Sender-receiver stopping games with finite horizon

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Abstract

We consider a sender-receiver stopping game with a finite horizon. At each stage, the sender observes the state of the world, which is modeled as a random variable that is uniformly distributed in a compact interval, and independent on different stages. After observing the state of the world, the sender sends a message to the receiver, suggesting either to quit or to continue. The receiver, after seeing the message, decides either to play quit - which ends of the game or to play continue - which takes the game to the next stage. Both players get a utility which is a function of the state of the world on the day the receiver quits.

The payoff functions of both the sender and the receiver are increasing in the state of the world. Hence, both prefer that the game ends when the state is 'high'. A strategy for the sender is called a threshold strategy if, after any history, the sender sends the message to quit if the state is higher than a particular threshold value and to continue if the state is lower than this threshold value. We show that there exists a Perfect Bayesian Equilibrium in which the sender plays a threshold strategy and the receiver plays according to the sender's suggestions. We also show that this is the unique Perfect Bayesian Equilibrium among all strategy profiles in which babbling does not occur at any stage. We also extend our model by introducing discounted payoff functions, arbitrary distributions on states and infinite horizon.

Model

We study a sender-receiver stopping game which is played over at most T stages. At stage t, play is as follows. First, a state of the world θ^t is drawn uniformly from the interval $I = [0, 1]$, independently of θ^1,...,θ^{t-1}. The sender learns θ^t, while the receiver only knows the distribution of θ^t. Next, the sender chooses a message $m^t \in \{m_c, m_q\}$ and sends it to the receiver. The message $m^t = m_c$ can be interpreted as a suggestion for the receiver to continue at this stage t and the message $m^t = m_q$ as a suggestion to quit. On seeing the message, the receiver chooses an action $a^t \in \{c, q\}$, where c stands for continue and q stands for quit, with the exception for $t = T$, where the receiver can only choose the action q. If the receiver quits, the game ends at stage t. Whereas if the receiver continues, the game proceeds to stage $t + 1$. If the game ends at stage t, then the sender receives the payoff $f(\theta^t)$ and the receiver receives the payoff $g(\theta^t)$. We assume that f and g are two strictly increasing functions from $[0, 1]$ to $\mathbb{R}_+$.

The model described above strongly connects to models examined in various areas of research. We mention dynamic versions of sender-receiver games (see for instance Renault, Solan, Vieille, 2013), as well as stopping games (see the survey Solan, Vieille, 2005), in particular secretary problems.

A history of the sender at stage t, assuming that the game has not ended yet, is the sequence $(\theta^1, m^1, \dots, \theta^{t-1}, m^{t-1}, \theta^t)$ of the past states and the messages sent by the sender, and the current state. Since the receiver does not observe the realization of the states, a history of the receiver at stage t is the sequence $h_r^t = (m^1, \ldots, m^{t-1}, m^t)$ of the messages sent by the sender.

A behavioral strategy of the sender is a mapping that to each possible history of the sender assigns a probability distribution on the message set $\{m_c, m_q\}$. A behavioral strategy of the receiver is a mapping that to each possible history of the receiver assigns a probability distribution on the action set $\{c, q\}$, with the exception that at stage T, probability 1 is placed on the action q.

We define special types of pure strategies for the sender and for the receiver. A pure strategy of the sender is called a threshold strategy, if there are $\alpha^1, \ldots, \alpha^T \in [0, 1]$ such that at each stage t, the strategy recommends the message m_c if $\theta^t < \alpha^t$ and the message m_q if $\theta^t > \alpha^t$. The obeying strategy for the receiver is the pure strategy that always 'obeys' the message sent by the sender. That is, the strategy always recommends the action c on seeing the message m_c and the action q on seeing the message m_q, except for stage T where action q is recommended regardless the message sent by the sender.

Results

[1] We construct a Perfect Bayesian Equilibrium (PBE) in which the sender uses a threshold strategy and the receiver uses the obeying strategy. The idea of the construction is as follows. First, we identify a strategy of the sender which is a best response to the the obeying strategy of the receiver. This strategy turns out to be a threshold strategy. Subsequently, we prove that the obeying strategy of the receiver is a best response to this particular threshold strategy of the sender. Hence this strategy pair is a PBE.

[2] Furthermore, we show that the above PBE is unique amongst all the behavioral strategy profiles in which there is no babbling at any stage. The rough idea of the proof is as follows. Suppose, that a strategy profile is a PBE in which there is no babbling. First, we deduce that in this PBE, the sender must play a threshold strategy and the receiver the obeying strategy. And then by using the result and techniques in [1], we can conclude that this PBE has to coincide with the PBE constructed in [1].

Extensions

We extend our model where the payoff of the players do not only depend on the realization of the state when the game ends, but also on the stage. In particular, we consider a special case in which the payoffs are discounted over time with a fixed discount factor. In this case, if the discount factor is large enough, we have the same existence and uniqueness results as before in [1] and [2].

We also consider the extension where instead of uniform distribution of the states over $[0, 1]$, we have an arbitrary distribution. In this case, the same results hold.

We are currently exploring the case where the states at each stage may not be independent, but they follow a Markov chain. This setting is even more general the one described before. We are currently also exploring the case where the the horizon is infinite.

In many of these extensions, the characterization of PBE remains valid, but existence may fail.

References

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