

STATUS QUO BIAS, AMBIGUITY AND RATIONAL CHOICE

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Abstract

Motivated by the extensive evidence about the relevance of status quo bias both in experiments and in real markets, a number of recent papers have studied this phenomenon from a decision-theoretic perspective. This paper focuses as well on this topic, albeit in the context of choice under uncertainty. Following the outline of the general analysis in Masatlioglu and Ok (2005), we develop an axiomatic framework that takes one's choice correspondence (over feasible sets of acts) as the primitive, and provide a characterization according to which the agent chooses her status quo act if nothing better is feasible for a given set of possible priors. If there are feasible acts that dominate the status quo in this sense, she chooses among these by using a single prior in the relative interior of her set of priors. We also show that this choice correspondence is rationalized by a set of ambiguity averse preference relations. Finally, we present two applications. First, we show that, in a financial choice problem, we can have the emergence of a risk premium even with risk neutral agents, as long as these agents abide by the rational choice model with status quo bias we develop here. Similarly, we show how the behavior of such agents would give rise to a gap between willingness to pay and willingness to accept, and offer an intuition for why this gap might diminish as the agent acquires more experience.

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1 Introduction

A striking amount of recent evidence has shown that individual decision makers often attach an additional value to their default options or status quo choice - this is dubbed the *status quo bias*.¹ Instances of this phenomenon have been noted, in numerous experiments that find a discrepancy between willingness to accept and willingness to pay of an individual, and in real markets of 401(k) plans, residential electrical services and car insurance.²

Of course, this evidence has not gone unnoticed in economic theory. In particular, explanations have been attempted by means of reference-dependent choice models with loss aversion, as in Tversky and Kahneman (1991) and Kahneman, Knetsch, and Thaler (1991). And, more recently, a number of papers have analyzed the status quo bias phenomenon from an axiomatic point of view without focusing on loss aversion. In particular, these papers take as primitives the choice functions or the preference relations, and assume at the outset, by means of behavioral axioms, the presence of status quo bias. In turn, they provide a characterization of various individual choice models that embody, per force, the status quo bias phenomenon. Among these papers are Masatlioglu and Ok (2005, 2006), and Sagi (2006).³

Following this second branch of the literature, the main focus of this paper is to present a characterization of a particular type choice correspondence with status quo bias. The main feature that distinguishes our approach from the existing literature is that we analyze the effects of the status quo bias in a rather specific setup: choice under uncertainty.⁴ That is, we focus on the choice among acts whose return depend on the state of the nature, the probability of which is *unknown* to the agent. Indeed, it is precisely in this sort of an environment that many real word examples of status quo bias are observed. For instance, pension and/or insurance plans, the choices of which are known to be affected by this form of bias, are almost always viewed as acts whose consequences are uncertain.

Consider an agent who has to choose an alternative from a given feasible set, and assume that this agent currently holds a default option (act) as her status quo choice. Her task is to decide if she should abandon her status quo

¹This term, coined by Samuelson and Zeckhauser (1988), has become rather standard in the literature. It is extremely close to the notion of *inertia* introduced by Bewley (1986) - more on this later- and it should be distinguished from the “endowment effect,” as discussed also by Tversky and Kahneman (1991) and Masatlioglu and Ok (2006).

²See, among others, Samuelson and Zeckhauser (1988), Kahneman, Knetsch, and Thaler (1991), Hartman, Doane, and Woo (1991), Madrian and Shea (2001) and Ameriks and Zeldes (2004).

³We refer to Masatlioglu and Ok (2006) for a more detailed discussion of the difference between these two strands of literature.

⁴The loss aversion literature already contains models of decision making under uncertainty. See, for instance, Tversky and Kahneman (1992) and Sugden (2003).

and, if so, in favor of which alternative. Suppose all feasible options in this situation have uncertain values. It is well known that the ambiguity aversion of the agent may then well kick in, thereby reducing her confidence in her ability to compare some alternatives.⁵ It seems quite reasonable that this situation might render her status quo particularly relevant for the agent. On the one hand, it is her default option, the most obvious candidate to choose on the face of the difficulties about comparing the feasible alternatives. On the other hand, her default option may be something that the agent is “familiar” with earlier experience, and hence she may feel “less worried” about making a mistake by staying with this choice. By contrast, the other options may be somewhat foreign and hence less attractive to her.⁶ Given this point of view, one might expect the agent to be rather cautious moving away from her status quo choice. But what does “cautious” mean in a setup with uncertainty? Following the classic works of Bewley (1986) and Gilboa and Schmeidler (1989), this may be modeled by means postulating the presence of a set of prior beliefs on the part of the agent, thereby viewing her deciding to abandon her status quo option by requiring some degree of dominance using this entire set. In fact, this is exactly what we find in this paper by means of an axiomatic approach.

This approach is not new the study of status quo bias. The idea that the presence of uncertainty might make the agent “confused”, and induce her stay with her status quo act is first proposed, to the best of our knowledge, by Bewley (1986). Bewley suggests that the presence of uncertainty might force the preference relation of the agent to become incomplete. Moreover, he emphasizes the role of the status quo in this context with the assumption of *inertia*, which says that the agent will remain with her status quo unless there is something better according to her incomplete preference relation - this postulate is quite similar to that of status quo bias. The difference between the present paper and Bewley (1986) is that the latter *assumes* an incomplete ordering and the inertia assumption, and derive the characterization of the behavior involving dominance for a set of priors. By contrast, the characterization that will be presented here posits the status quo bias behaviorally, and *derives* the choice behavior that can be represented in a way similar to that of Bewley (1986).⁷ Hence, the incompleteness of the preferences and the use of multiple priors are found here to be *consequences* of the status quo bias phenomenon itself. In this sense, we offer a behavioral justification for the incompleteness of the preferences of the

⁵This suggest that the agent’s preferences might be incomplete in this framework. This is the basis of the Knightian uncertainty model of Bewley (1986), with which, as we shall see, the present paper has a particularly strong connection.

⁶The point here is not that the agent has better information about the default option, but rather, that she has a psychological sense of familiarity about her status quo, in a similar vein of the explanations given to the phenomenon of home bias in financial investments in particular, and loss aversion, in general.

⁷More precisely, we show that the presence of status quo bias itself, together with some other axioms, *implies* that the choice about moving away from the status quo option can be described by means of an incomplete preference relation, which is itself modeled as in Bewley (1986) by means of multiple priors.

individual - this is something Bewley (1986) assumes at the outset. Furthermore, our analysis includes the presence of the case *without* a status quo, and it also addresses the issue, left undeveloped in the previous works, of what will the agent choose among many objects that are found to dominate the status quo option.⁸

Besides Bewley (1986), the present paper has also a strong connection to Masatlioglu and Ok (2005). In a way, one can think of the present work as a merging of the ideas behind these two papers. Indeed, we adopt the same axiomatic approach of Masatlioglu and Ok (2005). But, instead of focusing on the general (ordinal) case like they do, we focus on the more specific model of choice under uncertainty, and impose additional axioms that are reasonable in this framework. As a benefit of this additional structure, we obtain a tighter characterization. We find that, to abandon the status quo, the agent requires dominance over a set of priors on the state space; by contrast, Masatlioglu and Ok find that the agent requires dominance in an endogenous multi-utility space. This allows us to draw additional interpretations of this behavior, and work out a connection with concepts like ambiguity aversion. In addition, our structure allows us to deal with a possibly infinite prize space, as opposed to the case of Masatlioglu and Ok (2005), in which finiteness is required. Finally, the tighter representation we find allows us to apply our model to choices in which the latter paper, being so general, would have much less to say.

Put concisely, our analysis takes the choice correspondence of an individual as the primitive, but unlike the classical revealed preference theory, and like Masatlioglu and Ok (2005), allows the values of this choice correspondence to depend on an exogenously given feasible alternative, interpreted as the status quo choice of the agent. We impose three sets of axioms on this correspondence. First, we impose the Weak Axiom of Revealed Preference, both with and without status quo, but only as long as the status quo does not change across the considered problems. Second, we posit the properties of Continuity, Monotonicity and Affinity with and without status quo, provided that the status quo itself changes accordingly.

Third, we use a number of axioms that concern the connection between the choice of the agent across problems with distinct status quo points. These properties are borrowed from Masatlioglu and Ok (2005), and include the status quo bias axiom which states that if an option is chosen from a set when it is not the status quo, then it would be chosen uniquely when it *is* the status quo. Given these properties, (discussed in detail below), we find that a particularly interesting individual decision-model arises.

First, we find a utility function on the alternative space along with a prior belief on the state space such that the choice of the agent for a feasible set

⁸Note that, unless these alternatives are ordered according to the incomplete preference relation, the model of Bewley (1986) would have nothing to say about the choice among them.

without a status quo is formed upon the maximization of subjective expected utility, in concert with the standard Savagean viewpoint. But the situation is quite different for problems with a status quo. We find here a set of priors (containing the original prior of the agent in its relative interior) such that the agent chooses to remain with her status quo act unless a feasible act yields a higher level of expected utility than her status quo option with respect to *all* members of the set of priors. Finally, if some elements dominate the status quo act for all priors, the agent chooses among them, and only among them, using her single prior that she has used in the choice without status quo.

The interpretation of this decision-making model is fairly straightforward. If the agent has no status quo act, then she acts as a standard Savagean agent. If she has a status quo act, then she behaves as if she is “scared” of making the wrong move. She then starts entertaining a whole set of priors all around the original one, and requires dominance with respect to all of them for moving away from her default act. Thus, the very presence of her status quo bias induces the agent to act on the basis of a multi-prior decision making procedure. This link, in turn, allows us to relate two apparently distinct concepts: status quo bias and ambiguity aversion. Of course, going *from* ambiguity aversion *to* status quo bias is quite intuitive. An ambiguity averse agent might prefer to keep the status quo since this act might be the one she is “familiar with,” and hence she might well keep it unless she finds something better than her status quo even in the “worst possible case.” What we are able to show in the present work is that, conversely, one may go *from* status quo bias *to* ambiguity aversion. In our framework, an agent with status quo bias has a behavior that can be rationalized by the maximization of a group of preference relations which exhibit ambiguity aversion. Put differently, the presence of status quo bias renders the agent as ambiguity averse, thereby making the connection between these two concepts tighter.

We present two applications of the choice theory model developed here. First, we apply it to the decision-making in a financial market, and show that, if the choice behavior of an agent satisfies our axioms and the status quo is to choose an unambiguous act, then risk premia may emerge in such a market even if agents are risk neutral. Such a result, of course, cannot be obtained within the realm of the standard expected utility model. Moreover, if the agents were known to be risk averse, then our model predicts that the risk premium observed in the market would be much higher than the one predicted by the standard model. This might lead an economist that studies the market in question by means of the standard model to think that the risk aversion of the agent is implausibly high.⁹ We find here that the status quo bias phenomenon offers a simple way of explaining this situation.¹⁰

⁹In fact, the observation that only an implausibly high risk would justify the risk premium in real financial markets is well known. This phenomenon is famously called the “equity premium puzzle.”

¹⁰Moreover, our model can be used to show that the presence of a status quo that not

In our second application, we show how our model can be used to explain the frequently observed gap between willingness to pay and willingness to accept.¹¹ In fact, we prove that as long as the object traded has an uncertain value while money does not, then such a gap always exists, and its dimension depends positively on the “amount of ambiguity” of the value of the object. In turn, this might provide an intuition of why in market experiments we see the gap getting smaller and smaller with experience.¹²

The rest of the paper is organized as follows. Section 2 presents our axiomatic framework and our main characterization theorem. Section 3 analyzes the connection of our model with ambiguity aversion in some detail. Section 4 presents our two applications. Section 5 concludes. The proofs of the main results are relegated to the Appendix.

2 The Model

2.1 The Basic Framework

Our general setup is that of the standard Anscombe-Aumann model. We have a finite set S of possible states of the world and a set X of consequences, which is assumed to be a convex and compact subset of a Banach space. Let $\mathbb{P}(X)$ stand for the set of all Borel probability measures (lotteries) on X . Denote by \mathcal{A} the set of all acts,¹³ that is, the set of all functions $f : S \rightarrow \mathbb{P}(X)$. Of course, $f(s)$ denotes the consequence of act f in state $s \in S$. For any $f \in \mathcal{A}$ and $s \in S$, we denote by f^s the act that yields the consequence $f(s)$ at *every* state.¹⁴

We metrize \mathcal{A} by the product Prokhorov metric, and denote by \mathbf{A} the set of all nonempty closed subsets of \mathcal{A} . We fix the symbol \diamond to denote an object that does not belong to \mathcal{A} . By a *choice problem* we mean a list (A, τ) where $A \in \mathbf{A}$ and either $\tau \in A$ or $\tau = \diamond$. In the first case, the choice problem is said to be a *choice problem with status quo*. We denote by $\mathcal{C}(\mathcal{A})$ the set of all choice problems, and by $\mathcal{C}_{sq}(\mathcal{A})$ the set of all choice problems with status quo. By a *choice correspondence* on $\mathcal{C}(\mathcal{A})$ we mean a map $c : \mathcal{C}(\mathcal{A}) \rightarrow 2^{\mathcal{A}} \setminus \{\emptyset\}$ such that $c(A, \diamond) \neq A$ for all least one $A \in \mathbf{A}$.¹⁵

uncertain might induce the existence of a range of prices for which agents take neither a long nor a short position on a stock. This replicates the result found by Dow and da Costa Werlang (1992), albeit it is obtained by means of the status quo bias phenomenon instead of ambiguity aversion.

¹¹For a survey of such results, see Kahneman, Knetsch, and Thaler (1991), Camerer (1995) or Horowitz and McConnell (2002).

¹²For example, List (2003, 2004).

¹³For simplicity, we refer to as an *act* what is usually called a *horse race lottery*.

¹⁴That is, $f^s(t) := f(s)$ for any $t \in S$.

¹⁵This condition ensures that at least one of the choice correspondences considered in this

2.2 Axioms

Let c be a choice correspondence of $\mathcal{C}(\mathcal{A})$. The axioms that we wish to impose on c can be split into two groups. The first group consists of four axioms that are, respectively, none other than fairly straightforward extensions of the standard axioms of revealed preference, monotonicity, continuity and independence to our setting. In particular, our first axiom says simply that c satisfies the weak axiom of revealed preference across all choice problems with a *fixed* status quo.

Axiom 1 (WARP). For any $(A, \tau), (B, \tau) \in \mathcal{C}(\mathcal{A})$, if $f, g \in A \cap B$, $f \in c(A, \tau)$ and $g \in c(B, \tau)$, then $f \in c(B, \tau)$.

Our next axiom is a monotonicity property. Let us first define the preorder \succeq on \mathcal{A} as follows:

$$f \succeq g \text{ iff } f^s \in c(\{f^s, g^s\}, \diamond) \text{ for all } s \in S.$$

That is, we have $f \succeq g$ whenever what f returns is preferred to what g returns in *every* possible state.

Axiom 2 (Monotonicity). For any $f, g \in \mathcal{A}$ with $f \succeq g$, we have:

- (a) $f \in c(\{f, g\}, \diamond)$;
- (b) If $g \succeq f$ is false, then $f \in c(\{f, g\}, g)$;
- (c) If $g \succeq f$ is true, then, for any $h \in \mathcal{A}$,

$$h \in c(\{f, h\}, f) \text{ implies } h \in c(\{g, h\}, g)$$

and

$$f \in c(\{f, h\}, h) \text{ implies } g \in c(\{g, h\}, h)$$

The interpretation of part (a) is standard. If one act returns in *every* state something that the agent likes at least as much as what the other returns in that state (that is, $f \succeq g$), then she prefers the former to the latter. The rationale of part (b) is similar. If an act f returns something better at every state relative to an act g , and strictly better at some state (that is, $f \succeq g$ but not $g \succeq f$), then it is chosen over g even when g is the status quo. Finally, part (c) says that if the agent is indifferent on what two acts return in *all* possible states (that is, $f \succeq g \succeq f$), then their comparisons with other acts are identical. In particular, if f is “beaten” by an act h even when it is the status quo, then g should compare to h similarly, and if f is preferred to h even when h is the status quo, then g should also be chosen over h in the problem $(\{g, h\}, h)$.

paper are non-degenerate.

Since the main result of this paper is a utility representation type theorem, we need to demand some form of continuity from the choice correspondence at hand. This is articulated in our next postulate.

Axiom 3 (Continuity). $c(\cdot, \diamond)$ has the closed graph property. Moreover, for any $f, g \in \mathcal{A}$ such that $f \succeq g$ is false, and any $(f^n), (g^n) \in \mathcal{A}^\infty$,

- (a) if $g^n \rightarrow g$ and $g^n \in c(\{f, g^n\}, f)$ for all n , then $g \in c(\{f, g\}, f)$.
- (b) if $f^n \rightarrow f$ and $g \in c(\{f^n, g\}, f^n)$ for all n , then $g \in c(\{f, g\}, f)$.

The closed graph property is, of course, standard. In turn, parts (a) and (b) are the corresponding properties for choice problems with status quo, split into upper hemicontinuity (part (a)) and lower hemicontinuity (part (b)).

We next postulate the affinity of the choice correspondence with and without status quo. In what follows the expression $\lambda A + (1 - \lambda)h$ should be understood in the sense of Minkowski, that is, it equals the set $\{\lambda g + (1 - \lambda)h : g \in A\}$, for any $A \in \mathbf{A}$, $h \in \mathcal{A}$ and $0 \leq \lambda \leq 1$.

Axiom 4 (Affinity). For any $A \in \mathbf{A}$, $(g, h) \in A \times A$ and $0 < \lambda \leq 1$,

$$g \in c(A, \diamond) \text{ iff } \lambda g + (1 - \lambda)h \in c(\lambda A + (1 - \lambda)h, \diamond)$$

and

$$g \in c(A, f) \text{ iff } \lambda g + (1 - \lambda)h \in c(\lambda A + (1 - \lambda)h, \lambda f + (1 - \lambda)h).$$

The first part of this axiom is standard. When there is no status quo in her choice problem, the agent satisfies the classical independence axiom.¹⁶ The second part is a natural reflection of this property to the case of choice problems with status quo.¹⁷

Our assumptions so far were about relating the solutions of choice problems with either *fixed* or *varying* status quo acts. By contrast, our last set of axioms regards the connection between the choice *with* and *without* status quo. On this front we follow Masatlioglu and Ok (2005, 2006) here, and refer the reader to those papers for detailed discussions of these properties.

Axiom 5 (Dominance). For any $A \in \mathbf{A}$ and $(B, f) \in \mathcal{C}_{sq}(\mathcal{A})$ with $B \subseteq A$, if $\{g\} = c(B, f)$ and $g \in c(A, \diamond)$, then $g \in c(A, f)$.

The motivation for this axiom is simple. If an act g is revealed to be better than the status quo f , and if this act is also known to be a best choice in a

¹⁶The analogous property is used by Masatlioglu and Ok (2005) in the extension of their principal model to the case of risky choice problems.

¹⁷An analogous axiom is used also by Sagi (2006).

feasible set when there is no status quo, then this act is to be chosen from the latter feasible set even when the status quo is f . Put differently, if g is the best element in A without status quo, and moreover it is known to be better than f even if the latter is the status quo, then g should remain the choice from (A, f) .

Axiom 6 (Status Quo Irrelevance). For any $(A, f) \in \mathcal{C}_{sq}(\mathcal{A})$, if $\{g\} = c(A, f)$ and there does not exist a set $B \subseteq A$ with $B \neq \{f\}$ and $f \in c(B, f)$, then $g \in c(A, \diamond)$.

In words, if the status quo f is never chosen from any subset of a feasible set A (unless it is the only element of that set, of course), then it is revealed to be the worst alternative in A , and, as such, it should not affect the actual choices. Put differently, this property ensures that the status quo has a distorting effect on the choices only when it is chosen at least against one of the elements: otherwise, it is “irrelevant” and the choice with and without status quo are the same. To illustrate, consider the choice among the options {dine at a French restaurant, dine at an Italian restaurant, eat pet food}. The idea of this axiom is that, even if the option “eat pet food” is the status quo, it is so clearly dominated by the other two alternatives that it will not affect the final choice.

A caveat is perhaps called for here. By imposing Axioms 5 and 6, our theory is bound to focus on the mere status quo bias effect of the status quo, and *not* on its reference effect. In particular, if a status quo is “irrelevant”, that is, if everything else would be chosen over it even when it is the status quo, then the agent acts as if she had *no* status quo. Thus, Axiom 6 does not allow such (an undesirable) status quo to act as a reference point that may affect the final choice of the agent. To wit, a reference dependent theory would allow in the example above for the possibility that the status quo being “eat pet food” may make French cuisine more attractive than Italian cuisine. (This resonates better with intuition if we replace “eat pet food” with “eat left overs from previous night”, for instance.) In a similar vein, Axiom 5 rules out the possibility that a certain status quo might positively affect one element and not another, if the latter has ever been chosen against it. Again, this restricts the possibility of the agent to have a status quo that acts as reference point. This suggests that there is room for extending the present theory in a way that allows for status quo choices exerting reference effects. Indeed, precisely this sort of an extension is carried out by the recent work by Masatlioglu and Ok (2006). Our focus here, however, is to understand how a choice theory with status quo bias can be developed under uncertainty, so we adopt as our basic framework the simpler model developed by Masatlioglu and Ok (2005), in which reference effects of status quo alternatives are ignored.

Our final axiom is basic.

Axiom 7 (Status Quo Bias). For any $(A, f) \in \mathcal{C}(\mathcal{A})$, if $g \in c(A, f)$, then $\{g\} = c(A, g)$.

This axiom posits exactly the effect we aim to characterize, and hence has a special relevance for our work. It simply tells us that if the agent had chosen y when it is not the status quo, then it should be chosen uniquely from the same set when it is the status quo. This axiom, forcing the interpretation of the status quo as an object that exerts “attraction” toward itself, is relatively standard in the literature on status quo bias. Besides being adopted by Masatlioglu and Ok (2005), it is the axiom corresponding to that imposed by Sagi (2006) in terms of preferences, and is closely related to the *inertia* assumption of Bewley (1986) and of the many works with this approach.

2.3 The characterization of the choice correspondence

In this section we present our main result, but we need a final bit of notation for this. For any probability vector π on S and continuous function $u : X \rightarrow \mathbb{R}$, we define the map $U_{\pi,u} : \mathcal{A} \rightarrow \mathbb{R}$ by

$$U_{\pi,u}(f) := \sum_{s \in S} \pi(s) \mathbb{E}_{f(s)}(u),$$

where $\mathbb{E}_{f(s)}(u)$ denotes the expected value of u with respect to the probability measure $f(s)$. Clearly, $U_{\pi,u}(f)$ is simply the expected utility of an agent whose prior beliefs over the states of nature is π and whose preferences over the consequences (in X) are represented by the utility function u . Also, for any set Π of probability vectors on S , we denote by $\mathcal{D}_{\Pi,u}(f)$ the set of all $g \in \mathcal{A}$ such that $U_{\pi,u}(g) \geq U_{\pi,u}(f)$ for all $\pi \in \Pi$, where $>$ holds for at least one $\pi \in \Pi$. This is the set of all acts that dominate f in terms of all the expected utilities induced by the set Π of priors.

We can now state our main result.

Theorem 1. *A choice correspondence c on $\mathcal{C}(\mathcal{A})$ satisfies Axioms 1-7 if, and only if, there exist a non-constant continuous function $u : X \rightarrow \mathbb{R}$, a unique, compact, $(|S| - 1)$ -dimensional convex set Π of probability vectors on S and a probability vector ρ in the relative interior of Π , such that*

$$c(A, \diamond) = \operatorname{argmax}_{g \in A} U_{\rho,u}(g)$$

and

$$c(A, f) = \begin{cases} \{f\}, & \text{if } A \cap \mathcal{D}_{\Pi,u}(f) = \emptyset \\ \operatorname{argmax}_{g \in A \cap \mathcal{D}_{\Pi,u}(f)} U_{\rho,u}(g), & \text{otherwise} \end{cases}$$

for any $A \in \mathbf{A}$ and $f \in A$.

The agent whose choice correspondence satisfies Axioms 1-7 can thus be thought of as one with a utility function u over the prize space, a prior ρ and a

set Π of priors on S that are “around” ρ in the sense that ρ is in the relative interior of Π . When there is no status quo choice for her in a given choice situation, the agent evaluates the available acts on the basis of their subjective expected utilities (by using her ρ and u), just like a standard Savagean agent. When she has a status quo act, however, she acts as if she were “scared” of making the wrong move: in that case, she considers, rather, the whole set Π of priors, and before abandoning her status quo she wants to have a dominance relative to *all* of these priors. If there are no acts dominating the status quo in terms the expected utility values with respect to each of her priors $\pi \in \Pi$, then she gets conservative and stays with her status quo; otherwise, she chooses among the acts that dominate her status quo, and only among them, those acts that yield the highest expected utility relative to her *original belief* ρ . The status quo act f , thus, affects the ultimate choice in two distinct ways: (1) by inducing the agent to choose it if nothing dominates it for all priors (that is, when $\mathcal{D}_{\Pi,u}(f)$ does not contain any feasible act); and (2) by constricting the set of acts from which the agent may choose from.

To illustrate the choice procedure obtained in Theorem 1, consider the case in which there are two states and the choice between the acts f, g and h , whose utility returns are depicted in Figure 1. Here, $I(f, \pi)$ represents the set of elements with the same expected return of f , when this is computed using the prior π . The agent’s unique prior ρ , is the uniform distribution across the two states. So, if there is no status quo act, g would be the final choice of the agent from $\{f, g, h\}$. Now consider the case in which f is the status quo. Theorem 1 tells us that the agent would now have a entire set of priors Π all “around” ρ . Here, as an example, we assume that Π is computed by considering a constant “spread” around ρ in any possible direction. Let π_1 and π_2 be the extreme points of Π . Then, we see that if f is the status quo, then the agent would choose h from the set $\{f, g, h\}$. Indeed, g has a lower return than f when the expected value is computed using prior π_2 , that is, $g \notin \mathcal{D}_{\Pi,u}(f)$. Therefore, although g has a higher return than h for the original prior ρ , f being a status quo affects the choice of the agent by making him choose $h \in \mathcal{D}_{\Pi,u}(f)$. Again, we can interpret this as if the agent wanted to be “sure” not to make the wrong move from her status quo choice f , and hence opt for h , which is better for *all* of her priors, against g , which seems better for some priors, but worse for others. Notice, moreover, that if h were not available, then the agent would remain with f instead of moving to g , hence the status quo bias phenomenon.

Following this interpretation, and the fact that the agent has a unique original prior about the state of the world and then considers a neighborhood of other priors around it, a connection with several other works in the literature materializes. In particular, the behavior depicted in Theorem 1 is very much in the same vein with the one described by the theory of robust control preferences (Hansen and Sargent (2000), Hansen and Sargent (2001)), or by the more general theory of variational preferences (Maccheroni, Marinacci, and Rustichini (2006)). The former, in particular, characterize the behavior of the agent as

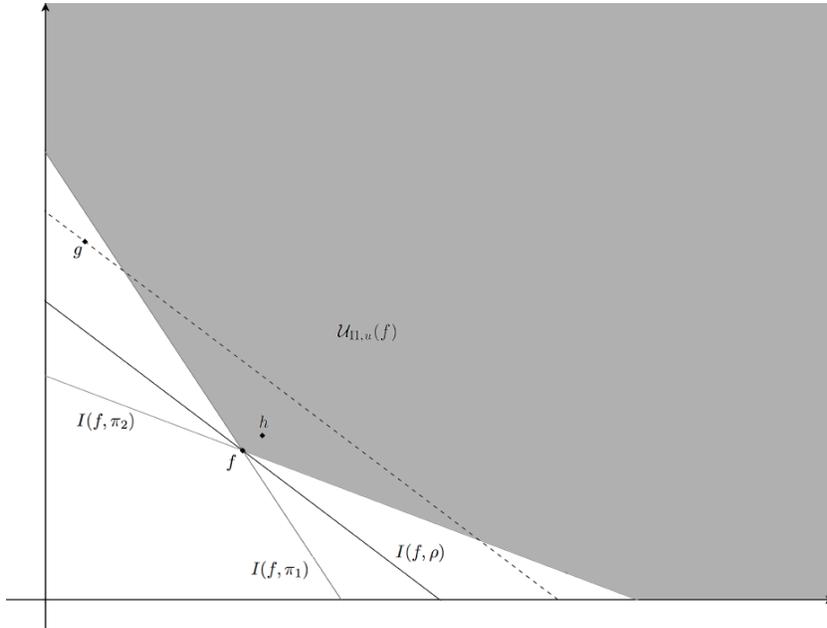


Figure 1: Example

if she had in mind not only a single prior, but also a neighborhood of priors around it, computed by considering a possible error in the choice of the single prior.¹⁸

Also worth noting is the close link of Theorem 1 to that of Masatlioglu and Ok (2005). After all, we have adopted here all of the axioms of that paper, albeit in a more specific framework, that allows us to impose additional (but standard) axioms. In brief, our analysis comes down to combining Masatlioglu-Ok's choice-with-status-quo model with the preferences-under-uncertainty framework of Anscombe-Aumann. This allows us to obtain a tighter characterization of the choice correspondence than that of the first model. Here we obtain a unique utility function and a convex compact set of prior beliefs, as opposed to multiple utility functions. Moreover, our framework enables us to work with a (possibly) infinite prize space, by contrast to Masatlioglu and Ok (2005).

Similarly, Theorem 1 is deeply connected with the Knightian Uncertainty model of Bewley (1986). In this well-known work, Bewley assumes an incomplete preference relation in a setup very similar to ours, and obtains a characterization of it by means of dominance in terms of expected utility for a compact and

¹⁸Of course, the representation they obtain is different since they consider ambiguity aversion, and hence model agents as if they took the worst of all possible priors in the set. The same is true for the comparison with the variational preferences. (More on this in Section 3.)

convex set of priors. Moreover, the author adds an additional assumption, *inertia*, quite similar to our status quo bias axiom, which says that the agent will keep his current endowment unless she has dominance with respect to her incomplete preference relation. What his work does not say, however, is what would the agent do to choose between two or more non-comparable acts that dominate the current endowment. By contrast, our work, although based on the same setup, takes choice correspondences as a primitive. On them, we impose similar axioms, including status quo bias, and obtain a characterization very similar to that of Bewley (1986) in terms of what would the agent do when deciding whether to keep the status quo act or to switch to another act. (Indeed, Bewley’s representation is a main ingredient of our proof.) From this point of view, our model could be seen as a behavioral justification of the previous one.¹⁹ But unlike that of Bewley, our model also predicts what would the agent do in case there were more than one act that dominate the status quo act for all possible priors. In this case, to choose between these dominating acts the agent uses a (complete) refinement of the incomplete relation, in particular, she uses her unique (original) prior in the relative interior of her set of priors. Finally, our model, as opposed to Bewley (1986), also considers the case of no status quo act, and provides a precise link between the problems with and without status quo acts.

3 Relation with Ambiguity Aversion

The characterization found in Theorem 1 by means of the status quo bias axiom is reminiscent of that found by Gilboa and Schmeidler (1989) from the ambiguity aversion axiom. In both cases the agent makes her choice according to a set of priors, and she does it with a strong degree of “pessimism.” Yet, the two characterizations also have some differences. As opposed to the result of Gilboa and Schmeidler (1989), in our choice model the agent considers the set of priors only when comparing elements with the status quo, while comparisons between other elements is performed according to a unique prior. Moreover, when performing the comparison with multiple priors, the agent requires dominance with respect to each and every prior, as opposed to comparing the lowest subjective expected utilities of the feasible act.²⁰

On the other hand, and this is the content of this section, there is in fact

¹⁹In fact, by looking at the proof of our Theorem, it is immediate to notice that we *obtain* from our axioms the incomplete preference relation that Bewley (1986) *assumes*: we will then use his result to characterize it. To be noticed, this incomplete preference relation is obtained from axioms reminiscent of inertia, which then can be seen from a behavioral point of view as a cause for incompleteness, and not as an additional requirement, as it seems to be in previous works.

²⁰In fact, dominance in all priors implies that the minimum value over priors is higher, but not viceversa.

a deeper connection between our axiomatization and the one with ambiguity aversion besides the reminiscence of the two characterizations. In particular, we show that a choice correspondence that satisfies Axioms 1-7 can be characterized by means of the maximization of a set of complete preference relations all but one endowed with ambiguity aversion. To formalize this point, recall that Gilboa and Schmeidler (1989) says that a preference relation \succeq on \mathcal{A} satisfies *ambiguity aversion* if, for any $f, g \in \mathcal{A}$ and $\alpha \in (0, 1)$, $f \sim g$ implies $\alpha f + (1 - \alpha)g \succeq f$. The following result clarifies how this notion is, in fact, embedded in our model.²¹

Proposition 1. *If a choice correspondence c on $\mathcal{C}(\mathcal{A})$ satisfies Axioms 1-7, then there exist a set of complete preference relations $\{\succeq_\diamond, \{\succeq_f\}_{f \in \mathcal{A}}\}$ such that:*

1. for all $(A, \tau) \in \mathcal{C}(\mathcal{A})$,

$$c(A, \diamond) = \text{MAX}(A, \succeq_\diamond) \quad \text{and} \quad c(A, f) = \text{MAX}(A, \succeq_f) \quad (1)$$

2. \succeq_\diamond satisfies the independence axiom.
3. \succeq_f satisfies ambiguity aversion for all $f \in \mathcal{A}$, and there exist $f, g, h \in \mathcal{A}$ such that $g \sim_f h$ and $\alpha g + (1 - \alpha)h \succ_f g$ for some $\alpha \in (0, 1)$.

Consider the case in which there are two states and the choice between the acts f, g and h , whose utility revenues are depicted in Figure 2.²² The choice model characterized in Theorem 1 maintains that if the agent had no status quo, she would be indifferent between g and h , and would strictly prefer both to f . Now consider the element $\frac{1}{2}g + \frac{1}{2}h$: by affinity, without status quo it will be clearly indifferent between g and h , and strictly preferred to f . Now, consider the problem $c(\{f, g, h\}, f)$ and notice that f is the unique choice, since both g and h return a lower expected utility with respect to at least one prior. Now examine instead the problem $c(\{f, g, h, \frac{1}{2}g + \frac{1}{2}h\}, f)$. Here $\frac{1}{2}g + \frac{1}{2}h$ is the unique choice, since, as is apparent from Figure 2, it is preferred to f for all priors. Hence, a mixture of non-dominating elements can be itself dominating, which is the basic intuition behind Proposition 1.²³

²¹In what follows, if \succeq stands for a preference relation of \mathcal{A} (that is, a reflexive and transitive binary relation on \mathcal{A}), then $\text{MAX}(A, \succeq)$ stands for the set of all maximal elements on A with respect to \succeq , that is, $\text{MAX}(A, \succeq) = \{f \in A : g \succ f \text{ for no } g \in A\}$, where \succ is the asymmetric part of \succeq .

²²The interpretation of Figure 2 is identical to that of Figure 1. In particular, ρ is the original prior of the agent and the convex hull of π_1 and π_2 constitutes her prior set Π .

²³This result might suggest that we could characterize the choice correspondence by separately characterizing each of the preference relations found in Proposition 1 using, for example, the results of Gilboa and Schmeidler (1989) or of Maccheroni, Marinacci, and Rustichini (2006). This, however, is not possible here. The reason is that neither C-Independence of Gilboa and Schmeidler (1989), nor Weak C-Independence of Maccheroni, Marinacci, and Rustichini (2006) are satisfied. To see this, consider any of these preference relations \succeq_f , and consider the act obtained by mixing the status quo f with some other unambiguous act x . The new act would not have the additional ‘‘status quo power’’ that f had, and therefore its ranking will be lower. Hence even these weaker forms of independence would not be satisfied.

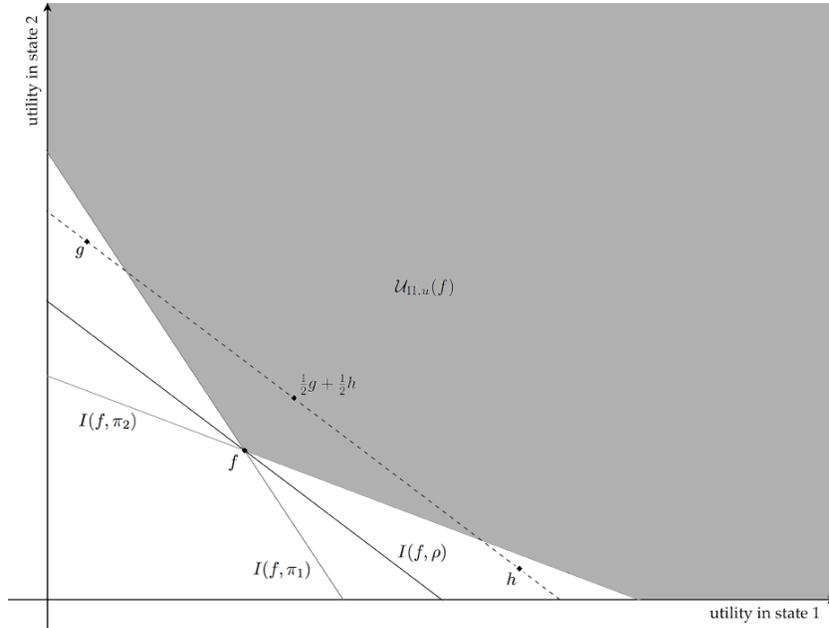


Figure 2: Example 2

We have therefore found a connection between two apparently distinct concepts: status quo bias and ambiguity aversion. Indeed, such a connection is intuitive in the direction *from* ambiguity aversion *to* status quo bias. In fact, one might think that an ambiguous averse agent might want to keep her status quo unless something better even for the worst “possible” prior is available.²⁴ What we have shown here is the less intuitive direction, i.e. *from* status quo bias *to* ambiguity aversion. That is, assuming status quo bias in a specific axiomatic structure implies that the agent’s behavior shows ambiguity aversion. We shall, in fact, exploit this connection in a number of applications below, showing that some interesting results that could be derived by assuming ambiguity aversion, can also be seen as consequences of the status quo bias phenomenon.

²⁴Yet, obtaining such a conclusion in our framework is not immediate, since we would then be in need of a dynamic model that consider the possibility that agents obtain some information specific on their status quo. A formal treatment of this direction of the connection is left for a further development.

4 Applications

4.1 Risk premium with risk neutral agents

Our first application of Theorem 1 aims to explain how risk premia can emerge in a financial market with *risk neutral* participants whose choice behavior abide by our axioms - status quo bias in particular. As is well known, a risk premium cannot possibly arise in the case of a market that consists of risk neutral standard expected utility maximizing agents without status quo bias. Consider an economy in which there is one representative agent, a government and a firm. There are two possible states of the world: s_1 , the “good state”, and s_2 , the “bad state”. The government issues a government bond, which is traded for the price p_b and yields, with certainty, B \$. The firm can issue a stock, priced at price p_{st} , which yields payoffs M \$ and m \$ respectively in the two states of the world, s_1 and s_2 , where $m < B < M$. The representative agent can choose whether to buy a stock, a bond, or not to invest. To keep the analysis simple, we assume that only one of these three options can be taken.²⁵ The agent’s choice behavior satisfies Axioms 1-7 and her status quo or default option is *not to invest in the market*.²⁶ We can therefore use the characterization found in Theorem 1. Moreover, we assume that she is *risk neutral*, and that her utility is the difference between the money return received in the realized state and the price payed.

More formally, define $S := \{s_1, s_2\}$ and $X := \mathbb{R}$. We focus on the choice behavior regarding three acts: buy the stock, st ; buy the bond, b ; keep the money uninvested, ni . For any given M, B, m, p_{st}, p_b , define them as: $b(s) := B - p_b$ for all $s \in S$; $ni(s) = 0$ for all $s \in S$; $st(s_1) := M - p_{st}$ and $st(s_2) := m - p_{st}$. Assume that the choice of the agent in this setup satisfies Axioms 1-7. Then, we know from Theorem 1 that there exist a utility function u on X , a single prior ρ and a full dimensional set of priors Π that rationalize her choice in the prescribed way. Define the agent’s utility function on X as $u(x) := x$, that guarantees risk neutrality.

We also assume that in the economy there are market analysts who agree on a probability distribution on the states $\hat{\pi}$ (on the basis of, say, a suitable regression analysis), with full support. Let us impose, again for simplicity, that $M \cdot \hat{\pi}(s_1) + m \cdot \hat{\pi}(s_2) = B$, that is, the expected payoff of the stock and the bond are the same with respect to the probability distribution declared by the analysts. This information is common knowledge in the market, and we assume, for illustrative purposes, that the agent maximizes expected utility according to the same prior $\hat{\pi}$ in the absence of a status quo option, that is, $\rho = \hat{\pi}$. In

²⁵That is, we assume that the agent can only buy *one* stock or *one* bond, and that she cannot buy both. Also, she is not allowed to sell any of these.

²⁶This seems to be the most natural choice in this environment. To be noted, we would get the same result if the status quo were to invest in the bond.

this sense, an agent without a status quo has the behavior prescribed in the standard model.²⁷ Finally, consider the set Π of priors, which we know to be not a singleton, and define $\underline{\pi}$ as the prior in Π which assigns the smallest probability to state s_1 being realized. Again for illustrative purposes, assume that $B > p_b > M\underline{\pi}(s_1) + m\underline{\pi}(s_2)$.²⁸ We will now analyze the choice of the agent in this environment.

First of all, notice that, since $B > p_b$, buying the bond is surely better than leaving the money uninvested. Therefore, the agent would never choose to leave the money uninvested if the bond was available. Notice also that, if the stock has the same price as the bond, the agent would *not* buy it. This is because, if $p_{st} = p_b$, we have $p_{st} = p_b > M\underline{\pi}(s_1) + m\underline{\pi}(s_2)$, which means that the expected return of buying the stock computed using prior $\underline{\pi}$ is negative, so the agent is better off by leaving the money uninvested. Therefore, since the stock is worse than the status quo of *not investing*, for at least one prior in the set Π , it will not be chosen by the agent. Instead, she would then buy the bond.

This implies that, if the stock is traded in the market, its price must be below that of the bond, and in particular, below $M\underline{\pi}(s_1) + m\underline{\pi}(s_2)$. If this is the case, then the agent would buy only the stock (and not the bond). The implication of this is that, although the bond and the stock have the same expected payoff according to the market analysts, the stock must be priced below the price of the bond for it to be sold, a result that we cannot have if we were considering standard expected utility maximizer agents.

Let us now compute the risk premium in this economy. First, notice in this case that the risk-free rate is equal to the expected return of the bond, that is $r_b := \frac{B-p_b}{p_b}$. At the same time, the expected return of the stock according to the market will be

$$r_s := \frac{M\hat{\pi}(s_1) + m\hat{\pi}(s_2) - p_t}{p_t} = \frac{B - p_t}{p_t}.$$

Since $p_s < p_b$, we then have $r_s > r_b$. This means that there is a *positive risk premium* in this economy, *even though the agent is risk neutral*. Obviously, such a case cannot materialize with “standard” expected utility maximizing agents.

We emphasize that risk premium emerges in a situation where the status quo “do not invest” is never chosen whatever the price of the stock and the set of priors are. This underlines an important feature of our model. The presence of the status quo might affect the final choice of a decision maker although it is not itself chosen. Hence, we might have a role for the status quo “do not invest” also

²⁷To be more precise, we also assume that there is no asymmetric information, and in particular, the agent has the same information as the market analysts. Also, market analysts are not “players”, and will always report their true prior.

²⁸Notice that it is *always* possible to find p_b that satisfies this condition, since the set of priors Π is full-dimensional and $\hat{\pi}$ is in its relative interior.

in more realistic situations, in which many people do actually choose to invest. Of course, the same qualitative effect would have been found if the status quo were buying the bond. Also, notice that this risk premium is larger the “larger” the set of priors is. In particular, it depends on the worst possible prior in the set Π . The more “pessimistic” the agent is, that is, the more she is scared of moving away from the status quo to an uncertain alternative, the higher the risk premium will be in the market.

Two additional characteristics of this result are worth emphasizing. The first one regards the fact that if an external observer (economist) studied this market but disregarded the role of the status quo, she would erroneously deduce that the agent is risk averse. In particular, it is easy to see that if the agent were *really* risk averse, then the observer would believe that she were even more risk averse, possibly attributing to the individual implausible levels of risk aversion. This situation is not unfamiliar to the macro-finance literature, where extremely high levels of risk aversion are required to justify the risk premium observed in financial markets; this is dubbed the *equity premium puzzle*. This emphasizes the relevance of considering the role of the status quo bias when analyzing decisions taken in this framework.

Moreover, it is worth emphasizing the connection of this result with the similar one found by assuming that the agent is ambiguity averse in the sense of Gilboa and Schmeidler (1989), or has preferences characterized by robustness in the sense of Hansen and Sargent (2000, 2001).²⁹ Indeed, in both of these cases we would obtain a similar risk premium arising in this environment even with risk neutral agents.³⁰ Our contribution is to obtain a “multiprior” explanation of high risk premia from the status quo bias phenomenon, and not from ambiguity aversion, showing therefore that this very explanation could be obtained with a completely different axiomatic framework.

4.2 The gap between willingness to pay and willingness to accept

We now apply our model to explain the so-called endowment effect, that is, the frequently observed gap between willingness to accept (WTA) and willingness to

²⁹For the case of ambiguity aversion, see Chen and Epstein (2002), Epstein and Wang (1994). For the case of robust control, precisely on this topic, see Barillas, Hansen, and Sargent (2007). Notice, however, that the models based on ambiguity aversion do not offer a term of comparison, i.e. a single prior to be used by the market to compute the risk premium. This is not the case here, since our characterization, as well as that of robustness, is such that we start from a *single* prior and a “neighborhood” of priors around it. Hence, we have a natural term of comparison emerging from the model.

³⁰Again, the same risk premium may arise, but the behavior might be different. In particular, concerning the choice between elements that both dominate the status quo, our characterization is such that the comparison is done according to a unique prior, while both ambiguity aversion and robustness would induce the use of multiple priors in this case as well.

pay (WTP).³¹ First, we wish to show that the presence of such a gap is not only consistent, but is in fact predicted by our model. Second, we argue that this methodology suggests an intuition of why these sorts of gaps may get reduced when traders obtain more experience, in accordance to market experiments.³² To the best of our knowledge, this behavior has not yet received a theoretical explanation.³³ In what follows we focus on a specific example, the trading of mugs, due to Knetsch (1989).

To analyze this problem in our framework, we consider the situation in which agents are uncertain about the value of mugs. This uncertainty can have many plausible interpretations: it can relate to how much the agent will like a mug that she has not evaluated “carefully enough,” about how much she will need it or ever use it, or it can relate to the price at which she will be able to sell the mug in the future, or both. In particular, the value of the traded object seems to be more clearly uncertain in the case of objects that the agent might want to sell in a later stage, such as the case of trading sports car, the one in which the diminishing of the gap with experience has been observed. Here the uncertainty about the real “value” of a mug for an agent seems particularly stringent for unexperienced traders, who should be aware of not being able to correctly forecast the price of the mug in the future, as opposed to experienced ones, who have a stronger training, and therefore confidence, in making predictions on the market. This suggests that the amount of uncertainty about the value of the mug could decrease with experience. What we will show next is that, when uncertainty decreases, the gap between WTA and WTP would decrease as well, obtaining therefore a possible explanation of the experimental evidence that shows exactly this phenomenon.

Let us now move to a more formal treatment. We first need to define the setup in which the model is applied. To this end, fix a space S and define

$$X := I \times A$$

where I is an interval of the form $[0, \alpha]$, with α being a number large enough, and A is a subset of \mathbb{R}_+ such that $0 \in A$ and $|A| \geq 3$. The interval I stands for the amounts of money that they can change hands during the trade, while A is the set of possible “values” that can be obtained from a mug, or from not having it: 0 if the agent does not own a mug, strictly positive otherwise.³⁴

Let us now define what we mean by owning a mug and not owning it. In

³¹For a survey of such results, see Kahneman, Knetsch, and Thaler (1991), Camerer (1995, pp. 665-670), or Horowitz and McConnell (2002).

³²For example, List (2003, 2004).

³³The model of Masatlioglu and Ok (2006) can indeed accommodate both the presence and the absence of the gap, but that paper does not offer an explanation about why this gap should decrease with experience.

³⁴This formulation is reminiscent of the one adopted by Masatlioglu and Ok (2006). In their case, however, there is no uncertainty on the value of the mug, and the set A is therefore equal to $\{0, 1\}$, to simply indicate whether it is owned or not.

this setup, a mug is be an object the benefits of which depend on the state of the world that is realized. In particular, define the function $b : S \rightarrow A \setminus \{0\}$ that indicates the outcome that the mug returns in the different states of the world: hence, a mug returns $b(s) \in A \setminus \{0\}$ in state $s \in S$. To guarantee the uncertainty on its value, let us assume that there exist $s, s' \in S$ such that $b(s) \neq b(s')$.

In this analysis we will focus on the choice behavior only regarding specific acts, that is, keeping or selling a mug for a range of prices. To this end, for any $w \in I$ define the act of “not owning” the mug and having w units of money as

$$a(w, 0)(s) := (w, 0)$$

Since there is no uncertainty about the value of money, this act is degenerate. By contrast, define

$$a(w, 1)(s) := (w, b(s)) , s \in S$$

This is the act that represents the situation of “owning” the mug and having w units of money.

Now, consider an agent with an endowment of $w_0 \in I$ and define the *willingness to accept* $wta(c)$ as the smallest amount of money that this agent endowed with a mug is willing to accept to “sell” it. Formally,

$$wta(c) := \inf\{d \geq 0 : a(w_0 + d, 0) \in c(\{a(w_0 + d, 0), a(w_0, 1)\}, a(w_0, 1))\} \quad (2)$$

Also, define the *willingness to pay*, $wtp(c)$ as

$$wtp(c) := \sup\{w_0 \geq e \geq 0 : a(w_0, 1) \in c(\{a(w_0, 1), a(w_0 + e, 0)\}, a(w_0 + e, 0))\} \quad (3)$$

This represents the maximum amount of money that an agent who does not own a mug is willing to give up in order to obtain it. We model this “payment” as a foregone gain rather than a loss, following the original discussion of Tversky and Kahneman (1991).³⁵

Now, we want to see how $wta(c)$ and $wtp(c)$ compare in our setup. Assuming that c satisfies Axioms 1-7, we represent c by means of u , ρ and Π as in Theorem 1. In addition, we assume that u is increasing in both components (which seems unexceptionable in this framework). We can write $wta(c)$ as

$$wta(c) = \inf\{d \geq 0 : u(w_0 + d, 0) \geq \mathbb{E}_\pi[u(w_0, b(s))] \forall \pi \in \Pi, > \exists \pi \in \Pi\}$$

which means that $wta(c)$ is such that

$$u(w_0 + wta(c), 0) \geq \max_{\pi \in \Pi} \mathbb{E}_\pi[u(w_0, b(s))].$$

³⁵This approach, justified also by empirical analysis, allows us to have a direct comparison with the previous works in the literature. Notice however that, if the utility function is additive in its two components and is not convex in money, then our result would hold even is we assumed payment as a loss.

Analogously, we have

$$wtp(c) = \sup\{w_0 \geq e \geq 0 : \mathbb{E}_\pi[u(w_0, b(s))] \geq u(w_0 + e, 0) \forall \pi \in \Pi, > \exists \pi \in \Pi\}$$

which means that $wtp(c)$ is such that

$$\min_{\pi \in \Pi} \mathbb{E}_\pi[u(w_0, b(s))] \geq u(w_0 + wtp(c), 0).$$

Since Π cannot be a singleton, and since there exist $s, s' \in S$ such that $b(s) \neq b(s')$, then

$$\begin{aligned} u(w_0 + wta(c), 0) &\geq \max_{\pi \in \Pi} \mathbb{E}_\pi[u(w_0, b(s))] \\ &> \min_{\pi \in \Pi} \mathbb{E}_\pi[u(w_0, b(s))] \\ &\geq u(w_0 + wtp(c), 0) \end{aligned}$$

Since u is increasing in both components, we find therefore $wta(c) > wtp(c)$, as is sought.

This shows that the gap between willingness to pay and willingness to accept emerges in our model as long as the value of the mug is uncertain for the agent. Moreover, this way of modeling the gap might provide an intuition of why this gap decreases as agents get more experienced.³⁶ It seems reasonable that the uncertainty a trader faces decrease as she gets more experienced, which can, in our model, be captured by positing that the set Π of priors shrinks with experience. But as Π shrinks, the difference $\max_{\pi \in \Pi} \mathbb{E}_\pi[u(w_0, b(s))]$ and $\min_{\pi \in \Pi} \mathbb{E}_\pi[u(w_0, b(s))]$ shrinks as well. Therefore, the gap between WTA and WTP will be smaller, which is exactly what is observed in the market experiments.

To conclude, notice that we have now applied our analysis on the gap between WTA and WTP on a specific example, that of the trade of the mugs, but of course it can be generalized to other cases. For example, we could apply our model in a similar manner to the study of an investment decision, and in particular to the choice on whether to buy an asset or sell it short. In fact, following the same steps we have followed here, one can prove that by means of our model we can show that the presence of an unambiguous status quo induces the existence of a range of prices for which agents do not take neither a long nor a short position on an uncertain stock. This replicates the result found by Dow and da Costa Werlang (1992), albeit it is obtained assuming status quo bias instead of ambiguity aversion.

³⁶In order to do this formally we would need a dynamic model, as opposed to the static one we have developed here. Nevertheless, we can get an immediate intuition what would happen in a dynamic setting, and it is on this intuition that we will focus here.

5 Conclusions

In this paper we have developed, axiomatically, a revealed preference model that rationalizes the choice behavior with status quo bias in a setup with uncertainty. In this model an agent acts as if she had multiple priors such that she wants weak dominance for all of them, and strict for at least one, in order to choose something different from the status quo. This model combines the classical Knightian Uncertainty model with the status quo bias phenomenon, and allows us to draw a connection between status quo bias and ambiguity aversion. We have also shown how this model may be used to “explain” phenomena like the equity premium puzzle and the endowment effect.

Appendix: Proofs

Proof of Theorem 1

We consider first the “only if” part of the assertion. Let c be a choice correspondence that satisfies Axioms 1-7. For any $f \in \mathcal{A}$ and preorder \succeq on \mathcal{A} , denote by U_{\succ} the strict upper contour set of f with respect to \succeq , that is, $U_{\succ}(f) := \{g \in \mathcal{A} : g \succ f\}$.

Claim 1. There exists a partial order \succeq and a completion \succeq^* such that $c(\cdot, \diamond) = \text{MAX}(\cdot, \succeq^*)$ and

$$c(A, f) = \begin{cases} \{f\}, & \text{if } f \in \text{MAX}(A, \succeq) \\ \text{MAX}(U_{\succ}(f) \cap A, \succeq^*), & \text{otherwise} \end{cases} \quad (4)$$

for all $(A, f) \in \mathcal{C}_{sq}(\mathcal{A})$

Proof. Apply Lemma 1 in Masatlioglu and Ok (2005). \square

It is easy to see that the partial order \succeq here must be of the following form:

$$f \succ g \Leftrightarrow f \in c(\{f, g\}, g)$$

Moreover, since $c(\cdot, \diamond) = \text{MAX}(\cdot, \succeq^*)$, standard arguments guarantees that, given the imposed axioms on c , the induced complete preference relation \succeq^* satisfies the requirements of the Anscombe-Aumann Expected Utility Theorem, and hence there exist a non-constant continuous function $u : X \rightarrow \mathbb{R}$ and a probability vector ρ on S such that

$$f \succeq^* g \Leftrightarrow \sum_{s \in S} \rho_s \mathbb{E}_{f(s)}(u) \geq \sum_{s \in S} \rho_s \mathbb{E}_{g(s)}(u) \Leftrightarrow U_{\rho, u}(f) \geq U_{\rho, u}(g),$$

for all $f, g \in \mathcal{A}$. Moreover, ρ is unique and u is unique up to positive affine transformations. Since $c(\cdot, \diamond) = \text{MAX}(\cdot, \succ^*)$, we have

$$c(A, \diamond) = \underset{g \in A}{\text{argmax}} U_{\rho, u}(g)$$

We now characterize the incomplete preference relation \succ . In order to do this, define \succ' as $\succ' := \succ$ and $\sim' := \{(f, g) \in \mathcal{A}^2 : f^s \sim^* g^s \text{ for all } s \in S\}$.

Claim 2. \succ' is a preference relation.

Proof. Notice that all we have to show is that \succ' satisfies transitivity. Notice also that it is an incomplete relation that is derived by adding elements to the incomplete preference relation \succ : all we have to show, then, is that adding this additional relations still keeps the transitivity. Since the relations we are adding are all of indifference, and the original ones are all strict, we have to show only two things. First, that if $f \succ' g$ and $g \sim' h$, then $f \succ h$, hence $f \succ' h$: to see it, notice that $f \succ' h$ must derive from $f \succ h$; but then Axiom 2.(c) tells us that $f \succ h$, hence $f \succ' h$. Same goes for showing that $f \succ' g$ and $f \sim' h$, then $h \succ g$, and this proves transitivity. \square

Now, it is immediate to see that $\succ' \neq \emptyset$ (from the fact that $\succ^* \neq \emptyset$ and Axiom 2.(b)), that \succ' satisfies Monotonicity (from Axiom 2.(b)), Upper and Lower Hemicontinuity (from Axiom 3.(a) and 3.(b)), Independence (from Axiom 4). We shall now show that \succ' satisfies the Partial Completeness Axiom. That is, if $\langle p \rangle \in \mathcal{A}$ is the constant act returning $p \in \mathbb{P}(X)$ in all states, the induced preference relation $\bar{\succ}$ on $\mathbb{P}(X)^2$ defined by $p \bar{\succ} q \Leftrightarrow \langle p \rangle \succ' \langle q \rangle$ for all $p, q \in \mathbb{P}(X)$, is complete and $\bar{\succ} \neq \emptyset$.

Claim 3. $\bar{\succ}$ satisfies the Partial Completeness Axiom. Moreover, $\bar{\succ} \neq \emptyset$.

Proof. Take $p, q \in \mathbb{P}(X)$. Notice that, since \succ^* is complete, then either $\langle p \rangle \succ^* \langle q \rangle$, or $\langle q \rangle \succ^* \langle p \rangle$ or $\langle p \rangle \sim^* \langle q \rangle$. If we are in one of the first two cases, then Axiom 2 and the representation of \succ^* tell us that we have $\langle p \rangle \succ' \langle q \rangle$ or $\langle q \rangle \succ' \langle p \rangle$. Else, if $\langle p \rangle \sim^* \langle q \rangle$, then by definition of \succ' we have $\langle p \rangle \sim' \langle q \rangle$. Hence, $\bar{\succ}$ is complete on unambiguous acts, and so $\bar{\succ}$ is complete. Finally, notice that since $\succ^* \neq \emptyset$, then there exists $p, q \in X$ such that $v(p) > v(q)$, hence $\langle p \rangle \succ^* \langle q \rangle$, which means that $\bar{\succ} \neq \emptyset$. \square

We then can apply Bewley's Expected Utility Theorem to characterize \succ' .

Claim 4. There exists a unique nonempty convex compact set Π of probability vectors on S such that, for all $f, g \in \mathcal{A}$,

$$f \succ' g \Leftrightarrow U_{\pi, u}(f) \geq U_{\pi, u}(g) \quad \text{for all } \pi \in \Pi$$

where u is the same as previously found using the Expected Utility Theorem.

Proof. First notice that \succ' satisfies all requirements of Bewley's Expected Utility Theorem. Therefore, there exists a $\psi \in \mathbf{C}(X)$ such that

$$f \succ' g \Leftrightarrow U_{\pi, \psi}(f) \geq U_{\pi, \psi}(g) \quad \text{for all } \pi \in \Pi$$

Consequently, we have:

$$\langle p \rangle \succeq' \langle q \rangle \Leftrightarrow \mathbb{E}_p(\psi) \geq \mathbb{E}_q(\psi)$$

for all $p, q \in \mathbb{P}(X)$. Now notice that \succeq' and \succeq^* agrees on constant acts due to Partial Completeness Axiom and to the fact that \succeq^* extends \succeq' . Hence, u that we have found with the Expected Utility Theorem must be a positive affine transformation of ψ . The assertion follows from this observation. \square

We have therefore a characterization of the preference relation \succeq . Recall the definition of $\mathcal{D}_{\Pi, u}(f)$ for $f \in \mathcal{A}$, $\mathcal{D}_{\Pi, u}(f) := \{g \in \mathcal{A} : U_{\pi, u}(g) \geq U_{\pi, u}(f) \text{ for all } \pi \in \Pi \text{ strictly for some}\}$. Notice that, given the characterization of our preference relation, then $U_{\succ}(f) = \mathcal{D}_{\Pi, u}(f)$ for all $f \in \mathcal{A}$. By using this last result, our characterization of \succeq^* and Lemma 1, we get to the characterization of the choice correspondence found in the Theorem.

We will now prove that the prior ρ used to evaluate the options without status quo is actually inside the set of priors Π and that the set Π is full dimensional in the simplex of $\mathbb{R}^{|S|}$, i.e. $\dim(\Pi) = |S| - 1$.

Claim 5. $\rho \in \Pi$

Proof. By contradiction. Say $\rho \notin \Pi$. Then, $\{\rho\} \cup \Pi \neq \Pi$. Now, notice that for any $f, g \in \mathcal{A}$, $U_{\pi, u}(f) \geq U_{\pi, u}(g)$ for all $\pi \in \Pi$, strictly for some, implies $f \succeq g$, which implies $f \succeq^* g$, hence $U_{\rho, u}(f) \geq U_{\rho, u}(g)$. This means that whenever the expected value is higher for all $\pi \in \Pi$, strictly for some, then it is higher for ρ as well. Moreover, take any $\hat{\pi} \in \Pi$, $\alpha \in (0, 1)$ and consider the prior $\pi' = \alpha\rho + (1 - \alpha)\hat{\pi}$. Now, whenever we have $U_{\pi, u}(f) \geq U_{\pi, u}(g)$ for all $\pi \in \Pi$, strictly for some, we know that we have $U_{\rho, u}(f) \geq U_{\rho, u}(g)$, which means that $U_{\pi', u}(f) \geq U_{\pi', u}(g)$. Define $\Pi' = \text{co}(\Pi \cup \rho)$, where $\text{co}(A)$ indicates the convex hull of the set A . By previous reasoning and by convexity of Π , we must have that whenever $U_{\pi, u}(f) \geq U_{\pi, u}(g)$ for all $\pi \in \Pi$, strictly for some, then we have $U_{\hat{\pi}, u}(f) \geq U_{\hat{\pi}, u}(g)$ for all $\hat{\pi} \in \Pi'$, strictly for some. Notice moreover that Π' is convex and compact.³⁷ But this contradicts the uniqueness of a convex, compact Π . \square

Claim 6. $\dim(\Pi) = |S| - 1$.

³⁷Because Π is compact, hence $\Pi \cup \rho$ is compact, and $\Pi \cup \rho \subset \mathbb{R}^{|S|}$ since we have a finite set of states: hence we can use the property that the convex hull of a compact subset of \mathbb{R}^n is compact and we are done.

Proof. Say, by contradiction, that this is not the case, which means, since Π is a subset of the simplex of $\mathbb{R}^{|S|}$, that $\dim(\Pi) < |S| - 1$. Notice that this implies that there does not exist $|S|$ linearly independent element in Π , which in turn means that there exists $x \in \mathbb{R}^{|S|}$, $x \neq 0$ such that $\sum_{s \in S} x_s \pi_s = 0$ for all $\pi \in \Pi$ ³⁸. Notice that this implies that for all $\lambda \in \mathbb{R}$, then $\sum_{s \in S} x_s(\lambda) \pi_s = 0$ for each $\pi \in \Pi$. Now, consider $y, z \in \mathbb{P}(X)$ such that $v(y) > v(z)$ (the existence of which have been discussed before). Consider any $\bar{\lambda} \in \mathbb{R}_{++}$ such that $\max_{s \in S} \bar{\lambda} |x_s| < v(y) - v(z)$ and define $\bar{x} := \bar{\lambda} x$. Now define two acts $f, g \in \mathcal{A}$ as follows:

$$f(s) := \frac{1}{2}y + \frac{1}{2}z \text{ for all } s \in S$$

and

$$g(s) := \frac{\bar{x}_s}{v(y) - v(z)}y + \left(1 - \frac{\bar{x}_s}{v(y) - v(z)}\right)\left(\frac{1}{2}y + \frac{1}{2}z\right)$$

First, notice that, since $\max_{s \in S} \bar{x}_s < v(y) - v(z)$, then both f and g are well defined. Then, notice that $f \neq g$ but $\sum_{s \in S} v(f(s))\pi_s = \sum_{s \in S} v(g(s))\pi_s$ for all $\pi \in \Pi$. Indeed,

$$\begin{aligned} \sum_{s \in S} v(g(s))\pi_s &= \sum_{s \in S} \pi_s \left[\frac{\bar{x}_s}{v(y) - v(z)}v(y) + \left(1 - \frac{\bar{x}_s}{v(y) - v(z)}\right) \left(\frac{1}{2}v(y) + \frac{1}{2}v(z)\right) \right] \\ &= \sum_{s \in S} \frac{\pi_s \bar{x}_s}{2} \left(\frac{v(y)}{v(y) - v(z)} - \frac{v(z)}{v(y) - v(z)} \right) + \sum_{s \in S} \pi_s \left(\frac{1}{2}v(y) + \frac{1}{2}v(z)\right) \\ &= \sum_{s \in S} v(f(s))\pi_s \end{aligned}$$

where the last passage follows from the fact that $\sum_{s \in S} x_s \pi_s = 0$ for all $\pi \in \Pi$.

Now consider any $(\lambda_n) \in \mathbb{R}_{++}^\infty$ such that $\lambda_n \rightarrow 0$ and define $(f^n) \in \mathcal{A}^\infty$ as $f^n(s) := \lambda_n y + (1 - \lambda_n)\left(\frac{1}{2}y + \frac{1}{2}z\right)$. Clearly, $f^n \rightarrow f$ and $\sum_{s \in S} v(f^n(s))\pi_s > \sum_{s \in S} v(f(s))\pi_s = \sum_{s \in S} v(g(s))\pi_s$ for all $\pi \in \Pi$. This implies that $f^n \in c(\{f^n, g\}, g)$ for each n . Moreover, $v(f(s)) \neq v(g(s))$ for some $s \in S$ (since $\bar{x} \neq 0$), so that we cannot have both $f \succeq g$ and $g \succeq f$. By Axiom 3.(b) therefore, since $f^n \rightarrow f$, we have $f \in c(\{f, g\}, g)$, hence $f \succ g$. But this is contradiction because we have $\sum_{s \in S} v(f(s))\pi_s = \sum_{s \in S} v(g(s))\pi_s$ for all $\pi \in \Pi$. \square

We now finally have to show that our prior ρ is in the relative interior of the set Π .

Claim 7. $\rho \in \text{rint}(\Pi)$

Proof. First notice that $\text{rint}(\Pi)$ is always non empty, and that if Π were a singleton, we would be done. Then, say that Π is not a singleton and proceed by

³⁸The reason for this is that, if there were at least $|S|$ linearly independent vectors in Π , then the system of equations $\sum_{s \in S} x_s \pi_s = 0$ for all $\pi \in \Pi$ will admit only one solution, $x = 0$. But since this is not the case, standard arguments in linear algebra points out that there is at least another non-zero solution.

contradiction assuming that ρ does not belong to $\text{rint}(\Pi)$: hence, ρ is a boundary point of Π relative to $\text{aff}(\Pi)$ (and also relative to $\mathbb{R}^{|S|}$, of course). Since Π is a convex subset of $\mathbb{R}^{|S|}$, then consider a supporting hyperplane of Π at ρ whose norm vector \bar{v} lies in $\text{span}(\Pi)$. So, we have $(\bar{v}, \bar{\alpha})$ such that $\sum_s \bar{v}_s \rho_s = \bar{\alpha}$. Moreover, since $\text{rint}(\Pi) \neq \emptyset$ and \bar{v} lies in $\text{span}(\Pi)$, then this is a non-trivial supporting hyperplane, i.e. there exist a $\pi \in \Pi$ such that $\sum_{s \in S} \bar{v}_s \pi_s > \bar{\alpha}$. Consider a normalization of the it, denoted (v, α) , such that all the elements of v are included in $[-1; 1]$, and such that $\alpha \in [-1; 1]$. Now, consider $x, y \in \mathbb{P}(X)$ such that $v(x) > v(y)$ (non-degeneracy of \succeq^* , proven before, guarantees their existence). Then, define the compound lotteries $z, w \in \mathbb{P}(X)$ as $z := \frac{2}{3}x + \frac{1}{3}y$ and $w := \frac{1}{3}x + \frac{2}{3}y$. Now, define the act (z, w, v) as $(z, w, v)(s) = zv_s + w(1 - v_s)$ ³⁹. Moreover, define the act (z, w, α) as the act returning $\alpha z + (1 - \alpha)w$ in every possible state. Then notice the following:

$$\begin{aligned} \sum_{s \in S} v((z, w, v)(s))\rho_s &= \sum_{s \in S} (v(x)\frac{v_s + 1}{3} + v(y)(1 - \frac{v_s + 1}{3}))\rho_s \\ &= v(x)\left(\frac{\sum_{s \in S} v_s \rho_s + \sum_{s \in S} \rho_s}{3}\right) + v(y)\left(\sum_{s \in S} \rho_s - \frac{\sum_{s \in S} v_s \rho_s + \sum_{s \in S} \rho_s}{3}\right) \\ &= v(x)\frac{\alpha + 1}{3} + v(y)\left(1 - \frac{\alpha + 1}{3}\right) \end{aligned}$$

Moreover, notice that

$$\sum_{s \in S} v((z, w, \alpha)(s))\rho_s = v(x)\frac{\alpha + 1}{3} + v(y)\left(1 - \frac{\alpha + 1}{3}\right)$$

This means that $(z, w, v) \sim^* (z, w, \alpha)$. It is easy to notice, moreover, that since $\sum_{s \in S} v_s \pi_s \geq \alpha$ for each $\pi \in \Pi$, then we must have $\sum_{s \in S} v((z, w, v)(s))\pi_s \geq \sum_{s \in S} v((z, w, \alpha)(s))\pi_s$ for all $\pi \in \Pi$. Now, consider $\pi' \in \Pi$ such that $\sum_{s \in S} v_s \pi'_s > \alpha$ ⁴⁰. Then notice that

$$\begin{aligned} \sum_{s \in S} v((z, w, v)(s))\pi'_s &= \sum_{s \in S} (v(x)\frac{v_s + 1}{3} + v(y)(1 - \frac{v_s + 1}{3}))\pi'_s \\ &= v(x)\left(\frac{\sum_{s \in S} v_s \pi'_s + 1}{3}\right) + v(y)\left(1 - \frac{1 + \sum_{s \in S} \pi'_s}{3}\right) \end{aligned}$$

while we have

$$\sum_{s \in S} v((z, w, \alpha)(s))\pi' = v(x)\frac{\alpha + 1}{3} + v(y)\left(1 - \frac{\alpha + 1}{3}\right)$$

³⁹Notice that this is an act that returns a lottery: in every state, it will return x with probability $\frac{2}{3}v(s) + \frac{1}{3}(1 - v(s))$ and y with probability $\frac{1}{3}v(s) + \frac{2}{3}(1 - v(s))$. It is immediate to see that, since $v(s) \in [-1; 1]$, these are well defined probabilities, even with $v_s < 0$.

⁴⁰We are sure that such a π' exists by a previous argument.

Since $v(x) > v(y)$ and $\sum_{s \in S} v_s \pi'_s > \alpha$, then

$$\sum_{s \in S} v((z, w, v)(s)) \pi'_s > \sum_{s \in S} v((z, w, \alpha)(s)) \pi'_s.$$

This means that we have $\sum_{s \in S} v((z, w, v)(s)) \pi \geq \sum_{s \in S} v((z, w, \alpha)(s)) \pi_s$ for all $\pi \in \Pi$, strictly for at least one π (namely, π'). Hence, $(z, w, v) \succ (z, w, \alpha)$. But since we had $(z, w, v) \sim^* (z, w, \alpha)$, this contradicts the fact that \succeq^* extends \succeq . \square

This concludes the proof of the “only if” direction. We now turn to the direction. The proof for WARP, Monotonicity, Affinity, Dominance and Status Quo Irrelevance is either standard or trivial: we are then left with Continuity and Status Quo Bias. As for Continuity, the closed graph property of $c(\cdot, \diamond)$ can be easily proved using continuity of u by standard arguments. Now consider part (a). Consider $(g^n) \in \mathcal{A}^\infty$, $g^n \rightarrow g$, $g^n \in c(\{g^n, f\}, f)$ for all n , and such that we do not have $f \supseteq g$. First notice that, if we had $g \supseteq f$, then Axiom 2.b would guarantee $g \in c(\{f, g\}, f)$, and we would be done: hence, assume this is not the case. Notice that, for all n , $g^n \in c(\{g^n, f\}, f)$ implies $U_{\pi, u}(g^n) \geq U_{\pi, u}(f)$ for all $\pi \in \Pi$, strictly for some. Hence, $g^n \rightarrow g$ implies $U_{\pi, u}(g) \geq U_{\pi, u}(f)$ for all $\pi \in \Pi$ for continuity of u . Now, if this inequality holds strictly for some $\pi \in \Pi$, then $g \in c(\{f, g\}, f)$ and we are done. We will now show that this must be the case. Say not, hence we have $U_{\pi, u}(g) = U_{\pi, u}(f)$ for all $\pi \in \Pi$. This means that $\sum_{s \in S} \pi_s (v(g(s)) - v(f(s))) = 0$ for all $\pi \in \Pi$. But this cannot happen since $v(g(s)) - v(f(s)) \neq 0$ (since we do not have $f \supseteq g$ and $g \supseteq f$), and we know, since $\dim(\Pi) = |S| - 1$, that the only $z \in \mathbb{R}^{|S|}$ such that $\sum_{s \in S} z_s \pi_s = 0$ for all $\pi \in \Pi$ is $z = 0$. For part (b), consider any $f, g \in \mathcal{A}$, $(f^n) \in \mathcal{A}^\infty$, $f^n \rightarrow f$, $g \in c(\{g, f^n\}, f^n)$ for all n , and we do not have $f \supseteq g$. Notice again that if we had $g \supseteq f$ the claim would follow from Axiom 2.b, so assume this is not the case. Then, we must have $U_{\pi, u}(g) \geq U_{\pi, u}(f^n)$ for all n and for all $\pi \in \Pi$, strictly for some, which implies $U_{\pi, u}(g) \geq U_{\pi, u}(f)$ for all $\pi \in \Pi$. Again, if there exist a $\pi \in \Pi$ such that the inequality holds strictly we are done, since it means that $y \in c(\{x, y\}, x)$. As we did for part (b), we shall now prove that this must be the case: say not, then we have $\sum_{s \in S} \pi_s (v(g(s)) - v(f(s))) = 0$ for all $\pi \in \Pi$, and that both $f \supseteq g$ and $g \supseteq f$ are false. We have already shown that this is a contradiction.

As for Status Quo Bias, suppose that we have $g \in c(A, f)$. If $f = g$ the claim is trivial. For $f \neq g$, we want to show that $\{g\} = c(A, g)$. Assume, by contradiction, that there exists a $h \in A, h \neq g$ such that $h \in c(A, g)$. Now, this implies that $h \in \mathcal{D}_{\Pi, u}(g)$; at the same time, $g \in c(A, f)$ implies $g \in \mathcal{D}_{\Pi, u}(f)$, and hence we have $h \in \mathcal{D}_{\Pi, u}(f)$: but then we must have $h \in c(A, f)$, since $h \in \mathcal{D}_{\Pi, u}(g)$, which means $U_{\rho, u}(h) = U_{\rho, u}(g)$. Thus, we have the following: $U_{\rho, u}(h) = U_{\rho, u}(g)$, $U_{\pi, u}(h) \geq U_{\pi, u}(g)$ for all $\pi \in \Pi$, and $U_{\bar{\pi}, u}(h) > U_{\bar{\pi}, u}(g)$ for some $\bar{\pi} \in \Pi$. Now, for compactness of Π and continuity of u , there must exist an $\epsilon > 0$ such that $U_{\pi', u}(h) > U_{\pi', u}(g)$ for all $\pi' \in B_\epsilon(\bar{\pi})$ where $B_\epsilon(\bar{\pi})$ is the

ball of ray ϵ around $\bar{\pi}$.⁴¹ Now, notice that with a simple application of Choquet Theorem, we could obtain ρ as the resultant of a strictly positive probability distribution μ on Π . Since $B_\epsilon(\bar{\pi})$ is open, we must have $\mu(B_\epsilon(\bar{\pi})) > 0$. But then we have a contradiction: we cannot have $U_\pi(h) \geq U_{\pi,u}(g)$ for all $\pi \in \Pi$, $U_{\pi'}(h) > U_{\pi',u}(g)$ for all $\pi' \in B_\epsilon(\bar{\pi})$ with $\mu(B_\epsilon(\bar{\pi})) > 0$, and $U_\rho(h) = U_{\rho,u}(g)$ (we must have $U_{\rho,u}(h) > U_{\rho,u}(g)$ since μ is a strictly positive distribution).

This concludes the proof of Theorem 1.

Q.E.D.

Proof of Proposition 1

Consider the following set of preference relations $\{\succeq_\diamond, \{\bar{\succeq}_f\}_{f \in \mathcal{A}}\}$ defined as follows:

$$f \succeq_\diamond g \Leftrightarrow f \in c(\{f, g\}, \diamond)$$

and

$$h \bar{\succeq}_f g \Leftrightarrow h \in c(\{f, g, h\}, f)$$

Clearly, \succeq_\diamond will be complete, while, for all $f \in \mathcal{A}$, $\bar{\succeq}_f$ might not be, since we can have $\{f\} = c(\{f, g, h\}, f)$ ⁴². We will consider a completion \succeq_f of it by following the ordering without the status quo for the cases in which it is incomplete. That is, for all $f \in \mathcal{A}$ define \succeq_f as:

- if $\{f\} \neq c(\{f, g, h\}, f)$, then

$$h \bar{\succeq}_f g \Leftrightarrow h \in c(\{f, g, h\}, f)$$

- if $\{f\} = c(\{f, g, h\}, f)$, then

$$h \succeq_f g \Leftrightarrow h \in c(\{h, g\}, \diamond)$$

It is straightforward to see that, for all $f \in \mathcal{A}$, \succeq_f is a preference relation and is complete.

We now will characterize these preference relations in terms of independence or ambiguity aversions. First consider \succeq_\diamond : because of affinity of the choice correspondence, it will satisfy independence. Then, consider the preference

⁴¹Otherwise, if such a ball didn't exist, then there must exist a sequence of probability vectors (π_m) in Π converging to $\bar{\pi}$ such that $U_{\pi_m,u}(h) = U_{\pi_m,u}(g)$. But then, for and continuity of u , $U_{\pi_m,u}(h) \rightarrow U_{\bar{\pi},u}(h)$ and $U_{\pi_m,u}(g) \rightarrow U_{\bar{\pi},u}(g)$, which implies $U_{\bar{\pi},u}(h) = U_{\bar{\pi},u}(g)$, a contradiction.

⁴²Notice that in our specification of the problem with status quo, to have a status quo we need to have the possibility to choose it again.

relations \succeq_f for some $f \in \mathcal{A}$. Let us use the characterization of the choice correspondence found in Theorem 1. We can have $h \sim_f g$ in two possible cases. Either $h, g \in c(\{f, g, h\}, f)$, or $\{f\} = c(\{f, g, h\}, f)$ and $h, g \in c(\{h, g\}, \diamond)$. Following Theorem 1, in the first case we will have that $h, g \in \mathcal{D}_{\Pi, u}(f)$ and $U_{\rho, u}(h) = U_{\rho, u}(g)$: since $h, g \in \mathcal{D}_{\Pi, u}(f)$, then $\alpha h + (1 - \alpha)g \in \mathcal{D}_{\Pi, u}(f)$ for any $\alpha \in (0, 1)$; moreover, $U_{\rho, u}(h) = U_{\rho, u}(g)$ implies $U_{\rho, u}(\alpha h + (1 - \alpha)g) = U_{\rho, u}(g)$ for any $\alpha \in (0, 1)$. Hence, in this case we must have $\alpha h + (1 - \alpha)g \sim_f h, g$. Consider now the other case, in which $\{f\} = c(\{f, g, h\}, f)$ and $h, g \in c(\{h, g\}, \diamond)$. Again by Theorem 1, $h, g \notin \mathcal{D}_{\Pi, u}(f)$ and $U_{\rho, u}(h) = U_{\rho, u}(g)$. Now, consider $\alpha h + (1 - \alpha)g$ for some $\alpha \in (0, 1)$ and notice that two possible cases can arise here. Either we have $\alpha h + (1 - \alpha)g \in c(\{\alpha h + (1 - \alpha)g, f\}, f)$, in which case we have that $\alpha h + (1 - \alpha)g \succ_f f$, and therefore $\alpha h + (1 - \alpha)g \succ_f h, g$. Or, we have that $\{f\} = c(\{\alpha h + (1 - \alpha)g, f\}, f)$, in which case, for the same reasoning as before, we have $\alpha h + (1 - \alpha)g \sim_f h, g$.

Finally, we have to show that for some $f, g, h \in \mathcal{A}$, $\alpha \in (0, 1)$ we have $f \sim_h g$ but $\alpha f + (1 - \alpha)g \succ_h f$. To see this, consider $x, y \in X$ such that $U(x) > U(y)$ (the existence of which has been previously discussed), and $s_1, s_2 \in S$ such that $\rho_{s_1} \geq \rho_{s_2}$ ⁴³. Now define $f, g \in \mathcal{A}$ as: $f(s_1) = y$, $f(s_2) = x$ and $f(s) = x$ for all $s \neq s_1, s_2$; $g(s_1) = \frac{\rho_{s_2}}{\rho_{s_1}}x + (1 - \frac{\rho_{s_2}}{\rho_{s_1}})y$, $g(s_2) = y$ and $g(s) = x$ for all $s \neq s_1, s_2$. Now notice that:

$$\begin{aligned} U_{\rho, u}(f) &= \sum_{s \neq s_1, s_2} u(x) + \rho_{s_2} u(y) + \rho_{s_1} \left(\frac{\rho_{s_2}}{\rho_{s_1}} u(x) + (1 - \frac{\rho_{s_2}}{\rho_{s_1}}) u(y) \right) \\ &= \sum_{s \neq s_1, s_2} u(x) + \rho_{s_1} u(y) + \rho_{s_2} u(x) = U_{\rho, u}(g) \end{aligned}$$

This means that f and g are indifferent without status quo. Consider $z \in \mathcal{A}$ such that $z := \frac{1}{2}f + \frac{1}{2}g$, where clearly $U_{\rho, u}(z) = U_{\rho, u}(f)$. Notice also that, since $\dim(\Pi) = |S| - 1$, then there exist $\pi_1, \pi_2 \in \Pi$ such that $U_{\pi_1, u}(z) > U_{\pi_1, u}(f)$ and $U_{\pi_2, u}(z) > U_{\pi_2, u}(g)$. Now define for each $\epsilon \in (0, 1]$ $f_\epsilon, g_\epsilon \in \mathcal{A}$ as: $f_\epsilon(s_1) = \epsilon x + (1 - \epsilon)y$ and $f_\epsilon(s) = f(s)$ for all $s \in S \setminus \{s_1\}$; $g_\epsilon(s_2) = \epsilon x + (1 - \epsilon)y$ and $g_\epsilon(s) = g(s)$ for all $s \in S \setminus \{s_2\}$. Now, notice that there exist ϵ_1, ϵ_2 small enough such that $U_{\pi_1, u}(z) > U_{\pi_1, u}(f_{\epsilon_1})$ and $U_{\pi_2, u}(z) > U_{\pi_2, u}(g_{\epsilon_2})$ for some $\pi_1, \pi_2 \in \Pi$. This means that $\{z\} = c(\{f_{\epsilon_1}, g_{\epsilon_2}, z\}, z)$, hence $f_{\epsilon_1} \sim_z g_{\epsilon_2}$ by our definition of \succeq_z . Now consider the act $\omega \in \mathcal{A}$ defined as $\omega := \frac{1}{2}f_{\epsilon_1} + \frac{1}{2}g_{\epsilon_2}$. Clearly we have $U_{\pi, u}(\omega) \geq U_{\pi, u}(z)$ for all $\pi \in \Pi$, strictly for some⁴⁴. But this implies that $\{\omega\} = c(\{z, \omega, f_{\epsilon_1}\}, z)$, hence $\omega \succ_z f_{\epsilon_1}$, and we are done.

Q.E.D.

⁴³By Monotonicity we already know that both will be positive.

⁴⁴This is immediate if we consider the definitions of z and ω : the latter is very similar to z but includes ϵ_1 and ϵ_2 probabilities to have something better in some states. Hence, it cannot return a lower expected utility for any prior, and will return a strictly higher one for some (namely, those assigning a positive probability to s_1 or s_2).

References

- AMERIKS, J. AND S. P. ZELDES (2004): “How Do Household Portfolio Shares Vary with Age?” Discussion Paper, Columbia University.
- BARILLAS, F., L. HANSEN, AND T. J. SARGENT (2007): “Risk-sensitive valuations of aggregate consumption fluctuations: risk or model uncertainty?” Mimeo, New York University.
- BEWLEY, T. F. (1986): “Knightian Decision Theory: Part I,” *Cowles Foundation discussion paper*, 807.
- CAMERER, C. (1995): “Individual decision making,” in *Handbook of Experimental Economics*, ed. by A. R. J. Kagel, Princeton University Press, Princeton, vol. 2.
- CHEN, Z. AND L. EPSTEIN (2002): “Ambiguity, Risk and Asset Returns in continuous time,” *Econometrica*, 70, 1403–1443.
- DOW, J. AND S. R. DA COSTA WERLANG (1992): “Uncertainty Aversion, Risk Aversion, and the Optimal Choice of Portfolio,” *Econometrica*, 60, 197–204.
- EPSTEIN, L. G. AND T. WANG (1994): “Intertemporal Asset Pricing under Knightian Uncertainty,” *Econometrica*, 62, 283–322.
- GILBOA, I. AND D. SCHMEIDLER (1989): “Maxmin expected utility with a non-unique prior,” *Journal of Mathematical Economics*, 18, 141–153.
- HANSEN, L. P. AND T. J. SARGENT (2000): “Wanting robustness in macroeconomics,” Mimeo, New York University.
- (2001): “Robust control and model uncertainty,” *American Economic Review*, 91, 60–66.
- HARTMAN, R., M. DOANE, AND C.-K. WOO (1991): “Consumer Rationality and the Status Quo,” *Quarterly Journal of Economics*, 106, 141–162.
- HOROWITZ, J. K. AND K. E. MCCONNELL (2002): “A Review of WTA/WTP studies,” *Journal of Environmental Economics and Management*, 44, 426–447.
- KAHNEMAN, D., A. AMOS, AND A. THALER (1991): “Anomalies: The Endowment Effect, Loss Aversion, and Status Quo Bias,” *Journal of Economic Perspectives*, 5, 193–206.
- KNETSCH, J. L. (1989): “The Endowment Effect and Evidence of Nonreversible Indifference Curves,” *American Economic Review*, 79, 1277–1284.
- LIST, J. A. (2003): “Does Market Experience Eliminate Market Anomalies?” *Quarterly Journal of Economics*, 118, 41–71.

- (2004): “Neoclassical Theory Versus Prospect Theory: Evidence from the Marketplace,” *Econometrica*, 72, 615–625.
- MACCHERONI, F., M. MARINACCI, AND A. RUSTICHINI (2006): “Ambiguity Aversion, Robustness, and the Variational Representation of Preferences,” *Econometrica*, 74, 1447–1498.
- MADRIAN, B. C. AND D. F. SHEA (2001): “The Power of Suggestion: Inertia in 401(k) Participation and Savings Behavior,” *Quarterly Journal of Economics*, 107, 1149–1187.
- MASATLIOGLU, Y. AND E. A. OK (2005): “Rational Choice with status quo bias,” *Journal of Economic Theory*, 121, 1–29.
- (2006): “Reference-Dependent Procedural Decision Making,” Mimeo, New York University.
- SAGI, J. (2006): “Anchored Preference Relations,” *Journal of Economic Theory*, 130, 283–295.
- SAMUELSON, W. AND R. ZECKHAUSER (1988): “Status quo bias in decision making,” *Journal of Risk and Uncertainty*, 1, 7–59.
- SUGDEN, R. (2003): “Reference-dependent subjective expected utility,” *Journal of Economic Theory*, 111, 172–191.
- TVERSKY, A. AND D. KAHNEMAN (1991): “Loss Aversion in Riskless Choice: A Reference-Dependent Model,” *Quarterly Journal of Economics*, 106, 1039–1061.
- (1992): “Advances in prospect theory: cumulative representation of uncertainty,” *Journal of Risk and Uncertainty*, 5, 297–323.