Vickrey versus Proxy Auctions: An Experimental Study

Eiichiro Kazumori[1]

Abstract

This article reports the results of the first laboratory experiments that compare proxy auctions (Ausubel and Milgrom (2002)) and Vickrey auctions for allocating packages of heterogeneous items. We find that (1) the seller's revenues in proxy auctions are higher in both cases of complements and substitutes, and (2) bidders omit bids for low-valued packages and thereby reduce prices in the Vickrey auction by reducing the externality imposed upon other bidders as measured by bids. The revenue ranking is repeated in a large scale experiment involving professional traders. *JEL classification:* D44: C92.

I. Introduction

What is the most effective way to allocate multiple heterogeneous

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objects? This question arises in many areas of economic policy and business applications. An important class of applications are the 'combinatorial auctions' (Cramton, et al. (2006)) used to allocate spectrum licenses, contracts for the operation of bus routes, and airport landing slots. Another class includes the matching procedures (Roth and Sotomayor (1990)) used to allocate workers to firms, students to schools, and medical interns to hospitals.

For the purpose of promoting an efficient allocation, the most well-known theoretical method is the Vickrey auction (Vickrey (1961), Clarke (1971), and Groves (1973)). Each bidder places a sealed bid for each package, and the seller chooses the allocation that maximizes the sum of the winning bids. A winning bidder pays the externality imposed upon other bidders, as measured by the submitted bids. Sincere bidding is a dominant strategy in a Vickrey auction and therefore the predicted equilibrium outcome is efficient.

Ausubel and Milgrom (2002) challenge the practicality of Vickrey auctions. Their principal argument is that the Vickrey auction payoff can be outside the core of the associated cooperative game. In particular, the seller's revenue can be low, so when objects are complements a coalition of the seller and the losing bidders can block the equilibrium.

They propose a ‘proxy auction’ as an alternative method. In a proxy auction, a bidder first reports his values of the various packages to a proxy agent, represented by a computerized algorithm. Then the proxy agent bids straightforwardly in ascending auctions on the bidder’s behalf. At each stage, the proxy agent bids to maximize the bidder’s potential profit, i.e., the difference between the bidder’s reported value and the bid, subject to the constraint that a bid must exceed the previous highest bid for the package. The seller retains the bids that maximize her total revenue, drawing from all submitted bids, including previously submitted bids. The proxy agents who did not win packages in the current round make new bids in the next round, if any profitable bids are possible. The auction ends at the stationary point of the process attained when no new bids are made by the proxy agents.

Ausubel and Milgrom (2002) establish the key theoretical results comparing these two auction designs. If the coalitional value function is bidder-submodular then sincere bidding is an equilibrium strategy of both auctions and predicted outcomes are equivalent. But if the coalitional value function is not bidder-submodular then the Vickrey theoretical payoff is not in the core and the seller’s payoff is less than her smallest core payoff. In contrast, proxy auctions have an equilibrium in ‘truncation strategies’ (defined by a profit target) such that the associated payoff is a bidder-optimal point in the core, and thus the seller’s revenue is higher than in the corresponding Vickrey payoff.

These mechanisms require large amounts of communication from bidders. Each bidder is asked to report a value for every package. The number of packages increases exponentially in the number of objects. Actual bidders have only limited resources and capabilities. Thus a question arises whether these mechanisms perform as predicted among real-world agents.

This paper reports laboratory experiments to add empirical data to theoretical analysis. We began with an experiment in a simple setting to identify and separate causality between mechanisms and outcomes. Then we examined an experiment in a more complex environment to check the robustness of the results obtained in the first experiment—following a recent discussion by Levitt and List (2006) and Roth (2006).

In our first experiment, each auction had three bidders and two objects. We compared the results of proxy and Vickrey auctions for each profile of values among the bidders, while randomizing the order of auctions to neutralize the effects of learning. Our findings are as follows: first, we found significant underbidding in Vickrey auctions. Second, we did not reject sincere bidding in proxy auctions when the coalitional value function is bidder-submodular. Third, when the Vickrey theoretical outcome is not in the core, 75% of the proxy outcomes were in the core in contrast to 13% in the Vickrey outcomes. Fourth, we found significant numbers of absent bids in both auctions. Fifth, the two auctions had statistically equal efficiency levels. Sixth, proxy auctions had significantly higher seller revenues regardless of whether the Vickrey theoretical outcome is in the core.

These experimental results suggest an existence of bid submission costs or communication constraints for bidders. The communication constraints may affect the mechanism performance through payment rules. Bidders with communication constraints do not place bids for low-valued packages since the potential profits are low and the marginal return from placing a bid is less than a marginal communication cost of placing an additional bid. This bidding behavior need not affect the revenue in a proxy auction since the revenue is the sum of the maximum bids when the auction stops. But it may reduce the revenue in Vickrey auctions by reducing bidders' opportunity costs as measured by submitted bids. Thus, a real-world friction of communication constraints reinforces the theoretical result about the revenue ranking of the two auction designs.

In the second experiment, we examined whether the insights obtained from the first experiment generalize to a more realistic environment. Each auction in the second experiment had 5 bidders and 16 objects. To simulate a scenario of actual transactions, the bidders and the objects were arranged in a rectangle and each bidder obtained superadditive values from a package of objects geographically close to his or her own location. In addition to university students, we invited professional traders in security markets and Internet auctions to participate in the experiment. As in the first experiment, a proxy auction had higher seller revenue in the second experiment. We also found evidence that bidders used truncation strategies.

Our experiment was apparently the first to compare proxy and Vickrey auctions. Prior experiments did not test proxy auctions. Ledyard, Porter, and Rangel (1997) compared various combinatorial mechanisms. Plott (1997) reported experiments comparing simultaneous ascending auctions and sequential auctions. Morgan (2002) studied Vickrey auctions and simultaneous ascending auctions without combinatorial bidding. Porter, Rassenti, and Smith (2003) examined simultaneous ascending auctions with and without package bidding and ‘combinatorial clock auctions'. Katok and Roth (2004) studied ascending and descending auctions for homogeneous objects. Salmon and Plott (2004) examined bidder behavior in simultaneous ascending auctions experimentally and empirically and found that bidders followed a straightforward bidding strategy. Chen and Takeuchi (2005) compare Vickrey and 'the iBEA auctions' using a human bidder and computerized agents. Brunner, Goeree, Holt, and Ledyard (2006) reported experiments on simultaneous ascending auctions with and without combinatorial bidding. But none of these papers examine proxy auctions.

We begin with a summary of the two auction mechanisms and the theoretical predictions. Section 3 then describes our first experiment and reports the results. In section 4, we explore an explanation of the results in the first experiment. Section 5 reports our second experiment. We conclude in section 6.

II. Two Auctions

We consider an environment where the seller $n = 0$ allocates L objects among N bidders $n = 1, ..., N$. We use $v_n(x)$ to designate bidder n's monetary value of a package $x \in \{0, 1\}^L$. Each bidder has quasi-linear preferences. The set of feasible allocations is defined by $X = \{(x_1,..,x_N) : \mathbf{0} \leq x_n \leq \mathbf{1}$ and $\mathbf{0} \leq \sum_n x_n \leq \mathbf{1}\}$ where $\mathbf{0} = (0, ..., 0)$ and $\mathbf{1} = (1,..,1)$. In the following subsections we describe briefly a Vickrey auction and a proxy auction. For more details see Milgrom (2004).

A. Vickrey Auction

In a Vickrey auction, a bidder reports a value for every package. Let $b_n(x_n)$ be n's bid for package x_n. The seller chooses the bid-maximizing allocation: $x^* \in \arg\max_{(x_1,..,x_N)\in X} \sum_n b_n(x_n)$. The distinctive feature of a Vickrey auction is that a bidder's payment (i.e., price) for the package he wins is the externality imposed upon the coalition of the other bidders. The coalition without n can create the value $\alpha_n = \max_{x\in X} \sum_{m\neq n} b_m(x_m)$. The coalition gets $\sum_{m\neq n} b_m(x^*_m)$ in the Vickrey allocation. The payment is $p_n = \alpha_n - \sum_{m\neq n} b_m(x^*_m)$.

	A	B	AB
1	5	5	12
2	10	0	10
3	0	10	10

Table 1. Example 1.

Consider example 1 with objects A and B and bidders $n = 1, 2$, and 3. Suppose the values are as in Table 1. For example, 1 has value 5 for A, 5 for B, and 12 for AB. In equilibrium, 2 wins A and 3 wins B. The most bidders 1 and 3 can get without 2 is 15. 1 and 3 get 10 with 2. Thus 2 pays 5. Similarly 3 pays 5. The equilibrium payoff vector is (the seller, bidder 1, bidder 2, bidder 3) $= (10, 0, 5, 5)$.

B. Proxy Auction

In a proxy auction, bidders report their values to proxy agents. Then the proxy agents bid straightforwardly in ascending auctions. The proxy agent computes the potential profit from each package given the minimum allowable bids determined from the results of the previous round. If the optimal potential profit is positive, the agent places a bid on the package with the optimal potential profit. The seller considers all the bids presented so far and chooses the bid-maximizing allocation. The outcome of the auction is a stationary point of this cumulative offer process.

Suppose that bidders reported the values in Table 1 to proxy agents. Let ε be a minimum bid increment. When the bidding starts at $(0, 0, 0)$, 1 places a bid ε for AB, 2 bids ε for A and 3 bids ε for B. The seller chooses bids from 2 and 3. In the next round, 1 comes back by bidding 2ε on AB. The auction ends at a price $(A, B, AB) = (6, 6, 12+\varepsilon)$ when 1 stops bidding. In the example above, the coalition of the seller and bidders 2 and 3 wins and the revenue is 12, which is the value of the coalition of the seller and bidder 1 that has the second highest value.

C. Comparison

Ausubel and Milgrom (2002) study properties of the two auction designs. A coalition S with the seller can create a value $w(S) = \max_{x \in X} \sum_{n \in S} v_n(x_n)$ by allocating objects among themselves. A coalitional value function is bidder-submodular if for each $0 \in S \subset S'$, $w(S \cup \{l\}) - w(S) \geq w(S' \cup \{l\}) - w(S')$. The core is the set of feasible and unblocked payoff vectors: $Core = \{\pi = (\pi_1,..,\pi_N) : w(L) = \sum_{n \in L} \pi_l$ and $w(S) \leq \sum_{n \in S} \pi_n$ for any $S\}$ where L is the coalition including every bidder. A payoff vector π is a bidder-optimal point in the core if there is no other $\pi' \neq \pi$ in the core for which $\pi'_n \geq \pi_n$ for every bidder n.

A substitutes condition, or bidder-submodularity of the coalitional value function, plays a key role. When the coalitional value function is bidder-submodular, the Vickrey theoretical payoff is the unique bidder optimal point in the core. Sincere bidding is an equilibrium strategy in proxy auctions and the equilibrium outcome is same as the Vickrey theoretical outcome. But when the coalitional value function does not satisfy bidder-submodularity, the Vickrey payoff may not be in the core and the seller revenue can be strictly less than any core payoffs. In Example 1, the Vickrey payoff $(10, 0, 5, 5)$ is blocked by a coalition of the seller and bidder 1, whose potential value is 12. In contrast, a proxy auction has an equilibrium in a profit-target strategy which implements a bidder optimal outcome in the core.

III. Experiment I

We begin with an experiment in a simple environment. Each auction had objects A and B and bidders 1,2, and 3. Let v_n^l be n's value of object $l = A, B$ and a package AB. Values for A and B were generated independently from the uniform distribution on [0,100]. The value for AB was $v_n^{AB} = v_n^A + v_n^B + \mu_n v_n^A v_n^B$ where $\mu_n = 0.04(x_n - 0.5)$ and $\{x_n\}$ were drawn independently from the uniform distribution on [0,1]. Each bidder knew other bidders' values. The minimum bid increment was 0.1. Bidders learned the outcome at the end of an auction.

We conducted experiments at Stanford University in August 2005. All bidders were students. We ran 40 auctions for 20 value profiles. The coalitional value function was bidder-submodular in 11 out of the 20 profiles. The order of auctions and values was randomized. We first explained values, the auctions, and examples. Then we answered questions

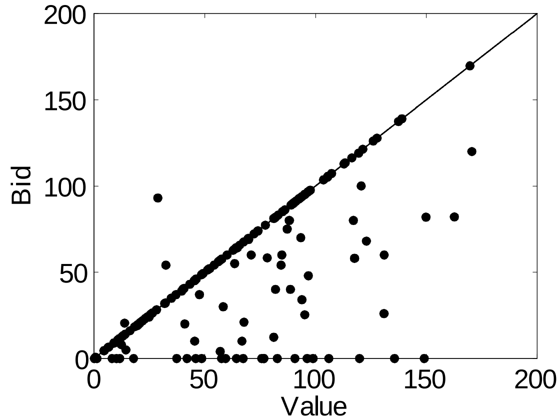

Figure 1: Vickrey Auction Bids

and conducted some practice runs before the auctions began. Bidders sent their bids to the auction server[2].

A. Bidding behavior

According to theory, Vickrey auctions have sincere bidding as an equilibrium in all cases. Figure 1 shows significant underbidding in the data of Vickrey auctions. 63% of bids were sincere, 34% of bids were underbids, and 4% of bids were overbids. The average value of a package was 68 and the average bid was 51. The p-value of the two-sided t-test of the zero mean difference between values and bid was 0.001.

Figure 2 displays the distribution of proxy bids. When the coalitional

[2] The auction program uses an algorithm by Wurman, Jie, and Cai (2004) and is available at http://users.kattare.com/~kazumori/cgi-local/auct-prog.cgi/ui.

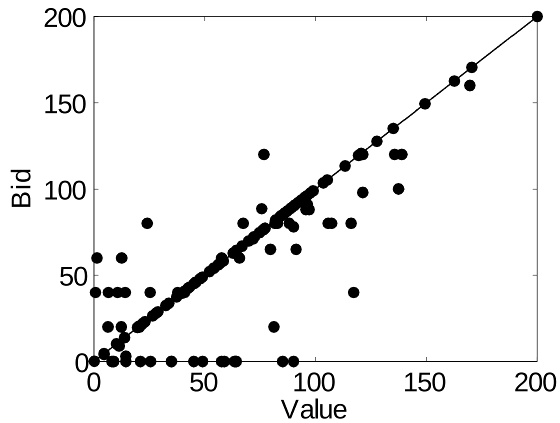

Figure 2: Proxy Auction Bids

bidding function is bidder-submodular, the average value of the package was 65 and the average bid was 64. The two-sided t test had p value 0.87 and did not reject the null hypothesis of sincere bidding. When the coalitional bidding function is not bidder-submodular, theory predicts that proxy auctions have an equilibrium with a bidder-optimal core outcome. We found 75% of outcomes were in the core in contrast to 13% of Vickrey auctions.

Although bidders were asked to place a bid on all three packages, bidders did not place bids for 15% of the packages in Vickrey auctions and 11% in proxy auctions. Absent bids occurred mainly for low valued packages. In Vickrey auctions, 27% of absent bids occurred for the highest valued packages, 34% were for the next to highest, and 40% were for the lowest valued packages in a bidder's value profiles. In proxy auctions, the ratios were 6%, 28%, and 67%.

B. Efficiency

Theoretically both proxy and Vickrey auctions have a fully efficient equilibrium in all cases. Figure 3 compares measures of efficiency attained by the two auction designs. We first measured efficiency by dividing the total values of the allocation by the total value of the efficient allocation. Proxy auctions had average efficiency 0.94. Vickrey auctions had average efficiency 0.91. The two-sided t test of a zero mean difference had p-value 0.38 and did not reject the null hypothesis. The result was robust when we measured efficiency by taking the difference between the value of the allocation and the value of a random allocation divided by the difference between the value of efficient allocation and the value of the random allocation. In that case, the p-value was 0.85.

C. Revenue

One of the key results in Ausubel and Milgrom (2002) is the revenue ranking between the two auction designs. Figure 4 shows the distribution of revenues for all cases when we measured the revenue by dividing with the theoretically predicted revenue from a Vickrey auction. Proxy

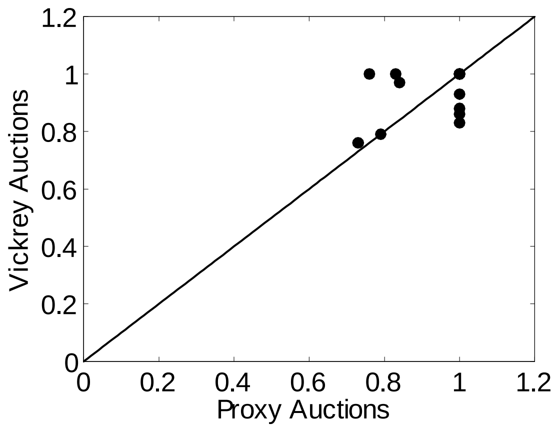

Figure 3: Efficiency Comparison

auctions had average revenue 1.2. Vickrey auctions had average revenue 0.82. Proxy auctions had higher seller revenue.[3]

When the coalitional value function is bidder-submodular, theory predicts that these two auctions have the same revenue. In contrast, we found that the average proxy revenue was 1.2 and the average Vickrey revenue was 0.85. The t test had p-value 0.003. 77% of proxy outcomes and 37% of Vickrey outcomes were in the core. When the coalitional value function was not bidder-submodular, the average proxy revenue was 1.2 and the average Vickrey revenue was 0.80. The t test had a p-

[3] The qualitative results were the same when we adjusted overbids in proxy auctions to the value of the packages on which bids were placed. We also conducted test by measuring revenue by the difference between the revenue and the value of a random allocation divided by the difference between the value of the allocation and the value of a random allocation. In that case, the two-sided t test of equal revenues had a p-value 0.01, in line with the result using the first measure.

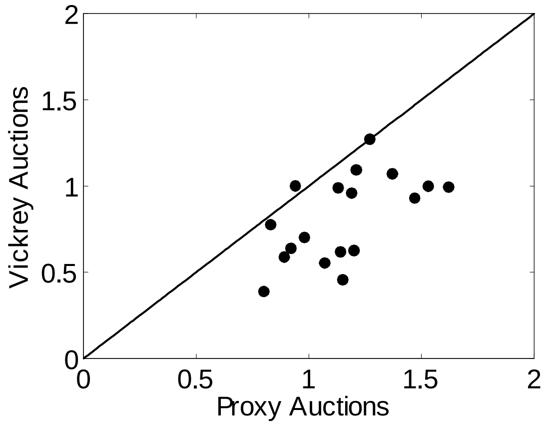

Figure 4: Revenue Comparison

value 0.004, in line with the theory. As noted, 75% of proxy outcomes were in the core, in contrast to 13% of Vickrey outcomes.

IV. Mechanism Performances with Communication Constraints

Absent bids in the first experiment, especially in Vickrey auctions, suggest an existence of bidder submission costs or communication constraints for bidders. In package auctions, bidders are asked to report values for all possible packages. Real-world bidders have limited resources for this task and bidding incurs some costs. As a result, bidders may not report values for every package.

This communication constraint can affect the relative performance of the two auction designs through payment rules. Bidders may reduce or forego their bids for low-value packages because the potential profit is low. In Vickrey auctions, truncation can reduce externalities upon other bidders as measured in bids, and hence the payments required from all winning bidders. On the other hand, in proxy auctions, low-valued packages are unlikely to matter when a proxy agent maximizes the potential profits.

We can illustrate this point by replacing 1's bid for A and B in Example 1 by b_1^A and b_1^B with $2 \leq b_1^A, b_1^B \leq 6$. The seller's revenue from a Vickrey auction is $b_1^A + b_1^B$ and the revenue from a proxy auction is 12. When bidder 1 reduces bids for A and B, it reduces the seller's revenue from a Vickrey auction but not from a proxy auction.

We now generalize this example to match the qualitative characteristics of the experimental results. Let $M_n \geq 1$ be the number of packages on which bidder n can report the value. Let $\{x_n^*\}$ be an efficient allocation. We first consider Vickrey auctions. Consider bids $\{b_n^V\}$ where bidder n bids sincerely on x_n^* and bids below the value for other packages. The allocation from $\{b_n^V\}$ is $\{x_n^*\}$. Bidder k's payoff is $\sum_n v_n(x_n^*) - \max_{x \in X} \sum_{n \neq k} b_n^V(x_n)$. Since the second term does not depend on bids by k, this payoff is the maximum payoff k can obtain. In this equilibrium, a bidder's equilibrium payment is less than the unconstrained Vickrey payment because of fewer reports.

For a proxy auction, let $\widehat{\pi}$ be a bidder-optimal core point given the original value $\{v\}$. By Ausubel and Milgrom (2002), $\widehat{v}_n(x_n) = \max\{0, v_n(x_n) - \widehat{\pi}_n\}$ is an equilibrium report by n with a payoff vector $\widehat{\pi}$. Let us describe a progress of a proxy auction with report $\{\widehat{v}_n\}$. Let $t = 1, ..., T$ be rounds of the auction. Let $\widehat{B}_n^t(x_n)$ be the maximum price by proxy agent n on a package x_n up to and including time t. Let $\{\widehat{x}_n^{*,t}\}$ be a provisional allocation at t. We set the initial value $\widehat{B}_n^0(x_n) = 0$ and $\widehat{x}_n^{*,0} = \phi$ for each n and x_n. We now describe the progress of an auction from time $t-1$ to t given $\{\widehat{B}_n^{t-1}(x_n)\}$ and $\{\widehat{x}_n^{*,t-1}\}$. Let $\underline{\widehat{B}}_n^t(x_n)$ be the lowest price n can bid for package x_n at round t defined by $\underline{\widehat{B}}_n^t(x_n) = \widehat{B}_n^{t-1}(x_n)$ for $x_n = \widehat{x}_n^{*,t-1}$ and $= \widehat{B}_n^{t-1}(x_n) + \varepsilon$ otherwise. Let $\widehat{\pi}_n^t(x_n) = \widehat{v}_n(x_n) - \underline{\widehat{B}}_n^t(x_n)$ be a potential profit by n on x_n. Define $\widehat{x}_n^t = \arg\max\{0, \max_{x_n} \widehat{\pi}_n^t(x_n)\}$ be the optimal potential profit package where we define $\widehat{x}_n^t = \phi$ if $\max_{x_n} \widehat{\pi}_n^t(x_n) \leq 0$. Let $\widehat{\pi}_n^{*t} = \max\{0, \max_{x_n} \widehat{\pi}_n^t(x_n)\}$ be the optimal potential profit. Then $\widehat{B}_n^t(x_n) = \underline{\widehat{B}}_n^t(x_n)$ if $x_n = \widehat{x}_n^t$ and $= \widehat{B}_n^{t-1}(x_n)$ else, and $\{\widehat{x}_n^{*,t}\} \in \arg\max \sum_n \widehat{B}_n^t(x_n)$.

Now let us define a profit-target strategy on the set of packages for which a proxy ever placed a bid in the auction. Let $\widehat{X}_n = \{\widehat{x}_n^t\}_{t=1,...,T}$ be the collection of packages on which n placed a bid in the auction. Define

$\overline{v}_n(x_n) = \max\{0, v_n(x_n) - \pi_n\}$ if $x_n \in \widehat{X}_n$ and no report otherwise. We now consider ascending proxy auctions with $\{\overline{v}_n\}$. As before, the initial conditions are $\overline{B}^0_n(x_n) = 0$ and $\overline{x}^{*,0}_n = \phi$ for each n and x_n. We now check whether $\overline{B}^t_n(x_n) = \widehat{B}^t_n(x_n)$ and $\overline{x}^{*,t}_n = \widehat{x}^{*,t}_n$ for each n, x_n, and t by induction. Take $t = 1$. $\underline{\overline{B}}^1_n = \varepsilon$ for all n and x_n. Thus $\underline{\overline{B}}^1_n(x_n) = \underline{\widehat{B}}^1_n(x_n)$. Then $\overline{\pi}^1_n(x_n) = \overline{v}_n(x_n) - \underline{\overline{B}}^1_n(x_n)$ has $\overline{\pi}^1_n(x_n) = \widehat{\pi}^1_n(x_n)$ if $x_n \in \widehat{X}_n$ and < 0 else. Then consider $\overline{x}^1_n = \arg\max\{0, \max_{x_n} \overline{\pi}^1_n(x_n)\}$. Since $\max\{0, \max \overline{\pi}^1_n(x_n)\} \leq \max\{0, \max \widehat{\pi}^1_n(x_n)\}$ and $\overline{\pi}^1_n(\widehat{x}_n) = \widehat{\pi}^1_n(\widehat{x}_n)$, $\widehat{x}^1_n \in \arg\max\{0, \max_{x_n} \overline{\pi}^1_n(x_n)\}$. Thus $\overline{B}^1_n(x_n) = \widehat{B}^1_n(x_n)$. Thus $\{\overline{x}^{1*}_n\} = \{\widehat{x}^{1*}_n\}$. For the second step of induction, assume $\overline{B}^{t-1}_n(x_n) = \widehat{B}^{t-1}_n(x_n)$ and $\overline{x}^{*,t-1}_n = \widehat{x}^{*,t-1}_n$. Then $\underline{\overline{B}}^t_n(x_n) = \underline{\widehat{B}}^t_n(x_n)$. Then $\overline{\pi}^t_n(x_n) = \widehat{\pi}^t_n(x_n)$ if $x_n \in \widehat{X}_n$ and < 0 else. Then, since $\widehat{x}^t_n = \arg\max\{0, \max \widehat{\pi}^t_n(x_n)\} = \arg\max\{0, \max \overline{\pi}^t_n(x_n)\}$, $\overline{\pi}^{t,*}_n = \widehat{\pi}^{t,*}_n$. Thus $\overline{B}^t_n(x_n) = \widehat{B}^t_n(x_n)$ and $\overline{x}^{*,t}_n = \widehat{x}^{*,t}_n$. We can check that $\overline{v}_n$ is an equilibrium strategy by applying the argument in Ausubel and Milgrom (2002). Thus we have:

Proposition 1 *For $M_n \geq T$, there exists an equilibrium of a proxy auction with a bidder-optimal core payoff outcome and a higher revenue than an equilibrium of a Vickrey auction.*

Example 2 shows that the communication requirement cannot be uniformly relaxed. In Table 2, a proxy outcome is that 1 wins A and 2 wins B with price 8 each. But it is impossible to implement this outcome with fewer than 4 bids. On the other hand, this bound does not depend on the number of objects and T can be controlled by the auction design through the minimum increment.

	A	B	AB
1	10	0	0
2	9	9	0
3	0	8	0

Table 2. Example 2.

These equilibrium outcomes match qualitative features of the experimental outcome: (1) bidders underbid in Vickrey auctions, (2) bidders did not place bids for low-valued packages, (3) the two auctions had the same efficiency level, (4) proxy auction outcomes are in the core, and (4) the seller's revenues were higher in proxy auctions regardless of whether the coalitional value function is bidder-submodular.

This communication problem is different from Nisan and Segal (2002) and Segal (2005) who characterize the amount of communication needed for a seller to compute an efficient allocation when the seller does not know the values of bidders. In the environment of this paper, bidders' values are common knowledge and a bidder does not need any communication to compute auction outcomes, as in Ausubel and Milgrom (2002).

V. Experiment II

In this section we examine the robustness of the results in the first experiment by extending the setting of the first experiment in two dimensions.[4] The first dimension is the number of objects in the allocation problem. Actual allocation problems typically have many objects. The FCC's auction No. 31 planned to offer 12 licenses. The National Residency Matching Program offers more than $20,000$ positions. We would like to see whether the results in the first experiment extend when bidders need to deal with many objects. The second dimension is bidder characteristics. The university students in the first experiment may not be representative of actual bidders, which could bias the result. In order to investigate the issue we ran one session with professional traders.

In the second experiment, each auction had 5 bidders and 16 objects. Bidder n had a base value $v_n^l, l = 1, ..., 16$. The average value of an object was 413, the mean was 400, and the standard deviation was 192. In the first set of value functions, the value function was additive. In the second and third cases, a bidder got additional benefits from packages in the area of interests displayed in Figure 5. Formally, the value of a package x_n is $v_n(x_n) = \sum_{l \in x_n} v_n^l + 0.01 \sum_{l,m \in x_n \text{ and } l,m \in A_n} \mu_n^l \mu_n^m v_n^l v_n^m$.

[4] Kazumori (2005) describes the details of the design of the second experiment.

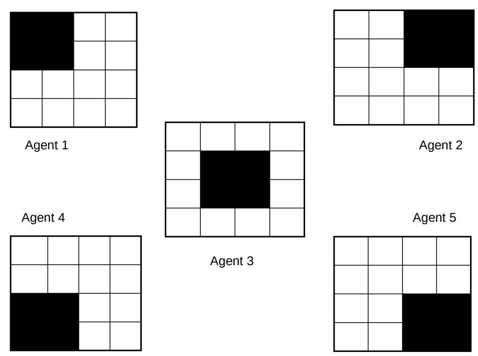

Figure 5: Areas of Interests

μ_n^l represents n's weight for object l. The values of $\{\mu_n^l\}$ were different in the second and the third cases to represent 'exposure' and 'threshold' problems of the sort described by Ausubel and Milgrom (2006) and Milgrom (2004).

In this experiment, we used a clock-proxy version of the proxy auction as in Ausubel and Milgrom (2006). The clock-proxy auction begins with clock rounds and ends with a proxy round. In our clock round, bidders place bids for each object and the price for an object with excess demand increases. The clock rounds end when there are no excess demands. Then bidders report values to proxy agents in the proxy rounds. The seller determines the final allocation from bids in both rounds.

The subject pool in the second experiments consisted of students and professional traders. Professional traders are defined to be ones who make or made their living through trading in securities markets or eBay markets. We used their resumes to check their qualifications. Bidders in a session were all students or all professionals. We analyzed the data from the last 30 auctions.[5]

A. Aggregate Outcomes

Figure 6 reports the revenues normalized by the theoretical revenue from Vickrey auctions. Proxy auctions had average revenue 0.53 and Vickrey auctions had lower average revenue of 0.35. The p-value of the two-sided t-test of equal revenue was 0.06. Thus the revenue ranking extended to the second experiment. Low revenue cases in Vickrey auctions were observed in the session with professional traders.[6]

As shown in Figure 7, efficiency levels were similar in the two auction designs. The average efficiency measure, normalized by the value of a fully efficient allocation, was 0.84 in proxy auctions and 0.79 in Vickrey auctions. The t test did not reject the null hypothesis of a zero mean difference. This result is also in line with the first experiment.

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[5] Kazumori (2005) earlier reported that proxy auctions had higher seller revenues than Vickrey auctions using a full set of data.

[6] Kazumori (2005) documented trading strategies by professional traders.

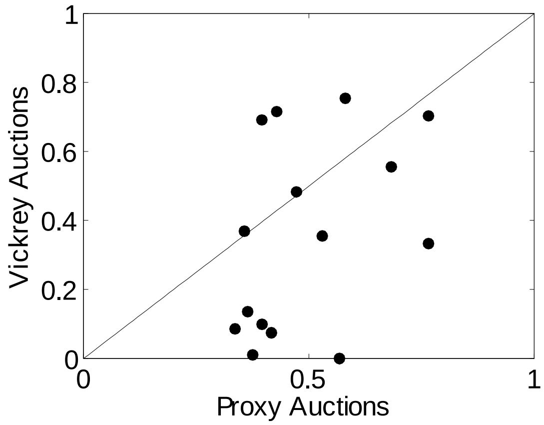

Figure 6: Revenue Comparison

B. Bidding Behavior

Bidders placed 801 bids in proxy auctions with average value 1822 and standard deviation 1417. In Vickrey auctions, bidders placed 331 bids with average value 1663 and standard deviation 1975.

If bidders had placed their sincere bids independently of their values in the additive value environment, then the number of bids less than the median 400 should have been equal to the number of bids more than 400. After assigning the bids at 400 uniformly to below or above 400, proxy auctions had 12% of bids equal or less than 400 and Vickrey auctions had 23% of bids equal or less than 400. Thus bidders placed fewer bids for less valuable packages.

Figures 8 and 9 show the distribution of bids and values in proxy and

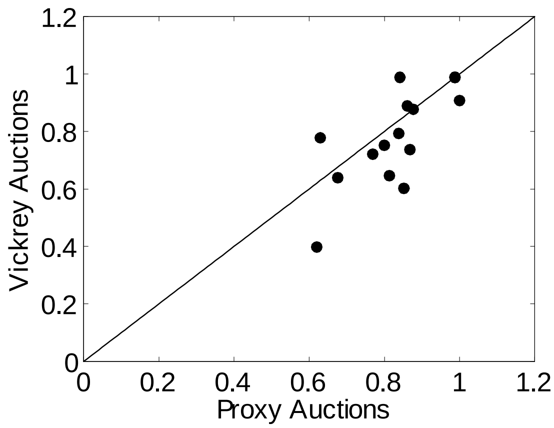

Figure 7: Efficiency Comparison

Vickrey auctions. In proxy auctions, 13% of bids were sincere, 76% were underbids, and 11% were overbids. In Vickrey auctions, 29% of bids were sincere, 56% were underbids, and 14% were overbids.

VI. Conclusion

Allocation of multiple heterogeneous objects is an important policy and business problem. Recent economic theory has challenged the practicality of Vickrey auctions and proposed proxy auctions as an alternative. This paper presents the result of the first experiment to test the revenue ranking of the two auction designs.

Our results yield several new findings. First, bidders did not place bids on every package. Second, proxy auctions had higher revenues re-

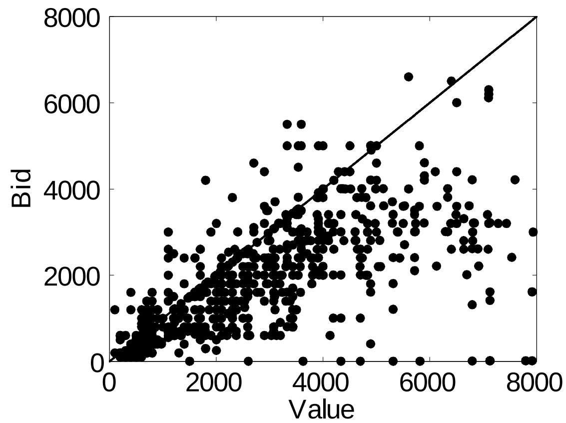

Figure 8: Proxy Auction Bids

gardless of whether the Vickrey theoretical outcome is in the core. We explained this departure from previous theory by considering a model in which bidders with communication constraints do not place bids for some packages, thereby reducing the seller's revenue from Vickrey auctions. Third, these result extended to a large scale experiment involving professional traders. We found robust performance of proxy auctions in both experiments.

We would like to see experiments in more complex environments to test further how well these results generalize. One particularly interesting case would be the case with incomplete information. We still expect that bidders will truncate bids and that this will again reinforce the revenue ranking of the two auction designs. More generally, experimental

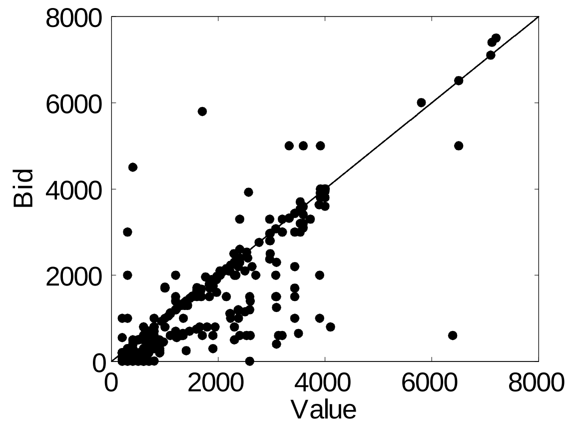

Figure 9: Vickrey Auction Bids

methods can complement theoretical analyses by verifying some predictions and by discovering departures from others that produce new ideas for future analysis.

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