

Patent Licensing and Technological Catch-up in an Asymmetric Duopoly*

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Abstract

We consider a model in which an outside inventor is the patentee of a cost-reducing technology that can be licensed to heterogeneous Cournot duopolists. As in most of the literature, we model the interaction between the inventor and the firms as a game in extensive form. We show that this game has no subgame-perfect equilibrium in which the least efficient duopolist becomes the sole licensee. Thus, in equilibrium, the technological distance between the firms, as measured by the difference in their costs, either increases or remains the same.

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1 Introduction

Starting with the seminal contributions of Kamien and Tauman (1984), Kamien and Tauman (1986), and Katz and Shapiro (1986), the theoretical literature on the licensing of cost-reducing technologies has flourished in the last decades. The models usually consider the interaction between an inventor, who is the patentee of a cost-reducing technology—or process innovation—, and potential licensees, who can adopt the technology. With few exceptions, most papers assume that the potential licensees are homogeneous firms operating in some given industry.¹

Little is known theoretically, however, about patent licensing in environments with heterogeneous potential licensees. In this paper, we consider a model in which potential licensees are heterogeneous Cournot duopolists, i.e. firms that are subject to different constant marginal costs

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¹One such exception is Stamatopoulos and Tauman (2009). See Remark 2 in section 4.

and compete in quantities. The inventor is an outsider to the industry and holds the patent of a technology that reduces the marginal cost of firms adopting it by the same additive amount. The interaction between the inventor and the firms is modeled as a game in extensive form in the spirit of the auction-licensing game introduced by Katz and Shapiro (1986) and reviewed by Kamien (1992). Basically, the inventor decides how many licenses to put for sale—either one or two—and allows the firms to bid competitively for the licenses.

We show that this game has no subgame-perfect equilibrium outcome in which the (*ex ante*) least efficient firm becomes the sole licensee. Therefore, unless the inefficient firm engages in R&D activity, technological catch-up in this industry is impossible, that is, in equilibrium either the efficient firm becomes relatively more efficient or the cost gap between the firms is unchanged. The result thus suggests a channel through which licensing may incentivize the search for, and development of new technologies. Indeed, relatively inefficient firms, according to the result, can only increase—or even maintain—their competitiveness if they engage in such activities.

The paper is organized as follows. In the next section, we describe the model and state the result referred to above. In section 3 we prove the result. Section 4 contains the concluding remarks.

2 The model

Consider a Cournot duopoly consisting of firms 1 and 2. Each firm $i \in \{1, 2\}$ produces output with a constant marginal cost technology. We denote by $c_i > 0$ the marginal cost of firm i , and we assume that $c_1 < c_2$. Firms face demand $D(p)$, with corresponding inverse demand given by $P(q) = \max\{0, \hat{P}(q)\}$, where $\hat{P}(\cdot)$ is a strictly decreasing, twice-differentiable log-concave function, and $\lim_{q \rightarrow \infty} P(q) = 0$.

An outside inventor holds the patent of a technology that reduces firms' costs by ε , that is, if firm i adopts the inventor's technology, then its marginal cost becomes $c_i - \varepsilon > 0$. The inventor's objective is to maximize licensing revenues by means of an auction (k, b) , where k is the number of licenses for sale in the auction and b is the minimum acceptable bid.

Let us now describe the patent licensing game Γ , involving the inventor and the two firms. The inventor moves first, announcing an auction policy (k, b) . Each firm then decides whether to participate in the auction, and, if it decides to participate, how much to bid. Firms offering the k highest bids win the auction—provided these bids are greater than b —and a winner pays to the inventor its own bid. After the auction, Cournot competition among the firms takes place. In this stage, firm i produces with cost $c_i - \varepsilon$ if it is a licensee, and with cost c_i if it is a nonlicensee.

The payoff to the inventor is given by the revenue he obtains in the auction, which is the sum of the winning bids. A licensee's payoff is given by its Cournot profit net of the payment to the inventor; a nonlicensee's payoff is simply its Cournot profit.

We say that *technological catch-up through licensing is possible* if there exists a subgame-perfect

equilibrium (SPE) of Γ in which the least efficient firm, namely, firm 2, becomes the sole licensee. If no such an equilibrium exists, we say that technological catch-up through licensing is impossible.²

We can now state our main result. Its proof is presented in the next section.

Proposition. *Technological catch-up through licensing is impossible.*

3 Proof of the Proposition

It is sufficient to show that if an SPE outcome of Γ has a sole licensee, then this licensee must be firm 1—if both firms become licensees in equilibrium, then the technological distance between them does not change; catch-up cannot happen.

Suppose then that firm i is the sole licensee in the Cournot stage of Γ . Under our assumptions, this Cournot subgame has a unique equilibrium.

Let $q_j(\{i\})$, for each $i, j \in \{1, 2\}$, denote the Cournot equilibrium quantity produced by firm j when firm i is the licensee.³ Similarly, denote by $\pi_j(\{i\})$ j 's equilibrium profit when i is the licensee. The following is a useful fact.

Lemma 1. *Let $p(\{i\})$ denote the Cournot equilibrium price when firm i is the sole licensee. Suppose that in this equilibrium $q_j(\{i\}) > 0$. Then, $p(\{i\})$ is the unique solution to*

$$p \cdot [1 - 1/2\eta(p)] = (c_1 + c_2 - \varepsilon)/2, \quad (1)$$

where $\eta(p) = -D'(p) \cdot (p/D(p))$ is the price elasticity of demand.

Proof. Equation (1) is obtained by adding both firms' first order conditions (assuming, of course, interior solutions), rearranging, and using the definition of $\eta(p)$. The assumptions on $P(\cdot)$ imply that $\eta(p)$ is increasing.⁴ Hence, the LHS of (1) is increasing in p , and the unique solution to this equation must be $p(\{i\})$. \square

²Observe that if either firm 1 becomes the sole licensee or both firms become licensees, then the relative inefficiency of firm 2, as measured by the difference between its marginal cost and that of firm 1, either increases or remains the same. Thus, in neither of these cases, firm 2 is able to catch-up with firm 1, in the sense of reducing the gap between its cost and that of firm 1.

³Obviously, we may have $i = j$. The curly braces in our notation stress the fact that equilibrium quantities depend on the *set* of licensees.

⁴To see this, observe first that log-concavity of the inverse demand implies that $P'(q)/P(q)$ is decreasing in q , and, therefore, increasing in p . But

$$\frac{P'(q)}{P(q)} = -\frac{1}{\eta(p)D(p)}.$$

Taking derivative w.r.t. p gives

$$\frac{1}{\eta(p)D(p)} \left(\frac{\eta'(p)}{\eta(p)} + \frac{D'(p)}{D(p)} \right) \geq 0 \iff \frac{\eta'(p)}{\eta(p)} \geq -\frac{D'(p)}{D(p)} > 0.$$

Thus, $\eta'(p) > 0$.

Now, given the announcement (k, b) with $k = 1$, firm i 's *willingness to pay for a license* is given by

$$w_i = \pi_i(\{i\}) - \pi_i(\{j\}).$$

In words, i 's willingness to pay for a license, when only one license is put for sale in the auction announced by the inventor, is the difference between i 's profit as the sole licensee and i 's profit as a nonlicensee when its rival j is a licensee.

Lemma 2. $w_1 > w_2$.

Proof. There are three cases to consider. These cases regard the effect of the adoption of the technology by a single firm on the industry structure. We consider each of these cases in turn.

Case 1 (Each firm as the sole licensee becomes a monopoly). Let π_i^m denote firm i 's monopoly profit when its marginal cost is $c_i - \varepsilon$. Then, for each $i \in \{1, 2\}$, we have $w_i = \pi_i^m$. The result follows by observing that $\pi_1^m > \pi_2^m$.

Case 2 (Only firm 1 as the sole licensee becomes a monopoly). We have $w_1 = \pi_1^m - \pi_1(\{2\})$ and $w_2 = \pi_2(\{2\})$. Thus,

$$\begin{aligned} w_1 - w_2 &= \pi_1^m - \pi_1(\{2\}) - \pi_2(\{2\}) \\ &= \pi_1^m - (p(\{2\}) - c_1) q_1(\{2\}) - (p(\{2\}) - c_2 + \varepsilon) q_2(\{2\}) \\ &> \pi_1^m - (p(\{2\}) - c_1 + \varepsilon) q_1(\{2\}) - (p(\{2\}) - c_2 + \varepsilon) q_2(\{2\}) \\ &= \pi_1^m - (p(\{2\}) - c_1 + \varepsilon) \cdot [D(p(\{2\})) - q_2(\{2\})] - (p(\{2\}) - c_2 + \varepsilon) q_2(\{2\}) \\ &= \pi_1^m - (p(\{2\}) - c_1 + \varepsilon) \cdot D(p(\{2\})) + \Delta c \cdot q_2(\{2\}) \\ &> 0, \end{aligned}$$

where $\Delta c = c_2 - c_1$. The first inequality follows from the fact that $\varepsilon > 0$; the third equality follows from the fact that $q_1(\{2\}) + q_2(\{2\}) = D(p(\{2\}))$ in equilibrium; and the last inequality follows from the fact that $\pi_1^m \geq (p(\{2\}) - c_1 + \varepsilon) \cdot D(p(\{2\}))$.

Case 3 (Neither firm as the sole licensee becomes a monopoly). Lemma 1 implies that, in the present case, $p(\{1\}) = p(\{2\})$. Using this fact, and noticing that it, in turn, implies that the aggregate output q satisfies $q(\{1\}) = q(\{2\})$, we have, for each $i \in \{1, 2\}$,

$$\begin{aligned} w_i &= (p(\{i\}) - c_i + \varepsilon) q_i(\{i\}) - (p(\{j\}) - c_i) q_i(\{j\}) \\ &= \varepsilon q_i(\{i\}) + (p(\{i\}) - c_i) \cdot (q_i(\{i\}) - q_i(\{j\})) \\ &= \varepsilon q_i(\{i\}) + (p(\{i\}) - c_i) \cdot \left(-\frac{(p(\{i\}) - c_i + \varepsilon)}{P'(q(\{i\}))} + \frac{p(\{j\}) - c_i}{P'(q(\{j\}))} \right) \\ &= \varepsilon \cdot (q_i(\{i\}) + q_i(\{j\})), \end{aligned}$$

where the third equality follows from firm i 's first order condition for profit maximization.

Hence,

$$\begin{aligned} w_1 - w_2 &= \varepsilon \cdot (q_1(\{1\}) - q_2(\{2\}) + q_1(\{2\}) - q_2(\{1\})) \\ &= -\frac{2\varepsilon\Delta c}{P'(q\{i\})} \\ &> 0. \end{aligned}$$

Finally, we observe that the case *only firm 2 as the sole licensee becomes a monopoly* is not possible: since firm 2 is relatively inefficient, if firm 2 as the sole licensee becomes a monopoly, then firm 1 as the sole licensee would also become a monopoly; this brings us back to Case 1 above. Therefore, the three cases above exhaust the possibilities and the proof of the lemma is complete. \square

It follows from Lemma 2 that whenever the inventor announces an auction policy (k, b) with $k = 1$ and $b = 0$, firm 1 wins the auction by offering a bid slightly above w_2 , or else we would be off the equilibrium path. If $k = 1$ and $b > 0$, then either firm 2 does not participate in the auction—take $b > w_2$ —or it does participate and is overbid by firm 1. Hence, in equilibrium, a sole licensee must be firm 1. These observations conclude the proof of the Proposition. \blacksquare

4 Concluding remarks

Remark 1 (Alternative licensing mechanisms). The Proposition also holds for alternative licensing mechanisms, namely, royalties, fixed fees and two-part tariffs.⁵ Indeed, under a royalty policy both firms become licensees, whereas under either fixed fee or two-part tariff policies an argument in the same lines as the one provided in the previous section holds, that is, one can show that firm 1 has higher willingness to pay for a license than firm 2.

Remark 2 (Firm-specific cost reduction). Clearly, the result depends on the assumption that adoption of the technology leads to the same *additive* cost reduction for each firm. Stamatopoulos and Tauman (2009) have studied the case in which the technology can be adopted only by the least efficient firm, and have provided conditions under which this firm becomes a licensee. Allowing cost reduction to be firm-specific is a natural extension of the traditional patent licensing model that can lead to many interesting research questions.

Remark 3 (Downstream Bertrand competition). Replacing downstream Cournot competition with Bertrand competition does not alter the Proposition. In fact, with Bertrand competition, firm 1 is the sole licensee in equilibrium and the inventor obtains a payoff equal to the difference between the post-invention and pre-invention profits of firm 1.

⁵With homogeneous firms, Kamien and Tauman (1986) have studied the royalty and fixed fee mechanisms; Sen and Tauman (2007), among others, have studied two-part tariffs.

Remark 4 (“Abstract” downstream competition). Let us briefly describe a patent licensing model that abstracts from the assumption that firms are Cournot competitors.⁶ Suppose that adoption of the technology by firms in the set $S \subseteq \{1, 2\}$ results in a profit equal to $\pi_i(S)$ to firm $i \in \{1, 2\}$. For each firm i we assume that $\pi_i(\cdot)$ satisfies

$$\pi_i(\{i\}) > \pi_i(\{1, 2\}) > \pi_i(\emptyset) > \pi_i(\{j\}),$$

i.e. becoming a licensee is always profitable for firm i ; being the sole licensee is the best outcome for i ; and not being a licensee, when its rival j is a licensee, is the worst outcome for i .

In this framework, firm 1 is *ex ante stronger than* firm 2 if $\pi_1(\emptyset) > \pi_2(\emptyset)$.⁷ Clearly, this condition says nothing about the possibility of firm 2 becoming the sole licensee. If

$$\pi_1(\{1\}) - \pi_1(\{2\}) > \pi_2(\{2\}) - \pi_2(\{1\}), \tag{2}$$

then firm 2 will never become the sole licensee. However, we cannot conclude from this that technological catch-up is impossible. Indeed, at this level of generality, we could have $\pi_2(\{1, 2\}) > \pi_1(\{1, 2\})$, so that firm 2 is stronger than firm 1 when both firms adopt the technology.

A model along these lines provides an alternative environment to study firm-specific cost reductions (see Remark 2).

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⁶Katz and Shapiro (1986) have considered such a model with homogeneous potential licensees.

⁷This, of course, corresponds to firm 1 producing with the smallest marginal cost in the Cournot formulation above.