

Endogenous Information Acquisition in an Investment Trading Game

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Abstract. In an investment trading game where the profitability of the new investment (the fundamental) is a random variable, entrepreneurs' higher-order beliefs about the future asset price of the realized investment enter in their investment decisions. On the other hand, the financial market uses the aggregate investment as a signal of the underlying fundamental. If agents have dispersed information, endogenous strategic complementarity in actions emerges owing to the information spillover and generates inefficiency in the economy. We introduce endogenous information acquisition and study what information is acquired and how it affects the equilibrium outcome. We also study the welfare properties of the equilibrium.

1. INTRODUCTION

In this paper we study information acquisition in an investment trading game where (i) entrepreneurs base their investment decisions on their expectation about both an unknown underlying economic fundamental and the price at which they may sell their capital to the financial markets in the future; and (ii) traders operating in the financial market use the aggregate investment to learn about the fundamental. According to Angeletos, Pavan, and Lorenzoni (2010) in a such framework with dispersed information and an exogenous information structure, the information spillover generates inefficiency, calling for policy intervention aimed to improve welfare. Differently from Angeletos, Pavan, and Lorenzoni (2010) we consider an endogenous information structure where agents not only take the investment decision according to the information available, but also before any investment decision is taken they choose how much costly attention to pay to an informative private signal. Our way of modeling information may be more appropriate: when new investment opportunities arise, it is quite plausible that agents cannot obtain good information quality unless they pay to acquire it.

Background and Motivation. Following the seminal work of Morris and Shin (2002) a growing literature has investigated the social welfare effects of different information structures in economies with strategic complementarity or substitutability in actions and incomplete information. In these economies, the value of an underlying economic fundamental is unknown. Greater access to information helps agents to choose actions closer to the realization of the fundamental. One key question is whether more public or private information is desirable in such economies. Public information is any signal about the unknown fundamental which is common across agents; private information instead is any signal about the unknown fundamental which is independent for each agent. Some papers focus on welfare analysis where the information available in the economy, either public and private, is exogenous and agents can only make decisions based on it, but cannot affect the information they get. Other research, instead, considers economies with an endogenous information structure. That is, when information is costly and agents have to acquire it, the information they obtain is endogenous because they can affect the information they get either by choosing which signals they purchase or by paying a cost in order to increase the precision of the signals.

A class of economies with strategic complementarity widely studied in the literature can be captured by the beauty contest game. In such a class of games, if the information structure is exogenous and public information is the only source of information, an increase of its precision is always beneficial for social welfare. Conversely, when agents can also access private information, public information may be detrimental for welfare (Morris and Shin, 2002). However, in a beauty contest framework where agents are allowed to choose the precision of their private signal, an increase in the precision of the public information is always welfare enhancing (Colombo and Femminis, 2008).

In the case of economies with investment complementarities, where the coordination between agents is both privately and socially valuable, better precision of public information always increases welfare, while the opposite may occur with an increase in the precision of private information (Angeletos and Pavan, 2004). However, when private information is not freely available to agents and it needs to be acquired, even if they optimally coordinate according to the information they get, the information acquired in equilibrium may be inefficient, i.e. less precise than optimal (Colombo, Femminis, and Pavan, 2014). Thus, according to whether strategic complementarities in actions are valuable only privately or also socially and whether the information structure is exogenous or endogenous, public and private information affect welfare differently.

Models with quadratic payoffs and strategic complementarity or substitutability have been extensively applied to: investment games (Angeletos and Pavan, 2004), monopolistic competition (Hellwig, 2005), financial market (Allen, Morris, and Shin, 2006), political leadership (Dewan and Myatt, 2008), Lucas-Phelps economy (Myatt and Wallace, 2014), Cournot competition (Myatt and Wallace, 2015).

Strategic complementarity may arise also endogenously as a result of an information spillover from one economic sector to another, such as in the case of real sector and financial market interacting with each other (Angeletos, Pavan, and Lorenzoni (2010); Goldstein, Ozdenoren, and Yuan (2013)). Angeletos, Pavan, and Lorenzoni (2010) model a two-way feedback between investment decisions and asset prices in a financial market with incomplete information about investment opportunities. They show that a beauty-contest may arise from the interaction between the real sector, that has to decide how much to invest, and the financial market interested in the price of the asset related to that investment. From a social point of view the equilibrium outcome is inefficient: the existence of an information spillover induces investors to react too much to the correlated signal (public information) and too little to the idiosyncratic signal (private information).

Research question, outline of the model and results. Within the framework of Angeletos, Pavan, and Lorenzoni (2010) a few questions arise: (i) What is the equilibrium outcome if agents were to buy the information they need? (ii) What is the optimal amount of private information acquired in equilibrium? (iii) What is the social value of the information acquired in equilibrium?

This paper addresses those questions, by introducing endogenous information acquisition in the model of Angeletos, Pavan, and Lorenzoni (2010). In particular the framework is the following. At the beginning of the period there is a new investment opportunity with unknown profitability (the fundamental). Two sectors operate in this economy: the real sector populated by entrepreneurs and the financial sector populated by traders. In the first period each entrepreneur has to decide how much to invest in the new project. In the second period before the profitability of the project is revealed, a fraction λ of entrepreneurs is hit by a liquidity shock and sells its capital to the financial sector. Information is incomplete in the sense that each agent in the economy does not know the profitability of the project, but it knows the prior distribution of this fundamental. In addition, entrepreneurs have free access to a public signal about the fundamental value of the project. Before they make their investment decision they can also acquire at some cost a private signal by paying attention to listen to it. The more they listen to the signal the more precise would be the signal acquired. In the financial

sector, traders also have free access to a public signal about the fundamental value and they also observe the aggregate capital invested in the economy.

We first characterize the benchmark economy, that is the case in which only the entrepreneurs have incomplete information about the fundamental, while traders are perfectly informed. We show that in this case there does not exist any information spillover between the real and financial sectors. We then characterize the equilibrium outcome in terms of the information acquired and how this information affects agents' actions in the incomplete information case, that is when there are information spillovers between the real and financial sectors.

Our results show that, relative to the benchmark economy, entrepreneurs pay less attention to private information implying that the equilibrium precision of the private signal is lower than in the benchmark case. While in terms of the investment decision, in line with the findings of Angeletos, Pavan, and Lorenzoni (2010), entrepreneurs put more weight on the public signal and less on the private signal than in the benchmark.

Another result is that under some parameters space of the precision of entrepreneurs' private signal, the precision of traders' public signal and the magnitude of the liquidity shock, entrepreneurs do not pay attention to the private signal.

2. LITERATURE REVIEW

In this section we provide a summary of the relevant literature to which our work is related. The paper is placed in the literature of social value of information in economies with strategic complementarity in actions. In fact, as in Morris and Shin (2002) we study the social values of information in a framework that resembles their beauty-contest game. The paper is also close to the investment game with strategic complementarities analyzed in Angeletos and Pavan (2004). However, our model differs from that because we do not assume strategic complementarities, but these emerge endogenously. As in the paper of Angeletos and Pavan (2007) we aim to study the efficient use of information and its social value, but we do this introducing costly information acquisition as in Colombo and Femminis (2008) and Colombo, Femminis, and Pavan (2014). The information acquisition in games with strategic complementarities is also studied by Hellwig and Veldkamp (2009) and Myatt and Wallace (2012) who move away from the social value of information acquisition and focus on the properties of the information acquired and how it affects the equilibrium.

This paper is closely related to the literature of feedback effects between two sectors (Angeletos, Pavan, and Lorenzoni (2010); Goldstein, Ozdenoren, and Yuan (2013)).

With these two papers we share in common the idea of the existence of feedback effects between two sectors that generate information spillovers and endogenous complementarity in actions.

Here below a review of the three strands of literature which our paper is related to: (i) social value of information in economies with strategic complementarity; (ii) endogenous information acquisition in coordination games; (iii) feedback effects between real and financial sector.

Social value of information in economies with strategic complementarities. The social value of information in economies with strategic complementarities has been studied considering either exogenous and endogenous information structure.

The pioneering work of Morris and Shin (2002) examined the social value of public information in a particular class of games with strategic complementarity: the beauty contest coordination game. It captures the Keynes's beauty contest metaphor of financial markets. In this game "... a large population of agents have access to public and private information on the underlying fundamentals, and aim to take actions appropriate to the underlying state. But they also engage in a zero-sum race to second-guess the actions of other individuals in which a player's prize depends on the distance between his action and the action of the others. The smaller is the distance, the greater is the prize." Morris and Shin (2002), page 1522. Agents' actions are thus driven by two motives: a fundamental and a coordination motive. The fundamental motive induces agents to take actions that match the underlying unknown state of the nature (the fundamental). The coordination motive induces agents to take actions that are close to the average of others' actions. Such coordination motives arise from the fact that agents' actions are strategic complements. If agents are fully informed about the fundamental, their actions maximize social welfare, which is defined as the normalized average of agents' utilities. However, when information is incomplete, the provision of public information has a different impact on social welfare. If the only source of information is the public one, social welfare increases as the public signal is more precise (i.e. has a lower variance). When agents have access also to private information, the better the precision of private information the more likely the increase in the provision of public information reduces social welfare.

Colombo and Femminis (2008) instead study a beauty-contest game with endogenous information structure. Public and private information are both costly and endogenous in the sense that agents choose the precision of the private information after observing the precision of the public information set by a public authority. The public authority finances the acquisition of public information via a lump sum tax, while the agents face

a cost to acquire the precision of the private information. The model shows that more precise public information is welfare enhancing when the marginal cost of public information is not higher than that of private information. This is so because an increase in the precision of public information reduces agents' incentive to acquire private information, thereby allowing agent to save private resources. Such a mechanism counterbalances the negative effect of the reduction in the precision of private information with the corresponding cost-saving of private resources. A similar result about the social value of public information may also be found in the paper of Angeletos and Pavan (2004). They find that, in economies with investment complementarities, social welfare increases with the precision of public information, while an increase in the precision of private information may reduce it. This result, which is in contrast with the work of Morris and Shin (2002), is due to the fact that, in investment complementarity games, strategic coordination between agents is both privately and socially valuable. They consider an economy with investment complementarities where the individual return to investment is increasing in the aggregate level of investment and where agents have access to two information sources about the underlying economic fundamental which is represented the productivity of the investment: one public (a common signal realization) and one private (an independent signal realization for each agent). More precise public information eases coordination between agents causing higher volatility of the equilibrium outcome generated by the common noise. Moreover, when both public and private information are available in the market, as the precision of public information increases the aggregate volatility increases due to the sensitivity of economic activity to common noise. Conversely, an increase in the precision of private information reduces the aggregate volatility.

A more comprehensive welfare analysis of information in coordination games is given in the paper of Angeletos and Pavan (2007). They analyze the efficiency of the exogenous information used by agents in economies that have externalities, strategic complementarity or substitutability and heterogeneous information. They first identify the efficiency of the equilibrium response to different sources of information. This efficiency benchmark captures what society can do as a whole with the only constraint that information cannot be transferred from one agent to another. Then they compare the efficiency benchmark with the equilibrium use of information to check the existence of possible inefficiencies and to determine the social value of information. Their results show that in economies where the equilibrium is efficient, welfare increases with the

precision of any type of information (private and public). In economies where the inefficiency arises because of incomplete information, more precision of the agents' forecast about fundamental (accuracy) increases welfare, while more correlation of forecast errors across agents (commonality) reduces welfare if the agents' private value of aligning decisions (the equilibrium degree of coordination) is higher than the optimal one (the optimal degree of coordination). Finally, in an economy where the equilibrium use of information is inefficient even under complete information, welfare may decrease with both the commonality and the accuracy of information.

In a related paper Colombo, Femminis, and Pavan (2014) examine the social value of public information in a large class of economies, allowing for either strategic substitutability or complementarity in actions, in cases where agents can choose the precision of the private information before taking any action. They first characterize the precision of private information acquired in equilibrium, which appears to be decreasing in the precision of public information. Then, they characterize the optimal precision of private information and show that the one acquired in equilibrium is typically inefficient, even in those economies in which the use of information is efficient (that is, the degree of coordination acquired in equilibrium is optimal). The paper also shows how the acquisition of private information may change the social value of public information, i.e., the comparative statics of equilibrium welfare with respect to the quality of public information.

Information acquisition in coordination games. This strand of research aims to understand to which information agents pay attention (acquire) and how the information acquired affects the equilibrium outcomes when information is costly. Here we discuss two papers: Hellwig and Veldkamp (2009) and Myatt and Wallace (2012). There are other papers that consider endogenous information acquisition in this class of games; some of them have been discussed in the previous paragraph (Colombo and Femminis (2008) and Colombo, Femminis, and Pavan (2014)). However here we examine only those two papers which are closely related to our way of modeling the information structure.

Hellwig and Veldkamp (2009) add a preliminary stage to a version of "beauty contest" game similar to the one of Morris and Shin (2002). In their model players first choose which signals to buy from a set of signals, and after having observed the chosen signals they choose an action. They examine how the choice of a player to acquire information before playing a strategic game depends on the information acquired by the other players. In their model signals can be purely private, purely public or correlated and agents observe heterogeneous exogenous signals before choosing their information.

Their model encompasses games with strategic complementarity and with strategic substitutability. Their main result is that strategic motives in information choice reflect the strategic motives in actions. If players' actions are strategic complements, information acquisition is also complementary. If actions are strategic substitutes, then information acquisition exhibits strategic substitutes as well. This result comes from the fact that the information acquired changes the covariance between the unobserved fundamental and the average of actions, which makes the information more or less valuable. Concerning information acquisition they show that there can be multiplicity of equilibria. The source of multiplicity is the acquisition of public signals: it is more valuable if others acquire it too. Uniqueness of equilibrium is guaranteed when signal realizations are uncorrelated, that is when players acquire private signals.

Myatt and Wallace (2012) focus on beauty contest models where the information is neither completely public nor completely private, and players have to choose among n sources of information. The precision of the common noise represents the accuracy of the signal while the precision of the idiosyncratic noise represents the clarity of the signal. The peculiarity of their model is that the degree of publicity of the information is endogenous: it depends on the costly attention paid by the agents to acquire the information. Choosing how much attention to pay to a particular information sources, players choose the precision of the private noise of each signal as well as its publicity (more attention implies higher correlation). This setting preserves uniqueness of equilibrium: agents pay attention only to a subset of the n signals. Agents acquire the clearest signals even if their accuracy is weak. Moreover, the number of signals players focus on reduces as the complementarity of action raises. Another equilibrium result of their model is that the information endogenously acquired becomes more public as the actions become more complementary.

Feedback effects between real sector and financial market. Angeletos, Pavan, and Lorenzoni (2010) and Goldstein, Ozdenoren, and Yuan (2013) show that, with dispersed information about an underlying economic fundamental, strategic complementarity in action emerges endogenously owing to information spillovers that arise when the agents of one sector use to agents' aggregate actions of another sector to learn about the unknown fundamental. In the paper "*Wall Street and Silicon Valley: A Delicate Interaction*" Angeletos, Pavan, and Lorenzoni (2010) focus on the information spillover that emerges when financial markets use real economic activity as a signal of the underlying fundamentals and study how the information spillovers impact agents' incentives when making their real investment decisions. The mechanism described in their paper captures situations in which financial markets use data about aggregate investment as

proxies for underlying economic fundamentals, like the profitability of a new technology or a new sector. At the same time, firms' incentives to invest increase with expected financial prices (this is because high financial prices raise the value of installed capital). The paper thus studies the positive and normative implications of this two-way feedback during periods of intense technological change, when information about the profitability of new technologies is widely dispersed. On the one hand entrepreneurs base their investment decision on two sources of information, one public (with common noise across entrepreneurs) and one private (with idiosyncratic noise), and on their expectations of the price which they may sell the capital. On the other hand traders use aggregate investment as a signal of the profitability of the new investment. Because high aggregate investment is "good news" for profitability, asset prices increase with aggregate investment. Because an entrepreneur's incentives to invest in turn increase with the financial market assessment of his capital, an entrepreneur is willing to invest more when he expects others to invest more. This endogenous complementarity induces entrepreneurs to rely more on common sources of information regarding profitability and less on idiosyncratic sources of information. Moreover, they show that these effects are symptoms of inefficiency: investment reacts too little to fundamental shocks (i.e shock related to the profitability) and too much to expectational shocks (i.e. variations in the noise). Such inefficiency originates in the dispersion of information, not in the fact that entrepreneurs care about financial prices.

Goldstein, Ozdenoren, and Yuan (2013) focus on the opposite information spillover, that is when a capital provider learns from the price in the financial markets to make an investment decision. Their model gives an explanation of trading frenzies in financial markets and allows them to investigate the effect of trading frenzies on real activities. In their model a capital provider decides how much capital to provide to a firm for the purpose of making new real investment. The decision of the capital provider depends on his information about the productivity of the proposed investments. In his decision, the capital provider uses two sources of information: his private information, and the information aggregated by the price of the firm's security that is traded in the financial market. The feedback effect that goes from the financial market to the real economy emerges because of the dependence of capital provision on financial market prices. On the other hand, speculators trade a security whose payoffs is correlated with the cash flow obtained from the firm's investment. Speculators make their trading decisions on the basis of two signals: one private and one public. When traders put large weight on the public signal relative to the private one, the price is strongly affected by the correlated information. Because prices work as an informative channel from the financial

market to the capital provider, a higher weight on the correlated information has also a stronger effect on the provision of capital and thus on the real value of the security traded. This mechanism increases the traders' incentives to put more weight on the correlated signal, leading to endogenous strategic complementarity and trading frenzies. They also show that this result is inefficient because there is always either too much or too little coordination between agents' actions, which creates excess volatility in the price and reduces the efficiency of investments.

3. THE MODEL

We introduce endogenous private information acquisition in the Angeletos, Pavan, and Lorenzoni (2010) model. Endogenous information acquisition is modeled similarly to Myatt and Wallace (2012).

Timing, information structure and key choices. We consider an economy with a real sector and a financial sector each operating for only one period and sequentially. The economy is populated by two types of agents: entrepreneurs and traders. Each type is of measure $1/2$, where entrepreneurs are indexed by $i \in [0, 1/2]$ and traders are indexed by $i \in (1/2, 1]$. Timing is divided in four periods, $t = \{0, 1, 2, 3\}$. In period $t = 0$ a new investment technology becomes available. The profitability of this new technology is uncertain and determined by the random variable $\theta \sim N(\mu, \sigma_\theta^2)$. Agents do not know θ , they know only its distribution.

In period $t = 1$ the real sector operates. Each entrepreneur i invests k_i unit of capital in the new technology. Investing in this technology costs $\frac{k_i^2}{2}$. Before deciding how much capital to invest, each entrepreneur has access to a public signal

$$\bar{x} = \theta + \eta, \quad (1)$$

where $\eta \sim N(0, \kappa^2)$ and η is independent of θ . The signal has thus precision $\pi_{\bar{x}} = \frac{1}{\kappa^2}$. By paying a cost $C(z_i)$ entrepreneurs have also the possibility to acquire a private signal

$$x_i = \theta + \epsilon_i, \quad (2)$$

where $\epsilon_i \sim N(0, \frac{\xi^2}{z_i})$ with ϵ_i independent of η and θ . The overall precision of the private signal depends on two different components: the exogenous precision $\pi_{x_i} = \frac{1}{\xi^2}$ and the endogenous precision z_i . Following an argument similar to Myatt and Wallace (2012) we refer to these elements of signal precision as the clarity and the attention paid to listen to the private signal, respectively. The way private information acquisition works is the following: each entrepreneur can pay attention $z_i \in R^+$ to listen to the signal and by doing so they can increase the total precision of the private signal. It

turns out that the private signal acquired in equilibrium has endogenous precision. $z_i = 0$ is taken to mean that the entrepreneur does not acquire the private signal. In this case the signal x_i is pure noise, that is $x_i \sim N(0, \infty)$.

In period $t = 2$ each entrepreneur is hit by a liquidity shock with probability λ , which forces them to sell the capital invested to the financial sector before the realization of θ . λ is common knowledge to both entrepreneurs and traders. We assume that only entrepreneurs hit by the shock sell their capital, the rest of them do not sell it. In this period the financial market starts to operate because some entrepreneurs sell their capital. The financial market is competitive and its market clearing price is denoted p . Traders observe only the fraction of the capital entrepreneurs sell to the traders. Because λ is known, by the law of large numbers, traders can infer the aggregate level of investment $K \equiv \int_0^1 k_i di$. The information available to traders about the profitability θ is given by a public signal $y = \theta + \omega$ with $\omega \sim N(0, \tau^2)$ where ω is independent of θ, η, ϵ_i . We denote its precision $\pi_y = \frac{1}{\tau^2}$. Moreover traders use the observation about aggregate capital to update their beliefs about θ .

Finally at $t = 3$ the fundamental value of θ is publicly revealed and production takes place assuming that each unit of capital delivers θ units of the consumption good.

Preferences and endowments. All agents receive an exogenous endowment e of the (nonstorable) consumption good in each period. Moreover, they are risk neutral and their discount rate is zero: preferences are given by $u_i = c_{i1} + c_{i2} + s_i c_{i3}$, where c_{it} denotes agent i 's consumption in period t , while s_i is a random variable that takes value 0 if the agent is an entrepreneur hit by a liquidity shock and value 1 otherwise.

The expected utility of entrepreneur i from investing k_i units of capital in the new technology, conditional on observing the signals \bar{x} and x_i with attention z_i is:

$$\mathbb{E}(u_i | \bar{x}, x_i) = \mathbb{E} \left[(1 - \lambda)\theta k_i + \lambda p k_i - \frac{k_i^2}{2} \middle| \bar{x}, x_i \right] - C(z_i). \quad (3)$$

The expected utility of a trader at $t = 2$ is:

$$\mathbb{E}(u_i | K, y) = (\mathbb{E}[\theta | K, y] - p)q_i, \quad (4)$$

where p is the market clearing price in the financial market and q_i is the the amount of capital bought by trader i .

4. EQUILIBRIUM

First we solve the last stage of the game, that is we consider a trader's expected utility at the time of trading. Notice that in this game we assume financial markets are

competitive, therefore traders do not act as strategic players. The market clearing price in the financial market is given by the traders' expectation of the fundamental: $p = E[\theta|K, y]$. We first solve for the market clearing price in the financial market, then we plug the optimal price function into the entrepreneur's expected utility and solve for optimal attention and investment. Before proceeding to solve the model we first define our equilibrium solution concept.

Definition 1. A linear REE (rational expectation equilibrium) is an individual information acquisition policy z_i and investment strategy $k(\bar{x}, x_i)$, an aggregate investment function $K(\theta, \eta)$ and a price function $p(\theta, \eta, \omega)$ that jointly satisfy the following conditions:

- i. $z_i \in \operatorname{argmax}_{z_i \geq 0} \left\{ \mathbb{E} \left[(1 - \lambda)\theta k_i + \lambda p k_i - \frac{k_i^2}{2} \right] - C(z_i) \right\}$
- ii. for all (\bar{x}, x_i) ,

$$k(\bar{x}, x_i) \in \operatorname{argmax}_{k_i} \left\{ \mathbb{E} \left[(1 - \lambda)\theta k_i + \lambda p k_i - \frac{k_i^2}{2} \mid \bar{x}, x_i \right] - C(z_i) \right\}$$
- iii. for all (θ, η) ,

$$K(\theta, \eta) = \int k(\bar{x}, x_i) d\Phi(\bar{x}, x_i | \theta, \eta);$$

with $\Phi(\bar{x}, x_i | \theta, \eta)$ joint cdf of \bar{x} and x_i , given θ and η ;

- iv. for all (θ, η, ω) ,

$$p(\theta, \eta, \omega) = \mathbb{E}[\tilde{\theta} | K(\theta, \eta), y];$$

- v. there exist scalars δ_0, δ_θ and δ_η such that, for all (θ, η) ,

$$K(\theta, \eta) = \delta_0 \mu + \delta_\theta \theta + \delta_\eta \eta.$$

Condition (i) requires that the information acquisition policy should be optimal, that is each entrepreneur chooses z_i to maximize his expected utility before any signal is observed. Condition (ii) requires that the entrepreneur's investment strategy is rational, taking as given the equilibrium price function. Condition (iii) defines the aggregate capital. Condition (iv) is just the market clearing condition in the financial markets. Condition (v) imposes linearity of the aggregate capital and therefore of the individual investment decision.

4.1. Benchmark.

In this section we characterize the equilibrium of the benchmark economy. The benchmark is an economy in which no uncertainty about the profitability of the asset (fundamental) exists on traders' side, that is traders can perfectly observe the value of θ . In such an environment, traders do not have to form any expectations about the profitability of the asset, therefore the market clearing price is equal to θ . This in turn, implies that entrepreneurs do not form expectations on traders expectations about the fundamental θ . Therefore asset prices do not affect the entrepreneur's expected utility.

Lemma 1. *In the benchmark economy, where uncertainty about the fundamental θ lies only on entrepreneurs' side, there does not exist any information spill-over between the real and financial sectors.*

Conditional on observing the public signal and the acquired private signal, an entrepreneur's expected utility reduces to

$$E(u_i|\bar{x}, x_i) = E\left(\theta k_i - \frac{k_i^2}{2} \mid \bar{x}, x_i\right) - C(z_i). \quad (5)$$

By differentiating the above pay-off function it is easy to see that optimal investment decision is $k_i(\bar{x}, x_i) = E(\theta|\bar{x}, x_i)$. By Bayesian updating, given the normality assumptions the above expectation is linear and implies that the entrepreneur's expectation about the fundamental is a weighted average of the information available to them.

$$k_i = \beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i}x_i. \quad (6)$$

Before any investment decision is taken, each entrepreneur has to choose the attention z_i he wants to pay to the private signal he acquires. This implies that the parameters β_0 , $\beta_{\bar{x}}$, and β_{x_i} depend on the entrepreneur's information acquisition policy. We can then solve the entrepreneur's sequential problem described above as if each entrepreneur simultaneously chose the attention to listen to the signal and the weight to assign to each information source. Notice that we can do so because each entrepreneur would not change his decision about z_i and the weights even after having observed the signal. Moreover, the information acquisition policy and thus the weights attached to each source of information are not revealed to anyone else, before any action is taken. Thus we can solve this two-stages entrepreneur's problem as if it were a one shot game.

From now on we restrict our analysis to a particular functional form of the cost of acquiring information. Given that each entrepreneur has to pay attention to at most one private signal, this restriction is almost without loss of generality.

Assumption 1. *Assume that the cost of acquiring information is equal to $C(z_i) = \frac{z_i}{2}$*

With this assumption we are imposing linearity of the cost of acquiring information, and the factor $\frac{1}{2}$ is needed just to simplify the algebra, but it does not affect any result.

Proposition 1. *In the economy without information spill-overs between the real and financial sectors the equilibrium information acquisition policy and individual investment strategy are unique.*

Under Assumption 1 they are characterized as follows

i. the information acquisition policy is:

$$z_i^B = \begin{cases} \frac{\sqrt{\pi_{x_i} - (\pi_\theta + \pi_{\bar{x}})}}{\pi_{x_i}} & \text{if } \pi_{x_i} > (\pi_\theta + \pi_{\bar{x}})^2 \\ 0 & \text{otherwise;} \end{cases}$$

ii. whenever $z_i^B > 0$ individual capital investment is $k_i = \beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i}x_i$,
where $\beta_0 = \frac{\pi_\theta}{\sqrt{\pi_{x_i}}}$, $\beta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\sqrt{\pi_{x_i}}}$ and $\beta_{x_i} = \frac{\sqrt{\pi_{x_i} - (\pi_{\bar{x}} + \pi_\theta)}}{\sqrt{\pi_{x_i}}}$;

iii. whenever $z_i = 0$ individual capital investment is $k_i = \beta_0\mu + \beta_{\bar{x}}\bar{x}$,
where $\beta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_{\bar{x}}}$ and $\beta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}}}$

Proof. Proof in the appendix. □

The above proposition clearly shows that in the benchmark economy individuals investment decision is not driven by asset prices, but only by their expectation about the fundamental. It also highlights an important feature of the benchmark economy: under what condition each entrepreneur acquires the private signal. First of all notice that the prior θ and the signal \bar{x} represent the total amount of public information that each entrepreneur has access to. Then we can define $\pi_\theta + \pi_{\bar{x}}$ as the entrepreneurs' overall precision of the public sources of information. Thus entrepreneurs acquire private information only if the value of the exogenous precision of the private signal is sufficiently high relative to the overall precision of their public information.

From Proposition 1 it follows that the aggregate capital is equal to $K(\theta, \eta) = \beta_0\mu + \beta_\theta\theta + \beta_\eta\eta$, with $\beta_\theta \equiv \beta_{\bar{x}} + \beta_{x_i}$ and $\beta_\eta \equiv \beta_{\bar{x}}$. The weight β_θ represents the response of aggregate capital to fundamental shocks, while β_η represents the response of aggregate capital to the correlated shock, that is the response to the common shock η . In other words the aggregate capital function tells us how much of the aggregate capital is driven by fundamental motive, which reflects the profitability of the investment, and how much of the aggregate capital is driven by the common noise, which represents the volatile part of aggregate capital. Notice that when the private signal is not acquired, that is $z_i^B = 0$, the fundamental shock and common shock have equal weight in determining aggregate capital.

4.2. Incomplete Information.

Traders' stage. The market clearing price in the financial market is $p = E[\theta|K, y]$. All the information traders have about the fundamental comes from the public signal y and the aggregate capital K . Let us assume for the moment that the aggregate investment, given the information available in the economy, takes the following linear

from $K(\theta, \eta) = \delta_0\mu + \delta_\theta\theta + \delta_\eta\eta$. We later verify that this corresponds to the true one. Observing the aggregate capital K is equivalent to observe the following signal:

$$s = \theta + \varphi\eta \equiv \frac{K - \delta_0\mu}{\delta_\theta}. \quad (7)$$

Conditional on the prior θ , the signal s has precision $\frac{\pi_{\bar{x}}}{\varphi^2}$, where $\varphi = \frac{\delta_\eta}{\delta_\theta}$. Thus, traders' expectation of θ conditional on K and y , using Bayesian updating is

$$E[\theta|K, y] = E[\theta|s, y] = \gamma_0\mu + \gamma_s s + \gamma_y y. \quad (8)$$

Accordingly, the price function is a linear function of the prior, the fundamental and the noise terms η and ω

$$p = \gamma_0\mu + \gamma_\theta\theta + \gamma_\eta\eta + \gamma_\omega\omega \quad (9)$$

where parameters $\gamma_0, \gamma_\theta, \gamma_\eta$ and γ_ω are calculated explicitly in appendix B.

Entrepreneur's stage. At $t = 1$ each entrepreneur has to choose how much attention to pay to acquire the private signal x_i and how much to invest. Given the normality of the prior and the signals and the quadratic payoffs, we can infer that individual investments can be expressed as a linear function of the information available to the entrepreneur:

$$k_i = \delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i \quad (10)$$

Using the fact that the signals are as given by equations (1) and (2) and substituting equations (9) and (10) into equation (3), the entrepreneur's problem reduces to choosing attention z_i and weights $\delta_0, \delta_{\bar{x}}$ and δ_{x_i} to maximize his unconditional expected utility.

Proposition 2. *Assume that $\pi_{x_i} > (\pi_\theta + \pi_{\bar{x}})^2$ holds. Under Assumption 1*

i. *any investment strategy with $z_i > 0$ is characterized as follows:*

$$\delta_0 = \frac{\pi_\theta}{\sqrt{\pi_{x_i}}} + \lambda \frac{\pi_\theta \varphi^2}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)}, \quad (11)$$

$$\delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\sqrt{\pi_{x_i}}} + \lambda \frac{\varphi \pi_{\bar{x}}}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)}, \quad (12)$$

$$\delta_{x_i} = \frac{\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)}{\sqrt{\pi_{x_i}}} - \lambda \frac{\varphi(\pi_{\bar{x}} + \varphi\pi_\theta)}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)}; \quad (13)$$

ii. *and the information acquisition policy is given by*

$$z_i = \xi \left\{ \frac{\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)}{\sqrt{\pi_{x_i}}} - \lambda \frac{\varphi(\pi_{\bar{x}} + \varphi\pi_\theta)}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)} \right\}, \quad (14)$$

where φ is an endogenous parameter that in equilibrium must satisfy:

$$\varphi = \frac{\delta_{\bar{x}}}{\delta_{\bar{x}} + \delta_{x_i}}; \quad (15)$$

Proof. Proof in appendix. \square

The above proposition simply characterizes the individual investment decision and the information acquisition policy when there exists information spill-over between the real and financial sectors. It is worth noticing that relative to the benchmark economy entrepreneurs put more weight on the prior and on the public signal and less on the private signal. Moreover, under information spill-over entrepreneurs pay less attention to private information than in the benchmark. Given the equilibrium attention paid to the private signal, it turns out that the equilibrium precision of the private signal is lower than in the benchmark case. This comes from the fact that the total precision of the private signal, which is $z_i \pi_{x_i}$, is endogenously determined in equilibrium by z_i . Thus, as entrepreneurs pay less attention to the private signal than in the benchmark case, they acquire a signal that in equilibrium is less precise.

It follows that the equilibrium value of aggregate capital is equal to

$$K = \delta_0 \mu + \delta_\theta \theta + \delta_\eta \eta, \quad (16)$$

where $\delta_\theta \equiv \delta_{\bar{x}} + \delta_{x_i}$ and $\delta_\eta \equiv \delta_{\bar{x}}$.

From equation (14) it is immediate to see that z_i may fail to be positive. Under some circumstances entrepreneurs may find it optimal not to acquire the private signal, i.e. $z_i = 0$. The following proposition shows the conditions under which entrepreneurs do not pay attention to the private signal.

Proposition 3. *Assume that (I) $\pi_{\bar{x}} \geq \pi_y$ and (II) $\pi_{x_i} > (\pi_\theta + \pi_{\bar{x}})^2$ hold. Denote $\hat{\lambda}$ the value of λ that satisfies equation (14) when $z_i = 0$. Then the following is true:*

- i. z_i is monotonically decreasing in φ .
- ii. $z_i = 0$ is an equilibrium information acquisition policy if and only if $\lambda \in [\hat{\lambda}, 1)$.

Proof. Proof in appendix B. \square

This proposition shows that when the probability of incurring a liquidity shock is high, entrepreneurs do not pay attention to the private signal about the profitability of the new project. This is true under the assumptions: (I) that entrepreneurs' public signal is as precise as traders' public signal or more (II) and that entrepreneurs' exogenous precision of private signal is sufficiently high with respect to the entrepreneurs' overall precision of public sources of information.

The first assumption is required to guarantee monotonicity of z_i in φ . This assumption is not particularly strong; it just requires that entrepreneurs' public signal is at least as informative as, i.e. same precision or higher, traders' public signal.

The second assumption is the condition under which entrepreneurs pay positive attention to the private signal in the benchmark economy. In this way, we are able to fully capture the effect of incomplete information on the information acquisition policy.

Thus, whenever the probability of a liquidity shock is above a certain threshold value, entrepreneurs are less concerned about learning the value of the fundamental and hence are not buying the private signal, and more concerned about the future asset prices. This is easy to see from equation (3), the conditional expected utility of an entrepreneur. Treating all the other parameters of that equation as fixed, the conditional expected value of θ is decreasing in λ , while the conditional expected value of asset prices is increasing in λ .

From the above proposition we know that entrepreneurs do not acquire information for high values of λ . However an issue that has not been addressed yet is when such $\hat{\lambda}$ exists. That is whether $\hat{\lambda} < 1$. Checking for $\hat{\lambda} < 1$ is very important for our equilibrium results. In fact, if $\hat{\lambda}$ fails to be less than 1, the set $(\hat{\lambda}, 1)$ is an empty set. This would imply that $z_i = 0$ if and only if $\lambda \in \emptyset$, which is equivalent to say that $z_i = 0$ is not an equilibrium. In the following paragraph we study the condition under which $\hat{\lambda} < 1$.

From the proof of Proposition 3 in appendix B, $\hat{\lambda}$ is defined as:

$$\hat{\lambda} = \frac{[\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_{\theta})](\pi_{\bar{x}} + \pi_{\theta} + \pi_y)}{\sqrt{\pi_{x_i}}(\pi_{\bar{x}} + \pi_{\theta})}. \quad (17)$$

By inspecting equation (17), it is straightforward to see that $\hat{\lambda}$ is lower than 1 if

$$\sqrt{\pi_{x_i}} < \frac{(\pi_{\bar{x}} + \pi_{\theta})^2}{\pi_y} + (\pi_{\bar{x}} + \pi_{\theta}). \quad (18)$$

Notice that under assumption (II) of Proposition 3 the RHS of equation (18) is higher than $\pi_{\theta} + \pi_{\bar{x}}$ and under assumption (I) $\pi_y \leq \pi_{\bar{x}}$. Moreover notice that the LHS of the same equation is decreasing in π_y . Thus, when the signal of the traders is pure noise, that is $\pi_y \rightarrow 0$, equation (18) always holds so long as $\pi_{x_i} < \infty$, which is always satisfied given that ξ^2 , which is the inverse of π_{x_i} , is positive.

On the contrary, when the signal of the traders is very precise, specifically, the maximum value it can take is $\pi_y = \pi_{\bar{x}}$, equation (18) holds for intermediate value of π_{x_i} , that is for $(\pi_{\bar{x}} + \pi_{\theta})^2 < \pi_{x_i} < \left[\frac{(\pi_{\bar{x}} + \pi_{\theta})^2}{\pi_{\bar{x}}} + (\pi_{\bar{x}} + \pi_{\theta}) \right]^2$. If instead the exogenous precision of the private signal is very high equation (18) no longer holds.

The above analysis suggests that, whether entrepreneurs acquire information or not in the case the probability of a liquidity shock is high, depends either on the precision of the traders' signal y and on the exogenous precision of entrepreneurs' private signal x_i . From now on rather than saying "the exogenous precision of the private signal" we refer to it as the clarity of the private signal. The following Corollary summarize these findings.

Corollary 1. *When the probability of a liquidity shock is high, that is $\lambda \in [\hat{\lambda}, 1)$:*

- i. *and the public signal of traders is pure noise, entrepreneurs do not acquire the private signal even if its clarity is very high;*
- ii. *and the public signal of traders is as precise as the public signal of entrepreneurs $\pi_{\bar{x}}$, entrepreneurs do not acquire the private signal when its clarity takes intermediate values;*
- iii. *as the precision of the public signal of traders increases, entrepreneurs acquire the private signal at smaller value of its clarity.*

These results shed light an important relation between the probability of a liquidity shock, the clarity of entrepreneurs' private signal and the precision of traders' public signal. In other words, we can say that for intermediate values of the clarity of the private signal, the probability of the liquidity shock matters to determine whether entrepreneurs optimally pay attention to the private signal. Notice that this range of intermediate values of the clarity is not fixed, but its upper bound is decreasing in the precision of traders' public signal. However, it is bounded above given that also the precision of traders' public signal is bounded above by assumption. We can then conclude the following.

One one hand, whenever the clarity of the private signal is bigger than the value of this bound, entrepreneurs always acquire the private signal no matter how precise is traders' public signal and how big is the probability of a liquidity shock. In this case the size of λ only determine how much attention is acquired in equilibrium. On the other hand, if the clarity of the private signal is too low, that is smaller than $\pi_{\theta} + \pi_{\bar{x}}$, the probability of a liquidity shock never affect entrepreneurs' decision to acquire the private signal. In fact, in this case entrepreneurs never acquire private information, no matter the magnitude of the liquidity shock.

In the case entrepreneurs do not acquire any private signal, the information available to entrepreneurs is fully conveyed to traders through aggregate capital. In such situations, the information spill-over is at its maximum, that is $\varphi = 1$, and the economy is characterized are as follows:

$$k_i = \delta_0 \mu + \delta_{\bar{x}} \bar{x}, \quad (19)$$

where $\delta_0 = \frac{\pi_\theta}{\pi_\theta + \pi_{\bar{x}}}$ and $\delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}}}$;

$$K = \delta_0\mu + \delta_\theta\theta + \delta_\eta\eta, \quad (20)$$

where $\delta_\theta = \delta_\eta \equiv \delta_{\bar{x}}$;

$$p = \gamma_0\mu + \gamma_\theta\theta + \gamma_\eta\eta + \gamma_\omega\omega, \quad (21)$$

where $\gamma_0 = \frac{\pi_\theta}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$; $\gamma_\theta = \frac{\pi_{\bar{x}} + \pi_y}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$; $\gamma_\eta = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$ and $\gamma_\omega = \frac{\pi_y}{\pi_\theta + \pi_{\bar{x}} + \pi_y}$.

Proposition 2 and 3 characterize the equilibrium under positive and zero information acquisition, respectively. As discussed above, under some parameters value in equilibrium there is information acquisition and under some others the private signal is not acquired in equilibrium and entrepreneurs rely only on their public information to set their investment decision. However, whether the equilibrium is unique or there exist multiple equilibria is still an issue. This means that: (i) under the same parameters space identified so far for the equilibrium with $z_i = 0$ we may also have one or more equilibria with $z_i > 0$; (ii) under the same parameters space identified so far for $z_i > 0$ we may have multiple equilibria with positive information acquisition; (iii) there might exist parameters space under which $z_i > 0$ is unique and other parameters space under which $z_i = 0$ is the only equilibrium.

However regardless of the uniqueness of the equilibrium, we can summarize the findings and the implications of this section in the following Corollary.

Corollary 2. *With respect to the benchmark economy, under incomplete information:*

- i. *entrepreneurs pay less attention to the private signal, that is $z_i^B > z_i$;*
- ii. *when the probability of a liquidity shock is high, no private signal is acquired in equilibrium for intermediate values of the clarity of the private signal.*
- iii. *individual investment responds less to the private signal and more to the public signal, that is $\delta_{x_i} < \beta_{x_i}$, $\delta_{\bar{x}} > \beta_{\bar{x}}$ and $\delta_0 > \beta_0$;*
- iv. *aggregate capital responds more to common shock and less to the fundamental shock, that is $\beta_\theta > \delta_\theta$ and $\beta_\eta < \delta_\eta$.*

The first two results are new with respect to the literature, while the last two results are in line with what Angeletos, Pavan, and Lorenzoni (2010) found. This means that whether the private signal is exogenously given or is acquired endogenously, the economy behaves qualitatively in the same way, in the sense that the information spillovers drive the incentives of entrepreneurs to rely more on public information and less on private information. However, the introduction of endogenous information acquisition in certain case may exacerbate the result. In all the situations in which

entrepreneurs do not pay attention to the private signal, the outcome in terms of individual investment and aggregate investment is worst than in the case of exogenous private information, either in the benchmark economy and in the incomplete information case.

In all the other cases in which entrepreneurs pay attention to the private signal, the strength of these effects might be attenuated. The next section analyses the strength of these effects by comparing an economy with exogenous private information to an economy with endogenous information acquisition.

5. QUANTITATIVE ASSESSMENT OF THE MODEL WITH RESPECT TO THE CASE OF EXOGENOUS INFORMATION

In this section we study the effect of information acquisition on the weights entrepreneurs give to their different signals, by comparing them with the equilibrium values found in the model with exogenous information of Angeletos, Pavan, and Lorenzoni (2010). In what follows, in order to compare the two models, we first describe briefly the key variable of their model and we adapt them to the notation used in our paper.

Entrepreneurs observe a public signal y and a private signal x , while traders observe a public signal ω . These three signals correspond respectively to \bar{x} , x_i and y in our model. In their benchmark economy the weights attached to the prior, the public signal and the private signal are denoted as δ_0 , δ_y and δ_x respectively; while in the incomplete information case the weights attached to these signals are denoted β_0 , β_y and β_x .

Let redefine those parameters as follows: $\delta_0 \equiv \beta_0^{ALP}$, $\delta_y \equiv \beta_{\bar{x}}^{ALP}$, $\delta_x \equiv \beta_{x_i}^{ALP}$, $\beta_0 \equiv \delta_0^{ALP}$, $\beta_y \equiv \delta_{\bar{x}}^{ALP}$ and $\beta_x \equiv \delta_{x_i}^{ALP}$.

In the benchmark, the individual investment decision of Angeletos, Pavan, and Lorenzoni (2010) model is then characterized as follows:

$$\beta_0^{ALP} = \frac{\pi_\theta}{\pi_\theta + \pi_{x_i} + \pi_{\bar{x}}} \quad (22)$$

$$\beta_{\bar{x}}^{ALP} = \frac{\pi_{\bar{x}}}{\pi_\theta + \pi_{\bar{x}} + \pi_{x_i}} \quad (23)$$

$$\beta_{x_i}^{ALP} = \frac{\pi_{x_i}}{\pi_\theta + \pi_{\bar{x}} + \pi_{x_i}} \quad (24)$$

By comparing equations (22)-(24) with the equations in part (ii) of our Proposition 1 we can understand how information acquisition affects the investment decision in the benchmark economy.

Proposition 4. $\forall \pi_{x_i}, \pi_\theta, \pi_{\bar{x}} \in R_+$ such that $\sqrt{\pi_{x_i}} - \pi_{x_i} > \pi_\theta + \pi_{\bar{x}}$ holds, then: (i) $\beta_0 > \beta_0^{ALP}$, (ii) $\beta_{\bar{x}} < \beta_{\bar{x}}^{ALP}$, (iii) $\beta_{x_i} > \beta_{x_i}^{ALP}$ and (iv) $\beta_0 + \beta_{\bar{x}} < \beta_0^{ALP} + \beta_{\bar{x}}^{ALP}$.

Proof. The proof is straightforward. Just compare the above equations with the solution of our benchmark economy. \square

This proposition simply shows that when information is costly and it is acquired endogenously, under certain parameters space of the precision of the prior, the precision of the public signal and the exogenous precision of the private signal, entrepreneurs put more weight on the prior and on the private signal and less on the public signal relative to the case in which the private information is exogenous. Moreover, overall the sum of the weights attached to the total amount of public information, that is the prior θ and the public signal \bar{x} , is lower with information acquisition than with exogenous information. These results suggest that under certain conditions, with endogenous information acquisition the benchmark economy performs better in terms of individual investment decision and therefore in terms of aggregate capital.

In the case of incomplete information, the individual investment decisions in Angeletos, Pavan, and Lorenzoni (2010) are characterized by the following equations:

$$\delta_0^{ALP} = \beta_0^{ALP} \left[1 - \frac{\lambda\varphi(\pi_{\bar{x}}(1-\varphi) - \varphi\pi_{x_i})}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)} \right] \quad (25)$$

$$\delta_{\bar{x}}^{ALP} = \beta_{\bar{x}}^{ALP} \left[1 + \frac{\lambda\varphi[\pi_{x_i} + (1-\varphi)\pi_\theta]}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)} \right] \quad (26)$$

$$\delta_{x_i}^{ALP} = \beta_{x_i}^{ALP} \left(1 - \frac{\lambda\varphi(\varphi\pi_\theta + \pi_{\bar{x}})}{\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)} \right) \quad (27)$$

First of all notice that the value of the parameter φ in the above equation may not be the same of that in equations (11)-(13), as its value is endogenous determined by the equilibrium values of $\delta_{\bar{x}}^{ALP}$, $\delta_{x_i}^{ALP}$ and $\delta_{\bar{x}}$, δ_{x_i} respectively.

6. CONCLUSIONS

This paper studies endogenous information acquisition in an investment trading game between a real sector and a financial sector, in which entrepreneurs first make their investment decision about a new research project and successively a fraction λ of this capital is traded in the financial market. The profitability (the fundamental value) of the project is unknown to either entrepreneurs and traders and the information they have access to are only noisy signals about profitability. These signals may be either public or private. Public information is freely available to every agent while private

information is agent specific and can be acquired only by entrepreneurs and only conditional on the fact that they pay attention to it. Information spillovers arise from the financial sector to the real sector from a two way feedback between entrepreneurs and traders. Entrepreneurs, by conveying a positive signal about the profitability of the new project, induce an increase in asset prices, which in turn raise their incentives to invest.

In line with the paper of Angeletos, Pavan, and Lorenzoni (2010) this effect creates endogenous complementarity in investment decisions, making entrepreneurs sensitive to high-order beliefs. As a consequence, the impact of fundamental shocks on aggregate capital is reduced, while common expectational shocks amplify their impact on aggregate capital.

The main difference from the paper of Angeletos, Pavan, and Lorenzoni (2010) is that with endogenous information acquisition of the private signal, the economy exhibits two different types of equilibrium. In one type of equilibrium entrepreneurs acquire the private signal and in the other type of equilibrium entrepreneurs rely only on public information. Moreover it is worth highlighting that the clarity of the private signal and the precision of traders' signal play an important role on entrepreneurs' decision to pay attention to the private signal.

For very low clarity of the private signal entrepreneurs never acquire it, either in the benchmark economy nor under incomplete information. For intermediate values of the clarity of the private signal, the probability of the liquidity shock matters to determine whether entrepreneurs optimally pay attention to the private signal; and this range depends on the precision of traders' public signal. Whenever instead the clarity of the private signal is very high, entrepreneurs always acquire the private signal no matter how precise is traders' public signal nor how big is the probability of a liquidity shock.

Current and future research is now in progress on the uniqueness of the equilibrium, the social value of information acquisition and the quantitative difference between this model of endogenous information acquisition and the model of Angeletos, Pavan, and Lorenzoni (2010) with exogenous private information.

APPENDIX A.

GENERALIZATION OF THE MODEL

In this section we characterize the equilibrium of such an economy using a more general and flexible information structure. Entrepreneurs' and traders' preferences and timing are as before. Traders' information structure does not change, that is traders observe a public signal $y = \theta + \omega$ with $\omega \sim N(0, \tau^2)$. We instead consider a different information structure on the entrepreneurs' side. The information structure we are considering is a simplified version of the one used by Myatt and Wallace (2012). We assume that entrepreneurs may get up to $n = 2$ different signals about the profitability of the new investment project. Both signals are costly to acquire and neither of the two is completely public nor completely private; precisely, by paying attention $z_{ji} \in \mathfrak{R}_+$ to listen to the signal j -th entrepreneur i observes

$$x_{ji} = \theta + \eta_j + \epsilon_{ji}, \quad (28)$$

with $\eta_j \sim N(0, \kappa_j^2)$ and $\epsilon_{ji} \sim N(0, \frac{\xi_j^2}{z_{ji}})$. We denote $\pi_{\eta_j} = 1/\kappa_j^2$ the precision of the correlated shock and $\pi_{\epsilon_j} = 1/\xi_j^2$ the exogenous precision of the idiosyncratic shock. Using the definition of Myatt and Wallace (2012), π_{η_j} represents the accuracy of the signal and π_{ϵ_j} represents its clarity. Moreover we assume that $\theta, \omega, \eta_i, \epsilon_{ji}$ are independently distributed. We denote z_i the overall attention paid by entrepreneur i to listen to the signals, that is $z_i = z_{1i} + z_{2i}$. If $z_{ji} = 0$, then it means that the entrepreneur does not acquire the signal j .

Entrepreneur's conditional expected utility is:

$$\mathbb{E}(u_i | x_{1i}, x_{2i}) = \mathbb{E} \left[(1 - \lambda)\theta k_i + \lambda p k_i - \frac{k_i^2}{2} \middle| x_{1i}, x_{2i} \right] - C(z_i). \quad (29)$$

Traders expected utility is:

$$\mathbb{E}(u_i | K, y) = (\mathbb{E}[\theta | K, y] - p)q_i, \quad (30)$$

where p is the market clearing price in the financial market and q_i is the the amount of capital k_i bought by trader i .

EQUILIBRIUM

Traders stage: $t = 2$.

By using backward induction, we first solve for the market clearing price $p = E(\theta | K, y)$ using our guess about aggregate capital, that is $K = \delta_0 + \delta_\theta \theta + \delta_{\eta_1} \eta_1 + \delta_{\eta_2} \eta_2$. Observing

K is equivalent to observing the signal s . Traders face a signal extraction problem:

$$s = \frac{K - \delta_0 \mu}{\delta_\theta} = \theta + \frac{\delta_{\eta_1}}{\delta_\theta} \eta_1 + \frac{\delta_{\eta_2}}{\delta_\theta} \eta_2 \quad (31)$$

Denote $\frac{\delta_{\eta_1}}{\delta_\theta} = \varphi_1$ and $\frac{\delta_{\eta_2}}{\delta_\theta} = \varphi_2$. Thus $VAR(s|\theta) = \varphi_1^2 \kappa_1^2 + \varphi_2^2 \kappa_2^2$. We denote $\pi_s = \frac{1}{VAR(s|\theta)}$ the precision of signal s , conditional on θ . Given the normality of the signals, the expected value of θ conditional on s and y , by Bayesian updating, corresponds to a linear combination of the signals, that is:

$$\mathbb{E}(\theta|s, y) = \gamma_0 \mu + \gamma_s s + \gamma_y y. \quad (32)$$

Therefore, the market clearing price can be written as follows:

$$p = \gamma_0 \mu + \gamma_\theta \theta + \gamma_{\eta_1} \eta_1 + \gamma_{\eta_2} \eta_2 + \gamma_\omega \omega, \quad (33)$$

where $\gamma_\theta = (\gamma_s + \gamma_y)$, $\gamma_{\eta_1} = \gamma_s \varphi_1$, $\gamma_{\eta_2} = \gamma_s \varphi_2$ and $\gamma_\omega = \gamma_y$. See appendix C.1 for detailed calculation of the parameters $\gamma_0, \gamma_\theta, \gamma_{\eta_1}, \gamma_{\eta_2}$ and γ_ω of equation (33).

Entrepreneur's stage.

At $t = 1$ each entrepreneur's problem is a two stage decision process:

- (i) whether to acquire the signals, and if so, how much attention $z_i = (z_{1i} + z_{2i}) \in R_+$ to pay to listen to them;
- (ii) how much to invest, $k_i \in R_+$ in the new project.

Given the normality of the signals and the prior, and quadratic payoffs, the entrepreneur's investment decision can be written as linear function of the information available to him

$$k_i = \delta_0 \mu + \delta_{x_{1i}} x_{1i} + \delta_{x_{2i}} x_{2i}. \quad (34)$$

Substituting equations (33) and (34) into equation (29), and using the information structure as given by equation (28) we obtain entrepreneur's ex-ante expected utility. Ex-ante expected utility is omitted in the main text, but can be found in the appendix C.2. From the ex-ante expected utility, i.e. equation (73), it is straightforward to see that entrepreneur's problem reduces to choosing $z_i, \delta_0, \delta_{x_{1i}}$, and $\delta_{x_{2i}}$ simultaneously. We can do so, because no-one observes z_i apart from entrepreneur i which implies that each entrepreneur would not change his mind. From now on we assume that the cost of acquiring information takes the form $C(z_i) = (z_{1i} + z_{2i})/2$. Given that the equilibrium we are looking for is a symmetric one, from now on we remove the subscript i to simplify the notation.

The following equations characterize the entrepreneur's information acquisition policy

and investment decision:

$$z_1 = \max \left\{ \begin{array}{l} \xi_1 \left[\frac{\pi_{\eta_1} [(1 + \pi_{\eta_2} \xi_2) - \xi_1 (\pi_{\theta} + \pi_{\eta_2})]}{\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta}} + \lambda \frac{\varphi_1 \pi_s (\pi_{\eta_2} + 1) - \pi_{\eta_1} (\pi_{\theta} + \varphi_2 \pi_s)}{(\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta})(\pi_{\theta} + \pi_s + \pi_y)} \right] \\ 0 \end{array} \right\}, \quad (35)$$

$$z_2 = \max \left\{ \begin{array}{l} \xi_2 \left[\frac{\pi_{\eta_2} [(1 + \pi_{\eta_1} \xi_1) - \xi_2 (\pi_{\theta} + \pi_{\eta_1})]}{\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta}} + \lambda \frac{\varphi_2 \pi_s (\pi_{\eta_1} + 1) - \pi_{\eta_2} (\pi_{\theta} + \varphi_1 \pi_s)}{(\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta})(\pi_{\theta} + \pi_s + \pi_y)} \right] \\ 0 \end{array} \right\}, \quad (36)$$

$$\delta_0 = \frac{\pi_{\theta} [1 + \xi_1 \pi_{\eta_1} + \xi_2 \pi_{\eta_2}]}{\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta}} - \lambda \frac{\pi_{\eta_2} (\pi_s + \pi_y - \pi_{\theta}) + \pi_{\theta} [\pi_s (\varphi_1 + \varphi_2) - \pi_{\eta_1}]}{(\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta})(\pi_{\theta} + \pi_s + \pi_y)} \quad (37)$$

$$\delta_{x_1} = \frac{z_1}{\xi_1} \quad (38)$$

$$\delta_{x_2} = \frac{z_2}{\xi_2} \quad (39)$$

See appendix C.3 for derivation of the above equations.

A.1. Benchmark. In this section we study the benchmark economy, that is when traders are informed about the value of θ . In this case the market clearing price is $p = \theta$. Conditional of observing the signal entrepreneur's expected utility reduces to:

$$E(u|x_1, x_2) = E\left[\theta k - \frac{k^2}{2} \mid x_1, x_2\right] - C(z) \quad (40)$$

As is standard in the literature, by the normality of the signals and quadratic payoff we consider entrepreneur's investment decision to be a linear combination of the signals available to him, that is

$$k = \beta_0 \mu + \sum_{j=1}^2 \beta_{x_j} x_j \quad (41)$$

Substituting equation (41) into equation (40) and taking expectation we obtain entrepreneur's ex-ante expected utility. The entrepreneur's problem thus reduces to choosing $z_1, z_2, \beta_0, \beta_{x_1}$ and β_{x_2} that maximize ex-ante expected utility. The following equations characterize the benchmark equilibrium of information acquisition policy and investment decision strategy:

$$z_1 = \max \left\{ 0, \frac{\xi_1 [\pi_{\eta_1} + \xi_2 \pi_{\eta_2} \pi_{\eta_1} - \xi_1 \pi_{\eta_1} (\pi_{\eta_2} + \pi_{\theta})]}{\pi_{\eta_1} + \pi_{\eta_2} + \pi_{\theta}} \right\}, \quad (42)$$

$$z_2 = \max \left\{ 0, \frac{\xi_2 [\pi_{\eta_2} + \xi_1 \pi_{\eta_1} \pi_{\eta_2} - \xi_2 \pi_{\eta_2} (\pi_{\eta_1} + \pi_{\theta})]}{\pi_{\eta_2} + \pi_{\eta_1} + \pi_{\theta}} \right\}, \quad (43)$$

$$\beta_0 = \frac{\pi_\theta[1 + \xi_1\pi_{\eta_1} + \xi_2\pi_{\eta_2}]}{\pi_{\eta_1} + \pi_{\eta_2} + \pi_\theta}, \quad (44)$$

$$\beta_{x_1} = \frac{z_{1i}}{\xi_1}, \quad (45)$$

$$\beta_{x_2} = \frac{z_{2i}}{\xi_2}. \quad (46)$$

A.1.1. *Information acquisition policy.* In this section we investigate whether and under which conditions entrepreneurs pay positive attention to either signal. In fact, equations (42) and (43) may not be simultaneously positive. For convenience we order the two signals according to their increasing clarity assuming that signal x_{1i} is less clear than x_{2i} , that is $\xi_1^2 > \xi_2^2$. Equation (42) is positive if $\xi_2 > \frac{\xi_1(\pi_{\eta_2} + \pi_\theta) - 1}{\pi_{\eta_2}}$ and equation (43) is positive if $\xi_2 < \frac{1 + \xi_1\pi_{\eta_1}}{\pi_{\eta_1} + \pi_\theta}$. A necessary condition in order to have both z_{1i} and z_{2i} positive is

$$\underline{\xi}_2 \equiv \frac{\xi_1(\pi_{\eta_2} + \pi_\theta) - 1}{\pi_{\eta_2}} < \frac{1 + \xi_1\pi_{\eta_1}}{\pi_{\eta_1} + \pi_\theta} \equiv \bar{\xi}_2. \quad (47)$$

Equation (47) is satisfied as long as $\xi_1 < \sigma_\theta^2$.

Thus assuming that $\xi_1 < \sigma_\theta^2$, a necessary condition for z_{1i} and z_{2i} to be simultaneously positive is $\xi_2 \in (\underline{\xi}_2, \bar{\xi}_2)$.

APPENDIX B. PROOFS

Proof of Proposition 1. As explained in Section 4, rather than solving the entrepreneur's problem in two steps - that is first he decides whether to pay attention to listen to a private signal and second conditional on observing the public and the private signal eventually acquired, he chooses how much to invest in the new project - we solve the problem simultaneously.

The entrepreneur's conditional expected utility, equation (5), is equivalent to the ex-ante entrepreneur's expected utility where each entrepreneur chooses z_i and k_i to maximize his payoff. By substituting equation (6) into equation (5) we obtain ex-ante expected utility

$$E(u_i) = E\left[\theta(\beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i}x_i) - \frac{(\beta_0\mu + \beta_{\bar{x}}\bar{x} + \beta_{x_i}x_i)^2}{2}\right] - C(z_i). \quad (48)$$

Taking into account the information structure as given by equations (1) and (2) and solving the expectation on the right-hand side of the above equation, ex-ante expected utility is equal to:

$$\begin{aligned} E(u_i) = & \beta_0\mu^2 + (\beta_{\bar{x}} + \beta_{x_i})(\sigma_\theta^2 + \mu^2) - \frac{\beta_0^2\mu^2}{2} - \frac{\beta_{\bar{x}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa^2) - \\ & - \frac{\beta_{x_i}^2}{2}(\sigma_\theta^2 + \mu^2 + \frac{\xi^2}{z_i}) - \mu^2\beta_0(\beta_{\bar{x}} + \beta_{x_i}) - \beta_{\bar{x}}\beta_{x_i}(\mu^2 + \sigma_\theta^2) - C(z_i). \end{aligned} \quad (49)$$

Under Assumption 1 the FOCs of the above equation with respect to β_0 , $\beta_{\bar{x}}$, β_{x_i} and z_i are:

$$\beta_0 = 1 - \beta_{\bar{x}} - \beta_{x_i} \quad (50)$$

$$\beta_{\bar{x}} = \frac{\sigma_\theta^2 + \mu^2 - \beta_0\mu^2 - \beta_{x_i}(\sigma_\theta^2 + \mu^2)}{(\sigma_\theta^2 + \mu^2 + \kappa^2)} \quad (51)$$

$$\beta_{x_i} = \frac{\sigma_\theta^2 + \mu^2 - \beta_0\mu^2 - \beta_{\bar{x}}(\sigma_\theta^2 + \mu^2)}{(\sigma_\theta^2 + \mu^2 + \xi^2/z_i)} \quad (52)$$

$$z_i = \xi\beta_{x_i} \quad (53)$$

Solving equations (50)-(53) we obtain:

$$\beta_0 = \frac{\xi}{\sigma_\theta^2} \quad (54)$$

$$\beta_{\bar{x}} = \frac{\xi}{\kappa^2} \quad (55)$$

$$\beta_{x_i} = \frac{\sigma_\theta^2\kappa^2 - \xi(\sigma_\theta^2 + \kappa^2)}{\sigma_\theta^2\kappa^2} \quad (56)$$

$$z_i = \xi\beta_{x_i} \quad (57)$$

Rewriting equations (54)-(57) as functions of signal precisions rather than signal variances yield the equations of Proposition 1

Derivation of parameters of equation (9). The parameters $\gamma_0, \gamma_\theta, \gamma_\eta, \gamma_\omega$ are derived as follows:

$$\begin{aligned}
E[\theta|s, y] &= E(\theta) + \frac{1}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2}} \left[\frac{1}{\varphi^2\kappa^2}(s - E(\theta)) + \frac{1}{\tau^2}(y - E(\theta)) \right] \\
&= \frac{\left[\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2} \right] \mu + \frac{1}{\varphi^2\kappa^2}s + \frac{1}{\tau^2}y - \mu \left[\frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2} \right]}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2}} \\
&= \frac{\frac{1}{\sigma_\theta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2}} \mu + \frac{\frac{1}{\varphi^2\kappa^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2}} s + \frac{\frac{1}{\tau^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi^2\kappa^2} + \frac{1}{\tau^2}} y \\
p &= \underbrace{\frac{\pi_\theta}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_0} \mu + \underbrace{\frac{\frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_\theta} \theta + \underbrace{\frac{\frac{\pi_{\bar{x}}}{\varphi}}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_\eta} \eta + \underbrace{\frac{\pi_y}{\pi_\theta + \frac{\pi_{\bar{x}}}{\varphi^2} + \pi_y}}_{\gamma_\omega} \omega \quad (58)
\end{aligned}$$

Proof of Proposition 2. Substituting (9) and (10) into equation (3) we transform the conditional expected utility into the following unconditional ex-ante expected utility

$$\begin{aligned}
E(u_i) &= E \left[\theta(1 - \lambda)(\delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i) + \right. \\
&\quad \left. + \lambda(\gamma_0\mu + \gamma_\theta\theta + \gamma_\eta\eta + \gamma_\omega\omega)(\delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i) + \right. \\
&\quad \left. - \frac{(\delta_0\mu + \delta_{\bar{x}}\bar{x} + \delta_{x_i}x_i)^2}{2} \right] - C(z_i) \quad (59)
\end{aligned}$$

Solving the expectation on the right hand side of equation (59) after substituting equation (1) and (2) into it, the entrepreneur's unconditional expected utility is

$$\begin{aligned}
E(u_i) &= \delta_0\mu^2[(1 - \lambda + \lambda(\gamma_0 + \gamma_\theta))] + (1 - \lambda)(\sigma_\theta^2 + \mu^2)(\delta_{\bar{x}} + \delta_{x_i}) + \\
&\quad + \delta_{\bar{x}}\lambda[\mu\gamma_0 + \gamma_\theta(\sigma_\theta^2 + \mu^2) + \gamma_\eta\kappa^2] + \delta_{x_i}\lambda[\mu\gamma_0 + \gamma_\theta(\sigma_\theta^2 + \mu^2)] + \\
&\quad - \frac{\delta_0^2\mu^2}{2} - \frac{\delta_{\bar{x}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa^2) - \frac{\delta_{x_i}^2}{2}(\sigma_\theta^2 + \mu^2 + \frac{\xi^2}{z_i}) + \\
&\quad - \mu^2\delta_0(\delta_{\bar{x}} + \delta_{x_i}) - \delta_{\bar{x}}\delta_{x_i}(\sigma_\theta^2 + \mu^2) - C(z_i). \quad (60)
\end{aligned}$$

Under Assumption 1, the FOCs with respect to $\delta_0, \delta_{\bar{x}}, \delta_{x_i}$ and z_i of the equation above are:

$$\delta_0 = (1 - \delta_{\bar{x}} - \delta_{x_i}) - \lambda[1 - \gamma_\theta - \gamma_0] \quad (61)$$

$$\delta_{\bar{x}} = \frac{(1 - \lambda)(\sigma_\theta^2 + \mu^2) + \lambda(\gamma_0\mu^2 + \gamma_\theta(\sigma_\theta^2 + \mu^2) + \gamma_\eta\kappa^2) - \delta_0\mu^2 - \delta_{x_i}(\sigma_\theta^2 + \mu^2)}{(\sigma_\theta^2 + \mu^2 + \kappa^2)} \quad (62)$$

$$\delta_{x_i} = \frac{(1 - \lambda)(\sigma_\theta^2 + \mu^2) + \lambda(\gamma_0\mu^2 + \gamma_\theta(\sigma_\theta^2 + \mu^2)) - \delta_0\mu^2 - \delta_{\bar{x}}(\sigma_\theta^2 + \mu^2)}{(\sigma_\theta^2 + \mu^2 + \frac{\xi^2}{z_i})} \quad (63)$$

$$z_i = \xi\delta_{x_i} \quad (64)$$

Solving equations (61)-(64) and rewriting them as function of precisions rather than variances we obtain

$$\delta_0 = \frac{\pi_\theta}{\sqrt{\pi_{x_i}}} + \lambda\gamma_0 \quad (65)$$

$$\delta_{\bar{x}} = \frac{\pi_{\bar{x}}}{\sqrt{\pi_{x_i}}} + \lambda\gamma_\eta \quad (66)$$

$$\delta_{x_i} = \frac{\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)}{\sqrt{\pi_{x_i}}} - \lambda[1 - (\gamma_\theta - \gamma_\eta)] \quad (67)$$

$$z_i = \xi \left\{ \frac{[\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)]}{\sqrt{\pi_{x_i}}} - \lambda[1 - (\gamma_\theta - \gamma_\eta)] \right\} \quad (68)$$

Equations (11)-(14) are obtained by substituting the value of parameters $\gamma_0, \gamma_\theta, \gamma_\eta$ into equations (65)-(68).

Proof of Proposition 3. We first show the monotonicity of equation (14). The first derivative of equation (14) with respect to φ is

$$\frac{\partial z_i}{\partial \varphi} = \frac{\lambda\pi_{\bar{x}}\sqrt{\pi_{x_i}}[\varphi^2(\pi_\theta + \pi_y) - \varphi 2\pi_\theta - \pi_{\bar{x}}]}{[\pi_{\bar{x}} + \varphi^2(\pi_\theta + \pi_y)]^2}$$

The above equation is negative so long as $\varphi^2(\pi_\theta + \pi_y) - \varphi 2\pi_\theta - \pi_{\bar{x}} < 0$. The quadratic equation has a minimum and two real roots as its discriminant is positive. Let's call $\underline{\varphi} = \frac{\pi_\theta - \sqrt{\pi_\theta^2 + \pi_{\bar{x}}(\pi_\theta + \pi_y)}}{\pi_\theta + \pi_y} < 0$ and $\bar{\varphi} = \frac{\pi_\theta + \sqrt{\pi_\theta^2 + \pi_{\bar{x}}(\pi_\theta + \pi_y)}}{\pi_\theta + \pi_y} > 0$ its roots. Therefore the quadratic equation is negative for $\varphi \in (\underline{\varphi}, \bar{\varphi})$ and positive for $\varphi \in (-\infty, \underline{\varphi}) \cup (\bar{\varphi}, +\infty)$. Given that the parameter φ is restricted to take values between $[0, 1]$, the quadratic equation is always negative $\forall \varphi \in (0, 1]$ if $\bar{\varphi} > 1$, that is if $\pi_{\bar{x}} + \pi_\theta > \pi_y$. Therefore under the assumption $\pi_{\bar{x}} \geq \pi_y$ equation (14) is monotonically decreasing in φ .

In order to prove part (ii.), let's first find the value $\hat{\lambda}$. Notice that if $z_i = 0$ then $\varphi = 1$ and equation (14) becomes:

$$0 = \sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta) - \hat{\lambda} \frac{\sqrt{\pi_{x_i}}(\pi_{\bar{x}} + \pi_\theta)}{\pi_{\bar{x}} + \pi_\theta + \pi_y}, \quad (69)$$

where $\hat{\lambda} = \frac{[\sqrt{\pi_{x_i}} - (\pi_{\bar{x}} + \pi_\theta)](\pi_{\bar{x}} + \pi_\theta + \pi_y)}{\sqrt{\pi_{x_i}}(\pi_{\bar{x}} + \pi_\theta)} > 0$ under the assumption $\pi_{x_i} > (\pi_\theta + \pi_{\bar{x}})^2$.

Now we prove the sufficient condition of part (ii), that is $z_i = 0$ is sufficient for $\lambda \in [\hat{\lambda}, 1)$.

From above we know that if LHS of (14) is zero, that is $z_i = 0$, then $\lambda = \hat{\lambda}$. From the definition of the information acquisition policy we know that z_i is bounded below by zero. That is $z_i = 0$ implies LHS of equation (14) ≤ 0 . Moreover notice that at $\varphi = 1$ equation (14) is decreasing in λ . Therefore it is straightforward to see that LHS of equation (14) ≤ 0 implies $\lambda \geq \hat{\lambda}(\varphi)$.

Now we show that $\lambda \in [\hat{\lambda}, 1)$ is necessary for $z_i = 0$ to be an equilibrium. That is $\forall \lambda \geq \hat{\lambda}$ $\varphi = 1$ and $z_i = 0$. Suppose $z_i = 0$ is not true. Then $\exists \lambda \geq \hat{\lambda}$ such that $z_i > 0$. But then $\varphi < 1$, given that z_i is monotonically decreasing in φ . This clearly contradicts our initial statement.

APPENDIX C. CALCULATION OF APPENDIX A

C.1. Market clearing price function. Equation (33) is derived by taking the expected value of θ conditional on K (or equivalently on s) and y :

$$p = E(\theta|s, y) = \frac{\frac{1}{\sigma_\theta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}} \mu + \frac{\frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}} s + \frac{\frac{1}{\tau^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}} y \quad (70)$$

$$p = \underbrace{\frac{\frac{1}{\sigma_\theta^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}}}_{\gamma_0} \mu + \underbrace{\frac{\frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}}}_{\gamma_\theta} \theta + \underbrace{\frac{\frac{\varphi_1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}}}_{\gamma_{\eta_1}} \eta_1 + \underbrace{\frac{\frac{\varphi_2}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}}}_{\gamma_{\eta_2}} \eta_2 + \underbrace{\frac{\frac{1}{\tau^2}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\varphi_1^2 \kappa_1^2 + \varphi_1^2 \kappa_2^2} + \frac{1}{\tau^2}}}_{\gamma_\omega} \omega \quad (71)$$

C.2. Entrepreneur's unconditional expected utility. Substituting equations (33) and (34) into equation (29), the conditional expected utility of entrepreneurs is:

$$E(u_i|x_{1i}, x_{2i}) = E \left[(1 - \lambda)(\delta_0 + \delta_{x_{1i}}x_{1i} + \delta_{x_{2i}}x_{2i})\theta + \lambda(\delta_0 + \delta_{x_{1i}}x_{1i} + \delta_{x_{2i}}x_{2i})(\gamma_0 + \gamma_\theta\theta + \gamma_{\eta_1}\eta_1 + \gamma_{\eta_2}\eta_2 + \gamma_\omega\omega) + \frac{(\delta_0 + \delta_{x_{1i}}x_{1i} + \delta_{x_{2i}}x_{2i})^2}{2} | x_{1i}, x_{2i} \right] - C(z_i) \quad (72)$$

Using equation (28) for $j = 1, 2$ entrepreneur's expected utility is equal to:

$$E(u_i) = (1 - \lambda) \left[\delta_0\mu + (\delta_{x_{1i}} + \delta_{x_{2i}})(\sigma_\theta^2 + \mu^2) \right] + \lambda\gamma_0(\delta_0 + \mu(\delta_{x_{1i}} + \delta_{x_{2i}})) + \lambda\gamma_\theta(\delta_0\mu + (\delta_{x_{1i}} + \delta_{x_{2i}})(\sigma_\theta^2 + \mu^2)) + \lambda\gamma_{\eta_1}\delta_{x_{1i}}\kappa_1^2 + \gamma_{\eta_2}\delta_{x_{2i}}\kappa_2^2 + \frac{\delta_0^2}{2} - \frac{\delta_{x_{1i}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa_1^2 + \frac{\xi_1^2}{z_{1i}}) - \frac{\delta_{x_{2i}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa_2^2 + \frac{\xi_2^2}{z_{2i}}) - \delta_0\delta_{x_{1i}}\mu - \delta_0\delta_{x_{2i}}\mu - \delta_{x_{1i}}\delta_{x_{2i}}(\sigma_\theta^2 + \mu^2) - C(z_i) \quad (73)$$

C.3. FOCs: Equilibrium characterization. Assuming cost of acquiring the signal is linear, with $C(z_i) = \frac{z_{1i} + z_{2i}}{2}$, the first order conditions with respect to z_{1i} , z_{2i} , δ_0 , $\delta_{x_{1i}}$ and $\delta_{x_{2i}}$ of equation (73) are:

$$z_{1i} = \delta_{x_{1i}}\xi_1 \quad (74)$$

$$z_{2i} = \delta_{x_{2i}}\xi_2 \quad (75)$$

$$\delta_0 = (1 - \lambda)\mu + \lambda\gamma_0 + \lambda\gamma_\theta\mu - \mu(\delta_{x_{1i}} + \delta_{x_{2i}}) \quad (76)$$

$$\delta_{x_{1i}} = \frac{(1 - \lambda + \lambda\gamma_\theta - \delta_{x_{2i}})(\sigma_\theta^2 + \mu^2) + \lambda\gamma_0\mu + \lambda\gamma_{\eta_1}\kappa_1^2 - \delta_0\mu}{\sigma_\theta^2 + \mu^2 + \kappa_1^2 + \frac{\xi_1^2}{z_{1i}}} \quad (77)$$

$$\delta_{x_{2i}} = \frac{(1 - \lambda + \lambda\gamma_\theta - \delta_{x_{1i}})(\sigma_\theta^2 + \mu^2) + \lambda\gamma_0\mu + \lambda\gamma_{\eta_2}\kappa_2^2 - \delta_0\mu}{\sigma_\theta^2 + \mu^2 + \kappa_2^2 + \frac{\xi_2^2}{z_{2i}}} \quad (78)$$

The solution of the system of equations (76)-(75), for $z_{1i}, z_{2i} > 0$ gives the following results:

$$z_{1i} = \frac{\xi_1 \left\{ \sigma_\theta^2(\kappa_2^2 + \xi_2) - \xi_1(\sigma_\theta^2 + \kappa_2^2) + \lambda \left[\sigma_\theta^2(\gamma_{\eta_1}\kappa_1^2 - \gamma_{\eta_2}\kappa_2^2 - \kappa_2^2(1 - \gamma_\theta)) + \gamma_{\eta_1}\kappa_1^2\kappa_2^2 \right] \right\}}{\sigma_\theta^2\kappa_2^2 + \kappa_1^2(\sigma_\theta^2 + \kappa_2^2)} \quad (79)$$

$$z_{2i} = \frac{\xi_2 \left\{ \sigma_\theta^2(\kappa_1^2 + \xi_1) - \xi_2(\sigma_\theta^2 + \kappa_1^2) + \lambda \left[\sigma_\theta^2(\gamma_{\eta_2}\kappa_2^2 - \gamma_{\eta_1}\kappa_1^2 - \kappa_1^2(1 - \gamma_\theta)) + \gamma_{\eta_2}\kappa_1^2\kappa_2^2 \right] \right\}}{\sigma_\theta^2\kappa_2^2 + \kappa_1^2(\sigma_\theta^2 + \kappa_2^2)} \quad (80)$$

$$\delta_0 = \mu \frac{\kappa_1^2\kappa_2^2 + (\xi_1\kappa_2^2 + \xi_2\kappa_1^2) - \lambda \left[\kappa_1^2\kappa_2^2(1 - \gamma_\theta + \gamma_{\eta_1} + \gamma_{\eta_2}) + \kappa_1^2\sigma_\theta^2\gamma_\theta \right]}{\sigma_\theta^2\kappa_2^2 + \kappa_1^2(\sigma_\theta^2 + \kappa_2^2)} + \lambda\gamma_0 \quad (81)$$

$$\delta_{x_{1i}} = \frac{\kappa_2^2(\sigma_\theta^2 - \xi_1) - \sigma_\theta^2(\xi_1 - \xi_2) + \lambda \left[\sigma_\theta^2[\gamma_{\eta_1}\kappa_1^2 - \gamma_{\eta_2}\kappa_2^2 - \kappa_2^2(1 - \gamma_\theta)] + \gamma_{\eta_1}\kappa_1^2\kappa_2^2 \right]}{\sigma_\theta^2\kappa_2^2 + \kappa_1^2(\sigma_\theta^2 + \kappa_2^2)} \quad (82)$$

$$\delta_{x_{2i}} = \frac{\kappa_1^2(\sigma_\theta^2 - \xi_2) - \sigma_\theta^2(\xi_2 - \xi_1) + \lambda \left[\sigma_\theta^2[\gamma_{\eta_2}\kappa_2^2 - \gamma_{\eta_1}\kappa_1^2 - \kappa_1^2(1 - \gamma_\theta)] + \gamma_{\eta_2}\kappa_2^2\kappa_1^2 \right]}{\sigma_\theta^2\kappa_2^2 + \kappa_1^2(\sigma_\theta^2 + \kappa_2^2)} \quad (83)$$

Equations (35)-(39) are derived by substituting the parameters value $\gamma_0, \gamma_\theta, \gamma_{\eta_1}$ and γ_{η_2} into the above equations and rewriting them in terms of precision rather than in terms of variances.

C.4. Benchmark economy.

$$E(u_i) = \beta_0\mu^2 + (\beta_{x_{1i}} + \beta_{x_{2i}} - \beta_{x_{1i}}\beta_{x_{2i}})(\sigma_\theta^2 + \mu^2) - \frac{\beta_{x_{1i}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa_1^2 + \frac{\xi_1^2}{z_{1i}}) - \frac{\beta_{x_{2i}}^2}{2}(\sigma_\theta^2 + \mu^2 + \kappa_2^2 + \frac{\xi_2^2}{z_{2i}}) - \frac{\beta_0^2}{2} - \beta_0\mu^2(\beta_{x_{1i}} + \beta_{x_{2i}}) - C(z_i) \quad (84)$$

The first order conditions of the above equation with respect to $z_{1i}, z_{2i}, \beta_0, \beta_{x_{1i}}$ and $\beta_{x_{2i}}$ are:

$$\beta_0 = 1 - \sum_{j=1}^2 \beta_{x_{ji}} \quad (85)$$

$$\beta_{x_{1i}} = \frac{\sigma_\theta^2 + \mu^2 - \beta_0 \mu^2 - \beta_{x_{2i}} (\sigma_\theta^2 + \mu^2)}{\sigma_\theta^2 + \mu^2 + \kappa_1^2 + \frac{\xi_1^2}{z_{1i}}} \quad (86)$$

$$\beta_{x_{2i}} = \frac{\sigma_\theta^2 + \mu^2 - \beta_0 \mu^2 - \beta_{x_{1i}} (\sigma_\theta^2 + \mu^2)}{\sigma_\theta^2 + \mu^2 + \kappa_2^2 + \frac{\xi_2^2}{z_{2i}}} \quad (87)$$

$$z_{1i} = \frac{\xi_1^2 \beta_{x_{1i}}}{\sqrt{2C_{z_{1i}}(z_i)}} \quad (88)$$

$$z_{2i} = \frac{\xi_2^2 \beta_{x_{1i}}}{\sqrt{2C_{z_{2i}}(z_i)}} \quad (89)$$

Assuming the cost to pay attention is $C(z_i) = \frac{z_{1i} + z_{2i}}{2}$, the solution of the system of equation (85)-(89) gives:

$$\beta_0 = \mu \frac{\kappa_1^2 \kappa_2^2 + (\xi_1 \kappa_2^2 + \xi_2 \kappa_1^2)}{\sigma_\theta^2 \kappa_2^2 + \kappa_1^2 (\sigma_\theta^2 + \kappa_2^2)} \quad (90)$$

$$\beta_{x_{1i}} = \frac{\kappa_2^2 (\sigma_\theta^2 - \xi_1) - \sigma_\theta^2 (\xi_1 - \xi_2)}{\sigma_\theta^2 \kappa_2^2 + \kappa_1^2 (\sigma_\theta^2 + \kappa_2^2)} \quad (91)$$

$$\beta_{x_{2i}} = \frac{\kappa_1^2 (\sigma_\theta^2 - \xi_2) - \sigma_\theta^2 (\xi_2 - \xi_1)}{\sigma_\theta^2 \kappa_2^2 + \kappa_1^2 (\sigma_\theta^2 + \kappa_2^2)} \quad (92)$$

$$z_{1i} = \xi_1 \beta_{x_{1i}} \quad (93)$$

$$z_{2i} = \xi_2 \beta_{x_{2i}} \quad (94)$$

Equations (42)-(46) are derived by rewriting the above equations in terms of precisions rather than in terms of variances.

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