Heterogeneous Nash Equilibria in Coordination Games Played on Metric Spaces

Nikhil Addleman

Institute for Mathematical Behavioral Science, University of California, Irvine

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Extended Abstract

The effect of local interaction on the behavior of a population of agents playing coordination games is investigated by introducing metric games. A metric game features populations of agents randomly located on metric spaces. An agent plays a single strategy against the entire population, receiving a payoff negatively related to its distance from each opponent. Population size, the functional form of payoff decay in distance, and the dimensionality of the space determine whether miscoordinated equilibria are possible.

Spatial structure affects both dynamic and in-equilibrium behavior of populations of interacting agents. Many results in this domain structure populations by placing agents on a social network or a graph where each agent interacts only with its neighbors, that is, plays against a particular subset of the population [2] [4] [5] [6] [7].

The simplest coordination game is a symmetric 2×2 game with zero off-diagonal payoffs and identical rewards for matching on either strategy. If an agent must pick a single strategy to play against all opponents, then for any population structure homogeneity of strategy is a Nash equilibrium. On networks, heterogeneous Nash equilibria may be made possible by disconnecting subsets of agents to a degree such that members of each group are incentivized to coordinate with their own group regardless of the behavior in the rest of the population. The spread of innovations and the adoption of technology have been modeled using coordination games on social networks

[3] [8]. Instead of a binary definition of proximity, in which two agents are either neighbors or disconnected, the present model introduces an alternative framework for considering (mis)coordination by exploiting the geometric structure provided by considering random arrangements of players on a metric space. The main feature of the model, "strength" of interactions decaying with distance, enables models where agents favor coordinating with others like them but don't disregard the rest of the population. Examples of problems that feature interactions of this type include the adoption of instant messaging applications (one needs to be able to chat with friends, but it also matters what everyone else in the country uses) and coordination in spatially distributed biological populations (trees must coordinate pollination and flowering periods).

A population of agents $\mathcal{I}$ is placed onto a metric space (X, d). Typical arrangements will be placing N agents onto the d-dimensional torus $\mathbb{T}^d_{[0,1)}$ or placing agents with uniform density throughout $\mathbb{R}^d$ using a Poisson point process [1]. The agents play a symmetric 2×2 game G with strategies $s = \{H, T\}$. In this case, G has payoff $v > 0$ if agents coordinate, and 0 otherwise.

A non-negative function $f : X \times X \to \mathbb{R}$, called the interaction strength is specified so that it is non-increasing in the distance between the points. We consider some natural functions such as $f(x, y) = e^{-\gamma d(x,y)}$ and $f(x, y) = \frac{1}{d(x,y)^\alpha}$ where γ and α are positive parameters.

Agent j's payoff from playing H is

$$\sum_{i \in \mathcal{I}, s_i = H} v f(i, j)$$

For small N and simple spaces, the probability that the agents are placed in such a way that a heterogeneous Nash equilibrium exists can be explicitly calculated. In order to study the effects of increasing the population size or dimensionality of the space, computer simulation is used.

The simulations converge to a Nash equlibrium by placing the population in space with random initial strategies, and then repeatedly giving a random agent the opportunity to change its strategy by best responding to the present state of the population until fixation at an equilibrium. These simulations have revealed some counterintuitive properties of the model. Even though the populations are fully connected, there are many conditions for which heterogeneous equilibria are not only possible but common. When

f is an inverse-square law, miscoordination is possible for any $N \geq 4$. In this case, as the population size increases miscoordination becomes almost inevitable. It is conjectured that for inverse-power laws, populations will almost surely converge to a heterogeneous equilibrium as $N \to \infty$.

References

[1] Adrian Baddeley, Imre Bárány, and Rolf Schneider. Spatial point processes and their applications. *LECTURE NOTES IN MATHEMATICS-SPRINGER-VERLAG-*, 1892:1, 2007.

[2] Glenn Ellison. Learning, local interaction, and coordination. *Econometrica: Journal of the Econometric Society*, pages 1047–1071, 1993.

[3] Andrea Montanari and Amin Saberi. The spread of innovations in social networks. *Proceedings of the National Academy of Sciences*, 107(47):20196–20201, 2010.

[4] Stephen Morris. Contagion. *The Review of Economic Studies*, 67(1):57–78, 2000.

[5] Martin A Nowak, Sebastian Bonhoeffer, and Robert M May. Spatial games and the maintenance of cooperation. *Proceedings of the National Academy of Sciences*, 91(11):4877–4881, 1994.

[6] Brian Skyrms. *The stag hunt and the evolution of social structure*. Cambridge University Press, 2004.

[7] György Szabó and Gabor Fath. Evolutionary games on graphs. *Physics reports*, 446(4-6):97–216, 2007.

[8] H Peyton Young. The diffusion of innovations in social networks. *The economy as an evolving complex system III: Current perspectives and future directions*, 267, 2006.