Partition Obvious Preference and Mechanism Design: Theory and Experiment

By LUYAO ZHANG AND DAN LEVIN*

Substantial experimental evidence shows that decision makers often fail to choose an available dominant strategy in tasks that requires forming hypothetical scenarios and reason state-by-state. Our proposed axiomatic approach, Partition Obvious Preference, formalizes such a deficiency in reasoning by weakening the Subjective Expected Utility Theory. We extend our approach to games and propose a new solution concept, partition dominant strategy, providing a theoretical explanation for the difference in dominant strategies and superior performance of dynamic mechanism over its strategic equivalent static implementation. Our new solution concept is a useful discovery for designers of markets and mechanisms as it enriches the class of mechanisms that perform better than those that only have a dominant strategy. We conduct a laboratory experiment to test and verify our theory and its implications.

Keywords: An Axiomatic Approach, Mechanism Design, Bounded Rationality

* Zhang: Ohio State University, 410 Arps Hall, 1945 N. High St, Columbus, OH 43210 (email: zhang.2625@osu.edu); Levin: Ohio State University, 433B Arps Hall, 1945 N. High St, Columbus, OH 43210 (email: Levin.36@osu.edu).

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Dominant strategy (DS) is optimal regardless of what other players contemplate or do. It is the strongest solution concept that game theory offers, and thus the most preferred implementation in the mechanism design literature (Maskin and Sjöström 2001). Yet, frequent deviations from DS have been found in experimental studies.
For example, Kagel, Harstad and Levin (1987) find significant deviations from the DS of bidding one’s value, mainly in the form of overbidding in *Second Price Sealed Bid* (SPSB) auctions with *affiliated private value*. Similar results emerge in SPSB auction with *independent private value* in experiments by Kagel and Levin (1993) and Harstad (2000). Garratt, Walker and Wooders (2011) show that even eBay sellers, who have substantial prior experience with auctions in the field don’t bid their value.\(^1\) Similar violations have been documented in other environments such as one-shot *Prisoner’s Dilemma* and *Public Goods* games (Dawes 1980; Dawes and Thaler 1988; Atiyeh 2000), *School Choice* (Chen and Sönmez 2006; Fack et al. 2015; Ding and Schotter 2015; Hassidim et al. 2016, 2017), *Matching Program* (Rees-Jone 2016), *Voting* (Esponda and Vespa 2014) and in the choice of *Health Insurance* (Bhargava et al 2015). Being the best response under any subjective beliefs implies that its violation cannot be explained by models that relax beliefs in equilibrium, such as Analogy-Based Expectation Equilibrium (Jehiel 2005), Level\(_k\) (Stahl and Wilson 1994, 1995; Nagel 1995; Crawford and Iriberri 2007) and Cursed Equilibrium (Eyster and Ragin 2005).

Several behavioral models use non-standard preferences, such as, *Joys of Wining* (Harrison 1989) or *Spite* motives (Morgan et al. 2003) to explain overbidding in *private-value, Second-Price* auctions.\(^2\) However, Kagel, Harstad and Levin (1987) also find a quick convergence to the DS in the strategically equivalent English auction, raising the question, “Where have all the joys or spites gone in English auctions?” Thus, a non-ad-hoc theoretical explanation to this robust experimental finding is still absent.\(^3\)

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1. Different from novice subjects in earlier experiments, they exhibited no greater tendency to overbid than to underbid.
2. *Other-regarding Preferences* (e.g., Cooper and Kagel 2009) such as *altruism* and *inequality* aversion, which were originally developed to explain the emergence of cooperation (Fehr and Schmidt 1999; Bolton and Ockenfels 2000), may be used to justify underbidding in auctions. *Anticipated Regret* (Filiz-Ozbay 2007; Engelbrecht-Wiggans 2007), another non-standard preference model proposed to explain overbidding in *First Price* auctions cannot account for the insincere bidding in *private-value Second Price* auctions, since bidding one’s value is a DS.
In a recent insightful paper, Li (2017) introduces *Obviously Strategy-Proof* (OSP) mechanisms implemented by a stronger solution concept than DS, the *obviously dominant strategy* (ODS). Li’s OSP mechanisms are a subset of DS mechanisms and can explain why OSP mechanisms (e.g., English auctions) can outperform “just” DS mechanisms (e.g., Second-Price auctions), while keeping the framework of standard game theory.

However, dominant strategy is optimal regardless of the players’, objective or subjective beliefs. Therefore, spotting DS does not require strategic thinking often required in games, reducing the task to optimization in individual decision making, which suggests that such deviations from available DS involve deeper violations of axioms in decision theory. On the other hand, the superior performance of ODS may imply players’ compliance with weaker axioms.

Our proposed model uses weaker axioms than those used before, and is motivated, in part, by making a psychological observation: decision makers’ inability to envision all hypothetical scenarios and reason state-by-state, which is essential in spotting a DS, but not an ODS. Thus, deviations from the DS in all aforementioned environments and convergences in the others are explained by such a deficiency. We formalize the deficiency by proposing *Partition Obvious Preference* (POP) using weaker axioms where a decision maker reasons by partitioning the *state-space* into events. The decision maker can reason event-by-event, but not state-by-state within each event. The coarser the partition, the more *bounded rational* the decision maker will be. We illustrate reasoning by partitions and why it matters by the following two examples.

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4 The decision maker who satisfies the axioms of *Subjective Expected Utility Theory* (Savage 1954; Anscombe and Autumn 1963; Fishburn 1970), or even weaker axioms proposed in *Ambiguity models* (Schmeidler and Gilboa 1989; Gilboa and Marinacci 2011), ought to adopt DS when it is available, due to the monotonicity axiom.

5 Such deficiency in contingent reasoning is supported by the psychology literature about the presence of *Disjunction Effect* (Shafir and Tversky, 1992) and findings in laboratory experiments (Charness and Levin 2009; Esponda and Vespa 2014; Levin et al. 2016).
Example 1: Consider decision problems in Figure 1 with Problem 1 on the left and Problem 2 on the right. Suppose there are two possible states, L and R; the decision maker has two available actions, U and D; and payoffs are given in the matrices. There are only two possible partitions, the finest \{L, R\} shown on the Top and the coarsest \{(L, R)\} on the bottom. Given the finest partition, any POP prefers U to D in both Problems 1 and 2, since the payoff of U is higher than that of D in any state. Given the coarsest partition, any POP prefers U to D in Problem 2, since any possible payoff of U (5, 4) is higher than that of D (2, 3). However, POP also allows the decision maker to prefer D to U in Problem 1 because some payoff of D (4) is better than one payoff of U (3). Note that U is an ODS in Problem 2, but just a DS in Problem 1. Thus, when there are only two states, the implication of our approach degenerates to that of Li’s (2016).

Example 2: Consider the decision problems in Figure 2, with Problem 3 on the left and Problem 4 on the right. Consider four possible states, \(\omega_1, \omega_2, \omega_3, \omega_4\); and consider two available actions, U and D, that the decision maker can take; any payoffs are given in the corresponding matrices. Given the finest partition, any POP prefers D to U in both Problems 3 and 4, since the payoff of D is higher than that of U in any state. In contrast, given the coarsest partition, POP also allows the decision maker to prefer U to D in both Problems 3 and 4, since we can find one payoff of U higher than that of D. Alternatively, consider the partition, \(\{B_1, B_2\}\), where \(B_1 = \{\omega_1, \omega_2\}\) and \(B_2 = \{\omega_3, \omega_4\}\); any POP prefers D to U in Problem 3,

\[
\begin{array}{c|cc}
\text{Problem 1} & \text{L} & \text{R} \\
\hline
\text{U} & 3 & 5 \\
\text{D} & 2 & 4 \\
\end{array}
\begin{array}{c|cc}
\text{Problem 2} & \text{L} & \text{R} \\
\hline
\text{U} & 4 & 5 \\
\text{D} & 2 & 3 \\
\end{array}
\]

\text{FIGURE 1. AN EXAMPLE OF TWO STATES}
since in either event $\mathcal{B}_1$ or $\mathcal{B}_2$, any payoff of D is higher than that of U. However, POP allows the decision maker to prefer U to D because in either $\mathcal{B}_1$ or $\mathcal{B}_2$, some payoff of U is higher than that of D.

<table>
<thead>
<tr>
<th>Problem 3</th>
<th>Problem 4</th>
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<tr>
<td></td>
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<tr>
<td>$\text{Partition}$</td>
<td>$\text{State}$</td>
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<td>U</td>
<td>20</td>
</tr>
<tr>
<td>D</td>
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</table>

**FIGURE 2. AN EXAMPLE OF FOUR STATES**

We develop *Partition Obvious Preference Theorem*, which subsumes the *Subjective Expected Utility Theorem* as an extreme case when the partition is the finest. Specifically, we envision that decision makers partition the state space into events, where for each event, they value each act as the weighted average of the most and least preferred outcomes, then form subjective expected probabilities over the partition, and choose the action that gives the highest subjective expected utility.

We extend our approach to games by defining *partition dominant strategy*, where at one polar case, when the partition is the finest, the *partition dominant strategy* coincides with the DS, and at the other, when the partition is the coarsest, it coincides with Li’s ODS. We show that a strategy is *partition dominant*, if and only if any POP prefers it to any deviating strategy at any reachable information set.

Our theory has three implications for mechanism design. First, when an implementation in ODS does not exist, the designer may still find an implementation using our new solution concept that is stronger than DS, i.e., with a partition coarser than the finest. We show further that as the state space becomes

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6 An implementation in *obvious dominant strategy* rarely exists. Li (2016) proves that no top trading cycle rule with more than three agents can be implemented by an *obviously dominant strategy*. Ashlagi and Gonczarowski (2016) show that for general preferences, no mechanism that implements a stable matching is *obviously strategy-proof*. Pycia (2017) finds that *Random Priority* is the unique mechanism that is *obviously strategy-proof*, *ex-post Pareto efficient*, and *symmetric*. Bade and Gonczarowski (2017) characterize a similar limitation in applications of *obviously strategy-proof* mechanisms in dictatorship mechanisms, house matching and multi-unit auctions.
larger, our “intermediate” solution concept grows in usefulness relative to the ODS. Second, when a DS is also partition dominant with respect to a coarser partition, the mechanism designer might be able to highlight that partition by a proper presentation and achieve a desirable outcome, not requiring people to reason in the finest partition. 7 Third, our work provides plausible explanations for the experimental observation that often, a dynamic mechanism performs better than its strategically equivalent static mechanism. A dynamic design enlarges (often strictly) the set of partitions so that the dominant strategy is also partition dominant. Thus, POP identifies a particular weakening of Savage’s framework that allows decision makers to deviate from DS (as in an SPSB auction), but insists on the use of an ODS (as in an English auction), as well as similar phenomena in other environments. Moreover, our theory accounts for the superior performance of a dynamic Ausubel (2004) Auction with a multiple-unit demand over its static implementation by a Vickrey (1961) auction, as documented in the experiment by Kagel and Levin (2009). Our approach does not evoke other elements of the decision makers’ preferences that are not explicit in the task.

Are mechanisms with a partition dominant strategy implementation easier for decision makers to understand? We study this question using a simple laboratory experiment. We compare a pair of mechanisms that implement the same allocation rule. Each of the mechanisms has equilibrium in DS, but not in ODS. However, while one of them implements the allocation rule with a partition dominant strategy given the represented partition, the other does not. Standard game theory and Li’s (2017) ODS concept do not suggest any performance differences between two mechanisms. We are interested in whether subjects play the DS at higher rates when

7 Kawagoe and Mori (2001) find a substantial decrease in departures from the dominant strategy in a Pivotal Mechanism when detailed payoff tables are provided. Their result suggests that a detailed presentation might facilitate subjects to spot the dominant strategy. Then, our theory provides insight into the specific design, presenting in partitions that might work. Chen and Zhang (2016) design an eye-tracking experiment to explore how subjects’ plays differ when they are facing different presentations, in terms of partitioning, of the same game.
it is also partition dominant. In addition, all subjects participate in a related individual decision task, where the optimal choice is either partition dominant or not. We found that subjects play the dominant strategy at higher rates in mechanisms with partition dominant strategies (not necessarily obviously dominant strategies), as compared to dominant strategy mechanisms that should implement the same allocation rule. Moreover, subjects choose dominated strategy at lower rates in decision tasks when the optimal choice is also partition dominant, than when it is not. The result is significant in pooled data, within-subject and cross-subject comparisons. A finding that subjects behave closer to predictions under partition dominant strategy is a useful discovery for designers of markets and mechanisms as it enriches the class of mechanisms that perform better than just those with DS.

What can we learn from POP when a DS does not exist? Applying the concept of Nash Equilibrium to POP, we propose Partition Nash Equilibrium to identify a set of mechanisms that are more robust than mechanisms with only Ex-post Nash Equilibria.

This paper is organized as follows. In Section 1, we present our Partition Obvious Preference Theorem, and we give three examples to illustrate how it works. In Section 2, we extend our results to games, define partition dominant strategy and partition obvious equilibrium, and discuss its applications to mechanism design. In Section 3, we describe our experimental design and analyze the results. We discuss additional related literature, future research, and conclude in Section 5.

I. Partition Obvious Preference

Denote by $\mathcal{X}$ a set of deterministic outcomes and by $\mathcal{Z}$ a set of distributions over $\mathcal{X}$ with finite supports, i.e., $\mathcal{Z}$ is a collection of random outcomes. Let $\Omega$ denote the state space of all states, $\omega, \omega \in \Omega$, and let $\mathcal{F}$ denote the set of all acts, $f: \Omega \rightarrow \mathcal{Z}$. 
For simplicity, we assume that $\Omega$ is finite. Denote by $\mathcal{O}(f)$ the set of all possible random outcomes given act $f$, and by $\mathcal{F}^c$ the set of constant acts in $\mathcal{F}$.

Definition 1. (partition) $\Sigma = (\mathcal{B}_k)_{k=1}^n$ is a finite partition of $\Omega$ if $\bigcup_{k=1}^n \mathcal{B}_k = \Omega$ and $\mathcal{B}_i \cap \mathcal{B}_j = \emptyset$, when $i \neq j$.

### A. The Axioms

The first two axioms are standard in decision theory.

AXIOM 1. (weak order) $\succeq$ is complete and transitive.

AXIOM 2. (non-degeneracy) there are $f, g \in \mathcal{F}$, such that $f \succ g$.

Denote by $\mathcal{O}(\mathcal{B})(f)$ the set of all possible outcomes induced by act $f$ given event $\mathcal{B}$. The next axiom generalizes the standard monotonicity axiom.

AXIOM 3. (partition monotonicity) for any $f, g \in \mathcal{F}$, if for each $\mathcal{B} \in \Sigma$, we have, for all $p \in \mathcal{O}(\mathcal{B})(f), q \in \mathcal{O}(\mathcal{B})(f)$, $p \succeq q$, then $f \succeq g$; In addition, if for a non-null event $\mathcal{B'} \in \Sigma$, we have for all $p \in \mathcal{O}(\mathcal{B'}) (f), q \in \mathcal{O}(\mathcal{B})(f)$, $p \succ q$, then $f \succ g$.

The partition monotonicity requires the decision maker to compare outcomes of two acts event-by-event, but not state-by-state within each event of the partition, as in the standard monotonicity axiom.

The following definition of mixed acts is standard in the literature.

Definition 2. (mixed acts) for any $f, h \in \mathcal{F}$, $\lambda \in [0, 1]$, and $\omega \in \Omega$, $[\lambda f + (1 - \lambda)h](\omega) \equiv \lambda f(\omega) + (1 - \lambda)h(\omega)$.

Definition 3. (partition measurable act) $\mathcal{F}^c(\Sigma)$ is the set of acts that is constant in each event $\mathcal{B}$ of the partition $\Sigma$. (The set of acts that is measurable with respect to the partition.)

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8 For simplicity, we only consider the case where each $\mathcal{B}_k$ is a non-null event.
Implication of Definitions 2 and 3: to understand the concept of mixed acts, it’s enough to reason by Partition $\Sigma$ for the $\Sigma$-measurable act, but reasoning in a finer partition is required for other acts.

The next two axioms generalize the continuity and independence axioms by imposing them on mixed acts of only partition measurable acts.

Axiom 4. (partition continuity) for any act $g \in F$ and any two acts $f, h \in F^c(\Sigma)$ such that $f > g > h$, there are $\lambda, \beta \in (0, 1)$ such that $\lambda f + (1 - \lambda)h > g > \beta f + (1 - \beta)h$.

Axiom 5. (partition independence) for any three acts $f, g, h \in F^c(\Sigma)$ and any $\lambda \in (0, 1], f > g$ implies that $\lambda f + (1 - \lambda)h > \lambda g + (1 - \lambda)h$.

Hence, partition continuity and partition independence are analogous to the continuity and independence axioms without request the decision maker to reason in an even finer partition.

Definition 4. (partition indifferent act) $F_e(\Sigma) = \{f \in F | p \sim q f or any B \in \Sigma and any p, q \in O_B(f)\}$ is the set of acts that generate indifferent outcomes in each event $B$ of the Partition $\Sigma$.

B. The Theorem

Theorem. (partition obvious preference) $\Sigma = (B_k)_{k=1}^n$ is a partition given exogenously. Let $\succeq$ be a binary relation defined on $F$. The following conditions are equivalent:

(i) $\succeq$ satisfies Axiom 1-5. (We call such preferences $\Sigma$—Obvious Preference)

(ii) there exists a non-constant affine function $u: \mathbb{Z} \rightarrow \mathbb{R}$, a probability function $P: \Sigma \rightarrow [0,1]$ and a function $\alpha: F \rightarrow [0,1]$ such that $\succeq$ is represented by the preference functional $V: F \rightarrow \mathbb{R}$ given by

\begin{equation}
V(f) = \sum_{k=1}^n V(f|B_k)P(B_k),
\end{equation}
where

\[ V(f|B) = \alpha(f) \max_{p \in \Omega_B(f)} u(p) + (1 - \alpha(f)) \min_{q \in \Omega_B(f)} u(q). \]

That is, for all \( f, g \in \mathcal{F} \), \( f \succeq g \) if and only if \( V(f) \succeq V(g) \).

Furthermore:
(a) The function \( u \) in (ii) is unique up to positive affine transformation;
(b) The probability function \( P \) is unique;
(c) \( \alpha \) is unique on \( \mathcal{F} \) with the exclusion of \( \mathcal{F}_e(\Sigma) \).

PROOF. See Appendix A.1.

C. Illustrative Examples

Example 3. Consider the following voting problem from experiments reported in Esponda and Vespa (2014). There is an urn with 5 red balls and 5 blue balls. One ball is randomly drawn and selected. Two computers “observe” the color of the selected ball and are programmed to vote by the same rule: if the selected ball is red, vote red; otherwise, vote blue or red with equal probability (1/2). The human subject must vote for either red or blue without observing the color of the selected ball. If the color chosen by a simple majority matches the color of the selected ball, the subject wins $2; otherwise, her payoff is $0. In Treatment A, subjects vote without any information about the actual votes of the computers. In Treatment B, the subjects know the votes of the two computers before casting her vote. In Treatment B, we only consider the case where the two computers vote differently; otherwise, the subject’s decision does not affect the outcome. Thus, it is not clear what a vote implies. Voting blue is a weakly dominant choice in both treatments. However, Esponda and Vespa (2014) observed that more than half of the subjects

\[9 \text{ For any } f \in \mathcal{F}(\Sigma), \text{ all } \alpha(f) \in [0,1] \text{ end up with the same } V(f). \]
voted red in Treatment A even after repetitions with feedbacks. In contrast, in Treatment B, the subjects converged quickly to voting blue, even without feedback. Denote each state by \( s = (\text{selected ball}, \text{votes of computer}) \). Now consider the partition by the votes of computer presented by two tables in Figure 4.

\[
\begin{array}{|c|c|c|}
\hline
\text{Selected Ball} & \text{Voted Red} & \text{Voted Blue} \\
\hline
\text{Your Choice} & \text{2 Red} & \text{1 Red, 1 Blue} & \text{2 Blue} \\
\hline
\text{Vote Red} (\alpha_r) & 2 & 0 & 2 \\
\text{Vote Blue} (\alpha_b) & 2 & 2 & 2 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{Selected Ball} & \text{Voted of computer} \\
\hline
\text{Your Choice} & \text{1 Red, 1 Blue} & \text{Blue} \\
\hline
\text{Vote Red} (\alpha_r) & 0 & \\
\text{Vote Blue} (\alpha_b) & 2 & \\
\hline
\end{array}
\]

FIGURE 3. THE VOTING EXPERIMENT

In Treatment A, by our POP Presentation, the Expected Utility of voting Red is

\[
V(red) = \frac{5}{8} \times 2\alpha_r + \frac{2}{8} \times 0 + \frac{1}{8} \times 2,
\]

and the Expected Utility of voting Blue is

\[
V(blue) = \frac{5}{8} \times 2\alpha_b + \frac{2}{8} \times 2 + \frac{1}{8} \times 2.
\]

The player strictly prefers to vote red when \( 5\alpha_r > 5\alpha_b + 2 \). For example, when \( \alpha_r = 0.9 \) and \( \alpha_b = 0.4 \), \( V(red) = 1.375 > 1.25 = V(blue) \). However, in Treatment B, any POP prefers voting blue to voting red.

Example 4. Consider the matrix game with incomplete information in Figure 4. There are three possible cases: A, B, and C. The computer randomly draws one case, which is unknown to both players, each with a probability of 1/3. Each player chooses between R and L and the payoff table is shown in three matrices. Each
decision maker is randomly matched with a player who is drawn from the pool of subjects who played the same game in pairs. The payoff of the decision maker thus depends on his choice, the case he is in, and the strategy chosen by his opponent in the past. The state space in this decision problem is a cross product of cases \{A, B, C\} and choices of the other player \{L, R\}. Given each case (A, B, or C) and each strategy of the opponent (R or L), choosing R always generates a higher payoff. Thus, any subjective utility maximizer would not be willing to pay for the non-instrumental\(^{10}\) information notifying which case they are in, at a positive price.

<table>
<thead>
<tr>
<th>Case A</th>
<th>Case B</th>
<th>Case C</th>
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<tbody>
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<td>27, 6</td>
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<tr>
<td></td>
<td></td>
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</tr>
<tr>
<td>The Other</td>
<td></td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>8, 25</td>
<td>6, 27</td>
</tr>
<tr>
<td>R</td>
<td>13, 13</td>
<td>11, 11</td>
</tr>
</tbody>
</table>

**FIGURE 4. THE MATRIX GAME WITH INCOMPLETE INFORMATION**

Now consider the partition by cases and POP with \(\alpha_L = \frac{2}{3}\) assigned to choice L and \(\alpha_R = \frac{1}{6}\) assigned to choice R.\(^{11}\) By our POP Presentation (1), the utilities of choosing L in States A, B, and C are as follows: \(V(L|A) = 20 \times \frac{2}{3} + 8 \times \frac{1}{3} = 16; V(L|B) = 22 \times \frac{2}{3} + 6 \times \frac{1}{3} = 16 \frac{2}{3}; V(L|C) = 18 \times \frac{2}{3} + 10 \times \frac{1}{3} = 15 \frac{1}{3}\). The expected utility of choosing L is thus \(V(L) = 16 \times \frac{1}{3} + 18 \times \frac{50}{3} + 16 \times \frac{46}{3} = 16\). The utilities of choosing R in States A, B, and C are \(V(R|A) = 25 \times \frac{1}{6} + 13 \times \frac{5}{6} = 15; V(R|B) = 27 \times \frac{1}{6} + 11 \times \frac{5}{6} = 13 \frac{2}{3}; V(R|C) = 23 \times \frac{1}{6} + 15 \times \frac{5}{6} = 16 \frac{1}{3}\). The expected utility of choosing R is thus \(V(R) = 15 \times \frac{1}{3} + 41 \times \frac{1}{3} + \frac{49}{3} \times \frac{1}{3} = 15\). Since

\(^{10}\) We call a piece of information “instrumental” in the case where this information can alter the optimal decision.

\(^{11}\) A similar argument follows for any \((\alpha_L - \alpha_R) \in \left(\frac{5}{18}, \frac{5}{9}\right)\) that the information of cases can vary optimal decisions for POP. See Appendix A.5 for details.
$V(L|C) < V(R|C)$ but $V(L) > V(R)$, the information regarding which case the decision maker is in can alter the optimal choice and thus it is instrumental for the bounded rational player we characterize. This example alerts us to the fact that non-instrumental information in theory might nevertheless be instrumental for bounded rational players, e.g., POP with a coarser partition.\textsuperscript{12}

The partition in our theorem is given exogenously. A natural question is: does there always exist a unique finest partition that can rationalize some POP? However, uniqueness is not guaranteed at least when the state space is finite.

Reconsider Problem 4 of Example 2 in the introduction in Figure 5. The choice of $U$ can be rationalized by $\Sigma = \{B_1, B_2\}$, where $B_1 = \{\omega_1, \omega_2\}$, $B_2 = \{\omega_3, \omega_4\}$; and $\Sigma' = \{B_1', B_2'\}$, where $B_1' = \{\omega_1, \omega_4\}$, $B_2' = \{\omega_2, \omega_3\}$. However, it cannot be rationalized by their joint, the coarsest common refinement: $\Sigma \vee \Sigma' = \{\omega_1, \omega_2, \omega_3, \omega_4\}$.

**FIGURE 5. UNIQUENESS IS NOT GUARANTEED**

<table>
<thead>
<tr>
<th>Event</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_1'$</th>
<th>$B_2'$</th>
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**II. Partition Dominant Strategy**

We extend our concept to dynamic games. We introduce the decision environment in dynamic games in Subsection A. We propose partition dominant strategy and relate it to POP in Subsection B. In Subsection C, we discuss the application of partition dominant strategy to mechanism design.

\textsuperscript{12} Zhang (2017) designed an experiment to compare the subjects’ demand for extra information when there is either just a DS or an ODS, and to explore how the behavioral difference across subjects are related to their performance on three cognitive tests of contingent reasoning and SAT scores.
We consider the extensive game form\textsuperscript{13} $\Gamma$ with imperfect information and perfect recall as defined in Osborne and Rubinstein (1994). Denote the set of strategy profiles by $S = S_i \times S_{-i}$ and the set of nature’s moves by $\Omega_N = \{\omega_n\}$. The domain of Player $i$’s uncertainty consists of moves of nature $\Omega_N$ and the strategy of other players $S_{-i}$. So, let $\Omega_i = S_{-i} \times \Omega_N$ denote the subjective state space of player $i$\textsuperscript{14}. At each terminal history, $h = (s_i, s_{-i}, \omega_n)$, Player $i$ is assigned a deterministic or random outcome in a set $Z$ defined in Section I. Thus, Player $i$’s preference over her own strategy set is characterized by a preference relation $\succ$ on the set of acts $\Omega_i \rightarrow Z$. The utility of Player $i$ at each terminal history, $u_i(s_i, s_{-i}, \omega_n)$, is thus determined by the utility function of lotteries in Theorem 1, $u_i: Z \rightarrow \mathbb{R}$. A dynamic game is then a tuple $G = \{\Gamma, Z, (u_i)_{i \in N}\}$.

For any information set $I \in \mathcal{I}$, denote by $S(I)$ the set of strategy profiles that reach $I$\textsuperscript{15}. The projections of $S(I)$ on $S_i$ and $S_{-i}$ are denoted by $S_i(I)$ and $S_{-i}(I)$; perfect recall implies that $S(I) = S_i(I) \times S_{-i}(I)$, and the set of available strategies for Player $i$ at information set $I$ is $S_i(I)$. Player $i$ who chooses an action $a \in A(I)$ at information set $I$, must restrict herself to a smaller set of strategies denoted by $S_i(I)[a] = \{s_i \in S_i(I) | s_i(I) = a\}$. We denote by $S_i(I)[a]^c = S_i(I) \setminus S_i(I)[a]$ the set of strategies from which the player is deviating by choosing $a$.

\textsuperscript{13} An extensive game form is a tuple $\Gamma = (N, \mathcal{Y})$, where $N$ is the set of players and $\mathcal{Y}$ is the game tree.

\textsuperscript{14} Bayesian Models in decision theory under uncertainty Savage (1954) and Solution Concepts in game theory (Nash 1950) originated independently. Aumann (1987) synthesizes the two viewpoints by Correlated Equilibrium. In his set-up, the state of the world in games is a specification of which strategy is chosen by each player. Esponda (2013) further defines the state space as the product of the strategy sets and the set of fundamentals to include both strategic and structural uncertainty. He further develops the Rationalizable Conjectural Equilibrium by adding certain restrictions to each player’s beliefs over states of the world in equilibrium. Siniscalchi (2016a, 2016b, 2016c) adopts a similar definition of the subjective state space in dynamic games in three of his recent works about structural rationality.

\textsuperscript{15} Formally, $\delta(I) = \{s \in \delta | \text{there exists } h \in I \text{ and } \omega_n \in \Omega_N \text{ such that } h \text{ is a subhistory of } (s, \omega_n)\}$
Upon reaching an information set $I \in \mathcal{I}_i$, Player $i$ must rule out moves of nature and strategies of other players that do not allow reaching information set $I$. We denote the conditioning event at information set $I$ by $[I]$, at which $I$ is reachable.\textsuperscript{16} Finally, denote by $\mathcal{I}(s_i) = \{I \in \mathcal{I} | s_i \in S_i(I)\}$, the set of information sets that is reachable by strategy $s_i$.

### B. Partition Dominant Strategy

Definition 5. (conditional partition-system) A conditional partition-system $\Sigma^G$ for Player $i$ in a dynamic Game $G$ is a collection of partitions, $\{\Sigma(I)\}_{I \in \mathcal{I}_i \cup \emptyset}$, such that

(i) $\Sigma(\emptyset) = \Sigma = \{\mathcal{B}_k\}_{k=1}^n$ is a partition of $\Omega_i$, 

(ii) for any $I \in \mathcal{I}_i$, $\Sigma(I) = \{[I] \cap \mathcal{B}_k\}_{k=1}^n$, from here on we denote $[I] \cap \mathcal{B}_k$ by $\mathcal{B}_k(I), k = 1, \ldots, n$.

Definition 6. (partition dominant strategy) In a dynamic Game $G$, a strategy $s_i^*$ is a $\Sigma$-dominant strategy for Player $i$, if for any information set, $I \in \mathcal{I}(s_i^*)$, any non-empty event, $\mathcal{B}(I) \in \Sigma(I)$ and any deviating strategy, $s_i' \in S_i[I(s_i^*)]$:

\begin{equation}
\inf_{(s_{-i}, \omega_n) \in \mathcal{B}(I)} u_i(s_i^*, s_{-i}, \omega_n) \geq \sup_{(s_{-i}, \omega_n) \in \mathcal{B}(I)} u_i(s_i', s_{-i}, \omega_n).
\end{equation}

Remark: When the partition is the coarsest, $\Sigma = \{\Omega_i\}$, $\Sigma$-dominant strategy coincides with Li’s (2016) obviously dominant strategy. Moreover, $s_i^*$ is $\Sigma$-dominant strategy if an only if it is obviously dominant conditioning on all $\mathcal{B} \in \Sigma$.

Definition 7. (dominant strategy)\textsuperscript{17} In a dynamic Game $G$, $s_i^*$ is a dominant strategy for Player $i$ if for any $s_i' \in S_i$, any state $(s_{-i}, \omega_n) \in \Omega_i$:

\begin{equation}
\inf_{(s_{-i}, \omega_n) \in \mathcal{B}(I)} u_i(s_i^*, s_{-i}, \omega_n) \geq \sup_{(s_{-i}, \omega_n) \in \mathcal{B}(I)} u_i(s_i', s_{-i}, \omega_n).
\end{equation}

\textsuperscript{16} Formally, $[I] = \{(s_{-i}, \omega_n) \in \Omega_{i-1} | \text{there exists } h \in I, s_i \in S_i \text{ such that } h \text{ is a sub-history of } (s_i, s_{-i}, \omega_n)\}$.

\textsuperscript{17} Li (2016, Def. 4) defines dominant strategy in a slightly different way. Note that, in our paper, nature’s moves $\Omega_N$ include both the chance moves and type randomizations in Li’s. Li defines a strategy as weekly dominant if its expected payoff with respect to chance moves is not smaller than that of any alternative strategy for any realized type. Our notion of
(4)  \[ u_i(s_i^*, s_{-i}, \omega_n) \geq u_i(s_i', s_{-i}, \omega_n). \]

Lemma 1. When the Partition \( \Sigma \) is the finest, Definitions 6 and 7 are equivalent.

PROOF: See Appendix A.2.

Proposition 1. In a dynamic Game \( G \), a strategy \( s_i^* \) is an \( \Sigma \)-dominant strategy for Player \( i \) if and only if for any \( \Sigma \)-Obvious Preference \( \succeq \), satisfies the following:

(5)  \[ s_i^* \in C(\succeq, S_i) = \{ s_i \in S_i | s_i \succeq s_i' \text{ for any } s_i' \in S_i[I] \} \text{ at any } [I], I \in \mathcal{I}_i. \]

PROOF: See Appendix A.3.

So, a strategy is \( \Sigma \)-dominant if and only if any \( \Sigma \)-Obvious Preference prefers it to any deviating strategy at any reachable information set. Hence, mechanisms with a strategy that is partition dominant in a coarser partition work for a larger set of preferences.

C. Applications to Mechanism Design

Our theory has three implications for mechanism design. First, if an implementation in ODS does not exist, we may still find an implementation in partition dominant strategy, a solution concept stronger than DS, with a partition that is coarser than the finest. We show by Example 5 that the usefulness of our “intermediate” concept grows relative to the ODS, as the state-space becomes larger.

Example 5. There are \( N \) possible states, \( \Omega = \{ \omega \}_{\omega=1}^{N} \) and two available actions, \( f \) and \( g \), where \( f \) is the DS, i.e., \( u(f(\omega)) \geq u(g(\omega)), \omega \in \Omega \). Denote by \( d(N) \) the number of pairs \( [f(w), g(w')] \), where \( w, w' \in \Omega, w \neq w' \), and \( u(f(w)) < u(g(w')) \). With \( N = 1 \), \( d(N) = 0 \), \( f \) is also an ODS. With \( N = 2 \), \( d(N) = 0 \text{ or } 1 \), \( f \) is obviously dominant in the former, but not in the latter case. However, as there dominance is stronger than Li’s, since our dominant strategy needs to be ex-post optimal, not only expected, given any realization of chance moves. However, Li’s Theorem 1 still holds even if he instead used our notion.
are only two possible partitions, our “intermediate” concept is not applicable. Now with \( N = 3, d(N) = 0, 1, 2, 3 \). \( f \) is not obviously dominant in cases where \( d(N) > 0 \). However, we can still find a partition coarser than the finest so that \( f \) is partition dominant when \( d(N) = 1, 2 \). With \( N = n, d(N) = 0, 1, 2, \ldots, \frac{N(N-1)}{2} \), \( f \) is not obviously dominant for \( d(N) > 0 \), but we can still find a partition coarser than the finest so that \( f \) is partition dominant when \( d(N) = 1, 2, \ldots, \frac{N(n-1)}{2} - 1 \).

Second, when a strategy is partition dominant, we may be able to highlight the partition by a proper presentation and achieve a more desirable outcome.

---

**FIGURE 6. TWO WAYS OF PARTITIONING**

Example 6. See two representations of the same game in Figure 6. There are two (exogenous) states of nature, L and R, and each player has two strategies, A and B. Payoffs are shown in the matrices. For each player, each player’s subjective state space is a cross-product of \( \{A, B\} \) and \( \{L, R\} \). In the top case, the payoffs are presented using partitioning by states of nature; in contrast, in the bottom case, the same payoffs are presented using partitioning by the strategy of the opponent. B is partition dominant based on the bottom partition but not the top. Thus, empirically, we might observe more convergence to the choice of dominant strategy, B, with the bottom presentation than with the top one.
Third, in general, a dynamic mechanism tends to perform better than its strategically equivalent static mechanism because as more information arrives and fewer possible states are left, inferior and less attractive alternative strategies are dismissed as their inferiority becomes clearer, and makes the desired choice more obvious. Specifically, following from Definition 6, if a strategy is $\Sigma$-dominant in a static game, then it is also $\Sigma$-dominant in its strategically equivalent dynamic games, but not vice versa. In other words, the dynamic variation weakly, and at times strictly, enlarges the set of partitions that makes the dominance obvious and thus help a subject who reasons in coarser partitions. For example, like Li’s (2016) theory, our theory predicts more sincere bidding in English auctions that is ODS, than in the SPSB auctions that is “just” DS.

<table>
<thead>
<tr>
<th>$v_i &lt; b_{\text{max}}^{\text{in}} &lt; v_i$</th>
<th>$v_i &lt; b_{\text{max}}^{\text{in}} &lt; v_i + c$</th>
<th>$v_i + c &lt; b_{\text{max}}^{\text{in}} &lt; v_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b(v_i) = v_i$</td>
<td>$u_i - b_{\text{max}}^{\text{in}}$</td>
<td>$u_i + c - b_{\text{max}}^{\text{in}}$</td>
</tr>
<tr>
<td>$b'(v_i) = v_i + c$</td>
<td>$u_i - b_{\text{max}}^{\text{in}}$</td>
<td>$u_i - b_{\text{max}}^{\text{in}}$</td>
</tr>
</tbody>
</table>

Example 7. In a SPSB auction, denote by $b(v_i) = v_i$ as bidding one’s value and by $b'(v_i) = v_i + c$ as overbidding by $c > 0$. As shown in Figure 7, upon choosing either strategy, the only case bidder $i$ wins the auction with positive payoff is when the highest bid of other bidders, denoted by $b_{\text{max}}^{\text{in}}$, is lower than $v_i$. Given the finest partition, one ought to realize that any amount of positive payoff would happen with the same probability, regardless of whether one bids $v_i$ or $v_i + c$. However, by a coarser partition, the bidder is allowed to think that overbidding would increase the probability of winning and thus increase the expected payoff. For example, with

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18 So, what matters the most is not the dynamic itself but the information updating and sequential choices allowed by it. For example, Kagel and Levin (2009) document the superior performance of a dynamic Ausubel auction with drop-out information over both the static Vickrey auction and the dynamic auction without dropout information.

19 In auctions with lottery prizes, Karni and Safra (1989) find that weaker assumptions are needed for sincere bidding to be optimal in English than in SPSB auctions, but they focus on weakening the axioms of risk preferences.
the partition \( \{ v < b_{-i}^{max} < v_i + c, v_i + c < b_{-i}^{max} < \bar{v} \} \), consider a POP that assigns \( \alpha' = 1 \) to \( b'(v_i) = v_i + c \) and \( \alpha = 0 \) to \( b(v_i) = v_i \). Then \( V(b) = 0 < V(b') \). Such a POP will prefer to overbid although it is a dominated strategy. However, in an English auction, any POP prefers sincere bidding.

Our theory also accounts for the superior performance of a dynamic Ausubel (2004) auction with a multiple-unit demand over its static implementation (Vickrey 1961), a phenomenon documented in the experiment by Kagel and Levin (2009). In brief, when we restrict the strategy set to cut-off strategies, the Ausubel auction is strategically equivalent with Vickrey auction. However, there exist partitions, by the “clinching price”, coined by Ausubel, so that sincere bidding is partition dominant in the Ausubel auction, but not in the Vickrey auction. We provide a detailed argument in Appendix A.4.

Elmes and Reny (1994) prove that if two finite extensive form games with perfect recall share the same normal form, then we can get one game from another by three kinds of transformations in finite steps. It raises the questions: why in general, among two games that share the same normal form, one has a certain partition dominant strategy but the other does not; which transformation breaks the nice property of partition dominance. We show by the following example that one of their transformation, called “ADD,” is critical for answering this question.

Example 8: In Figure 8, we get extensive form Game B from Game A by Elmes and Reny’s “ADD” transformation. Clearly, the two games share the same normal

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20 If we consider a larger set of strategies, where one bidder’s active units of demand can depend on not only the clock price, but also on other bidders’ active units of demand, then in the Ausubel auction with dropout information, sincere bidding is not even a dominant strategy (see Ausubel 2004). However, we argue that to consider such strategies, it also requires higher cognitive ability because bidders need to consider more contingencies. The bottom line is: once we allow the bounded rationality of contingent reasoning, our theory can resolve the discrepancy between the theory and experimental evidence: Ausubel auctions with feedback perform better than Vickrey’s, not as a weaker solution concept, but a stronger solution concept.

21 See Page 12 of Elmes and Reny’s paper for the definition of “ADD” transformation.
Partitioning by moves of nature, L is a *partition dominant strategy* for Player 2 in Game A but not in Game B.

**D. Partition Obvious Equilibrium**

Applying the concept of Nash Equilibrium to POP, we propose *Partition Equilibrium*. Following the notation in Aumann (1976), we denote by \( \varepsilon(\omega) \) the event in Partition \( \Sigma \) that contains state \( \omega \).

Definition 8. (*partition Nash equilibrium*) In a dynamic Game G, a strategy profile \( s^* \) is an \( \Sigma \)-Equilibrium if for any Player \( i \), at any information set \( I \in \mathcal{I}(s_i^*) \), for any deviating strategy \( s_i' \in S_i(I)[s_i^*(I)]^c \) and any move of nature \( \omega_n \in \Omega_N \):

\[
\begin{align*}
    u_i(s_i^*, s_{-i}^*, \omega_n) \geq \sup_{(s_{-i}(\omega_n) \in \varepsilon(s_{-i}^*(\omega_n)(I))}} u_i(s_i', s_{-i}, \omega_n).
\end{align*}
\]

Since any equilibrium payoff can be viewed as a constant act, a strategy profile is an \( \Sigma \)-Equilibrium if and only if any \( \Sigma \)-Obvious Preference prefers any realized equilibrium payoff to any deviating strategy at any reachable information set conditioning on the event that contains the realized state.\(^{23}\) When the partition is the finest \( \Sigma = \Omega_i \), \( \Sigma \)-Equilibrium coincides with *Ex-post Equilibrium*, and when the partition is the coarsest \( \Sigma = \{\Omega\} \), it coincides with *Obvious Nash Equilibrium*.

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\(^{22}\) That is, if the state of the world is \( \omega \), then the player is informed of the element \( \varepsilon(\omega) \) of \( \Sigma \) that contains \( \omega \).

\(^{23}\) The proof is similar to that of Proposition 1.
In one “intermediate” case, when partitioning by strategies of opponents $\Sigma = \{s_{-i} \times \Omega_N | s_{-i} \in S_{-i}\}$, $\Sigma$-Equilibrium coincides with *Obvious Ex-post Equilibrium* (Li 2017b).

When multiple *Ex-post Nash Equilibria* exist, $\Sigma$-Equilibrium can serve as a criterion for refinement, as in the following stag-hunt game with imperfect information. When nature moves Left (Right), the payoff is shown in the matrix on the Left (Right). There are two *Ex-post Nash Equilibria*, (Stag, Stag) and (Hare, Hare). There is no *Obvious Nash Equilibrium* or *Obvious Ex-post Equilibrium*. However, there exists a unique $\Sigma$-Equilibrium, (Stag, Stag), when partitioning by moves of nature, $\Sigma = \{S_{-i} \times \omega_n | \omega_n \in \Omega_N\}$

$$
\begin{array}{c|cc}
| & Stag & Hare \\
\hline
\text{Stag} & 10,10 & 0,8 \\
\text{Hare} & 8,0 & 8,8 \\
\end{array}
\begin{array}{c|cc}
| & Stag & Hare \\
\hline
\text{Stag} & 6,6 & 0,4 \\
\text{Hare} & 4,0 & 4,4 \\
\end{array}
$$

*FIGURE 8: STAG-HARE GAME WITH INCOMPLETE INFORMATION*

In scenarios where a DS mechanism does not exist, *Partition Nash Equilibrium* can be used to identify sets of mechanisms that are more robust than mechanisms with just *Ex-post Nash Equilibria*. For example, in some interdependent value settings, a generalization of a Vickrey auction achieves efficiency with *Ex-post Nash Equilibrium* (Crémer and Mclean 1988; Dasgupta and Maskin 2000; Bergemann and Morris 2008). In such settings, there might exist a dynamic auction.

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24 In the setting of Li (2017b), type randomizations are the only type of nature’s moves.
that achieve efficiency with our stronger solution concept, *Partition Nash Equilibrium*.\(^{25}\)

### III. Laboratory Experiment

Can mechanisms with *partition dominant strategies* significantly reduce deviations from *dominant strategies*? Our laboratory experiment provides a straightforward test: We compare a pair of mechanisms that both implement the same allocation rule by an equivalent DS, that in both is not an ODS. However, one mechanism implements the allocation rule with a *partition dominant strategy* respect to the represented partition but the other does not. Standard game theory and Li (2017)’s ODS concept do not suggest any performance difference between both mechanisms. We study whether subjects play the DS at higher rates when it is also *partition dominant*. In addition, all subjects also participate in an individual decision task, where the optimal choice is either *partition dominant or not*. This will allow us to investigate whether the subjects’ behavior in games is consistent with their non-strategic choices.

#### A. Experiment Design

**Random Serial Dictatorship.** Consider the following variation of the random serial dictatorship experiment in Li (2017). The subjects are randomly assigned into a group of four, and each of them may receive one of four money prizes. There are two cases, L and R. Each group will be either in Case L or R, selected with a probability of \(\frac{1}{2}\), but the groups will not know to which of these two cases they are assigned. The total value of prizes for a group is \(T_L = 10\) in Case L and \(T_R = 22.5\) in Case R. In both cases, four prize values are drawn *uniformly* at random and

\(^{25}\) In the scenario of single-object auctions, Li (2017b) proposes *Obvious Auctions* that achieve efficiency with his *Obvious Ex-post Equilibrium*. 
without replacement, from the set \( \{0.1T, 0.2T, 0.3T, 0.4T\} \), where \( T \) is the total value.

At the start of each game, the subjects observe the value of four prizes in both Cases L and R. They are also assigned and informed of a priority score, which is drawn uniformly from integers 1 to 10. There are two games, S and D.

In Game S, each player is asked simultaneously to submit a list that ranks her preferences over the four prizes. The players are then processed sequentially, from the highest to the lowest priority score. Ties in priority score are broken randomly. Each player is assigned the highest-ranked prize on her list among the prizes that have not yet been assigned to players with higher priorities who selected earlier.

In Game D, the players take turns to select a prize in order of their priority score, from the highest to the lowest. When a player takes her turn, she is shown the prizes that have not yet been taken and is asked to pick one of them.

In both games, the players are paid the monetary value of the prize assigned based on the case their group is in. In both Games S and D, truthful reporting – ranking prizes with respect to the monetary values in either state – is the DS but not an ODS in either game. Note that, truthful reporting is partition dominant (by Cases L and R) in Game D, but not in Game S. Therefore, if subjects are capable of reasoning by partitioning, we ought to observe more truthful reporting in Game D than in Game S.

**The Decision Task.** Consider the following individual choice problem. There are six prizes worth 0, 2, 4, 6, 8 and 10. They are randomly, without replacement, inserted inside six Boxes (one in each box): A, B, C, D, E and F, and cannot be seen from the outside. Another six prizes worth: 0, 2, 4, 6, 8 and 10, are written on six Stickers (one on each sticker): A, B, C, D, E and F, and are visible. A subject is randomly assigned, with a probability of \( \frac{1}{2} \), to Case L or Case R, but they will not be informed whether they are in Case L or Case R. There are two
decision tasks, S and D. The subjects’ payoff depends on the case they are in, and on their choices.

In Decision S, the subject is asked to first choose one of the stickers, and then pick one box with unknown value. Then the subject sees the monetary value in the box she picks.

In Decision D, the subject is asked to first pick one box. And then is shown the monetary value inside the box. Then she is asked to choose a sticker.

In both decision tasks, S and D, if the case is L, the subject is assigned the lower monetary value between the one in the box and the one on the sticker; if the case is R, the subject is assigned the higher monetary value between these two. Picking a sticker with a monetary value other than the highest possible, 10, is a dominated strategy (/choice), but not an ODS in both tasks. Again, neither the standard game theory nor Li’s approach would suggest behavioral difference between the two tasks, but in contrast our approach does. Consider the partition by Case L and Case R. Picking a sticker with a prize of 10 is a partition dominant strategy in Decision D but not in Decision S. Thus, Our POP predicts more choices of stickers with a prize of 10 in Decision D than in Decision S.

B. Treatments

We adopt a crossover design (Piantadosi 2005) as shown in Table 1 below. The treatments are across subjects. Each treatment consists of 4 tasks, Game S, Game

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26 They are dominated by picking the prize of 10 and the same box.

27 For example, the highest prize the subject can get by picking a sticker with 8 on it, is 10. It is higher than the lowest prize the subject can get by picking a sticker with 10, which is 0.

28 In Decision D, after the subject see the value inside the box (x ≤ 10): in Case L, the lowest prize by picking a sticker with 10 on it is x; the highest prize by picking a sticker with any other value is at most x. In Case R, the lowest prize by picking a sticker with 10 on it is 10; the highest prize by picking a sticker with any other value is at most 10. Thus, picking a sticker with 10 on it is obviously dominant in both cases. The argument does not follow in Decision S. For example, in Case L, the lowest prize by picking a sticker with 10 is 0, lower than the highest prize of picking 8 (that is 8).
D, Decision S and Decision D, and each task will repeatedly be played for 10 rounds. In Treatment 1, the subjects first play 10 rounds of Game S followed by 10 rounds of Game D, and then Decision S for the first 10 rounds, followed by 10 rounds of Decision D. In Treatment 2, the order is reversed. The instructions for each task are given immediately before that task. There is no information feedback until the end of the experiment. At the start of each game of the experiment, the subjects are randomly assigned into groups of four. These groups persist throughout the experiment. Consequently, each group’s play can be regarded as a single independent observation in the statistical analysis. Our design allows us to compare each subject’s behavior in Game (Decision) S with those in Game (Decision) D, controlling for sequential order effects. Moreover, using the data for only the 1st game and decision tasks in both treatments, we are also able to compare across subjects how players behave differently in Game (Decision) S and Game (Decision) D.

| Table 1—The Crossover Design |
|-------------------------------|-------------------|
|                               | Game Tasks        | Decision Tasks   |
|                               | 1st Game          | 2nd Game        | 1st Decision   | 2nd Decision   |
| Treatment 1                   | Game S            | Game D          | Decision S     | Decision D     |
| Treatment 2                   | Game D            | Game S          | Decision D     | Decision S     |

C. Administrative Detail

The subjects were paid $5 for participating in the experiment, in addition to their profits or losses from every round of the experiment. On average, they received a total of $16.19, including the participation payment.

We conducted the experiment in January 2017 at the Ohio State University Experimental Economics Laboratory, using z-Tree (Fischbacher 2007). We

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29 In each task, at the end of Round 10, we randomly select a round and add to the subjects’ earnings the payment they receive in that round. Azrieli, Chambers and Healy (2016) prove that such random problem section mechanism is the only incentive compatible mechanism assuming monotonicity.
recruited subjects from the student population using the ORSEE online recruiting system. We administrated 7 sessions, where each session involved 3-5 groups. Each session lasted about 60 minutes. The data was collected from a total of 108 subjects in 27 groups of 4, with 13 groups in Treatment 1 and 14 groups in Treatment 2. 48% of subjects are female and 52% are male.

D. Result

To compare the subjects’ behavior in Game S to their behavior in Game D, we report the proportions, in the pooled data, of the games that do not end in the DS outcome. In Game S, 36.58% of the games did not end in the DS outcome, where in Game D this percentage is just 3.33%. Tables 2, 3 and 4 show the empirical frequency of non-DS outcomes by Games and by 5-round blocks, in the pooled data, the within-subject, and the cross-subject comparison. Deviations from the DS outcome happen almost 10 times more frequently in Game S than in Game D, and these differences are highly significant in both early and late rounds of the pooled data, the within-subject, and the cross-subject comparison. In Game S, 31.85% of the submitted erroneous rank-order lists, and in Game D, this percentage is just 1.11%.

<table>
<thead>
<tr>
<th></th>
<th>Game S</th>
<th>Game D</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds 1-5</td>
<td>36.11%</td>
<td>3.70%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Rounds 6-10</td>
<td>37.04%</td>
<td>2.96%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>p-value</td>
<td>0.987</td>
<td>0.135</td>
<td></td>
</tr>
</tbody>
</table>

Notes: For each group, for each 5-round block, we record the error rate. When comparing Game S to Game D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using the Wilcoxon matched-pairs signed-ranks test.
TABLE 3 PROPORTIONS NOT ENDING IN DS OUTCOMES (Within-Subject Comparison)

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Game S</th>
<th>Game D</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>31.92%</td>
<td>1.54%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>6-10</td>
<td>36.92%</td>
<td>1.54%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>p-value</td>
<td>0.650</td>
<td>1.000</td>
<td></td>
</tr>
</tbody>
</table>

TABLE 4 PROPORTIONS NOT ENDING IN DS OUTCOMES (Cross-Subject Comparison)

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Game S (Treatment 1)</th>
<th>Game D (Treatment 2)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>31.92%</td>
<td>5.71%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>6-10</td>
<td>36.92%</td>
<td>4.29%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>p-value</td>
<td>0.650</td>
<td>0.080</td>
<td>0.824</td>
</tr>
</tbody>
</table>

Notes: We use the data for only the 1st game in both treatments, Game S in Treatment 1 and Game D in Treatment 2, for Cross-Subject Comparison.

To compare subject behavior in Decision S and Decision D, we display the proportion of dominated choice. In Decision S of the pooled data, 23.80% of choices are dominated strategies. In Decision D of the pooled data, 0.83% of choices are dominated strategies. Tables 5, 6 and 7 report the empirical frequency of dominated choice, by Decision Tasks and by 5-round blocks, in the pooled data, the within-subject, and the cross-subject comparison. Dominated choice happens more frequently in Decision S than in Decision D, and these differences are highly significant in both early and late rounds of the pooled data, the within-subject and the cross-subject comparison. 30

TABLE 5 PROPORTIONS OF DOMINATED CHOICE (POOLED DATA)

<table>
<thead>
<tr>
<th>Rounds</th>
<th>Decision S</th>
<th>Decision D</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-5</td>
<td>25.74%</td>
<td>1.11%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>6-10</td>
<td>21.85%</td>
<td>0.56%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>p-value</td>
<td>0.014</td>
<td>0.473</td>
<td></td>
</tr>
</tbody>
</table>

30 Shorrer and Sóvágó (2017) documented significant evidence in choices of dominated strategies in a field experiment of the Hungarian college admissions process. Our experiment results thus point to a direction of solving this problem: we could design a variation of the current mechanism that implements truthful reporting in partition dominant strategies.
Notes: For each group, for each 5-round block, we record the error rate. When comparing Decision S to Decision D, we compute p-values using a Wilcoxon rank-sum test. When comparing early to late rounds of the same game, we compute p-values using Wilcoxon matched-pairs signed-ranks test.

TABLE 6 PROPORTIONS OF DOMINATED CHOICE (Within-Subject Comparison)

<table>
<thead>
<tr>
<th></th>
<th>Treatment 1</th>
<th></th>
<th>Treatment 2</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Decision S</td>
<td>Decision D</td>
<td>p-value</td>
<td>Decision D</td>
</tr>
<tr>
<td>Rounds 1-5</td>
<td>23.85%</td>
<td>1.54%</td>
<td>&lt;0.001</td>
<td>Rounds 1-5</td>
</tr>
<tr>
<td>Rounds 6-10</td>
<td>20.00%</td>
<td>0.38%</td>
<td>&lt;0.001</td>
<td>Rounds 6-10</td>
</tr>
<tr>
<td>p-value</td>
<td>0.143</td>
<td>0.312</td>
<td></td>
<td>p-value</td>
</tr>
</tbody>
</table>

Notes: When comparing Decision S to Decision D within subjects, we compute p-values using Wilcoxon matched-pairs signed-ranks test.

TABLE 7 PROPORTIONS OF DOMINATED CHOICE (Cross-Subject Comparison)

<table>
<thead>
<tr>
<th></th>
<th>Decision S (Treatment 1)</th>
<th>Decision D (Treatment 2)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rounds 1-5</td>
<td>23.85%</td>
<td>0.714%</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>Rounds 6-10</td>
<td>20.00%</td>
<td>0.714%</td>
<td>&lt;0.001</td>
</tr>
</tbody>
</table>

Notes: We use the data for only the 1st decision task in both treatments, Decision S in Treatment 1 and Decision D in Treatment 2, for cross-subject Comparison.

In summary, subjects play the dominant strategy at much higher rates in mechanisms with partition dominant strategies, as compared to dominant strategy mechanisms that should implement the same allocation rule. Moreover, subjects choose dominated strategy at much lower rate in decision tasks when the optimal choice is also partition dominant than when it is not. In Appendix B, we display alternative statistical analyses that yield similar results. We also found a significant negative correlation between the priority score and the deviation from dominant strategies in Game S.\(^{31}\) See Appendix B online for details.\(^{32}\)

\(^{31}\) Hassidim, Romm and Shorrer (2016) also find negative correlation between the priority score and the deviation from the dominant strategy in Serial Random Dictatorship by revisiting data from one of the treatments in Li’s (2017) experiment.

\(^{32}\) As a side result, we found that women are more likely to choose dominated strategies in both Game and Decision S. But we are aware that the gender difference we found might be due to other correlated factors, which is beyond the scope of the current paper.
IV. Discussion and Future Research

A. Other Related Literature

Our paper contributes to the broad literature on limited human cognition and its impact on economic decisions. It develops and illustrates a distinct approach by focusing on a deficiency in contingent reasoning under uncertainty, other than incorrect probability judgement (e.g., Kahneman and Tversky 2000)\textsuperscript{33} and previous reported violations of Expected Utility Theory.\textsuperscript{34}

Our approach does not necessarily contradict other psychological explanations for choosing a dominated strategy. A decision maker who has a POP might be also affected by certain “non-optimal” heuristics in scenarios where reasoning the optimal choice is beyond his or her bound of rationality. For example, Shafire, Simonson and Tversky (1993) proposed a theory of reason-based choice to explain violations of dominant strategies in the prisoner dilemma games and other choice tasks.\textsuperscript{35} that leads them to cooperation. However, such a decision can also be rationalized by our POP,\textsuperscript{36} as the player somehow believes that by cooperating, even in a single shot game, the other player is more likely to reciprocate. Another example is Arad (2014)’s choice experiment, where participants select one of the following money prizes–\{16, 17, 18, 19, 21, 23\} and write their selection on a

\textsuperscript{33}The subjects’ inability to make correct probability judgment in a risk environment includes overestimation of small probabilities, failure of Bayesian updating and representative bias, conjunction fallacy, etc.

\textsuperscript{34}To accommodate experimental evidences of the violation of Expected Utility Theory, especially the violation of the independent axiom (Allais 1953), several theories were introduced as alternatives. Prominent in those are Rank-dependent utility models (Quiggin 1982; Yaari 1987; Hong et al. 1987); Green and Jullien 1989)), Betweenness Conforming theories (Chew and MacCrimmon 1979; Fishburn 1983; Dekel 1986; Gul 1991), Prospect Theory (Kahneman and Tversky 1979) and Regret Theories (Bell 1982; Loomes 1982).

\textsuperscript{35}They claim that people’s choices are guided by reasons and some players may follow the golden rule, a variant of what is mentioned in both Kant’s Categorical Imperative and Confucianism in ancient China, “treat others as you wish to be treated.”

\textsuperscript{36}Note that in the prisoner’s dilemma game, there is no ODS. Thus, a player who cannot reason state-by-state may fail to recognize the DS.
sticker. Next, subjects are asked to place the sticker on one face of a six-face standard die and roll it. The participants win that prize only if the die falls with the sticker faces up. Selecting any other prize than 23 is dominated, since the participant could put the prize of 23 on the same face as the other prize she considers. However, 31% of the subjects did not choose the prize of 23 in the experiment. Arad argues that those subjects are affected by magical thinking: greediness or tempting-fate would increase the likelihood of adverse outcomes. Such behavior can also be captured by POP preference that assign a larger $\alpha$ or a higher subjective probability of winning the prize by choosing a lower prize (being less greedy). Explanations such as reason-based choice, magical thinking as well as joys of winning, spite-motive, and inequality aversion offer specific explanations in specific environments for DS violations. Our approach provides a more unified theoretical foundation. It thus also serves to bridge the gulf between the rational and the psychological narratives.

The general idea that a decision maker has a coarse vision of the state-space appears in research in psychology (see e.g., Tversky and Koehler 1994), ambiguity and non-additive probabilities (Schmeidler 1989; Epstein et al. 2007; Ghirardato 2001; Mukerji 1997; Ahn and Ergin 2010), formation of subjective state-space (Dekel et al. 2001), and growing awareness (Karni and Marie-Louise 2013). However, these papers mainly focus on the formation of subjective probabilities over the state-space, while ours first proposes a weakening of the monotonicity axiom based on such a coarser understanding of the state-space.

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37 Using the magical concerns of punishment for greediness to explain the choice of dominated strategies dates to the Newcomb paradox (Nozick, 1969).

38 As in Ozdenoren and Peck (2008), the decision maker may behave as if playing “against” the less than benevolent nature and think their choices may affect nature’s move as in a sequential game.
Our POP can also be interpreted from the perspective of procedural rationality, similar to, but richer than, how Osborne and Rubinstein (1998) motivate their \( S(K)-equilibrium \). We allow a decision maker to have a coarse understanding of the relationship between the other players’ choice, state of the world, her own choice and the outcome. The level of coarseness is modeled by a partition of their subjective state-space.\(^{39}\) Decision makers know the set of outcomes that each of their actions induces, given any event of the partition; however, they do not distinguish in detail, in which state each outcome would come out. Thus, rather than following substantive rationality (Simon 1976) right away, optimizing given a belief over the state-space, they adopt the following procedure: They first associate one outcome with each of their actions, in each event of the partition, by sampling, literately or virtually, in a certain way. They then follow substantive rationality in the reduced problem by optimizing a given a belief over the partition. Our approach thus retains the tight system of axioms that have dominated classical economics but also consider the actual processes of cognitions that have prevailed in psychology.

\(\text{B. Future Research and Conclusion}\)

Li (2017) points out that it is often difficult to find an ODS implementation when the allocation rule is implementable by a DS mechanism. Can we still come up with better implementation in such cases? In other words, could we find a mechanism that works for POP with coarser partitions? We have already shown in this paper, by examples, that such an improvement is possible. We leave those for future

\(^{39}\) Osborne and Rubinstein (1998) only consider the coarsest partition. In their model, decision makers first associate one outcome with one action, by sampling and then choosing the one that has the best outcome.
research to characterize the necessary and sufficient conditions needed for implementation in *partition dominant strategies*.\(^{40}\)

In this paper, we discuss partitions over subjective state-space as it is given exogenously. It is beyond the scope of this paper to propose a general theory that accounts for the formation of partitions.\(^{41}\) Clearly, more work is needed to analyze such dynamics.

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\(^{40}\) Zhang and Kwon (2017) prove a *revelation principle of exclusion* that substantially simplifies the procedure of searching for mechanisms with *partition dominant strategies*. The principle shows that we can without loss of generality restrict attention to the, newly proposed, *direct mechanisms of exclusions*.

\(^{41}\) Zhang and Kwon (2017) show that by empirical observations, we cannot distinguish bounded rational agents who cannot spot a DS from rational agents who lack trust in the designer when monitoring is limited. The coarseness of the partition characterizes both the level of bounded rationality and limitations in Monitoring. It directs to a way of endogenizing the partition: optimization on the levels of monitoring given the tradeoff between audit cost and achieving desirable outcomes.


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