The two-stage rules for multi-issue allocation situations

Abstract

In a bankruptcy problem, there is an estate to be divided among a number of claimants, whose total claim exceeds the estate available. The multi-issue allocation situations are used to model bankruptcy-like problems in which the estate is divided not on the basis of a single claim for each agent, but several claims resulting from so-called issues. These situations were studied by Calleja et al. (2005), Ju et al. (2007), and Lorenzo-Freire et al. (2005), among others.

Many real situations can be modeled by means of multi-issue allocation situations. We can consider, for example, the allocation of the budget of the European Union. Each year the European Union must allocate money to different issues, such as agriculture and natural resources, freedom, security and justice, citizenship programmes, external relation actions, ... Moreover, the countries which are members of the European Union have different necessities on these issues.

This paper is based on the extension of bankruptcy rules to the case of multi-issue allocation situations. We consider extensions of well-known bankruptcy rules such as the proportional rule, the constrained equal awards rule, and the constrained equal losses rule. These extensions are obtained by means of a procedure in two stages where first the estate is divided among the issues by means of a bankruptcy rule and then the quantity obtained for each issue is divided among the agents using another bankruptcy rule.

We study these rules from an axiomatic point of view. In this way, we introduce suitable properties and characterize some of the two-stage rules with them. Although some of these two-stage rules have been studied previously (see Lorenzo-Freire et al. (2005), for example), we pursue a more ambitious program, surveying it from most of the perspectives that have found to be fruitful in previous literature on
standard bankruptcy problems. Moreover, while in Lorenzo-Freire et al. (2005) the bankruptcy rule used in both stages is the same, we consider that the two bankruptcy rules can be different.

In this way, we characterize the two-stage constrained equal awards rule, which is the two-stage rule where we use in both stages the constrained equal awards rule for bankruptcy problems, using properties based on equal treatment (cf. O’Neill (1982)), composition up (cf. Young (1988)), invariance under claims truncation (cf. Curiel et al. (1987)), conditional full compensation (cf. Herrero et al. (2001,2002)), claims monotonicity (cf. Thomson (2001)), super-modularity (cf. Dagan et al. (1997)), and composition down (cf. Moulin (2000)). Similarly to the case of the constrained equal awards rule for bankruptcy problem, we obtain dual characterizations of the two-stage constrained equal losses rule.

Finally, we characterize the two-stage proportional rule and other two-stage rules using properties based on the property of no advantageous transfer (cf. Chun (1988)), among others.

Notice that although these characterizations are extensions of characterizations for bankruptcy rules, there are many technical and conceptual innovations.

References
Lorenzo-Freire, S., Casas-Méndez, B., Hendrickx, R., 2005. The


