{k}_j} c_j dF_j(k_j) + \int_0^{\overline{k}_j} c_j dF_j(k_j)\right) + 1 - F_j(\overline{k}_j).$$

We know that $\frac{dU_j}{dz_j}|_{z=1/2} > 0$. Also, at $z = 1$, only the compromise region remains (lemma 4). Hence $\frac{dU_j}{dz_j}|_{z=1} < 0$. If $\frac{dU_j}{dz_j}$ is decreasing in z (when $z_i = z_j = z$), then we have unique gamble equilibrium. The last part of the equation for net benefit is clearly decreasing in z (since from lemma 4 it follows that $\overline{k}_j$ increases in z). From lemma 4 we know $1-\overline{k}_j = \underline{k}_j$. Since F_j is symmetric around $1/2$, we have $f_j(\overline{k}_j) = f_j(\underline{k}_j)$. Also since $\underline{k}_j = 1 - \overline{k}_j$, we have $\frac{d\overline{k}_j}{dz} = -\frac{d\underline{k}_j}{dz}$. Therefore,

$$\frac{d}{dz}\left(\int_0^{\underline{k}_j} c_j dF_j(k_j) + \int_0^{\overline{k}_j} c_j dF_j(k_j)\right) = (c(\overline{k}_j) - c(\underline{k}_j))f_j(\overline{k}_j)\frac{d\overline{k}_j}{dz} > 0.$$

Therefore, $\frac{dU_j}{dz_j}$ is decreasing in z. This implies that there is a unique symmetric gamble equilibrium. ■

9 Online Appendix

Deal with this later*

9.1 Asymmetric discount factors

Proposition 12 *Agents with different time preferences never settle right away if public opinion is uniformly distributed.*

Proof. Suppose that $r_i \neq r_j$. Then in the second stage, any agent takes her minimum acceptable share $z_i k_i$ plus the Rubinstein share $v_i^R = r_j/(r_i + r_j)$ of the left over $(1 - z_i k_i - z_j k_j)$ (Note that for equal time discounting $r_i = r_j$, $v_i^R = 1/2$). Therefore, if both agents compromise in the second stage, they each get

$$v_i^c(\mathbf{z}, \mathbf{k}) = z_i k_i + v_i^R(1 - z_i k_i - z_j k_j).$$

Suppose that players made compatible initial demands $\mathbf{z} = (z_i, z_j)$ at the first stage. Without loss of generality, suppose that agent j makes an initial demand that is weakly less than the Rubinstein share; i.e., $z_j \leq v_j^R$, and $z_i \geq v_i^R$. Then it is easy to check that

$$\overline{k}_j = \underline{k}_j = \frac{(1 - z_i)/v_j^R}{z_i/v_i^R + z_j/v_j^R} = \frac{1}{1 + \frac{v_j^R}{v_i^R}\frac{z_i}{z_j}} \leq \frac{1}{2}.$$

If agent j marginally increases her demand, then bargaining will proceed to the second stage and the outcome will depend on the realized public opinion. The marginal net benefit from such a deviation by agent j is given by (see equation (NB))

$$1 - \overline{k}_j - \int_0^{\underline{k}_j} c_j dk_j$$

The above simplifies to $1 + ln(1 - \underline{k}_j) > 0$. In other words, agent j can gain by deviating. This implies that compatible demands do not constitute an equilibrium. ■

9.2 One-sided costly compromise

Suppose only one party suffers a cost of compromise. In particular, suppose that $k_j = 0$ - i.e., backing down is not costly at all for agent j. In this case, as well, there cannot be any settle equilibrium, and welfare loss is inevitable. To see, why note that if the two agents demand half of the surplus, then for any public opinion k_i, i has a higher revoking cost than j. If agent i increases her demand marginally, almost surely agent j has to surrender. Thus, agent i will be tempted to deviate and take a gamble. The following proposition characterizes the equilibrium in this setting.

Proposition 13 *If only agent i is subject to public opinion while she backs down from her initial demand, then there is a unique equilibrium, in which agent i demands $z_i^* \in (1/2, 1)$ such that*

$$H_i(z_i^*) := \frac{1}{z_i^*} + ln\left(\frac{1 - z_i^*}{z_i^*}\right) = 0$$

and agent j demands $1/2$. Thus, welfare loss in inevitable.

Proof. Suppose that the initial demand $z = (z_i, z_j)$. Under one-sided public opinion, $v_i^c(z, k) = (1 + z_i k)/2$ and $v_j^c(z, k) = (1 - z_i k)/2$.

1. If k is so large that $(1 + z_i k)/2 \geq z_i$ - i.e., $k \geq (2z_i - 1)/z_i =: \overline{k}$, then agent j surrenders. This gives $U_i^{D_i}(z) = z_i$ to agent i and $U_j^{D_i}(z) = 1 - z_i$ to agent j.

2. If k is so small that $(1 - z_i k)/2 \geq z_j$ - i.e., $k \leq (1 - 2z_j)/z_i =: \underline{k}$, then agent i surrenders. This gives

$$U_i^{D_j}(z) = z_i + (z_i + z_j - 1)\frac{ln(1 - \underline{k})}{\underline{k}}$$

to agent i and $U_j^{D_j}(z) = z_j$ to agent j.

3. If $k \in [\underline{k}, \overline{k}]$, then both agents compromise. This gives

$$\begin{aligned} U_i^C(z) =& \frac{1}{\overline{k} - \underline{k}} \int_{\underline{k}}^{\overline{k}} \left(\frac{1 + z_i k}{2} - \frac{k}{1 - k}\left(z_i - \frac{1 + z_i k}{2}\right)\right) dk \\ =& \frac{1}{\overline{k} - \underline{k}} \int_{\underline{k}}^{\overline{k}} \frac{1}{1 - k}\left(\frac{1 - z_i k}{2}\right) dk \\ =& \frac{z_i}{2} + \frac{1 - z_i}{2}\left(\frac{ln(1 - \underline{k}) - ln(1 - \overline{k})}{\overline{k} - \underline{k}}\right) \end{aligned}$$

to agent i and

$$U_j^C(z) = \frac{1}{2}\left(1 - z_i \frac{\overline{k} + \underline{k}}{2}\right)$$

to agent j.

Suppose, for contradiction, that there is a settle equilibrium $(z_i, z_j) = (z, 1-z)$. If $z > 1/2$, then $\underline{k} = \overline{k} > 0$. Thus, there is a positive probability that public opinion will lean against i and i will surrender. Since backing down is not at all costly for her, agent j will be better off by marginally increasing her demand. Thus, for a settle equilibrium to exists $z \leq 1/2$. But if $z \leq 1/2$, then agent i knows that regardless of the public opinion tomorrow, she does not need to compromise (Since $\overline{k} < 0$). Hence, she will be better off by marginally increasing her demand. Thus, inefficiency must arise in equilibrium.

Imagine that the agents demand some z. For this to be an equilibrium, no agent must have incentive to deviate. Consider agent j. Her expected payoff is

$$\begin{aligned} U_j(z) = & \left(1 - \tfrac{2z_i - 1}{z_i}\right)(1 - z_i) + \left(\tfrac{1-2z_j}{z_i}\right) z_j + \tfrac{z_i + z_j - 1}{z_i}\left(1 - z_i \tfrac{z_i - z_j}{z_i}\right) \\ = & \tfrac{(1-z_i)^2}{z_i} + \tfrac{(1-2z_j)z_j}{z_i} + \tfrac{z_j^2 - (1-z_i)^2}{z_i}. \end{aligned}$$

$\partial U_j(z)/\partial z_j = 0$ implies that

$$1 - 4z_j + 2z_j = 0 \;\Rightarrow\; z_j = \frac{1}{2}.$$

Thus, agent j's optimal initial demand is half the surplus regardless of agent i's demand. Agent i, on the other hand, gets expected payoff

$$\begin{aligned} U_i(z) = & (1 - \overline{k})z_i + \underline{k}\left(z_i + (z_i + z_j - 1)\tfrac{ln(1-\underline{k})}{\underline{k}}\right) \\ & + (\overline{k} - \underline{k})\left(\tfrac{z_i}{2} + \tfrac{1-z_i}{2}\tfrac{ln(1-\underline{k}) - ln(1-\overline{k})}{\overline{k} - \underline{k}}\right). \end{aligned}$$

$$\frac{\partial U_i(z)}{\partial z_i} = 1 - \frac{\overline{k} - \underline{k}}{2} + \frac{1}{2}ln(1 - \underline{k}) + \frac{1}{2}ln(1 - \overline{k}) + \frac{\partial U_i(z)}{\partial \overline{k}}\frac{\partial \overline{k}}{\partial z_i} + \frac{\partial U_i(z)}{\partial \underline{k}}\frac{\partial \underline{k}}{\partial z_i}.$$

Note that lemma 2 holds true. Therefore, at $z_j = 1/2$,

$$\begin{aligned} \frac{\partial U_i(z)}{\partial z_i} = & 1 - \tfrac{\overline{k} - \underline{k}}{2} + \tfrac{1}{2}ln(1 - \underline{k}) + \tfrac{1}{2}ln(1 - \overline{k}) \\ = & 1 - \tfrac{z_i + z_j - 1}{z_i} + \tfrac{1}{2} \cdot ln\left(\tfrac{z_i + 2z_j - 1}{z_i} \cdot \tfrac{1 - z_i}{z_i}\right) \\ = & \tfrac{1}{2z_i} + \tfrac{1}{2} \cdot ln\left(\tfrac{1-z_i}{z_i}\right). \end{aligned}$$

Therefore agent i demands z_i such that

$$H_i(z_i) := \frac{1}{z_i} + ln\left(\frac{1 - z_i}{z_i}\right) = 0.$$

Note that $H_i(1/2) = 2$ and $H_i(1) = -\infty$. Also, the $H_i(.)$ is decreasing in z_i for $z_i \geq 1/2$. Hence, there is a unique $z_i \in (1/2, 1)$ that solves $H_i(z_i) = 0$. ■