Quality Disclosure on Online Marketplaces

Sangjun Yea*

The Ohio State University

April 6, 2017

Abstract

We analyze duopoly firms’ quality disclosure incentive when they sell a horizontally and vertically differentiated product in an online marketplace. Vertical characteristic of a product, say quality, is common to all consumers but is privately known to its producer while horizontal characteristic of both products is known to all consumers. We assume that the online marketplace can observe the realized quality of both products that are sold through it and can send unverifiable messages regarding the product information to consumers with no costs. We show that there exists a set of equilibria where both firms use a cutoff strategy for quality disclosure decision and the online platform employs a communication rule sending informative messages that consumers use to learn about which product’s quality is higher. Indeed, sending “comparative messages” in equilibrium is payoff-dominant for the platform among all possible informative equilibria in the interim subgame where no firm discloses quality. We also show that firms which are on an informative platform withhold information more than when they are on a “non-informative platform”. Comparative statics and welfare comparison between “comparative platform” and “non-informative platform” are provided.

1 Introduction

Firms pay substantial costs to disclose product information. Promotion through advertisement or quality certification from trusted institutes often involve a huge amount of expenditure. Even

*e-mail : yea.2@osu.edu
though it is natural that producers have more information on their products than consumers do, consumers tend not to believe producers’ claims about quality unless the message is costly. This is mainly due to the fact that consumers know that a firm’s interest is not aligned with their benefits and thus the firm wants to exaggerate product information. As many studies have argued, when the conflict of interest between informed party and uninformed party is large, cheap and non-verifiable message from informed party is not considered to be credible to uninformed party (Crawford and Sobel 1982). Thus, firms with private information on important characteristic of their product frequently rely on costly devices for verification or on raising credibility in the message itself.

However, the emergence of marketplaces based on online platforms has significantly changed the way that product information is revealed to consumers. Online platforms have two important features that might affect information disclosure decisions made by firms. First, online platforms have technologies to aggregate information from their users. By analyzing users’ purchasing patterns or collecting evaluation of a product from users who purchased it, online marketplaces such as Amazon and Google Play can be informed about the quality of products sold on their platform as well as the producers who sell through the platform. Second, online platforms have communication technologies to provide their users with curated information. Consumers accessing an online marketplace to purchase items are exposed to many messages such as recommendations, five-star ratings and consumer reviews, and their purchasing decisions are affected by these factors.

When online marketplaces are able to collect data and transmit information at negligible costs, not only are they an intermediary in a market, but they also play a role as an informed expert who observes the realized product quality and sends this information to consumers to maximize its own profits. Indeed, unlike the firms who earn sales profits directly from transactions with consumers, online marketplaces have no reason to favor a particular firm, and in fact have incentives that may be partially aligned with consumers’ welfare. Contemplating this incentive of online marketplace, there may be a room for informative communication between consumers and platform.

In this paper, we focus on the questions related to communication between an informed platform and uninformed consumers: Are there any communication rules under which a platform credibly sends informative messages to consumers without verification? To what extent does the platform
communicate the information it has available? What is the most profitable communication policy for the platform?

We also address questions about duopoly firms’ quality disclosure incentives when both firms sell their products on an online platform. When the quality information provided by an online sales platform is credible to consumers, quality disclosure incentives of firms can be influenced by the extent of information communicated via that platform. This raises further questions. Does the informative communication by the platform crowd out firms’ voluntary quality disclosure? What is the relationship between a firm’s quality disclosure and informativeness of the platform’s message?

We analyze the above questions by setting up a theoretical model, which extends Levin, Peck and Ye (2009) by introducing a platform. There are duopoly firms in the economy where each firm produces a product to serve a continuum of consumers located along a Hotelling line. It is assumed that consumers know about their own horizontally differentiated tastes on each product while vertically differentiated products’ qualities are common but unobservable among all consumers. The true quality of a product is privately revealed to the firm who produces it. After observing its product quality, each firm simultaneously decides whether to reveal the quality information by incurring a fixed cost. Once a firm decides to disclose, the realized quality of a product is revealed to all consumers. That is, the disclosing firm cannot lie about the quality.

A novel feature of our model is that platform arranges transactions between consumers and firms as an intermediary and also acts as an informed expert who can send messages regarding products’ quality to influence consumers. We explicitly define the platform’s payoff by assuming that it collects a portion of sales proceeds from both firms. The functional form of the platform’s payoff incentivizes the platform to distinguish both products’ quality as much as possible. Accordingly, the platform’s communication rule in equilibrium is directly related to its profit maximization behavior. After an announcement on the products’ quality is made by the platform, consumers have some expectation on the quality. Then, firms compete in price.

With this model, we show that in an equilibrium, message that compares product qualities, called by “comparative message”, can be credibly transmitted to consumers. This feature can be attributed to the fact that the platform has a state-independent preference and the relevant state
space is multidimensional. As shown in Chakraborty and Harbaugh (2010), under this setting, there exists a set of cheap talk equilibria where an informed sender can send informative messages to an uninformed decision maker. We also prove that among all possible informative equilibrium communication rules, sending two different messages depending on the realized state, that is, saying either “firm 1’s product is better than or equal to that of firm 2” or “firm 2’s product is better than that of firm 1”, is a payoff-dominant communication rule for the platform on the equilibrium path where an informative cheap talk is feasible. A necessary condition for sending informative messages in equilibrium is that the platform should get the same payoff irrelevant of the messages it sends. This implies that for any informative equilibrium communication rule we can always find a payoff-equivalent informative equilibrium which uses two distinct messages in communication. Since the platform can get the best payoff when the differential between the expected quality of the two products is the largest, the best communication policy that the platform can have with just two messages is to inform which product has the higher quality.

Then focusing on firms’ quality disclosure incentive, we show that firms who sell through a platform which sends comparative messages, denoted by “comparative platform”, have less incentive to disclose quality compared to when they are under a platform with non-informative messages. Intuitively, the marginal type of firm who is indifferent between disclosure and non-disclosure under the non-informative platform finds it more profitable not to disclose under the comparative platform because given that both firms are not disclosing, the marginal firm’s product is perceived to be of higher quality under the comparative platform than under the non-informative platform. In the following comparative statics analysis, we see that an increase in the cost of disclosure or the share of profits collected by the platform ambiguously affects firms’ quality disclosure incentive under the comparative platform. The most striking result that we find is that under the comparative platform, an increase in the disclosure cost might lead the firms to reveal their product quality more when the threshold level of disclosing strategy is high enough. This unintuitive result can be explained by the fact that the platform providing a quality comparison when both firms do not reveal quality reduces the size of marginal change in relative benefit of disclosure over non-disclosure so significantly that the overall marginal change in the expected relative benefit of disclosure has
a negative sign when the threshold level is high enough. Finally, we prove that platform’s ex-ante expected profit is higher under the comparative strategy than under a non-informative strategy if and only if the firms’ cost of disclosure is above some threshold level. This result is based on the observation that even if the platform’s ex-ante expected profit is higher under the comparative strategy than under a non-informative strategy at the same level of quality disclosure, the firms’ equilibrium threshold level of quality disclosure is so high under the comparative platform compared to the one under a non-informative platform that the ex-ante expected profit is smaller under the comparative platform. However, when the cost of disclosure is high enough, then firms are unlikely to reveal quality regardless of platform’s strategy, and the comparative platform can have a higher ex-ante expected profit because consumers can get imprecise but meaningful information about the differential in qualities under the comparative platform. This intuition also can be applied to the consumer welfare analysis and the result is similar to the case of platform’s ex-ante expected profit: ex-ante consumer welfare is also higher under a comparative platform if and only if the firms’ cost of disclosure is above some threshold level.

This paper contributes to the quality disclosure literature by considering the possibility that firms may rely on a costless but conditional disclosure implemented by an online platform, which can substitute costly and verifiable disclosure. In line with previous studies (e.g., Viscusi 1978, Jovanovic 1982, Cheong and Kim 2004, Guo and Zhao 2009, Levin, Peck and Ye 2009), disclosure friction described as costs still plays a role in our model regarding firms’ withholding information. However, our model additionally claims that the non-disclosure incentive of the firm is reinforced by the existence of an informative platform. Thus, the firms are less likely to disclose quality compared to those firms in the previous studies.

Also, although our paper analyzes duopoly firms, the reason for doing so here differs from the previous literature that has analyzed the same. Other studies set up duopoly models in order to show that firms’ decision in the competition stage shapes the incentive of quality disclosure (e.g., Board 2009, Hotz and Xiao 2013). However, the present model uses duopoly firms because having multiple products is a crucial element in constructing a credible communication rule employed by the platform. That the form of information transmission using soft information is based on a
specific market structure is a unique feature of our model.

Another strand of literature in quality disclosure has analyzed the situation where a firm can also signal product quality by using means other than disclosure. In particular, price may serve to signal quality in these papers (e.g., Fishman and Hagerty 2003, Daughety and Reinganum 2008, Caldieraro, Shin and Stivers 2011, Janssen and Roy 2015). If different prices fully separate unobservable quality types in price competition, disclosure is redundant and price-signaling might substitute for disclosure. As Janssen and Roy (2015) and Caldieraro et al. (2011) show, there might exist a trade-off in some cases between choosing to disclose, which brings about a perception of higher quality but also more competition in expectation, and choosing not to disclose, which allows for avoidance of competition and benefiting from price distortion. When the former effect is dominated by the latter one, non-disclosure outcome arises in equilibrium. Instead of considering the price as an alternative means of information disclosure, we introduce an independent player which sends unverifiable messages to consumers. When the platform transmits some credible quality signal through cheap talk, this soft information partially substitutes for the use of hard information stemming from firm’s disclosure as we have seen in the above. However, the mechanisms are very different.

This paper also pertains to a growing literature of comparative advertisement. Anderson and Renault (2009) assumes that a firm may utilize its rival’s matching value information in the form of comparative message and shows that a lower quality firm uses comparative advertising against a higher quality firm when the quality difference is large enough. Emons and Fluet (2012) studies duopoly firms’ disclosure strategy when each firm can observe the other firm’s quality and disclose the difference in quality levels by using costly comparative advertising.

There are two distinctions between our model and these studies. First, they assume comparative advertising only contains verifiable information due to legal restrictions. The verifiability assumption does not limit the set of feasible messages which are credibly sent to consumers, even though the firm is intrinsically a biased sender. On the contrary, in our model, comparative messages are generated by a platform whose interest is not particularly aligned with any firm, and such impartiality enables it to send an informative message to consumers even though the mes-
sage itself is unverifiable. Second, we show that there exists an equilibrium under which sending comparative message is the payoff-dominant communication policy among all feasible equilibrium communication rules in the interim subgame. Therefore, our model rationalizes the use of comparative message as an endogenous choice under a more generalized domain of communications, while precedent studies consider the comparative advertising as an option to choose which is exogenously given.

Lastly, this paper is closely related to persuasive cheap talk models under a market environment (e.g., Chakraborty and Harbaugh 2010, Chakraborty and Harbaugh 2014). By formalizing a platform as a biased expert who has a state-independent preference under multidimensional state space, as assumed in the above papers, we draw out the informative role of online sales platform and fit it in the context of quality disclosure. As far as I know, our model is the first to approach the problem where the quality disclosure of duopoly firms and the persuasive cheap talk of a biased expert are interrelated in a framework.

2 Model

2.1 Framework

We consider a duopoly model where two firms compete in a platform. A sales platform, denoted by A, consists of a continuum of consumers who are uniformly distributed over an interval [0,1]. All transactions between two firms and consumers are processed by the platform. We assume that firm 1 is located at 0 and firm 2 is located at 1 in [0,1]. Consumer’s location, which is a point in the interval [0,1], describes the consumer’s horizontal taste difference on both firms’ products. Specifically, we assume that the consumer whose location is at x has the following preference:

\[
 u(x; q_1, q_2, p_1, p_2) =
\begin{cases} 
 \phi + q_1 - p_1 - x & \text{if purchases at 1} \\
 \phi + q_2 - p_2 - (1 - x) & \text{if purchases at 2}
\end{cases}
\]

In consumer’s utility function, \( \phi \) is a common value parameter, and \( p_i \)'s are prices charged by each firm \( i \), where \( i = 1, 2 \). We denote firm \( i \)'s product quality by \( q_i \) and assume that each firm \( i \)
draws \( q_i \) from a uniform distribution which is defined on \([0, 1]\). Since \( q_i \) is privately known to firm \( i \), no one but firm \( i \) can observe \( q_i \). As \( x \) increases, a consumer with taste \( x \) more intensely prefers 2 to 1. Each consumer is risk-neutral and has a unit demand. We assume that \( \phi \) is so large that in equilibrium all consumers purchase either from 1 or 2.

Both firms’ marginal cost of production is normalized to 0. Each firm gets the payoff of \((1 - \alpha)\), where \( \alpha \in (0, 1) \), fraction of gross profits after selling its product, and the rest of the sales proceeds is transferred to the platform. Therefore, platform \( A \) collects \( \alpha \) fraction of total sales profits, \( \alpha(\pi^G_1 + \pi^G_2) \), as its own payoff.

With the use of information technology that platform employs, it is assumed that the platform can observe the true quality of both firms’ products and provide consumers with relevant information by choosing an element in a rich set of messages, \( M \), without incurring costs. In the following section, we describe the timeline of the play and suggest a relevant definition of equilibrium in detail.

### 2.2 Timing of the Model

The timing of game is as follows. In the first stage, nature independently draws each firm’s quality, \( q_i \), from a uniform distribution on \([0, 1]\) and each firm \( i \) privately observes \( q_i \). After observing \( q_i \), firm \( i \) decides whether or not to disclose it publicly. Each firm’s disclosure strategy is represented by a function \( d_i \) where \( d_i: [0, 1] \to \{0, 1\} \). Here “\( d_i = 1 \)” represents “firm \( i \) discloses” and “\( d_i = 0 \)” represents “firm \( i \) does not disclose”. By abusing notation, let us denote the firms’ disclosure decisions by \( d = (d_1, d_2) \in \{(0, 0), (0, 1), (1, 0), (1, 1)\} \). We assume that the cost of disclosure for both firms is the same and fixed at \( c \), where \( c \in [0, \infty) \). Following the quality disclosure literature, it is assumed that disclosing false quality is impossible. That is, once a firm \( i \) discloses its product quality, then \( q_i \) is revealed truthfully. After disclosure decisions are simultaneously made by both firms, in the second stage, the true quality of firms’ products, \( q = (q_1, q_2) \), is revealed to platform \( A \) and platform sends a costless message, \( m \in M \), to all consumers. Platform’s strategy can be represented by a set of functions \( \{v_d\}_{d \in \{(0, 0), (0, 1), (1, 0), (1, 1)\}} \) where \( v_d: [0, 1]^2 \to M \) denotes a mapping from the realized quality to a message under given disclosure outcome, \( d \). Given each firm’s
disclosure strategy, $d_i$ where $i = 1, 2$, it is useful to define the inverse image of disclosure outcomes, denoted by $D^{-1}(n_1, n_2) \equiv \{(q_1, q_2) : (d_1(q_1), d_2(q_2)) = (n_1, n_2)\}$. In the next stage, following the announcement of the platform, both firms simultaneously choose prices. Consumers update their belief on each firm’s product quality after observing firms’ quality disclosure and platform’s message. Consumers’ posterior belief can be described by a mapping $\mu : M \times d \rightarrow \mathcal{P}[0, 1]^2$ where $\mathcal{P}[0, 1]^2$ denotes the set of probability density functions over $[0, 1]^2$. Then they make a purchasing decision based on the prices and the evaluation on both products’ quality. Payoffs for all players are realized at the end of the game. Figure 1 describes the timeline.

To characterize a feasible set of equilibria for studying platform’s communication and its effect on duopoly firms’ quality disclosure incentives, we mainly focus on a set of Perfect Bayesian Equilibrium (PBE) which is refined as follows:

**Definition 1** A duopoly equilibrium with an informative platform is a symmetric PBE where

1. There is a threshold level, $q^D$, such that
   
   (a) Firm $i$ discloses its product quality $q_i$ if and only if $q_i > q^D$
   
   (b) and if firm $i$ does not disclose its quality, beliefs about $q_i$ are independent of both firms’ pricing strategies.
2. And on the equilibrium path, if possible, platform sends informative messages about the quality of products. That is, there exist at least two distinct messages such that consumers have different expectation on both products’ quality depending on the message they receive.\footnote{The posterior belief that is induced by a message is the same among the consumers who receive it.}

3 Main Results

3.1 Price Competition

We consider a set of PBE and solve from the last stage by using the backward induction argument. Since price does not signal product quality, consumers form an expectation \((\tilde{q}_1, \tilde{q}_2)\) on the quality of both firms’ products by observing message, \(m\), from platform and disclosed quality, \(\{d_iq_i\}_{i=1,2}\), which are revealed by the firms’ decision, \(\{d_i\}_{i=1,2}\). Given the expected quality \((\tilde{q}_1, \tilde{q}_2)\) and prices \((p_1, p_2)\), there exists an indifferent consumer whose location is at \(x^*\). Since the expected utility from either firm’s product is identical to this consumer, the location \(x^*\) is determined by the following equality:

\[
\phi + \tilde{q}_1 - p_1 - x^* = \phi + \tilde{q}_2 - p_2 - (1 - x^*)
\]

\[
\iff x^* = \frac{1}{2} + \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2 + p_2 - p_1).
\]

Firms’ gross profits when they charge \((p_1, p_2)\) are given by:

\[
\pi_1^G(p_1, p_2) = p_1x^* = p_1 \left[ \frac{1}{2} + \frac{1}{2}(\tilde{q}_1 - \tilde{q}_2 + p_2 - p_1) \right]
\]

\[
\pi_2^G(p_1, p_2) = p_2(1 - x^*) = p_2 \left[ \frac{1}{2} + \frac{1}{2}(\tilde{q}_2 - \tilde{q}_1 + p_1 - p_2) \right]
\]

Since we assume that both firms transfer a fraction of their gross profits to platform with a portion \(\alpha \in (0, 1)\), the total net profit for firm \(i\) is given by \(\pi_i(p_i, p_{-i}) = (1 - \alpha)\pi_i^G(p_1, p_2) - c_{di}\). The optimal price for firm \(i\) solves the following maximization problem:

\[
p_i^* \in \arg \max_{p_i} \pi_i(p_i, p_{-i}) = \arg \max_{p_i} \pi_i^G(p_i, p_{-i})
\]
Thus,

\[ p_1^* = 1 + \frac{1}{3} (\tilde{q}_1 - \tilde{q}_2) \]
\[ p_2^* = 1 + \frac{1}{3} (\tilde{q}_2 - \tilde{q}_1) \]  

(1)

Note that the optimal price depends on the consumers’ expectations on qualities \((\tilde{q}_1, \tilde{q}_2)\). As a firm’s product has higher quality advantage over the other firm’s product in consumers’ expectations, the firm can charge a higher price. In equilibrium, the firm selling a product with higher perceived quality wants to raise price rather than to expand demand by cutting its price. Since both firms’ products are horizontally differentiated, the price-cutting strategy to make marginal consumer surely purchase its product reduces the profits gained from loyal consumers. With these equilibrium prices, we have the following equilibrium market share and profits as a function of expected quality:

\[ x^*(\tilde{q}_1, \tilde{q}_2) = \frac{1}{2} + \frac{1}{6} (\tilde{q}_1 - \tilde{q}_2) \]
\[ \pi_1^G(\tilde{q}_1, \tilde{q}_2) = p_1^* x^* = \frac{1}{18} (3 + \tilde{q}_1 - \tilde{q}_2)^2 \]
\[ \pi_2^G(\tilde{q}_1, \tilde{q}_2) = p_2^* (1 - x^*) = \frac{1}{18} (3 + \tilde{q}_2 - \tilde{q}_1)^2 \]  

(2)

Given the Hotelling line competition with full market coverage assumption, note that each firm can secure some positive profits for any consumers’ belief on quality. This is because the differentiated attributes of two products reduce price competition pressure between two firms even when both products’ quality is revealed. In the following section, we characterize a set of cheap talk equilibria in which informative message is transmitted by the platform when two firms use a symmetric cutoff strategy in the previous stage.

### 3.2 Platform’s Information Transmission

In this section, we analyze platform’s equilibrium communication rule under which informative messages about quality may be sent to consumers. To show that there exists such a set of equilibria, first we define the platform’s payoff function explicitly. We assume that platform collects a fixed
fraction (denoted by $\alpha$) of sales proceeds which are earned by both firms:

$$\pi_A = \alpha(\pi_1^G(\tilde{q}_1, \tilde{q}_2) + \pi_2^G(\tilde{q}_1, \tilde{q}_2))$$

$$= \alpha(p_1^*x^* + p_2^*(1-x^*))$$

$$= \alpha\left(\frac{1}{18}(3 + \tilde{q}_1 - \tilde{q}_2)^2 + \frac{1}{18}(3 + \tilde{q}_2 - \tilde{q}_1)^2\right)$$

$$= \alpha + \frac{1}{9}\alpha(\tilde{q}_1 - \tilde{q}_2)^2$$

(3)

In the second equation in above expression (3), the payoff of a platform is a convex combination of $\alpha p_1^*$ and $\alpha p_2^*$ with the weight of $x^*$ and $(1-x^*)$, respectively. Note each firm’s equilibrium price is equally distant from 1 and is increasing in the difference between its own perceived quality and that of the other firm. Also, the firm whose quality is perceived as a higher one gets more demand. Thus, the platform has an incentive to induce consumers to differentiate the quality of the products in order to maximize $\alpha(p_1^*x^* + p_2^*(1-x^*))$. Indeed, the payoff of the platform is a function of the difference in the expected quality of two products.

After observing the true state of the quality $q = (q_1, q_2)$ and firms’ disclosure decisions $d = (d_1, d_2)$, platform employs a communication rule $v_d : [0, 1]^2 \rightarrow M$, which might influence consumers’ expectation. Formally, Bayesian consumers’ belief update based on platform’s communication rule and firms’ disclosure decisions, is described by $\mu : M \times d \rightarrow P[0, 1]^2$ where $P[0, 1]^2$ denotes the set of probability density functions over $[0, 1]^2$. Here, the belief function $\mu(m, d)$ assesses a posterior belief over $[0, 1]^2$ given $v_d(\cdot)$, $\{d_i q_i\}_{i=1,2}$, and $D^{-1}(d)$. To simplify the notation, we denote the expected quality following the updated belief, when $m$ and $d$ are given, by $E\mu(m, d)$. According to the expected quality of both products and the prices charged, each consumer maximizes utility by making a purchasing decision. Given this belief update rule and the purchasing decision by consumers, platform $A$ chooses a communication rule $v_d : [0, 1] \times [0, 1] \rightarrow M$ such that for each $q$, if $v_d(q) = m$, then $m \in \arg\max_{m' \in M} \pi_A(E\mu(m', d(q)))$.

---

As we mentioned before, it is assumed that the realized quality of both firms’ products which are sold through the platform can be observed by the platform with its technology. Also, it is assumed that the platform can report unverifiable messages to consumers without paying costs.

We assume that all consumers arrive at the same posterior with a belief update by receiving upon the same message.

Formally, $E\mu(m, d) \equiv \int_{[0,1]^2} q d\mu(m, d) = E[q|v_d^{-1}(m), \{d_i q_i\}_{i=1,2}, D^{-1}(d)]$. 

12
The following proposition states the necessary and sufficient conditions for a platform to have an informative communication strategy on the equilibrium path.

**Proposition 2** Given the disclosure outcome \( d \in \{(0,0), (0,1), (1,0), (1,1)\} \), and consumers’ belief update \( \mu(m,d) \), platform’s equilibrium communication rule, \( v_d(q) \), sends informative messages on the equilibrium path if and only if (i) there exist \( m, m' \in \bigcup_{q \in D^{-1}(d)} v_d(q) \) such that \( E\mu(m,d) \neq E\mu(m',d) \), (ii) for all \( q \in D^{-1}(d) \), \( \pi_A(E\mu(v_d(q),d)) \) is a constant, denoted by \( K(d) \), and (iii) for all \( m' \in M \setminus \bigcup_{q \in D^{-1}(d)} v_d(q) \), \( K(d) \geq \pi_A(E\mu(m',d)) \).

**Proof.**

(\( \leftarrow \)) By (i), there exist at least two distinct messages, given by \( v_d(q) \), which induce different expectations to consumers. Thus, some messages that arise under this strategy are informative. For each \( q \in D^{-1}(d) \), \( \pi_A(E\mu(v_d(q),d)) \geq \pi_A(E\mu(v_d(q'),d)) \) for all \( q' \in D^{-1}(d) \) by (ii). Thus, the platform does not have an incentive to deviate by sending \( v_d(q') \) instead of sending \( v_d(q) \). Also, the platform does not deviate to sending \( m' \) by (iii). Therefore, on the equilibrium path following firms’ disclosure decision \( d \), any \( v_d(q) \) satisfying above three conditions has equilibrium property, and informative messages are sent under \( v_d(q) \).

(\( \Rightarrow \)) Suppose not. If all messages given by \( v_d(q) \) always induce the same expectation to consumers, then those messages are not informative. Thus, there exist at least two distinct messages, which induce different beliefs. If \( \pi_A(E\mu(v_d(q),d)) \) is not a constant for all \( q \in D^{-1}(d) \), without loss of generality, there exist \( q' \) and \( q'' \) in \( D^{-1}(d) \), such that \( \pi_A(E\mu(v_d(q'),d)) < \pi_A(E\mu(v_d(q''),d)) \). Then, instead of sending \( v_d(q') \), platform sends \( v_d(q'') \) at \( q' \) to maximize profits. Thus, \( v_d(q) \) is not an equilibrium strategy. If \( \pi_A(E\mu(m',d)) > K(d) \) for some \( m' \in M \setminus \bigcup_{q \in D^{-1}(d)} v_d(q) \), then sending \( v_d(q) \) is not a best-response at \( q \) for all \( q \in D^{-1}(d) \). Thus, \( v_d(q) \) is not an equilibrium strategy.

Proposition 2 claims that in equilibrium any messages associated with the information on the true quality should give the same payoff to the platform even if they might induce different expectations to consumers. From the fact that the platform’s profit is not state-dependent but depending only on the consumers’ expectation on the quality, if there were a message inducing an expectation which leads to a higher profit than other messages, then it would be always sent by the platform.
Then, in this case the consumers’ original interpretation of the message should change because the profitable message is also sent in other states, thereby invalidating the equilibrium property of the given strategy. Thus, all messages that are sent on the equilibrium path should give the same payoff to the platform. Since the payoff from any message is the same, the platform does not have an incentive to misrepresent the message by which consumers infer in which region the true state lies in.

Figure 2: Iso-profit Curves of Platform

Figure (2) describes the iso-profit curves of platform. From the shape of indifference curves, we can observe that two distinct expectations on quality, say \((\tilde{q}_1^1, \tilde{q}_2^1)\) and \((\tilde{q}_1^2, \tilde{q}_2^2)\), give the same payoff to the platform if and only if \(|\tilde{q}_1^1 - \tilde{q}_1^2| = |\tilde{q}_2^1 - \tilde{q}_2^2|\). In other words, these expectations must be equally distant from the diagonal line.

Now let us assume that in equilibrium both firms use a symmetric cutoff strategy with the same threshold, \(q^D\), when making disclosure decision.\(^6\) That is, each firm \(i\) discloses its product quality \(q_i\) if and only if \(q_i\) is above the threshold level \(q^D\). Accordingly, it is believed for consumers that unrevealed product’s quality is uniformly distributed on \([0, q^D]\) on the equilibrium path. Then, on the equilibrium path, platform \(A\) can send informative messages only when no firm discloses.

\(^6\)Indeed, we will show that such a cutoff strategy arises in equilibrium.
product quality in the previous stage.

**Corollary 3** Under a duopoly equilibrium with an informative platform, a platform cannot send informative messages on the equilibrium path if at least one firm discloses.

**Proof.** When both products’ quality is revealed to consumers by firms’ disclosure \((d = (1, 1))\), 
\[
E\mu(m, d) = E[q|v_d^{-1}(m), \{q_i\}_{i=1,2}, D^{-1}(1,1)] = (q_1, q_2)
\]

is not depending on \(m\). Thus, the platform cannot send informative messages by Proposition 2. Now we consider the case where only one firm discloses quality \((d \in \{(0,1), (1,0)\})\). Without loss of generality, we only consider the case of \(d = (1,0)\). In this case, 
\[
E\mu(m, d) = E[q|v_d^{-1}(m), q_1, D^{-1}(1,0)].
\]

By the definition of duopoly equilibrium with an informative platform, we have 
\[
D^{-1}(1,0) = [q^D, 1] \times [0, q^D].
\]

This is because firm \(i\) discloses its product quality if and only if 
\[
q_i > q^D
\]
on the equilibrium path. Thus, 
\[
E\mu(m, d) = E[q|v_d^{-1}(m), q_1, 0 \leq q_2 \leq q^D] = (q_1, \tilde{q}_2(m)).
\]

Now suppose that there exist \(m_1, m_2 \in \bigcup_{q \in D^{-1}(d)} v_d(q)\) such that \(m_1 \neq m_2\) and 
\[
E\mu(m_1, d) \neq E\mu(m_2, d).
\]

That is, \(\tilde{q}_2(m_1) \neq \tilde{q}_2(m_2)\) and 
\[
0 \leq \tilde{q}_2(m_1), \tilde{q}_2(m_2) \leq q^D.
\]

Without loss of generality, let us assume that 
\[
\tilde{q}_2(m_1) < \tilde{q}_2(m_2) (\leq q^D < q_1).
\]

Then, 
\[
\pi_A(E\mu(m_1, d)) = \alpha + \frac{1}{2} \alpha(q_1 - \tilde{q}_2(m_1))^2 > \alpha + \frac{1}{2} \alpha(q_1 - \tilde{q}_2(m_2))^2 = \pi_A(E\mu(m_2, d)).
\]

Since \(\pi_A(E\mu(m, d))\) is not constant for all \(m \in \bigcup_{q \in D^{-1}(d)} v_d(q)\), by Proposition 2 the platform cannot send informative messages in equilibrium. We can similarly prove that the platform cannot send informative messages in equilibrium when \(d = (0,1)\). 

Intuitively, once at least one firm discloses quality in the previous stage, platform cannot induce distinct expectations to consumers by different messages because it is known to consumers that now the platform’s interest is not aligned with consumers’ interest and the platform might want to misrepresent the information about the unrevealed quality for its own profit. As a result, platform only “babbles” on the unrevealed quality and has no influence on consumers’ belief. The unique cheap talk equilibrium on the equilibrium path where at least one firm discloses is the “babbling equilibrium”. “Babbling equilibrium” is characterized by prescribing the platform send a message \(m_0\) for every state \(q\) and consumers interpret this message to be “meaningless”, thereby resulting in no belief update from the message.

In the following Corollary, we prove that there exists an informative equilibrium if no firm reveals quality in the previous stage.
Corollary 4 Under a duopoly equilibrium with an informative platform, a platform can send informative messages on the equilibrium path if no firm discloses. In particular, there exists an equilibrium under which comparative messages are sent credibly to consumers who understand these messages as which firm’s product is of higher quality.

Proof. When \( d = (0,0) \), consumers’ prior beliefs on \((q_1, q_2)\) is distributed on \([0, q^D] \times [0, q^D]\) uniformly. Since the iso-profit curve of platform is parallel to the diagonal in \([0, q^D] \times [0, q^D]\) and the payoff level of the iso-curve increases as it gets farther away from the diagonal, the platform’s payoff is only determined by the distance between the point of an induced belief and the diagonal. This implies that platform is indifferent between inducing \((\tilde{q}_1, \tilde{q}_2)\) and inducing \((\tilde{q}_2, \tilde{q}_1)\) by two different messages. Let us construct a communication rule, \( v_d(q) \), by which two messages are corresponding to the realization of two elements of a partition of \([0, q^D] \times [0, q^D]\). With an abuse of notation, we identify these messages with their representing element of the partition, denoting by \( m_1 = \{(q_1, q_2) \in [0, q^D] : q_1 \geq q_2\} \) and \( m_2 = \{(q_1, q_2) \in [0, q^D] : q_1 < q_2\} \), respectively. Note that \( m_1 \) induces the expectation \((\frac{2q^D}{3}, \frac{2q^D}{3})\) and \( m_2 \) induces the expectation \((\frac{q^D}{3}, \frac{2q^D}{3})\) when each message is sent. Under this communication rule, platform does not have an incentive to misrepresent where the true state lies either in \( m_1 \) or in \( m_2 \) because both \( E_{\mu}(m_1, d) = (\frac{2q^D}{3}, \frac{2q^D}{3}) \) and \( E_{\mu}(m_2, d) = (\frac{q^D}{3}, \frac{2q^D}{3}) \) give the same payoff to the platform. For all other \( m \in M \setminus \{m_1, m_2\} \), let us suppose that \( E_{\mu}(m, d) = (\frac{q^D}{3}, \frac{q^D}{3}) \). That is, consumers consider all messages but \( m_1 \) and \( m_2 \) to be meaningless. Given this equilibrium belief update by consumers, \( v_d(q) \) satisfies three conditions in Proposition 2. Therefore, when no firm reveals product quality, there exists an equilibrium communication rule which provides consumers with informative messages in the form of quality comparison. This communication strategy is graphically described in Figure 3.

Following the construction in Corollary 4, when no firm reveals quality, we can construct other informative communication rules which use a set of messages that \( (i) \) represent each element of a partition on \([0, q^D] \times [0, q^D]\) and \( (ii) \) induce different expectations on the quality but \( (iii) \) they result in the same payoff to the platform. However, some informative communication rule, as in
Example 5, cannot give higher payoff than the “babbling platform” under which the same message is sent in all states, thereby always giving the same expectation on quality, \((q^D, q^D)\). The following communication rule cannot benefit consumers even though it is informative.

**Example 5** Suppose that no firm discloses quality. Then, platform A can send informative messages about average quality of two products sold in the platform: there exists \(N\) messages \(\{m_k\}_{k=1}^{N}\) representing \(N\) elements of a partition of \([0, q^D] \times [0, q^D]\), \(m_k = \{(q_1, q_2) \in [0, q^D] : \frac{b_k-1}{2}q^D \leq \frac{q_1+q_2}{2} < \frac{b_k}{2}q^D\}\) with \(b_0 = 0 < b_1 < b_2 < \cdots < b_{N-1} < b_N = 2\) where each message \(m_k\) induces a belief \((q^k, q^k)\) such that \(\frac{b_k-1}{2}q^D < q^k < \frac{b_k}{2}q^D\) in equilibrium. Due to the shape of the iso-profit curve of the platform, any expected belief of consumers which is placed on the diagonal in \([0, q^D] \times [0, q^D]\) induces the same payoff of \(\alpha\) for the platform. Since the center of mass in each partition element \(m_k\) is positioned on the diagonal of \([\frac{b_k-1}{2}q^D, \frac{b_k}{2}q^D] \times [\frac{b_k-1}{2}q^D, \frac{b_k}{2}q^D]\), every message sent to consumers induces a conditional expectation on the diagonal, thereby giving the same payoff of \(\alpha\) to the platform. Thus, the platform does not have an incentive to misrepresent the information about the realized partition element. As the number of partition elements increases, the information about the average quality of two products becomes more accurate. However, an increase in the accuracy of information does not increase consumers’ surplus in this case because consumers’ updated belief
with more accurate information does not affect either the prices charged by firms or the perception on the quality differential.

Let us call the platform who sends comparative messages in equilibrium by *comparative platform* (CP). It can be shown that given \( d = (0, 0) \) and \( q^D \), the communication strategy of CP constitutes a payoff-dominant equilibrium among all possible informative equilibria. In the following proposition, we prove that the expected payoff of any informative communication rule is between the expected payoff of CP and the expected payoff of babbling platform (BP).\(^9\)

**Proposition 6** If both firms follow the same cutoff strategy regarding disclosure decision and neither firm discloses its quality, the expected payoff of a platform which uses an informative communication rule cannot be above the expected payoff of comparative platform and cannot be below the expected payoff of babbling platform.\(^10\)

**Proof.** Consider a platform’s informative communication rule constructed by using a set of partition elements, which are indicated by a set of messages \( \{m_k\}_{k=1}^N \) with \( \bigcup_{k=1}^N m_k = [0,q^D] \times [0,q^D] \). In equilibrium, platform must be indifferent in sending a truthful message corresponding to a partition element which contains the realized state. Thus, the conditional expected payoff of the platform must be constant regardless of the messages to be sent; \( \pi_A(E[q|m_k]) = \alpha + \frac{1}{9}\alpha(E[q|m_k] \cdot (1,-1))^2 = \bar{k} \) for all \( k \in \{1,\ldots,N\} \). This implies that every \( \{E[q|m_k]\}_{k=1}^N \) lies on the same level of iso-profit curve of the platform.

If all \( \{E[q|m_k']\}_{k=1}^N \) lie on the diagonal, this communication rule gives the lowest equilibrium payoff because, for any informative equilibrium rule which induces a set of beliefs \( \{E[q|m_k]\}_{k=1}^N \),

\[
\pi_A(E[q|m_k]) = \alpha + \frac{1}{9}\alpha(E[q|m_k] \cdot (1,-1))^2 \\
\geq \alpha + \frac{1}{9}\alpha \cdot 0 \\
= \alpha + \frac{1}{9}\alpha((q',q') \cdot (1,-1))^2 \\
= \alpha + \frac{1}{9}\alpha(E[q|m_k'] \cdot (1,-1))^2
\]

\(^9\)We sometimes use the expression of “a platform with non-informative messages” to refer to BP.

\(^10\)In the proof of Proposition 6, we use the notation of \( E[\hat{q}|m] \) for simplicity in place of \( E\mu(m,d) \) because \( d \) is fixed at \((0,0)\).
\[ = \pi_A(E[\tilde{q}|m_{k'}]) \text{ for all } k \text{ and } k'. \]

Note that the babbling communication rule does not change the consumers’ prior and thus, induces an expected belief on the diagonal. Therefore, the babbling communication rule gives the lowest expected payoff which is the infimum of the set of platform’s expected payoffs that can be attained by all informative communication rules.

Now we consider the case where all \( \{E[\tilde{q}|m_k]\}_{k=1}^N \) lie off the diagonal. Then, we can separate \( \{m_k\}_{k=1}^N \) into two groups according to whether its position is either above or below the diagonal. Without loss of generality, we assume that \( \{E[\tilde{q}|m_k]\}_{k=1}^i \) are placed above the diagonal and \( \{E[\tilde{q}|m_k]\}_{k=i+1}^N \) below the diagonal, and we denote \( \bigcup_{k=1}^i m_k \) and \( \bigcup_{k=i+1}^N m_k \) by \( M^+ \) and \( M^- \), respectively. Then for given informative rule, we can construct an informative equilibrium strategy with two-element partition, \( \{M^+, M^-\} \), which induces the same expected payoff as the payoff under the original rule for each state. Indeed, the induced expectations according to the two-element partition strategy are \( E[\tilde{q}|M^+] \) and \( E[\tilde{q}|M^-] \), and they are located on the same level of the iso-profit curve where the original equilibrium’s induced expectations are located. Obviously, \( E[\tilde{q}|M^+] = \sum_{k=1}^i E[\tilde{q}|m_k]Pr(m_k|M^+) \) is above the diagonal and \( E[\tilde{q}|M^-] \) is below the diagonal. From the fact that the conditional expected payoff of a platform must be constant regardless of the messages to be sent in an informative equilibrium, we can conclude that the original equilibrium strategy and the new equilibrium strategy give the same expected payoff to the platform. Thus, we can replicate any arbitrary informative equilibrium with a two-element partition informative equilibrium which is payoff-equivalent to the duplicated one.

In the following, we claim that \( Pr(M^+) = Pr(M^-) = \frac{1}{2} \) must hold for any two-element partition, \( M^+ \) and \( M^- \), in any informative communication rule. This is from the fact that \( E[\tilde{q}|M^+] Pr(M^+) + E[\tilde{q}|M^-] Pr(M^-) = E[\tilde{q}] = (\frac{q^D}{2}, \frac{q^P}{2}) \) holds for any \( Pr(M^+) \) and \( Pr(M^-) \) and the fact that the distances between the diagonal line and the two points \( E[\tilde{q}|M^+] \) and \( E[\tilde{q}|M^-] \) are positive and equivalent, i.e., \( \frac{|E[\tilde{q}|M^+]|(1,-1)|}{\sqrt{2}} = \frac{|E[\tilde{q}|M^-]|(1,-1)|}{\sqrt{2}} > 0 \). Indeed,

\[
Pr(M^+)E[\tilde{q}|M^+] \cdot (1,-1) + Pr(M^-)E[\tilde{q}|M^-] \cdot (1,-1) = (\frac{q^D}{2}, \frac{q^P}{2}) \cdot (1,-1)
\]
\[
\Rightarrow \Pr(M^+) E[\tilde{q}|M^+] \cdot (1, -1) = -\Pr(M^-) E[\tilde{q}|M^-] \cdot (-1, 1)
\]
\[
\Rightarrow \Pr(M^+) |E[\tilde{q}|M^+]| \cdot (1, -1) = \Pr(M^-) |E[\tilde{q}|M^-]| \cdot (1, -1)|
\]
\[
\Rightarrow \Pr(M^+) = \Pr(M^-) = \frac{1}{2}.
\]

Now let us assume that a payoff-dominant equilibrium communication rule is replicated by an informative communication rule with a two-element partition \(\{M^{+*}, M^{-*}\}\) in the sense that they give the same payoff for all states. Since this rule attains the highest expected payoff among all informative equilibrium communication rules, its partition must solve the following maximization problem.

\[
(M^{+*}, M^{-*}) \in \arg \max_{(M^{+}, M^{-})} \Pr(M^+) \pi_A(E[\tilde{q}|M^+]) + \Pr(M^-) \pi_A(E[\tilde{q}|M^-])
\]
\[
\Leftrightarrow \arg \max_{(M^{+}, M^{-})} \alpha + \frac{1}{9} \alpha (|E[\tilde{q}|M^+] \cdot (1, -1)|)^2
\]
\[
\Leftrightarrow \arg \max_{(M^{+}, M^{-})} E[\tilde{q}|M^+] \cdot (-1, 1)
\]
\[
s.t. \Pr(M^+) = \Pr(M^-) = \frac{1}{2}
\]

The first equivalence in the objective function is from the fact that \(\frac{|E[\tilde{q}|M^+] \cdot (1, -1)|}{\sqrt{2}} = \frac{|E[\tilde{q}|M^-] \cdot (1, -1)|}{\sqrt{2}}\) and the constraint. The second equivalence is from the fact that \(E[\tilde{q}|M^+]\) is above the diagonal and by using the affine transformation of the objective function with a strictly increasing function.

In the following, we prove that an equilibrium rule with a partition \(\{U, D\}\) is a payoff-dominant equilibrium rule where \(U\) denotes the above-the-diagonal region and \(D\) denotes the below-the-diagonal region. In other words, we can claim that \(E[\tilde{q}|U] \cdot (-1, 1) \geq E[\tilde{q}|M^+] \cdot (-1, 1) > 0\) for any \(M^+\) such that \(\Pr(M^+) = \frac{1}{2}\).

Suppose not. Then, there exists \(M^{+*}\) such that \(\Pr(M^{+*}) = \frac{1}{2}\) and \(E[\tilde{q}|M^{+*}] \cdot (-1, 1) > E[\tilde{q}|U] \cdot (-1, 1) > 0\). Since \(M^{+*} \neq U\), it is implied that \(U \cap M^{+*}\) and \(D \cap M^{+*}\) are both non-empty.

By the fact that

\[
Pr(U \cap M^{+*}|M^{+*}) = \frac{Pr(U \cap M^{+*})}{Pr(M^{+*})}
\]

20
\[
\frac{Pr(U \cap M^{++})}{Pr(U)}
= Pr(U \cap M^{++}|U)
\]

and that

\[
Pr(D \cap M^{++}|M^{++}) = \frac{Pr(D \cap M^{++})}{Pr(M^{++})} = \frac{Pr(M^{++}) - Pr(U \cap M^{++})}{Pr(U)} = \frac{Pr(U) - Pr(U \cap M^{++})}{Pr(U)} = \frac{Pr(U - M^{++})}{Pr(U)} = Pr(U - M^{++}|U),
\]

We get the following:

\[
E[\tilde{q}|M^{++}] = Pr(U \cap M^{++}|M^{++})E[\tilde{q}|U \cap M^{++}]
+ Pr(D \cap M^{++}|M^{++})E[\tilde{q}|D \cap M^{++}]
= Pr(U \cap M^{++}|U)E[\tilde{q}|U \cap M^{++}]
+ Pr(U - M^{++}|U)E[\tilde{q}|D \cap M^{++}].
\]

Note that \(E[\tilde{q}|D \cap M^{++}] \cdot (-1,1) < 0 < E[\tilde{q}|U - M^{++}] \cdot (-1,1)\) because \(E[\tilde{q}|D \cap M^{++}]\) is placed below the diagonal and \(E[\tilde{q}|U - M^{++}]\) is placed above the diagonal. Thus,

\[
E[\tilde{q}|M^{++}] \cdot (-1,1) = Pr(U \cap M^{++}|U)E[\tilde{q}|U \cap M^{++}] \cdot (-1,1) + Pr(U - M^{++}|U)E[\tilde{q}|D \cap M^{++}] \cdot (-1,1)
< Pr(U \cap M^{++}|U)E[\tilde{q}|U \cap M^{++}] \cdot (-1,1) + Pr(U - M^{++}|U)E[\tilde{q}|U - M^{++}] \cdot (-1,1)
= E[\tilde{q}|U] \cdot (-1,1)
\]
This is contradictory to the assumption that \( E[\tilde{q}|M^+]|\cdot(-1,1) > E[\tilde{q}|U]|\cdot(-1,1) > 0 \). Therefore, the informative equilibrium using communication strategy with the partition \( \{U,D\} \) is a payoff-dominant equilibrium. Obviously, this is comparative platform's equilibrium communication rule.

Proposition 6 claims that sending comparative messages is the most profitable form of informative communication when no firm discloses its product quality given that each firm uses a symmetric cutoff strategy for quality disclosure.

It is notable that the firms' quality disclosure decision in the previous stage is affected by the communication strategy that is employed by the platform. In the following section, we characterize a set of symmetric equilibrium cutoff strategies under CP, which are represented by some threshold \( q^D_{CP} \), and compare them to the symmetric equilibrium cutoff strategy under BP, which is represented by \( q^D_{BP} \).

### 3.3 Quality Disclosure by Duopoly Firms

Now we analyze duopoly firms' equilibrium disclosure strategy. As a benchmark case, we first consider the case where both firms sell their products on BP. Given BP’s communication rule, platform sends meaningless messages in all states and consumers have no means to update their beliefs other than firms’ quality disclosure decision. When firms use a symmetric cutoff strategy with a threshold, \( q^D_{BP} \), consumers’ expectation on the unrevealed quality of a product is \( \frac{q^D_{BP}}{2} \) as in Levin, Peck, and Ye (2007). Proposition 7 characterizes the equilibrium threshold level, \( q^D_{BP} \). To simplify notation, we hereafter denote \( \frac{c}{1-a} \) by \( \delta \). In the following, the term “cost of disclosure” refers to the normalized cost \( \delta \).

**Proposition 7** In a platform with babbling communication rule (BP), there exists an equilibrium where both firms use a symmetric cutoff strategy with the threshold, \( q^D_{BP} \), such that

\[
q^D_{BP} = \begin{cases} 
\frac{-5}{3} + \frac{1}{3} \sqrt{25 + 216\delta} & \text{if } \delta \in (0, \frac{13}{72}] \\
1 & \text{if } \delta > \frac{13}{72}
\end{cases}
\]
**Proof.** When both firms do not disclose information, BP induces consumers to believe both firms’ products are identical in quality. In this case, the profit for each firm is equal to \((1 - \alpha)^{1/2}\). By a similar argument to Levin, Peck, and Ye (2009), we get \(q_{BP}^{D}\) straightforwardly. The following equation holds at \(q_1 = q_{BP}^{D}\):

\[
(1 - \alpha)q_{BP}^{D} \left[ \frac{1}{18} (3 + q_1 - \frac{q_{BP}^{D}}{2})^2 \right] + (1 - \alpha) \int_{q_{BP}^{D}}^{1} \frac{1}{18} (3 + q - q_{BP}^{D})^2 dq - c
\]

\[= (1 - \alpha)q_{BP}^{D} \frac{1}{2} + (1 - \alpha) \int_{q_{BP}^{D}}^{1} \frac{1}{18} (3 + \frac{q_{BP}^{D}}{2} - q)^2 dq
\]

\[
\Leftrightarrow \quad q_{BP}^{D} \left[ \frac{1}{18} (3 + q_1 - \frac{q_{BP}^{D}}{2})^2 \right] + \int_{q_{BP}^{D}}^{1} \frac{1}{18} (3 + q - q_{BP}^{D})^2 dq - \delta
\]

\[= q_{BP}^{D} \frac{1}{2} + \int_{q_{BP}^{D}}^{1} \frac{1}{18} (3 + \frac{q_{BP}^{D}}{2} - q)^2 dq.
\]

The LHS in (4) denotes the expected payoff of quality disclosure for the firm with quality \(q_1\) and the RHS in (4) denotes the expected payoff of non-disclosure for the firm with quality \(q_1\). Since the marginal firm with \(q_{BP}^{D}\) is indifferent between disclosure and non-disclosure, the LHS and the RHS should be equal at \(q_{BP}^{D}\). Note that the LHS in (4) is strictly increasing in \(q_1\) and RHS and LHS in (4) cross at most once. Thus, \(q_{BP}^{D}\) is uniquely determined. By plugging \(q_{BP}^{D}\) in \(q_1\) and solving for \(q_{BP}^{D}\), we get the following:

\[
q_{BP}^{D} = \begin{cases} 
-\frac{5}{3} + \frac{1}{3} \sqrt{25 + 216\delta} & \text{if } \delta \in (0, \frac{13}{72}] \\
1 & \text{if } \delta > \frac{13}{72}
\end{cases}
\]  

\[(5)\]
By rearranging (4), we get the following expression.

\[ q_{BP} \cdot \left[ \frac{1}{18} \left( 3 + q_{BP} - \frac{q_{BP}^{2}}{2} \right)^{2} - \frac{1}{2} \right] + (1 - q_{BP}) \cdot \left[ \int_{q_{BP}}^{1} \frac{1}{18} \left( 3 + q_{BP} - q \right)^{2} - \frac{1}{18} \left( 3 + \frac{q_{BP}^{2}}{2} - q \right)^{2} \right] dq \]

(a) \( q_{BP} \)'s expected benefit of disclosure over non-disclosure when the other firm does not disclose.

(b) \( q_{BP} \)'s expected benefit of disclosure over non-disclosure when the other firm discloses.

\[ = \delta \]

(c) Cost of disclosure

(6)

That is, equation (6) shows that \( q_{BP} \) equates the expected benefit of disclosure over non-disclosure to the cost of disclosure. The first term (a) in the LHS consists of the marginal type \( q_{BP} \) firm’s probability of winning the other firm, \( Pr^{W}(q_{BP}) \), and the expected benefit of disclosure over non-disclosure when winning, \( \pi^{W}(q_{BP}) \). It is obvious that both the winning probability and the relative benefit of disclosure increases as \( q_{BP} \) increases. Therefore, the first term (a) in the LHS in (6) is strictly increasing in \( q_{BP} \).

The second term (b) in the LHS consists of the marginal type \( q_{BP} \) firm’s probability of losing by the other firm, \( Pr^{L}(q_{BP}) \), and the expected benefit of disclosure over non-disclosure when losing, \( \pi^{L}(q_{BP}) \). Note that in this case the losing probability decreases, but the relative benefit of disclosure increases, as \( q_{BP} \) increases. Thus, the effect of an increase in \( q_{BP} \) on the term (b) is not obvious and the direction of the change depends on the level of \( q_{BP} \). Note that

\[ \Delta(b) \simeq \Delta Pr^{L}(q_{BP}) \cdot \pi^{L}(q_{BP}) + Pr^{L}(q_{BP}) \cdot \Delta \pi^{L}(q_{BP}) \]  

(7)

Intuitively, if \( q_{BP} \) is near 0, then the level of relative benefit of disclosure is almost zero because there is a negligible difference between the perceived quality when not disclosing, \( q_{BP}^{2} \), and the disclosed quality, \( q_{BP} \). On the other hand, the marginal increase in the relative benefit of disclosure, \( \Delta \pi^{L}(q_{BP}) \), is bounded below by some positive number even though \( q_{BP} \) is very low. Thus, the effect of the second term of RHS in (7) dominates the effect of the first term when \( q_{BP} \) is very low, thereby resulting that \( \Delta(b) > 0 \) for \( q_{BP} \simeq 0 \).
If $q_{BP}^D$ is near 1, then the probability of losing is negligible while the level of relative benefit of disclosure is high. As $q_{BP}^D$ increases, the relative benefit of disclosure is taken away in proportion to the change in $q_{BP}^D$. Overall, the effect of the first term of RHS in (7) dominates the effect of the second term when $q_{BP}^D$ is very high, thereby resulting that $\Delta(b) < 0$ for $q_{BP}^D \simeq 1$.

By the direct calculation of $\Delta^2(b)$, we can conclude that $\mathbf{(b)}$ in (6) is inverse U-shaped in $q_{BP}^D$. Indeed,

$$
\Delta^2(b) \simeq \Delta^2 P_L(q_{BP}^D) \cdot \pi_L(q_{BP}^D) + 2 \Delta P_L(q_{BP}^D) \cdot \Delta \pi_L(q_{BP}^D) + P_L(q_{BP}^D) \cdot \Delta^2 \pi_L(q_{BP}^D)
$$

$$
= -2 \Delta \pi_L(q_{BP}^D) + P_L(q_{BP}^D) \cdot \Delta^2 \pi_L(q_{BP}^D) = -2 \left( \frac{1}{36} q_{BP}^D + \frac{5}{36} \right) + (1 - q_{BP}^D) \frac{1}{36} = -\left( q_{BP}^D + \frac{3}{12} \right) < 0.
$$

To sum up, when $q_{BP}^D$ is small, an increase in $q_{BP}^D$ increases both terms (a) and (b) of LHS in (6). This implies that if the cost of disclosure, $\delta$, increases in the region where $q_{BP}^D$ is very small, $q_{BP}^D$ must be also increasing to raise the expected benefit of disclosure over non-disclosure in equilibrium. When $q_{BP}^D$ is close to 1, an increase in $q_{BP}^D$ increases the term (a) while this change decreases the term (b). Thus, in this case, the overall change depends on the relative size of $|\Delta(a)|$ and $|\Delta(b)|$ and it can be shown that $|\Delta(a)| > |\Delta(b)|$ in the neighborhood of $q_{BP}^D = 1$. This implies that as $q_{BP}^D$ increases near 1, the marginal increase in the expected relative benefit of disclosure when the other firm does not disclose always dominates the marginal decrease in the expected relative benefit of disclosure when the other firm discloses. Therefore, if the cost of disclosure, $\delta$, increases in the region where $q_{BP}^D \in (0,1)$, then $q_{BP}^D$ must increase to raise the expected relative benefit of disclosure in equilibrium.

It is important to note that the equilibrium threshold level $q_{BP}^D(\delta)$ is increasing in $\delta$ due to the non-trivial marginal change in the expected relative benefit of disclosure when $q_{BP}^D$ increases. Specifically, we checked that the term (a) in (6) increases so largely, as $q_{BP}^D$ increases, that the overall marginal change in the expected relative benefit of disclosure is positive even though it is partly offset by the marginal change in the term (b) in some region. If it were not the case, that is, $|\Delta(a)| < |\Delta(b)|$ when $q_{BP}^D(\delta)$ is near 1, then in equilibrium $q_{BP}^D(\delta)$ should decrease in response.
to an increase in $\delta$ to raise the expected relative benefit of disclosure.

It is obvious that if the platform’s informative communication rule allows the marginal firm with the threshold quality $q^D(\delta)$ to enjoy higher profits when no firm discloses, compared to the non-informative communication rule, then the expected relative benefit of disclosure when the other firm does not disclose under the informative platform is lower than the one under the non-informative platform. Then, in some informative equilibrium it is probable to have that $|\Delta(a)| < |\Delta(b)|$ when $q^D(\delta)$ is near 1, which implies that $q^D(\delta)$ is decreasing in $\delta$. Indeed, it can be shown that this “reversal” happens when the platform sends comparative messages in equilibrium. In proposition 8, we fully characterize a set of symmetric equilibrium cutoff strategies of firms under comparative platform and show the comparative statics results including $q^D_{CP}(\delta)$’s decrease with respect to an increase in $\delta$ under some parameter values.

**Proposition 8** In comparative platform (CP), there exists a set of duopoly equilibria with CP where both firms use a symmetric cutoff strategy with threshold, $q^D_{CP}$. Specifically, there exist two distinct values $\delta_1$ and $\delta_2$ ($\delta_1 < \delta_2$) \( ^{11} \) such that

1. For $\delta \in [0, \tilde{\delta}_2]$, there exist at most two symmetric cutoff strategy equilibria.
   
   (a) For $\delta \in [0, \tilde{\delta}_1)$ and $\delta = \tilde{\delta}_2$, there exists a unique symmetric cutoff strategy equilibrium.
   
   (b) For $\delta \in [\tilde{\delta}_1, \tilde{\delta}_2)$, there exist two symmetric cutoff strategy equilibria.
   
   (c) In (a) and (b), $q^D_{CP}(\delta)$ is characterized by the following equation:
   
   $$h(q^D_{CP}, q^D_{CP}; \delta) = -\frac{5}{72} q^D_{CP}^2 + \frac{5}{36} q^D_{CP} - \frac{1}{162} q^D_{CP}^3 - \delta = 0,$$
   
   where $q^D_{CP} \in [0, 1]$. \( ^{9} \)

   (d) Let us define $q^-(\delta) \equiv \min \{ q^* : h(q^*, q^*; \delta) = 0 \}$ and $q^+(\delta) \equiv \max \{ q^* : h(q^*, q^*; \delta) = 0 \}$. Then, $q^-(\delta)$ is continuous and strictly increasing on $[0, \tilde{\delta}_1]$, and $q^+(\delta)$ is continuous and strictly decreasing on $[\tilde{\delta}_1, \tilde{\delta}_2]$. That $q^-(\delta) = q^+(\delta)$ holds only when $\delta \in [0, \tilde{\delta}_1)$ and $\delta = \tilde{\delta}_2$.

2. For $\delta \in (\tilde{\delta}_2, \infty)$, $q^D_{CP}(\delta) = 1$.

\( ^{11} \) $\delta_1 = \frac{41}{648}$ and $\delta_2 = \frac{5(23\sqrt[3]{345} - 405)}{1728}$. 

26
Proof. Without loss of generality, let us consider the case where firm 2 follows a cutoff strategy with threshold $q^* > 0$. That is, firm 2 does not disclose quality if and only if its product quality is less than or equal to $q^*$. When $q_1$ is realized, firm 1’s expected payoff of revealing its quality $q_1$ is

$$
(1 - \alpha)q^* \left[ \frac{1}{18} (3 + q_1 - \frac{q^*}{2})^2 \right] + (1 - \alpha) \int_{q^*}^{1} \frac{1}{18} (3 + q_1 - q_2)^2 dq_2 - c. \tag{10}
$$

The first term in the above expression denotes the expected payoff of firm 1 when firm 2 does not reveal the quality. The second term denotes the expected payoff of firm 1 when firm 2 reveals the quality.

If firm 1 does not reveal its quality $q_1$, then the expected payoff is

$$
(1 - \alpha) \int_{0}^{q_1} \frac{1}{18} (3 + \frac{2q^*}{3} - \frac{q_2}{3})^2 dq_2 + (1 - \alpha) \int_{q_1}^{q^*} \frac{1}{18} (3 + \frac{2q^*}{3} - \frac{2q_2}{3})^2 dq_2 + (1 - \alpha) \int_{q^*}^{1} \frac{1}{18} (3 + \frac{q_2}{2} - q_2)^2 dq_2 \tag{11}
$$

if $q_1 \leq q^*$,

$$
(1 - \alpha) \int_{0}^{q^*} \frac{1}{18} (3 + \frac{2q^*}{3} - \frac{q_2}{3})^2 dq_2 + (1 - \alpha) \int_{q^*}^{1} \frac{1}{18} (3 + \frac{q_2}{2} - q_2)^2 dq_2 \tag{12}
$$

if $q_1 > q^*$.

In expression (11), the first term denotes firm 1’s expected payoff when firm 2’s realized product quality is below $q_1$. The second term denotes the expected payoff when firm 2’s realized product quality is above $q_1$ but is below the cutoff level $q^*$. The last term denotes the expected payoff when $q_2$ is above the threshold level and discloses its quality. In expression (12), we consider the off-the-equilibrium case where $q_1$ is above the threshold level but firm 1 does not reveal its quality.\(^{13}\)

\(^{12}\)If firm 1 reveals its product quality when it is below $q^*$, at this off-the-equilibrium path we assume that platform sends a “babbling” message about the firm 2’s product quality as long as firm 2’s quality is not revealed. Indeed, this suggested cheap talk equilibrium strategy at the off-the-equilibrium path supports the on-the-equilibrium path strategy in a duopoly equilibrium with an informative platform in question. In the subgame where $q_1$ belongs to $[\frac{1}{4}q^*, \frac{2}{3}q^*]$ and only $q_1$ is revealed, we can construct an informative cheap talk equilibrium where two messages as to $q_2$ can be sent to consumers depending on the level of $q_2$.

\(^{13}\)We assume that the platform still sends a comparative message which induces consumers to expect that firm 1’s product quality is higher than that of firm 2 when neither firm discloses at this off-the-equilibrium path. Also, if firm 2 only reveals its quality when both $q_1$ and $q_2$ are above the threshold, we assume that platform “babbles” on the unrevealed quality of firm 1’s product and consumers interpret this message as a meaningless one. This off-the-
After dividing expressions (10), (11), and (12) by \((1 - \alpha)\), we define the modified expressions as a functions of \(q_1\) as follows:

\[
f(q_1, q^*; \delta) \equiv q^* \left[ \frac{1}{18} (3 + q_1 - \frac{q^*}{2})^2 \right] + \int_{q^*}^{1} \frac{1}{18} (3 + q_1 - q_2)^2 dq_2 - \delta
\]

\[
= \frac{1}{18} q_1^2 + \frac{1}{18} q^* + \frac{5}{18} - \frac{1}{3} q^* + \frac{1}{18} (6 - q^*)q_1 - \frac{1}{54} q^* + \frac{19}{54} + \frac{1}{6} q^* - \frac{1}{2} q^* \\
+ \frac{1}{18} \left(3 - \frac{1}{2} q^*\right)^2 q^* - \delta
\]

\[
g_1(q_1, q^*) \equiv \int_{0}^{q_1} \frac{1}{18} (3 + \frac{2q^*}{3} - \frac{q^*}{3})^2 dq_2 + \int_{q^*}^{1} \frac{1}{18} (3 + q^* - \frac{2q^*}{3})^2 dq_2 + \int_{q^*}^{1} \frac{1}{18} (3 + \frac{q^*}{2} - q_2)^2 dq_2
\]

\[
= \left(\frac{1}{18} \left(3 + \frac{1}{3} q^*\right)^2 - \frac{1}{18} \left(3 - \frac{1}{3} q^*\right)^2\right)q_1 - \frac{1}{54} q^* + \frac{1}{54} + \frac{1}{2} \left(- \frac{1}{3} - \frac{1}{18} q^*\right) (-q^* + 1) \\
+ \frac{1}{18} \left(3 + \frac{1}{2} q^*\right)^2 (1 - q^*) + \frac{1}{18} \left(3 - \frac{1}{3} q^*\right)^2 q^*
\]

for \(q_1 \leq q^*\)

and

\[
g_2(q_1, q^*) \equiv \int_{0}^{q^*} \frac{1}{18} \left(3 + \frac{2q^*}{3} - \frac{q^*}{3}\right)^2 dq_2 + \int_{q^*}^{1} \frac{1}{18} \left(3 + \frac{q^*}{2} - q_2\right)^2 dq_2
\]

\[
= \frac{1}{648} q^* + \frac{1}{8} q^* + \frac{5}{36} q^* + \frac{19}{54}
\]

for \(q_1 > q^*\).

Note that \(g_1(q_1, q^*)\) is strictly increasing in \(q_1\) when \(q^* > 0\) and \(g_1(q_1, q^*) = g_2(q_1, q^*)\) at \(q_1 = q^*\). This is from the fact that \(\frac{\partial g_1(q_1, q^*)}{\partial q_1} = \frac{1}{18} \left(3 + \frac{1}{3} q^*\right)^2 - \frac{1}{18} \left(3 - \frac{1}{3} q^*\right)^2 > 0\). Now we define a non-decreasing continuous function \(g(q_1, q^*)\) as follows:

equilibrium path strategy of platform supports the on-the-equilibrium path strategy in the duopoly equilibrium we are considering.
\( g(q_1, q^*) = \begin{cases} 
  g_1(q_1, q^*) & \text{if } 0 \leq q_1 \leq q^* \\
  g_2(q_1, q^*) & \text{if } q^* < q_1 \leq 1 
\end{cases} \) \hspace{1cm} (13)

In the following, we show that \( h(q_1, q^*; \delta) \equiv f(q_1, q^*; \delta) - g(q_1, q^*) \) is strictly increasing in \( q_1 \in [0, 1] \), and moreover, \( h(q_1, q^*; \delta) \) crosses 0 at most once.

First, it can be easily shown that

\[
\frac{\partial h(q_1, q^*; \delta)}{\partial q_1} = \begin{cases} 
  \frac{1}{7}q_1 + \frac{5}{18} - \frac{2}{7}q^* & \text{if } 0 \leq q_1 \leq q^* \\
  \frac{1}{7}q_1 + \frac{5}{18} & \text{if } q^* < q_1 \leq 1 
\end{cases}
\]

\[
= \begin{cases} 
  \frac{2q_1 + 4(1-q^*)}{18} & \text{if } 0 \leq q_1 \leq q^* \\
  \frac{1}{7}q_1 + \frac{5}{18} & \text{if } q^* < q_1 \leq 1
\end{cases}
\]

\[
> 0.
\]

Thus, \( h(q_1, q^*; \delta) \) is strictly increasing in \( q_1 \). Also, by the fact that \( h(0, q^*; \delta) = -\frac{1}{648}q^*(4q^2 - 63q^* + 90) - \delta < 0 \) for any \( \delta \geq 0 \), we can conclude that \( h(q_1, q^*; \delta) \) crosses 0 at most once.

From the above results, we have the following interpretation. When \( q^* > 0 \), firm 1 with the lowest quality never reveals its product quality. For any other qualities, firm 1 discloses \( q_1 \) once \( h(q_1, q^*; \delta) \) has a positive value. That is, if there exists \( \tilde{q} \in (0, 1) \) such that \( h(\tilde{q}, q^*; \delta) = 0 \), then firm 1 discloses if and only if \( q_1 \) is above \( \tilde{q} \). If \( h(q_1, q^*; \delta) \) is non-positive for all \( q_1 \in [0, 1] \), firm 1 prefers not to disclose its quality in any case.

Since we find a set of symmetric equilibria, in such equilibria where firm 2’s cutoff strategy is given with a threshold \( q^* \), the best response of firm 1 is to use a cutoff strategy with the same threshold \( q^* \). Thus, \( q^* \) can be characterized by the equation \( h(q^*, q^*; \delta) = -\frac{5}{72}q^* + \frac{5}{36}q^* - \frac{1}{102}q^3 - \delta = 0 \) with \( q^* \in (0, 1] \). It can be easily verified that \(-\frac{5}{72}q^* + \frac{5}{36}q^* - \frac{1}{102}q^3 - \delta = 0\) has the minimum value of 0 and the maximum value of \( \frac{5(23\sqrt{777} - 405)}{1728} \approx 0.0642 \). A sufficient and necessary condition for the existence of the solution is, therefore, \( \delta \in [0, \frac{5(23\sqrt{777} - 405)}{1728}] \). In other cases where \( \delta > \frac{5(23\sqrt{777} - 405)}{1728} \), it is implied that \( h(q, 1; \delta) \leq h(1, 1; \delta) < 0 \) for any \( q \in (0, 1] \). Therefore, no firm has an incentive to disclose its product quality in equilibrium. We consider this non-disclosure equilibrium as the
symmetric cutoff equilibrium with the threshold \( q^* = 1 \) because it is strategically equivalent.

For \( \delta \in \left[ 0, \frac{5(23\sqrt{345} - 405)}{1728} \right] \), the number of solutions for the equation \( h(q^*, q^*; \delta) = 0 \) is determined by the level of \( \delta \). Indeed, it is easily shown that \( h(q^*, q^*; \delta) \) is inverse-U shaped in the relevant region, implying that there exist at most two symmetric equilibria. The exact number of solutions can be identified by the number of intersections between \( y = -\frac{5}{72}q^*^2 + \frac{5}{36}q^* - \frac{1}{144}q^*^3 \) and \( y = \delta \). From the direct calculation, we get 1(a), 1(b), 1(c), and 2 in Proposition (8).

Finally, by the inverse function theorem, \( q^{-} (\delta) \) and \( q^{+} (\delta) \) are continuous and differentiable for all \( \delta \in [0, \bar{\delta}_2) \). Specifically, the sign of \( \frac{dq^+ (\delta)}{d\delta} = \left( \frac{dh(q^*(\delta), q^*(\delta); \delta)}{dq^+} \right)^{-1} \) determines the direction of change of \( q^{-} (\delta) \) and \( q^{+} (\delta) \) with respect to \( \delta \). Thus, we get 1(d).

From Proposition 8, firstly we observe that \( q_{CP}^{D} (\delta) \) converges to 0 as \( \delta \) goes to 0, implying that unraveling result holds when there is no cost to quality disclosure (Grossman and Hart 1980, Grossman 1981). Second, the equilibrium threshold level \( q_{CP}^{D} (\delta) \) is strictly increasing in \( \delta \) when \( q_{CP}^{D} (\delta) < q_{CP}^{D} (\bar{\delta}_2) \). Higher cost raises the equilibrium threshold level because the force of unraveling is offset by the disclosure cost when \( q_{CP}^{D} (\delta) \) is low. Third, more important and striking result in our model is that for some intermediate level of costs where the equilibrium threshold level \( q_{CP}^{D} (\delta) \) is characterized between \( q_{CP}^{D} (\bar{\delta}_2) \) and 1, it is possible that \( q_{CP}^{D} (\delta) \) is decreasing in \( \delta \). That is, firms are more likely to disclose with higher disclosure costs. To explain this result, we rearrange the equation \( h(q_{CP}^{D}, q_{CP}^{D}; \delta) = f(q_{CP}^{D}, q_{CP}^{D}; \delta) - g(q_{CP}^{D}, q_{CP}^{D}) = 0 \) as follows:

\[
\begin{align*}
q_{CP}^{D} \cdot \left[ \frac{1}{18} \left( 3 + q_{CP}^{D} - \frac{q_{CP}^{D}}{2} \right)^2 - \frac{1}{18} \left( 3 + \frac{2q_{CP}^{D}}{3} - \frac{q_{CP}^{D}}{3} \right)^2 \right] \\
+ (1 - q_{CP}^{D}) \cdot \left[ \int \frac{1}{18} \left( 3 + q_{CP}^{D} - q \right)^2 - \frac{1}{18} \left( 3 + \frac{q_{CP}^{D}}{2} - q \right)^2 dq \right] \\
= \delta
\end{align*}
\]

Notice that the functional forms in (6) and (14) are only different in the terms of the expected relative benefit of disclosure when the other firm does not disclose, denoted by (a). In other words,
Figure 4: Determination of Cutoff Thresholds under BP and CP

(a) $\delta = 0.057$

(b) $\delta = 0.12$

(a) in (6) is $\frac{1}{18}(3 + q_{BP}^D - \frac{q_{BP}^D}{2})^2 - \frac{1}{2}$ while (a) in (14) is $\frac{1}{18}(3 + q_{CP}^D - \frac{q_{CP}^D}{2})^2 - \frac{1}{18}(3 + \frac{2q_{CP}^D}{3} - \frac{q_{CP}^D}{3})^2$. This implies that given the same level of threshold, the expected benefit of disclosure over non-disclosure when the other firm does not disclose is higher under the non-informative platform than under the comparative platform. This is because with probability 1 the threshold type of firm is perceived to be of higher quality under the comparative platform when both firms do not disclose while the same threshold type of firm is perceived to be of equal quality under the non-informative platform when both firms do not disclose. As we show in the following, the reduced relative benefit of disclosure, due to the informative messages from the comparative platform when no firm discloses, is not growing sufficiently fast to overcome the marginal decrease in (b) as the threshold type increases. This will reverse the direction of change in $q_{CP}^D(\delta)$ with respect to $\delta$ when $q_{CP}^D(\delta)$ is high.

In conclusion, when $q_{CP}^D$ is low, both terms in the LHS in (14) are strictly increasing in $q_{CP}^D$. Thus, if the cost of disclosure increases, then $q_{CP}^D(\delta)$ should be increasing to raise the expected benefit of disclosure over non-disclosure as we observed in the non-informative platform’s case. Still, when $q_{CP}^D$ is high, it is not obvious whether the marginal change in the LHS in (14) with respect to a change in $\delta$ is positive. By the direct calculation, it can be shown that $|\Delta(a)| < |\Delta(b)|$ when $q_{CP}^D(\delta)$ is between $q_{CP}^D(\bar{\delta}_2)$ and 1. Therefore, under the parameter region where $q_{CP}^D(\delta)$ is
above $q_{CP}^D(\delta_2)$, if the cost of disclosure increases, then $q_{CP}^D(\delta)$ should be decreasing to raise the expected relative benefit of disclosure. Figure 4 describes the equilibrium threshold level of cutoff strategy under BP and CP. The red line represents the expected relative benefit of disclosure under BP while the blue line represents the expected relative benefit of disclosure under CP. The equilibrium threshold level arises at the intersection of each line and the horizontal line which denotes the level of disclosure cost, $\delta$.

As we noted earlier, under CP, if both firms do not disclose their quality, then for firm with $q_i$, with probability $\frac{q_i}{q_{CP}^D} \cdot (1 - \frac{q_i}{q_{CP}^D})$, the firm’s product is advertised as a higher (lower) one whose perceived quality to consumers is $2q_{CP}^D(\frac{q_{CP}^D}{3})$. Then, the marginal firm who is indifferent between disclosure and non-disclosure under BP now finds it more profitable not to disclose its quality under CP because when the firm does not reveal its quality, its product is certainly advertised as a better product by the CP’s comparative message. Therefore, CP always reduces duopoly firms’ quality disclosure incentive compared to the case of BP. This intuition is proved in Proposition 9.

**Proposition 9** For all $\delta \in [0, \infty)$, the equilibrium threshold for disclosure under BP is less than or equal to the equilibrium threshold for disclosure under CP. $q_{BP}^D(\delta)$ is equal to $q_{CP}^D(\delta)$ only when $\delta \in \{0\} \cup \left(\frac{13}{72}, \infty\right)$.

**Proof.** By rearranging above characterization equations (6) and (14) for each platform, we get

$$\frac{5}{36} q_{BP}^D(\delta) + \frac{1}{24} q_{BP}^D(\delta)^2 = \delta, \text{ where } q_{BP}^D(\delta) \in [0, 1]$$

and

$$\frac{5}{36} q_{CP}^D(\delta) - \frac{5}{72} q_{CP}^D(\delta)^2 - \frac{1}{162} q_{CP}^D(\delta)^3 = \delta, \text{ where } q_{CP}^D(\delta) \in [0, 1].$$

When $\delta = 0$, the unique solution for both (15) and (16) is 0. Thus, $q_{CP}^D(0) = q_{BP}^D(0) = 0$. Let us define $f(q) = \frac{5}{36} q + \frac{1}{24} q^2$ and $g(q) = \frac{5}{36} q - \frac{5}{72} q^2 - \frac{1}{162} q^3$. By (15) and (16), for any $\delta$, $f(q_{BP}^D(\delta)) = g(q_{CP}^D(\delta)) = \delta$. Now we define $h(q) \equiv f(q) - g(q)$. Note that $h(q) > 0$ and $h'(q) > 0$ for all $q \in (0, 1)$. Thus, whenever $q_{CP}^D(\delta) > 0$,

$$0 < h(q_{CP}^D(\delta)) = f(q_{CP}^D(\delta)) - g(q_{CP}^D(\delta)) = f(q_{CP}^D(\delta)) - f(q_{BP}^D(\delta))$$
\[ \Leftrightarrow f(q_{BP}^D(\delta)) < f(q_{CP}^D(\delta)). \]

Since \( f'(q) > 0 \) for all \( q > 0 \), we get \( q_{BP}^D(\delta) < q_{CP}^D(\delta) \). This result holds for \( \delta \in (0, \frac{5(23\sqrt{345}-405)}{1728}] \) because under these values, there exist solutions for (15) and (16), and \( q_{CP}^D(\delta) > 0 \). If \( \delta \in (\frac{5(23\sqrt{345}-405)}{1728}, \frac{13}{72}] \), \( q_{CP}^D(\delta) = 1 \) while \( q_{BP}^D(\delta) \) is strictly increasing below 1. For \( \delta \in [\frac{13}{72}, \infty) \), \( q_{CP}^D(\delta) = q_{BP}^D(\delta) = 1 \).

In the following proposition, we compare the ex-ante expected payoffs of BP and CP. We show that the platform might not want to provide information on quality comparison in the ex-ante perspective when disclosure cost is so low that the firms in BP disclose their product quality with a high probability. However, when the disclosure cost is so high that the firms in BP disclose their product quality with a low probability, platform might want to provide information on quality comparison to evoke enough level of quality differential in consumers’ perception. Indeed, it is shown in Proposition 10 that there exists a unique parameter value, \( \bar{\delta}_3 \), such that a platform prefers to employ comparative communication rule ex-ante if and only if \( \delta > \bar{\delta}_3 \).

**Proposition 10** The ex-ante expected payoff for BP is higher than or equal to the ex-ante expected payoff for CP if and only if \( \delta \leq \bar{\delta}_3 \).\(^{14}\) A platform is indifferent over both communication rules only when \( \delta \in \{0, \bar{\delta}_3\} \).

**Proof.** For a given \( \delta \), let us denote the ex-ante expected payoff of BP by \( \Pi_A^{BP}(\delta) \) and that of CP by \( \Pi_A^{CP}(\delta) \). To get an expression for \( \Pi_A^{BP}(\delta) \), we consider following cases:

1. BP’s expected profit when both firms disclose:

\[
\Pi_I^{BP} = \alpha \int_0^{q_{BP}^D(\delta)} \int_{q_{BP}^D(\delta)}^1 (1 + \frac{1}{9}(q_1 - q_2)^2) dq_1 dq_2
\]

2. BP’s expected profit when only one firm discloses:

\[
\Pi_I^{BP} = \alpha \int_0^{q_{BP}^D(\delta)} \int_{q_{BP}^D(\delta)}^1 (1 + \frac{1}{9}(\frac{q_{BP}^D(\delta)}{2} - q_1)^2) dq_1 dq_2 + \alpha \int_0^{q_{BP}^D(\delta)} \int_{q_{BP}^D(\delta)}^1 (1 + \frac{1}{9}(\frac{q_{BP}^D(\delta)}{2} - q_2)^2) dq_2 dq_1
\]

\(^{14}\)\( \bar{\delta}_3 = \frac{5}{108} \sqrt{9} + \frac{1}{72} \sqrt{7}. \)
3. BP’s expected profit when neither firm discloses:

\[ \Pi_{BP}^{III} = \alpha \int_0^{q_{BP}(\delta)} \int_0^{q_{BP}(\delta)} (1 + \frac{1}{9}(q_{BP}^2/2 - q_{BP}^2)^2) dq_1 dq_2 \]

Thus, the ex-ante expected payoff of BP is :

\[ \Pi_{A}^{BP}(\delta) = \Pi_{BP}^{I} + \Pi_{BP}^{II} + \Pi_{BP}^{III} \]

\[ = \alpha \left[ -\frac{1}{54}(q_{BP}^3) + \frac{55}{54} \right] \]

Similarly, we get an expression for \( \Pi_{A}^{CP}(\delta) \) by considering the following cases:

1. CP’s expected profit when both firms disclose:

\[ \Pi_{I}^{C} = \alpha \int_{q_{CP}^I}^{1} \int_{q_{CP}^I}^{1} (1 + \frac{1}{9}(q_1 - q_2)^2) dq_1 dq_2 \]

2. CP’s expected profit when only one firm discloses:

\[ \Pi_{II}^{C} = \alpha \int_{q_{CP}^I}^{1} \int_{q_{CP}^I}^{1} (1 + \frac{1}{9}(q_{CP}^I - q_2)^2) dq_2 dq_1 + \alpha \int_{q_{CP}^I}^{1} \int_{q_{CP}^I}^{1} (1 + \frac{1}{9}(q_{CP}^I - q_2)^2) dq_2 dq_1 \]

3. CP’s expected profit when neither firm discloses:

\[ \Pi_{III}^{CP} = \alpha \int_0^{q_{CP}^I} \int_0^{q_{CP}^I} (1 + \frac{1}{9}(q_{CP}^I - q_1)^2) dq_1 dq_2 \]

Thus, the ex-ante expected payoff of CP is :

\[ \Pi_{A}^{CP}(\delta) = \Pi_{I}^{CP} + \Pi_{II}^{CP} + \Pi_{III}^{CP} \]

\[ = \alpha \left[ \frac{1}{81}(q_{CP}^4) - \frac{1}{54}(q_{CP}^3) + \frac{55}{54} \right] \]

34
Now we compare $\Pi_A^{BP}(\delta)$ and $\Pi_A^{CP}(\delta)$ for each $\delta \in [0, \infty)$.

$$
\Pi_A^{BP}(\delta) - \Pi_A^{CP}(\delta) = \alpha \left[ -\frac{1}{54} (q_{BP}(\delta))^3 + \frac{55}{54} \right] - \alpha \left[ \frac{1}{81} (q_{CP}(\delta))^4 - \frac{1}{54} (q_{C}^c)^3 + \frac{55}{54} \right] \\
= \alpha \left[ -\frac{1}{54} (q_{BP}(\delta))^3 - \frac{1}{81} (q_{CP}(\delta))^4 + \frac{1}{54} (q_{BP}(\delta))^3 \right].
$$

By

$$
q_{BP}(\delta) = \begin{cases} 
-\frac{5}{3} + \frac{1}{3} \sqrt{25 + 216\delta} & \text{if } \delta \in [0, \frac{13}{72}] \\
1 & \text{if } \delta > \frac{13}{72},
\end{cases}
$$

and the relationship

$$
\delta = -\frac{1}{162} (q_{CP}(\delta))^3 - \frac{5}{72} (q_{CP}(\delta))^2 + \frac{5}{36} q_{BP}(\delta)
$$

where $\delta \in [0, \frac{5(23\sqrt{3} - 405)}{1728}]$.

We have the following expression for $\delta \in [0, \frac{5(23\sqrt{3} - 405)}{1728}]$:

$$
\Pi_A^{BP}(\delta) - \Pi_A^{CP}(\delta) = \alpha \left[ -\frac{1}{54} (q_{BP}(\delta))^3 - \frac{1}{81} (q_{CP}(\delta))^4 + \frac{1}{54} (q_{CP}(\delta))^3 \right] \\
= \alpha \left[ -\frac{1}{54} \left( \frac{5}{3} + \frac{1}{3} \sqrt{25 + 216(-\frac{1}{162} (q_{CP}(\delta))^3) - \frac{5}{72} (q_{CP}(\delta))^2 + \frac{5}{36} q_{BP}(\delta)} \right)^3 \\
- \frac{1}{81} (q_{CP}(\delta))^4 + \frac{1}{54} (q_{BP}(\delta))^3 \right].
$$

Now it can be verified that $\Pi_A^{BP}(\delta) > \Pi_A^{CP}(\delta)$ for all $q_{BP}(\delta) \in (0, 1)$.\footnote{Let us define $f(q) \equiv -\frac{1}{81} q^4 + \frac{1}{54} q^3 - \frac{1}{54} \left( \frac{5}{3} + \frac{1}{9} \sqrt{-12q^3 - 135q^2 + 270q + 225} \right)^3$. We want to show that $f(q) > 0$ for all $q \in (0, 1)$. By using Taylor’s theorem, we have $\sqrt{-12q^3 - 135q^2 + 270q + 225} < \sqrt{-135q^2 + 270q + 225} = 15 + 9q - \frac{25}{9} q^2 + \frac{108}{27} q^3 + R_3(q) \leq 15 + 9q - \frac{108}{27} q^2 = \frac{108}{27} q^2$ for $q \in (0, 1)$. Thus, for $q \in (0, 1)$,

$$
f(q) = \frac{108}{27} q^2 + 176 \frac{3375}{3125} q^2 - 56 \frac{1875}{3125} q^2 + 32 \frac{1225}{3125} q^2 - \frac{32}{1225} q^2.
$$

Let $g(q) \equiv -\frac{13}{405} q^4 - \frac{14}{225} q^3 + \frac{176}{3375} q^2 - \frac{56}{1875} q^2 + \frac{32}{1325} q^4 - \frac{32}{15625} q^2$. Then $g'(q) = -\frac{14}{225} + \frac{352}{3375} q + \frac{352}{1325} q^4 - \frac{32}{15625} q^2$. Then $g''(q) = -\frac{13}{405} q^4 + \frac{176}{3375} q^2 - \frac{128}{3125} q^2$, and $g^{(3)}(q) = -\frac{112}{1225} + \frac{678}{3125} q - \frac{384}{3125} q^2$. Note that $g^{(3)}(q) = -\frac{112}{1225} + \frac{708}{3125} q - \frac{384}{3125} q^2 < 0$ for $q \in [0, 1]$ implying $g'(q)$ is strictly decreasing in $q$ in $[0, 1]$. By evaluating $g''(q)$ at $q = 1$, we have $g''(1) = \frac{112}{1225} > 0$.

35
When \( \delta \in \left( \frac{5(23\sqrt{345} - 405)}{1728}, \infty \right) \), firms under CP never disclose in equilibrium and the ex-ante expected payoff of CP equals to \( \Pi_{A}^{CP}(\delta) = \frac{82}{81} \alpha \). By the fact that \( \Pi_{A}^{BP}(\delta) \) is strictly decreasing in \( q_{BP}^{D}(\delta) \) and that \( \Pi_{A}^{BP}(\delta)|_{q_{BP}^{D}(\delta)=1} < \Pi_{A}^{CP}(\delta)|_{q_{BP}^{D}(\delta)=1} < \Pi_{A}^{BP}(\delta)|_{q_{BP}^{D}(\delta)=0} \), there exists a unique \( q^{*} \in (0, 1) \) such that \( \Pi_{A}^{BP}(\delta)|_{q_{BP}^{D}(\delta)=q^{*}} = \Pi_{A}^{CP}(\delta)|_{q_{BP}^{D}(\delta)=1} \). By direct calculation, we get \( q^{*} = \frac{1}{3} \sqrt[3]{9} (\approx 0.693) \). Since \( q_{BP}^{D}(\delta) \) is strictly increasing in \( \delta \) by (17), there exists a unique \( \tilde{\delta}_{3} < \frac{13}{72} \) such that 
\[-\frac{5}{3} + \frac{1}{3} \sqrt{25 + 216 \tilde{\delta}_{3}} = \frac{1}{3} \sqrt[3]{9}. \]
By direct calculation, we get \( \tilde{\delta}_{3} = \frac{5}{108} \sqrt[3]{9} + \frac{1}{72} \sqrt[3]{3} (\approx 0.116) \). For all \( \delta \in \left( \frac{5(23\sqrt{345} - 405)}{1728}, \tilde{\delta}_{3} \right) \), \( \Pi_{A}^{BP}(\delta) > \Pi_{A}^{CP}(\delta) \) still holds. However, for all \( \delta > \tilde{\delta}_{3} \), \( \Pi_{A}^{BP}(\delta) < \Pi_{A}^{CP}(\delta) \).

At \( \delta = \tilde{\delta}_{3} \) and \( \delta = 0 \), both communication rules give the same ex-ante expected payoffs to the platform.

Figure 5: Equilibrium Ex-ante Profits under BP and CP

![Equilibrium Ex-ante Profits under BP and CP](image)

Figure 5 illustrates the results in Proposition 10. First note that for any fixed threshold level, \( q \), the ex-ante expected payoff of CP is higher than that of BP. This is due to the fact that by which implies that \( g''(q) > 0 \) in \([0, 1]\) and that \( g'(q) \) is strictly increasing in \([0, 1]\). By evaluating \( g'(q) \) at \( q = 1 \), we have \( g'(1) = -\frac{443}{81075} \), which implies that \( g'(q) < 0 \) for all \( q \in [0, 1] \) and that \( g(q) \) is strictly decreasing in \([0, 1]\). Since \( g(1) = -\frac{443}{81075} > 0 \), \( g(q) > 0 \) for all \( q \in [0, 1] \). Thus, \( f(q) > g(q)q^{4} > 0 \) for all \( q \in (0, 1) \). With \( f(1) > 0 \), we can conclude that for all \( q \in [0, 1] \), \( f(q) > 0 \).
sending comparative messages when no firm reveals quality, CP can induce a quality differential in consumers’ perception while BP cannot differentiate both products. Since inducing a quality differential is always profitable to a platform, CP would have higher ex-ante expected payoff than BP if the threshold level for quality disclosure were the same in both platforms.

However, as we proved in Proposition 9, $q_{BP}^D(\delta)$ is always lower than or equal to $q_{CP}^D(\delta)$ at any $\delta$. Note that a marginal increase in the threshold level of quality disclosure reduces the ex-ante expected payoffs of both platforms because a higher threshold level implies that the probability of transmitting precise information on quality differential is replaced by the probability of transmitting no information or imprecise information on quality differential in both BP and CP. Therefore, the two different aspects as to ex-ante expected payoffs of BP and CP complicate the payoff comparison exercise.

Proposition 10 shows that at any given $\delta$ such that $q_{CP}^D(\delta) \in (0,1)$, comparative messages sent by CP crowds out firms’ quality disclosure incentives to a great extent and it reduces CP’s ex-ante expected payoff up to the point where BP’s ex-ante expected payoff is higher even though the ex-ante expected payoff curve of CP is placed higher than that of BP. This relation might be reversed when firms in BP have a very high level of threshold for quality disclosure due to the high level of disclosure cost. In this situation, the probability of transmitting information on quality differential, which is provided only by firms in BP, is very low. Then, a platform prefers sending comparative messages for inducing imprecise quality differential in consumers’ mind to sending non-informative messages.

As long as the platform’s message which induces a perception on quality differential is credible to consumers, it is obvious that such a message is also beneficial to consumers. Thus, the intuition on the comparison of ex-ante expected consumer surplus under BP and CP is based on the same structure as the case of platform’s ex-ante expected payoff. Indeed, there exists a threshold cost at which the ex-ante expected consumer surplus under BP and CP switch their rank, and that threshold level is the same with the threshold cost in Proposition 10.

**Proposition 11** The ex-ante expected consumer surplus under BP is higher than or equal to the ex-ante expected consumer surplus under CP if and only if $\delta \leq \bar{\delta}_3$. The ex-ante expected consumer surplus
surplus is the same under both platforms only when \( \delta \in \{0, \tilde{\delta}_3\}\).

**Proof.** First we calculate the expected consumer surplus under BP, denoted by \(CS_B^P(\delta)\). We consider the following distinct cases:

1. \( q_1, q_2 > q_{BP}^D(\delta) \)

\[
\begin{align*}
u(q_1, q_2) &= \int_0^{x^*(\tilde{q}_1, \tilde{q}_2)} (\phi + q_1 - p_1^*(\tilde{q}_1, \tilde{q}_2) - x)dx + \int_{x^*(\tilde{q}_1, \tilde{q}_2)}^1 (\phi + q_2 - p_2^*(\tilde{q}_1, \tilde{q}_2) - (1 - x))dx \\
&= \int_0^{\frac{1}{2} + \frac{1}{6}(q_1 - q_2)} (\phi + q_1 - 1 - \frac{1}{3}(q_1 - q_2) - x)dx + \int_{\frac{1}{2} + \frac{1}{6}(q_1 - q_2)}^1 (\phi + q_2 - 1 - \frac{1}{3}(q_2 - q_1) - (1 - x))dx \\
&= \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} + \frac{1}{36}(q_1 - q_2)^2
\end{align*}
\]

By taking the integral over the region where the utility function is defined,

\[
CS_I^B = \int_{q_{BP}^D(\delta)}^1 \int_{q_{BP}^D(\delta)}^1 u(q_1, q_2)dq_1dq_2
= \int_{q_{BP}^D(\delta)}^1 \int_{q_{BP}^D(\delta)}^1 \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} + \frac{1}{36}(q_1 - q_2)^2dq_1dq_2
\]

2. \( q_1 > q_{BP}^D(\delta), q_2 \leq q_{BP}^D(\delta) \)

\[
\begin{align*}
u(q_1, q_2) &= \int_0^{x^*(\tilde{q}_1, \tilde{q}_2)} (\phi + q_1 - p_1^*(\tilde{q}_1, \tilde{q}_2) - x)dx + \int_{x^*(\tilde{q}_1, \tilde{q}_2)}^1 (\phi + q_2 - p_2^*(\tilde{q}_1, \tilde{q}_2) - (1 - x))dx \\
&= \int_0^{\frac{1}{2} + \frac{1}{6}(q_1 - q_{BP}^D(\delta))} (\phi + q_1 - 1 - \frac{1}{3}(q_1 - q_{BP}^D(\delta)) - x)dx \\
&\quad + \int_{\frac{1}{2} + \frac{1}{6}(q_1 - q_{BP}^D(\delta))}^1 (\phi + q_2 - 1 - \frac{1}{3}(\frac{q_{BP}^D(\delta)}{2} - q_1) - (1 - x))dx \\
&= \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} + \frac{1}{36}q_1^2 + \frac{q_{BP}^D(\delta)}{18}q_1 - \frac{5}{144}(\frac{q_{BP}^D(\delta)}{2})^2 - \frac{1}{6}q_1q_2 + \frac{q_{BP}^D(\delta)}{12}q_2
\end{align*}
\]

38
By taking the integral over the region where the utility function is defined,

\[ CS_{BP}^{II} = \int_{q_{BP}^D(\delta)}^{1} \int_{0}^{q_{BP}^D(\delta)} u(q_1, q_2) dq_2 dq_1 \]

\[ = \int_{q_{BP}^D(\delta)}^{1} \int_{0}^{q_{BP}^D(\delta)} (\phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} + \frac{1}{36} q_1^2 + \frac{q_{BP}^D(\delta) q_1}{18} - \frac{5}{44} (q_{BP}^D(\delta))^2 \]

\[ - \frac{1}{6} q_1 q_2 + \frac{q_{BP}^D(\delta)}{12} q_2) dq_2 dq_1 \]

3. \( q_2 > q_{BP}^D(\delta), \ q_1 \leq q_{BP}^D(\delta) \)

By symmetry, the expected surplus in this case is the same as the one in case 2. That is,

\[ CS_{BP}^{III} = CS_{BP}^{II} \]

4. \( q_1, q_2 \leq q_{BP}^D(\delta) \)

\[ u(q_1, q_2) = \int_{x^*(\tilde{q}_1, \tilde{q}_2)}^{x^*(\tilde{q}_1, \tilde{q}_2)} (\phi + q_1 - p_1^*(\tilde{q}_1, \tilde{q}_2) - x) dx + \int_{x^*(\tilde{q}_1, \tilde{q}_2)}^{1} (\phi + q_2 - p_2^*(\tilde{q}_1, \tilde{q}_2) - (1 - x)) dx \]

\[ = \int_{0}^{\frac{1}{2}} (\phi + q_1 - 1 - x) dx + \int_{\frac{1}{2}}^{1} (\phi + q_2 - 1 - (1 - x)) dx \]

\[ = \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} \]

By taking the integral over the region where the utility function is defined,

\[ CS_{BP}^{IV} = \int_{0}^{q_{BP}^D(\delta)} \int_{0}^{q_{BP}^D(\delta)} u(q_1, q_2) dq_1 dq_2 \]

\[ = \int_{0}^{q_{BP}^D(\delta)} \int_{0}^{q_{BP}^D(\delta)} \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} dq_1 dq_2 \]

We add up these surpluses and get,

\[ CS_A^{BP}(\delta) = CS_{BP}^{II} + CS_{BP}^{IV} + CS_{BP}^{III} + CS_{BP}^{IV} \]

\[ = \int_{0}^{1} \int_{0}^{1} \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} dq_1 dq_2 + \int_{q_{BP}^D(\delta)}^{1} \int_{q_{BP}^D(\delta)}^{1} \frac{1}{36} (q_1 - q_2)^2 dq_1 dq_2 \]

\[ + 2 \int_{q_{BP}^D(\delta)}^{1} \int_{0}^{q_{BP}^D(\delta)} \left( \frac{1}{36} q_1^2 + \frac{q_{BP}^D(\delta) q_1}{18} - \frac{5}{44} (q_{BP}^D(\delta))^2 \right) dq_1 dq_2 \]

\[ + \frac{1}{6} q_1 q_2 + \frac{q_{BP}^D(\delta)}{12} q_2) dq_2 dq_1 \]
\[
\phi - \frac{161}{216} - \frac{1}{216}(\hat{q}_{BP}(\delta))^3
\]

Now we calculate the expected consumer surplus under CP, denoted by \(CS_{IIV}^{CP}(\delta)\). Note that the only difference between CP and BP is that the former gives higher expected utilities to consumers by providing with *imprecise* information about quality when neither firm discloses. The following expression describes the consumers’ average utility when the true quality of the firm 1’s product is higher than that of the firm 2’s product but no firm reveals its quality.

\[
\begin{align*}
    u(q_1, q_2) &= \int_{x^*(\tilde{q}_1, \tilde{q}_2)}^{x^*(\hat{q}_1, \hat{q}_2)} (\phi + q_1 - p_1^*(\tilde{q}_1, \tilde{q}_2) - x)dx + \int_{x^*(\hat{q}_1, \hat{q}_2)}^{1} (\phi + q_2 - p_2^*(\tilde{q}_1, \tilde{q}_2) - (1 - x))dx \\
    &= \int_{0}^{\frac{1}{2} + \frac{1}{18}q_{CP}^{P}(\delta)} (\phi + q_1 - 1 - \frac{q_{CP}^{P}(\delta)}{9} - x)dx + \int_{\frac{1}{2} + \frac{1}{18}q_{CP}^{P}(\delta)}^{1} (\phi + q_2 - 1 + \frac{q_{CP}^{P}(\delta)}{9} - (1 - x))dx \\
    &= \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4} + \frac{q_{CP}^{P}(\delta)}{18}(q_1 - q_2) - \frac{5}{324}(q_{CP}^{P}(\delta))^2
\end{align*}
\]

By symmetry, we have the following consumer surplus when neither firm has disclosed product quality and platform announces a comparative message on the level of those quality.

\[
\begin{align*}
    CS_{IIV}^{CP} &= 2 \int_{0}^{q_{CP}^{D}(\delta)} \int_{0}^{q_{CP}^{D}(\delta)} u(q_1, q_2)dq_2dq_1 \\
    &= \int_{0}^{q_{CP}^{D}(\delta)} \int_{0}^{q_{CP}^{D}(\delta)} \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4}dq_2dq_1 \\
    &\quad + 2 \int_{0}^{q_{CP}^{D}(\delta)} \int_{0}^{q_{CP}^{D}(\delta)} \frac{q_{CP}^{D}(\delta)}{18}(q_1 - q_2) - \frac{5}{324}(q_{CP}^{P}(\delta))^2dq_2dq_1 \\
    &= \int_{0}^{q_{CP}^{D}(\delta)} \int_{0}^{q_{CP}^{D}(\delta)} \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4}dq_2dq_1 + \frac{1}{324}(q_{CP}^{P}(\delta))^4
\end{align*}
\]

By adding up all surpluses, we get the following expression.

\[
\begin{align*}
    CS^{CP}(\delta) &= CS_{I}^{CP} + CS_{II}^{CP} + CS_{III}^{CP} + CS_{IIV}^{CP} \\
    &= \int_{0}^{1} \int_{0}^{1} \phi + \frac{1}{2}(q_1 + q_2) - \frac{5}{4}dq_1dq_2 + \int_{q_{CP}^{D}(\delta)}^{1} \int_{q_{CP}^{D}(\delta)}^{1} \frac{1}{36}(q_1 - q_2)^2dq_1dq_2 \\
    &\quad + 2 \int_{q_{CP}^{D}(\delta)}^{1} \int_{0}^{q_{CP}^{D}(\delta)} (\frac{1}{36}q_1^2 + \frac{q_{CP}^{P}(\delta)}{18}q_1 - \frac{5}{144}(q_{CP}^{P}(\delta))^2 - \frac{1}{6}q_1q_2 + \frac{q_{CP}^{P}(\delta)}{12}q_2)dq_2dq_1
\end{align*}
\]
Now we compare the ex-ante expected consumer surplus under two distinct platforms. By the direct calculation, we can verify that the sign of \(CS_A^{BP}(\delta) - CS_A^{CP}(\delta)\) coincides with the sign of \(\frac{1}{4\alpha}(\Pi_A^{BP}(\delta) - \Pi_A^{CP}(\delta))\) as follows:

\[
\begin{align*}
CS_A^{BP}(\delta) - CS_A^{CP}(\delta) &= \phi - \frac{161}{216} \left( q_{BP}(\delta) \right)^3 - (\phi - \frac{161}{216} \left( q_{BP}(\delta) \right)^3 + \frac{1}{324} \left( q_{CP}(\delta) \right)^4) \\
&= -\frac{1}{216} \left( q_{BP}(\delta) \right)^3 + \frac{1}{216} \left( q_{CP}(\delta) \right)^3 - \frac{1}{324} \left( q_{CP}(\delta) \right)^4 \\
&= \frac{1}{4} \left( -\frac{1}{54} \left( q_{BP}(\delta) \right)^3 + \frac{1}{54} \left( q_{CP}(\delta) \right)^3 - \frac{1}{81} \left( q_{CP}(\delta) \right)^4 \right) \\
&= \frac{1}{4\alpha} \left( \Pi_A^{BP}(\delta) - \Pi_A^{CP}(\delta) \right)
\end{align*}
\]

From Proposition 10, we have the same result about the expected consumers’ surplus.

If we assume that the cost of disclosure is fully transferred to the society, then from Proposition 10 and Proposition 11, we have the following corollary about social welfare. Since this result is obvious by the definition of social welfare, we state it without a proof.

**Corollary 12** Let us assume that the the cost of disclosure is fully transferred to the society, then in the ex-ante perspective, social welfare under BP is higher than or equal to social welfare under CP if and only if \(\delta \leq \bar{\delta}_3\). Both are the same only when \(\delta \in \{0, \bar{\delta}_3\}\).

## 4 Conclusion

In this paper, we consider the duopoly firms’ quality disclosure incentive when both firms are selling their products in an informative sales platform by which informative messages about quality are disseminated to consumers. We mainly focused on a comparative platform’s case where the communication rule employed in the platform is to indicate which product is better than the other. Under the comparative platform, we showed that firms are less likely to reveal quality because they more rely on the platform’s informative message when compared to the benchmark case with
Our model can be extended to the case with more than two firms because in any multidimensional state space, a biased expert with state-independent preference can send informative messages to uninformed decision makers. In this case, as an informative equilibrium communication rule, platform might send messages in the form of ranking which only exhibits the relative positions of quality. Under the informative platform, the main result that a firm’s quality disclosure incentive is reduced compared to the case of uninformative platform would still hold.

Firm’s symmetric equilibrium disclosure strategy in our analysis is based on the assumption that online marketplace equally treats each firm and collects the same fraction of sales profits from them. By the symmetry assumption which is reflected on the platform’s payoff function, platform can send symmetric messages in the relevant informative equilibrium, thereby bringing about firm’s symmetric quality disclosure decision. This strong symmetry assumption limits the robustness of our model and requires more flexible modeling to further study the interaction between informative communication in online platforms and quality disclosure incentives of firms.
References


