A Supernatural Reputation*

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Abstract

I study how someone can successfully sustain the reputation of having a special and secret ability to predict the future. Rational agents believe that psychics, financial experts or political advisers have a special ability to predict the future even when they do not, because, in their eyes, the data that would be generated by someone with such abilities is the same as the one generated by someone who only pretends to have a special predicting ability. Experts have an incentive to pretend to have secret special predicting skills as this increases the number of people who are willing to pay for their advice. Furthermore, I argue that an expert who claims to have supernatural powers may actually be better for society than an honest expert, who recognizes that he has no special skill but simply access to better data.

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1 Introduction

Throughout the history of civilization, there have always been a number of institutions that have claimed to have some secret special skill or knowledge that enables them to predict the future. The oracles of Ancient Greece claimed to have information directly provided by the Gods, certain psychics claim that their information comes from the stars or from the cards, financial experts often claim to have a special intuition to understand financial markets, political advisors sometimes claim that they have a special understanding of the political world because they have a special ability to understand people. A lot of the times, one would suspect, these assertions are false. One might suspect that psychics do not actually learn anything from looking at a crystal ball or that financial experts do not really have a sixth sense about the functioning of the market. Nevertheless, it is often the case that people believe the claims that these "experts" make despite there being an abundance of evidence about their ability to predict the future.

In a recent poll from 2005 in the United States, a group of people was asked whether they believed in a number of phenomena of a supernatural nature. A total of 75% of the respondents claimed to believe in at least one of the phenomena presented. In particular, 31% of the people surveyed believed in telepathy, 26% believed in clairvoyance (the power of the mind to know the past and predict the future), 25% believed in astrology and 21% believed that it is possible to communicate mentally with someone who has died.\footnote{According to a Gallup poll from June, 2005.} At first glance, it might seem surprising that so many people have these beliefs. One would think that if some expert claims to be able to predict the future because she has the ability to communicate with the dead, by looking at that expert's track record - whether her advice was actually correct in the past - one should be able to get a pretty clear picture as to whether she does have special powers or not. So then, if one takes the skeptic view and assumes that experts do not have special powers, why are there so many people who believe they do?

In this paper, I put forward a theory that explains how an expert with no special powers might be able to sustain the lie that she does in the long run, even when an unlimited amount of evidence about the quality of her advice is available.

Typical explanations for why people believe that there are those who have supernatural powers involve assuming that they have some type of behavioral bias: they do not process information as a rational agent would. I follow a different approach. I assume that all agents have rational expectations and are able to access an arbitrarily large set of evidence. What this implies is that the only way that experts can sustain the lie that they have some secret power is to have the evidence back it up. In a way, the lie they tell must be empirically indistinguishable from the truth, so that an agent who observes the data and updates his beliefs about the world using Bayes' rule cannot rule out the hypothesis that the expert
indeed has some secret power.

My theory builds on two arguments made by historians, in the context of trying to explain why the oracles of Ancient Greece, in particular the oracle of Delphi, were able to survive as long they did. The first argument is that the advice given by the oracle was ambiguous, so that the oracle was never proved wrong. Even if true, this cannot be the whole story. If oracles only gave advice which did not have any informational value there would be no reason for clients to spend money to hear it. The second theory is that those employed by the oracle would try and obtain information from the client (or his entourage) in order to better inform the advice given by the oracle. The powerful and wealthy clients of the oracle of Delphi would, often times, have to stay for several days in the city of Delphi waiting to be received by the priests. During this time, several of the citizens of Delphi, affiliated with the oracle, would try and learn more about the circumstances of the client and report back to the oracle. Once again, this does not appear to be the whole story because this strategy would, at best, leave the oracle with the same information as the client. Therefore, the advice given by the oracle would not give the client any additional information. But if the oracle is simply telling its clients what they already know, the clients would not want to pay for the privilege.

I add two additional insights to these arguments. First, I argue that it is likely that the problems of the different clients that the oracles had were somewhat correlated. The information that the oracle obtains from past clients is useful when dealing with future clients. For example, imagine that state $A$ is considering invading state $B$ and asks the oracle for advice. The oracle, through its past dealings with state $B$, might know that a war will be damaging for both states and, as a result, might give the (good) advice of restrain. The same logic applies to modern experts. Imagine, for example, that someone consults a psychic in order to obtain marital advice. The psychic is likely to have advised many people with similar problems in the past and, as a result, is likely to have gathered some expertise on the matter, which allows her to actually give good advice to the client. A financial expert, who is asked whether a certain firm will be successful in the future, might have had access to relevant information about that firm through her past dealings with other clients and, as a result, might be very capable of giving good advice. Provided this past information is sufficiently relevant, it might even be possible for the expert to give as good advice as someone who actually has special powers, whether that means a special intuition or an ability to talk with the Gods.

While this theory explains why it is possible for an expert to sustain a lie that she has special powers it begs the question of why there is any need to lie. Why do experts lie and

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2See Fontenrose (1978).

3There is the famous story about the king of Lydia who asked the oracle of Delphi whether he should attack the Persian empire. The oracle responded that if he went to war he would destroy a great empire. The king of Lydia did choose to wage war and was defeated and his own empire was destroyed. Nevertheless, the prediction of the oracle was correct if one was to think of Lydia as that great empire.
say they have special powers? If they do provide good advice - as good advice as if they had special powers - why not just confess to the clients that the reason they are able to give good advice is their access to data and not a sixth sense or an ability to talk with the Gods? The answer is that, by lying, the expert attracts more clients. The argument is made clearer by the following example.

1.1 Example

Consider an infinite sequence of periods, where, in each period, there is a short lived agent (he) who must make a decision \( a_t \in \{L, R\} \). The goal of the agent is to match the random state of the period \( \omega_t \in \{L, R\} \). If he does, he receives a payoff of 1, if he does not his payoff is 0. Before making the decision each agent observes a signal \( s_t \in [0, 1] \). Assume that \( s_t \overset{iid}{=} U(0,1) \) for all \( t \geq 0 \), while

\[
\Pr\{\omega_t = L|s_t, s_{t-1}\} = \begin{cases} 
  s_t & \text{if } s_{t-1} \geq \frac{1}{2} \\
  1 - s_t & \text{if } s_{t-1} < \frac{1}{2} 
\end{cases}
\]

for all \( (s_t, s_{t-1}) \in [0,1]^2 \) and for any \( t \geq 1 \). Notice that, for the agent of period \( t \), observing only his signal \( s_t \) does not help him because

\[
\Pr\{\omega_t = L|s_t = s\} = \frac{1}{2} \text{ for all } s \in [0,1]
\]

However, if \( s_{t-1} \) is also known, the agent is able to do better at least as long as \( s_t \neq \frac{1}{2} \). So, at least when \( s_t \neq \frac{1}{2}, \) there is an interest by agent \( t \) to access the information held by agent \( t - 1 \).

To facilitate the exchange of information, assume that there is a long lived agent called the expert (she). Assume first that the only thing that the expert does is to collect all the signals of her past clients and give as good advice as possible to any client who might consult her in exchange for a fixed price \( c > 0 \). The first problem she faces is the first client, because there is no advantage for him to pay \( c \) to learn nothing. To bypass this problem, assume that the expert is in possession of \( s_0 \). Given this, consider the problem of agent \( t = 1 \). Notice that the expected net benefit of consulting the expert and gaining access to \( s_0 \) when \( s_1 = s \) is equal to

\[
g(s) \equiv \max\{s, 1-s\} - \frac{1}{2}
\]

which is represented by the red line in Figure 1. Therefore, agent 1 will only consult the expert if \( s_1 \) is sufficiently different from \( \frac{1}{2} \). In fact, for any \( c > 0 \), there is an interval \( I \in [0,1] \) around \( \frac{1}{2} \) for which, for any time period \( t \), agent \( t \) prefers not to consult the expert if \( s_t \in I \). Assume that each agent observes which of the preceding agents consulted the expert and, if they did, whether they were able to match the state. In this case, at the moment that some
agent chooses not to consult the expert (which happens at least when $s_t \in I$), no one else
will ever consult the expert again. So, even if $c$ is small, the chain of clients who consult the
expert will break almost surely. This is bad for the expert, who will not be able to attract
as many clients as she would like, and is bad for the agents because, if $c$ is small enough,
every agent would be better off (ex-ante) if everyone blindingly chose to consult the expert
at every period, no matter what.

Imagine that, at period $t = 0$, the expert announces to the agents that the world is not
what they think it is. In particular, the lie that the expert tells is that $\omega_t$ does not depend
on $s_t$ and $s_{t-1}$ but rather only on some signal $\eta_t \in \{L, R\}$ that only she observes in every
period. In particular, the expert says that $\Pr\{\eta_t = L\} = \frac{1}{2}$ for all $t$ and that

$$
\Pr\{\omega_t|\eta_t\} = \begin{cases}
\frac{3}{4} & \text{if } \omega_t = \eta_t \\
\frac{1}{4} & \text{if } \omega_t \neq \eta_t
\end{cases}
$$

for all $t \geq 1$.

Agents are not assumed to blindly believe the expert. What is assumed instead is that
they place a probability $\pi \in (0, 1)$ on the lie being true and then, based on the past success of
the expert, update their beliefs. In this way, the lie that the expert tells must be empirically
indistinguishable from the truth, in order to keep the lie going.

Assume that $\pi = \frac{1}{2}$ and continue to assume that $s_0$ is observed by the expert. Consider
agent $t = 1$. If $s_1 = s$, his expected net benefit of accessing the expert’s information is now
given by

$$
\hat{g}(s) \equiv (1 - \pi) g(s) + \pi \frac{1}{4}
$$

which is represented by the green line in Figure 1. This means that, provided $c \leq \frac{1}{8}$, agent
$t = 1$ will consult the expert no matter what his signal is. This is not particularly impressive
because it is not hard to come up with a lie which convinces the first agent to come in. The
only thing necessary is to make the lie sufficiently appealing. The real issue is to make the
lie believable and still appealing.

Consider what happens at period $t = 2$. The agent at period $t = 2$ is assumed to observe
that agent $t = 1$ consulted the expert and, because he did, whether he was able to match the
state. So, at period $t = 2$, he will update his belief about the lie. In order to do so, he must
calculate the probability that agent $t = 1$ was successful under the two hypothesis: that the
world is such that $\omega_t$ depends on $s_t$ and $s_{t-1}$ and that the world is such that $\omega_t$ depends on
$\eta_t$. However, notice that, under both hypothesis, the probability of success is the same and is
equal to $\frac{3}{4}$. In particular, in the world where $\omega_t$ is correlated with $s_t$ and $s_{t-1}$, the probability
of success is given by
\[ \int_{0}^{1} g(s) \, ds = \frac{3}{4} \]

Therefore, at period \( t = 2 \), the belief will be the same as in period \( t = 1 \). Basically, the lie that is being told promises a success rate that is, on average, the same as in the real world, so that it becomes empirically indistinguishable from it. So, at period \( t = 2 \), the agent will again consult the expert, which will generate the same belief for period \( 3 \) and so on.

Simply put, the expert lies because it attracts more clients. If the expert did not lie, at period \( t = 1 \), the agent would sometimes be very willing to consult her (if \( s_1 \) is close to \( 1 \) or \( 0 \)) but some other times would be very unwilling (if \( s_1 \) is close to \( \frac{1}{2} \)). The problem for the expert is that, when the agent is very unwilling, he will not consult her, which will have multiplying effects going forward, because, as a consequence, no future agent would consult her. So, it is in the expert’s interest to increase the agent’s willingness to consult when he is more willing. A lie which makes the past less relevant accomplishes exactly that as one can see in Figure 1, where the green line is "flatter" than the red line. The argument is somewhat related with Kamenica and Gentzkow (2011): the fact that agents have rational expectations only forces the average success rate of the "lie" to be the same as the truth in order for the lie to last, but not each conditional success rate. The expert has additional degrees of freedom that do not affect how the agents form beliefs that she is able to explore to her benefit.
Furthermore, notice that by being dishonest the expert not only makes herself better off but also the agents. In particular, if $c$ is small, the expert is able to tell a lie that gets every agent to consult her, which increases everyone’s utility ex-ante (except agent $t = 1$). In fact, the main result of the paper, which I discuss in section 3, is that, if all agents would always be better off by committing to consulting the expert before observing their own signal, it is possible to build a lie which accomplishes just that and is empirically indistinguishable from the truth.

In section 4, I discuss the consequences of assuming that the initial prior that the lie is believed by the agents is "small". In section 5, I discuss the case where the price is endogenous and, in section 6, I conclude.

1.2 Related Literature

One of the things that I do in this paper is to show how an expert can create a lie which is empirically indistinguishable from the truth, and in that sense, lasts forever. There is a literature, initiated by Foster and Vohra (1998) among others, which discusses the somewhat related problem of whether it is possible to design a test that empirically validates someone’s theory. Imagine that an expert claims to know the distribution of some random variable. An uninformed agent would like to have a way to test whether the claim of the expert is correct. Is there such a test? The challenge is that, while the expert might be lying and, may, in fact, have no idea about the distribution of the random variable, knowing the test allows her to strategically tailor her predictions in order to pass. The general message of this literature seems to be that it is virtually impossible to design such tests, at least if one considers only tests which only use past predictions of the expert. While the two problems are fundamentally alike - whether it is possible that someone makes up a lie which lasts forever despite there being an unlimited amount of data available - the two approaches are different. In the above literature, the test is announced from the outset, the expert knows what it is and then makes her predictions in order to pass it, without being concerned about how her actions influence the uninformed agent’s beliefs. In my paper, by contrast, the only things that matter are the beliefs of the uninformed agents because those are what determine whether they choose to consult the expert. In this sense, my approach fits more with the literature on reputations in repeated games, which I discuss below. The expert in my paper creates a lie which enables her to generate a reputation that such a lie is true. Nevertheless, the message of my paper is very much aligned with this literature on the empirical testing of theories: it is at least possible for an expert to put forward a false theory about the world and never be disproved despite there being unlimited evidence.

\[4\text{See Dekel and Feinberg (2006), Olszewski and Sandroni (2007) and Olszewski and Sandroni (2008).}\]
There is an extensive literature on reputations in Economics. The general idea of this literature is that "normal" agents can create reputations of being of a "commitment" type. Cripps, Mailath and Samuelson (2004) consider a model where a long-run agent faces an infinite sequence of short-lived agents in a context with imperfect monitoring. They show that, for any equilibrium, the normal long-lived agent is unable to sustain the belief that he is the commitment type, so that there are no permanent reputations. Several of the papers that followed describe conditions under which the lie that the normal type is the commitment type may last asymptotically: for example, if the type of the long-run player is not permanent (Mailath and Samuelson (2001) and Ekmeci, Gossner and Wilson (2012); or if the access to past data is either costly (Liu (2011)) or limited (Ekmekci (2011) and Hu (2016)).

In my paper, the reputation is not about the long-lived agent's type but about the world, i.e. the distribution of the public signal. The basic idea of the reputation literature is that, if it is conceivable that a commitment type exists, a normal type agent can sustain the belief that he is the commitment type through his actions and become better off as a result. The only difference in my paper is that, instead of the long-lived agent using the fact that it is conceivable that a commitment type exists, he uses the fact that it is conceivable that an alternative world exist. In a way, I replace the commitment type by a false world, and then study a similar question: can the long-lived agent do better by building up a reputation that the false world is true?

2 Model

2.1 Description of the game and the payoffs

Consider a model with infinitely many periods. In each period $t \geq 1$, there is a short lived agent $t$ (he) who must take an action $a_t \in \{L, R\}$. The agent receives a payoff of 1 if his action matches the state of the world $\omega_t \in \{L, R\}$, which is random and unobservable. Otherwise, the agent receives no payoff. Let

$$u_t = \begin{cases} 
1 & \text{if } a_t = \omega_t \\
0 & \text{if } a_t \neq \omega_t
\end{cases}$$

If $u_t = 1$, I refer to the agent as having been successful. Before deciding $a_t$, each agent $t$ is assumed to observe a private signal $s_t \in [0, 1]$.

There is a long-lived agent called the expert (she). The expert is assumed to observe a private signal $\eta_t \in \{L, R\}$ in every period $t$ in addition to have access to more past information.
than the agent (see the section on information). Each agent may choose to consult the expert
before deciding $a_t$ (but after observing $s_t$) at a cost $c > 0$. Let

$$b_t = \begin{cases} 
1 & \text{if agent } t \text{ consults the expert} \\
0 & \text{if agent } t \text{ does not consult the expert}
\end{cases}$$

If the agent consults the expert, he sends a message $m_t \in [0, 1]$ to the expert. After
observing $m_t$, the expert responds by sending a recommendation $r_t \in \{L, R\}$ to the agent.
The agent is then free to follow the advice of the expert or not. Figure 1 displays the timing
of the events within each period.

The distribution of all the random variables $\{\omega_t, s_t, \eta_t\}_{t=1}^{\infty}$ depends on a single random
variable $\theta \in \{T, F\}$, where $T$ stands for "true" and $F$ stands for "false", which is realized
at the beginning of period 0 and is only observed by the expert. Let $\pi \in [0, 1)$ denote the
probability that $\theta = F$.

The interpretation of this modelling choice is the following. At period 0, the expert
observes the "true" world, i.e. he becomes aware of the distribution of $\omega_t, s_t$ and $\eta_t$. However,
she is able to tell a "lie", i.e. she is able to claim that the world works in a different way. At
period 0, the public belief is that there is a probability $\pi$ that such alternative world is real.
Therefore, if $\theta = T$, the distribution of $\{\omega_t, s_t, \eta_t\}_{t=1}^{\infty}$ is the true one, whereas if $\theta = F$, it is
the one that the expert says it is at the beginning of period 0.

Finally, it is assumed that the expert wants to maximize her discounted expected profit:

$$E_{\{b_t\}_{t=0}^{\infty}} \left[ \sum_{t=0}^{\infty} \delta^t b_t c \right]$$

where $\delta \in (0, 1)$. 

Figure 2: Timing within any period $t$
2.2 Information

The public history $h^t$ available at the beginning of period $t$ is

$$h^t = \{b_{\tau}, b_{\tau}u_{\tau}\}^{t-1}_{\tau=1}$$

and is observable by every agent $t' \geq t$. In words, each agent $t$ observes whether the agents that came before him consulted the expert and, if so, whether they were successful. In addition to the public history $h^t$, agent $t$ observes $s_t$ before deciding whether to consult the expert. If he chooses to consult the expert, he also observes her report $r_t$ before deciding $a_t$.

At the beginning of period $t$ (or at the end of period $t-1$), the expert is assumed to have observed i) which of the previous agents consulted with her, ii) their messages, iii) her own past recommendations, iv) whether the past agents who consulted with her were successful and v) all past private signals. Formally, let

$$\hat{h}^t = \{b_{\tau}, b_{\tau}m_{\tau}, b_{\tau}r_{\tau}, b_{\tau}u_{\tau}, \eta_{\tau}\}^{t-1}_{\tau=1}$$

so that, at the beginning of period $t$, the expert observes $\hat{h}^t$. During the period, she is assumed to observe $\eta_{\tau}$, which is assumed to be realized before the agent has had the chance to consult with the expert, and message $m_t$ if agent $t$ chooses to consult her. Finally, recall that the expert is always informed about $\theta$.

2.3 The True World

If $\theta = T$, it is assumed that, for all $t \geq 1$, each $s_t \sim U(0, 1)$, while $\omega_t$ depends only on $s_t$ and $s_{t-1}$. In particular, I assume that there is a function $p : [0, 1]^2 \rightarrow [0, 1]$ such that

$$\Pr\{\omega_t = L|s_t, s_{t-1}\} = p(s_t, s_{t-1})$$

for all $t$, $s_t \in [0, 1]$ and $s_{t-1} \in [0, 1]$.

As for $\eta_t$, it is assumed to be independent of any $\omega_{t'}$ and $s_{t'}$ for any $t' \geq 0$, i.e. it is completely uninformative for any agent.

In the real world, the only information an agent at period $t$ should care about is contained in his own signal and in the previous agent’s signal. The benefit of the expert in this context is that she aggregates information, i.e. she makes past information available to current agents. Finally, notice that the assumption that $s_t \sim U(0, 1)$ is without loss of generality because function $p$ has no restrictions.
2.4 The False World

If \( \theta = F \), it is assumed that, for all \( t \geq 1 \)

\[
\Pr \{ \omega_t = L \} = \frac{1}{2} \text{ and } s_t \sim U(0,1)
\]

As for the signal \( \eta_t \) that the expert observes, it is assumed that, for all \( t \geq 1 \),

\[
\Pr \{ \eta_t = \omega_t | \omega_t, s_t, h^t \} = \frac{1}{2} + \lambda_t^{h^t}(s_t)
\]

where \( \lambda_t^{h^t}(s_t) \in [0, \frac{1}{2}] \) for all \( s_t \in [0,1] \) and any history \( h^t \).

The main difference from the false world to the true world is that, in the false world, \( s_{t-1} \) does not influence directly \( \omega_t \). So, while in the real world, the agent is concerned with learning about \( s_{t-1} \), in the false world, he is concerned with learning about \( \eta_t \) (because he already observes \( h^t \)). In the false world, the expert is helpful not because he makes past information available but because he has access to a relevant and private source of information herself.

This formulation for the false world allows the lie to be dependent of the public history and of the agent’s signal. If \( \lambda \) is independent of the history, I say that the lie is "simple". In Appendix A, I study simple lies in more detail. In the example, the lie was not only independent of the history but also of the agent’s signal, as the expert promised a constant success rate of 75%. Lies that are dependent on the public history, while perhaps more farfetched, seem compatible with what one might expect to hear from any number of experts. A psychic might claim that her ability to predict the future is correlated with a number of world events like her previous amount of successes or a financial expert might claim that the accuracy of her model that predicts stock returns depends on her history of past successful predictions.

Finally, notice that, for all \( h^t \) and \( s_t \in [0,1] \),

\[
\Pr \{ \omega_t | s_t, h^t \} = \Pr \{ \eta_t | s_t, h^t \} = \frac{1}{2} \text{ for all } \omega_t \in \{L, R\} \text{ and } \eta_t \in \{L, R\}
\]

so that

\[
\Pr \{ \omega_t = L | s_t, h^t \} = \frac{1}{2}
\]

Therefore, an agent who is born at period \( t \) and only observes public history \( h^t \) and his own signal \( s_t \), is indifferent between choosing \( L \) and \( R \) if he believes that the false world has been realized. So, the structure of the lie makes the opportunity cost of consulting the expert in the false world as small as possible. If that same agent had access to \( \eta_t \), the probability of success would be \( \frac{1}{2} + \lambda_t^{h^t}(s_t) \) so that \( \lambda_t^{h^t}(s_t) \) represents how much the agent would be willing to pay to access the signal of the expert in the false world.
2.5 Period \( t = 1 \)

Notice that, if \( \theta = T \), the expert faces a problem immediately at period 1. Recall that, if every agent is aware that they are in the real world, the only reason why they would want to consult the expert would be to access past information that previous agents have provided the expert with. But at period 1, it would be known that the expert has no information from past periods, because it is assumed that there are no past periods. As a result, agent \( t = 1 \) would never consult the expert and, as a chain reaction, no one would ever consult the expert.

To avoid this, I assume that the expert holds \( s_0 \) (so that there is no agent \( t = 0 \)), in addition to assuming that there is no \( u_0 \) (or that it is not observable). In this way, the agent at \( t = 1 \) knows only that \( s_0 \) is known by the expert.

2.6 Strategies and Equilibrium Concept

For any \( t \), agent \( t \)'s strategy is made of three components: i) a choice of whether to consult the expert, ii) a choice of the message to relay to the expert should he consult her and iii) what action to take. Let \( \tilde{m}_t^h(s_t) \in [0, 1] \) denote the choice of agent \( t \) of what message to send to the expert after public history \( h_t \) and signal \( s_t \), should he choose to consult her. I say that agent \( t \) reports truthfully at history \( h_t \) if \( \tilde{m}_t^h(s_t) = s_t \) for any \( s_t \in [0, 1] \). Agent \( t \) is said to follow a truthful reporting strategy if he reports truthfully after any public history.

The expert’s strategy is simply a choice on what recommendation to give, for any (private) history \( \tilde{h}_t \), message \( m_t \in [0, 1] \) sent by agent \( t \), signal \( \eta_t \in \{L, R\} \) and \( \theta \in \{L, R\} \).

A perfect Bayesian equilibrium (PBE) is a strategy for the expert and a strategy for each agent such that i) the expert chooses her recommendation optimally given her beliefs and the agents’ strategies, ii) each agent makes each decision optimally given their beliefs and the other agents and expert’s strategies and iii) beliefs are updated according to Bayes’ rule whenever possible.

3 Analysis

3.1 If there is no lie

Assume first that \( \pi = 0 \) so that the expert is honest. In this case, every agent knows that \( \theta = T \): everyone knows that the world is the true world.
Let

\[ v(s) = \int_0^1 \max \{ p(s, x), 1 - p(s, x) \} \, dx \]

and notice that it represents the expected benefit of consulting the expert for agent 1 when \( s_1 = s \) and the expert is as helpful as possible, i.e. the expert reports \( L \) if and only if

\[ \Pr \{ \omega_1 = L | s_0, s_1, \theta = T \} \geq \frac{1}{2} \text{ for all } (s_0, s_1) \in [0, 1]^2 \]

Agent 1’s opportunity cost of consulting the expert is given by

\[ q(s) = \max \left\{ \int_0^1 p(s, x) \, dx, 1 - \int_0^1 p(s, x) \, dx \right\} \]

Finally, let

\[ g(s) \equiv v(s) - q(s) \]

It follows that, if agent 1 draws some signal \( s_1 = s \) such that

\[ g(s) < c \]

the agent will not consult the expert, no matter how the expert decides her report. Let

\[ I^c = \{ s \in [0, 1] : g(s) < c \} \]

and notice that the probability that agent 1 does not consult the expert is at least \( \int_{s \in I^c} ds \).

**Proposition 1** If \( p \) is such that

\[ \int_{s \in I^c} ds > 0 \]  \hspace{1cm} (1)

then

\[ b_t \to^{a.s.} 0 \]

in any PBE.

**Proof.** See Appendix B. \( \blacksquare \)

Notice that once the chain of agents who consults the expert is broken, it is never restored: if an agent does not consult the expert, there is no advantage whatsoever to consulting the expert for the following agent. Therefore, if condition (1) holds, and agent 1 does not consult the expert, no one else will. If \( s_1 \) is such that the agent does consult the expert, it is possible
that agent 2 is interested in consulting the expert as well. However, on average, he is less inclined to do so than agent 1 was because the expert will have less relevant information at period 2 than at period 1 (recall that $s_0$ is assumed to be known by the expert). As a result, even if agent 1 consults the expert, there is a positive probability that agent 2 prefers not to. In proposition 1, I show that, for any PBE, the probability that the chain is broken at period $t + 1$ given that it was not broken until period $t$, for any $t$, is larger than some $\varepsilon > 0$. Therefore, the probability that agent $t$ consults the expert converges to 0 as $t$ becomes increasingly large.

### 3.2 Main Result

The problem of the expert is to find a lie, or more specifically $\lambda$, which enables her to do better than what she was able to do if she was honest. In this section, I find sufficient conditions on function $p$ for which there is such a lie which not only makes the expert better off, but actually maximizes her profit, i.e. gets every agent to consult her in every period. Moreover, such conditions are less restrictive than condition (1). In other words, as the example of section 1.1 illustrates, it is possible that condition (1) holds but, at the same time, there is a lie which makes every agent consult the expert in every period.

My approach is the following: I select $\lambda$ so that a PBE with the following properties exists:

- **A**: every agent consults the expert in every period for any signal,
- **B**: the public belief that $\theta = F$ is always $\pi$,
- **C**: the expert gives as good advice as she is able to on the path of play,
- **D**: the expert gives uninformative advice off the path of play,
- **E**: every agent reports truthfully.

The path of play of the described profile is the path of public histories for which every preceding agent has consulted the expert. Let $H^t$ be the set of all public histories on the path of play at period $t$. It follows that

$$h^t \in H^t \iff b_\tau h^t = 1 \text{ for all } \tau < t$$

where $b_\tau h^t$ denotes the choice of agent $\tau$ of whether to consult the expert consistent with history $h^t$.

Property $C$ is a consequence of property $A$ in that, if the expert gives as good advice as she is able to, she obtains the maximum possible profit so any deviation from this behavior would not be strictly beneficial to her. Formally, to "give as good advice as she is able to"
on the path of play means that, for any private history $h^t \in H^t$, the choice of the report of the expert is $L$ if and only if

$$\Pr \left\{ \omega_t = L[h^t, m_t, \eta_t, \theta] \right\} \geq \frac{1}{2}$$

for all $m_t \in [0, 1]$, $\eta_t \in \{L, R\}$ and $\theta \in \{T, F\}$.

Property $D$ ensures that no agent consults the expert off the path of play. Basically, it says that in any private history $h^t$ not consistent with any $h^t \in H^t$, the expert reports $L$ with a probability of 50%, for any $m_t \in [0, 1]$, $\eta_t \in \{L, R\}$ and $\theta \in \{T, F\}$. The expert does not want to deviate from this for the same reason that, in standard static cheap talk games, there is always a babbling equilibrium.\(^6\)

Finally, agents want to report truthfully (property $E$) because it helps the expert give better advice.

The challenge then becomes one of finding such a lie for which, whenever properties $C, D$ and $E$ hold, then properties $A$ and $B$ also hold. Before stating the result, it is necessary to introduce additional notation.

Consider some agent $t$ at some public history $h^t \in H^t$ after observing some signal $s_t = s$. At this point, the agent will have a belief about $\theta$. If $\theta = F$, the agent’s expected benefit of consulting the expert is equal to $\frac{1}{2} + \lambda_t^h(s)$. If, however, $\theta = T$, the expected benefit of the agent, which I denote by $v_t^h(s)$, will depend on what the previous agent has chosen to do. By property $A$, agent $t$ is able to infer that the previous agent would have consulted the expert for any $s_{t-1}$. However, observing whether agent $t - 1$ was successful will allow agent $t$ to update his beliefs about $s_{t-1}$ so that, in general, he will not believe that $s_{t-1}$ is distributed uniformly. One can recursively define $v_t^h$ as follows:

$$v_{1}^h (s) = v (s) \text{ for all } s \in [0, 1]$$

while, for any $h^t \in H^t$ and any $t > 1$,

$$v_t^h (s) = \frac{1}{\int_{0}^{1} d_t^h (x) \max \{p(s, x), 1 - p(s, x)\} dx}$$

\(^6\)See Crawford and Sobel (1982).
where

\[ d^h_t(x) = \begin{cases} 
\frac{v^h_{t-1}(x)}{\int_{0}^{1} v^h_{t-1}(x) \, dx} & \text{if } u_{t-1}|h^t = 1 \\
0 & \text{if } u_{t-1}|h^t = 0 \\
\frac{1 - v^h_{t-1}(x)}{\int_{0}^{1} (1 - v^h_{t-1}(x)) \, dx} & \text{if } u_{t-1}|h^t = 0 
\end{cases} \]

The density function \( d^h_t \) represents the beliefs that agent \( t \) has about \( s_{t-1} \) when at history \( h^t \in H^t \). In an analogous way, define \( q^h_t(s) \) to denote the expected payoff for agent \( t \) of not consulting the expert if \( \theta = T \), conditional on any history \( h^t \in H^t \) and on \( s_t = s \), which can be written as

\[ q^h_t(s) = \max \left\{ \frac{1}{\int_{0}^{1} d^h_t(x) p(s, x) \, dx}, \frac{1}{\int_{0}^{1} d^h_t(x) (1 - p(s, x)) \, dx} \right\} \]

where

\[ d^h_t(x) = 1 \text{ for all } x \in [0, 1] \]

Finally, let

\[ g^h_t(s) = v^h_t(s) - q^h_t(s) \]

for all \( h^t \in H^t \) and for all \( t \).

**Proposition 2** If there is \( k > 0 \) such that, for all \( h^t \in H^t \) and for all \( t \),

\[ \int_{0}^{1} g^h_t(s) \, ds \geq c + k \tag{2} \]

then there is \( \lambda \) and \( \pi^c \in (0, 1) \) such that, for every \( \pi \geq \pi^c \) there is a PBE with properties A through E.

**Proof.** Let

\[ \lambda^h_t(s_t) = \int_{0}^{1} v^h_t(s) \, ds - \frac{1}{2} \]

so that the expected success rate after history \( h^t \) is the same in the true world and in the false world. This implies that, provided property A holds, the belief that \( \theta = F \) is kept constant and equal to \( \pi \). For each agent to want to consult the expert in every period, it must be that

\[ \pi \lambda^h_t(s_t) + (1 - \pi) g^h_t(s) \geq c \tag{3} \]
Notice that

\[ \pi \lambda_t^{h^t}(s_t) \geq \pi \int g_t^{h^t}(s) \, ds \geq \pi (c + k) \]

As a result, it follows that, for every \( \pi \geq \bar{\pi}^c \) condition (3) holds, where

\[ \bar{\pi}^c \geq \frac{c}{c + k} \]

Notice that, by definition, \( g \) is completely independent of the lie that the expert tells. So, what condition (2) states is that, on the path of play, the average expected net benefit of consulting the expert if \( \theta = T \), is always larger than \( c \). To better understand condition (2), consider the following thought experiment. Suppose that each agent must decide whether to consult the expert \textit{before} observing their own signal. What condition (2) implies is that, in such a case, if it was known that \( \theta = T \), each agent would prefer to consult the expert for any history where the previous agents also did so. So, in this hypothetical scenario, there would be no need for the expert to lie. Every agent would consult the expert in every period, the expert would be maximizing her profit and all agents would be better off than if there was no expert. What the lie does is it transforms the actual problem of the expert into this hypothetical problem. It averages out the problem of each agent.

As an illustration consider the example of section 1.1. In that case, \( g_t^{h^t} \) is independent of \( h^t \) so that, conditional on all previous agents consulting the expert, the problem of each agent is independent of \( t \). The lie that the expert tells is that the success rate is independent of the agent’s signal, so as to average out the agent’s willingness to pay. The larger \( \pi \) is, the flatter the expected net benefit of consulting the expert becomes (see Figure 3). If it becomes sufficiently flat, then the decision of the agent becomes the same as if he had not observed his signal and so, if (2) holds, he chooses to consult the expert.

4 Small Prior

Proposition 2 states that, provided condition (2) holds, by setting

\[ \lambda_t^{h^t}(s_t) = \frac{1}{2} \int_0^t \nu_t^{h^t}(s) \, ds - \frac{1}{2} \]

there is always some threshold \( \bar{\pi}^c \) such that if \( \pi \geq \bar{\pi}^c \), the lie lasts forever and everyone consults the expert. Naturally, one concern the reader might have is whether threshold \( \bar{\pi}^c \) is too large. There are a few observations that are important in that regard.
Figure 3: When $\pi$ increases, the expected net benefit of consulting the expert flattens.

1. Threshold $\pi^c$ decreases with $c$.

Think of $\pi^c$ as the smallest threshold allowed by Proposition 2. The first thing to note is that $\pi^c$ depends on $c$ and so, it will be smaller as $c$ itself becomes smaller. In the example of section 1.1, if the expert was honest, he would eventually be left without any clients for any $c > 0$. However, there is always some $c$ for which $\pi > \pi^c$ even if $c$ is small. So, at least if $c$ is small, the expert is able to do strictly better by being dishonest.

2. The lie of proposition 2 is not optimal.

A second observation has to do with the lie that the expert tells. In proposition 2, the lie the expert is assumed to tell promises a constant success rate (conditional on $h^t$) for any signal $s_t$. However, it is possible for the expert to tell a different lie which might be sustainable and attract every agent to the expert even if $\pi < \pi^c$. In other words, the lie of proposition 2 might not be optimal. In fact, the "best" lie $\hat{\lambda}$ for the expert is still sustainable but maximizes the willingness to consult the expert when the agent is less willing. Formally, for any $h^t \in H^t$, let $\hat{\lambda}^{h^t}$ maximize

$$
\min_{s \in [0,1]} \pi \lambda_t^{h^t}(s) + (1 - \pi) g_t^{h^t}(s)
$$

subject to

$$
\int_0^1 \hat{\lambda}^{h^t}_t(s) \, ds = \int_0^1 v_t^{h^t}(s) \, ds - \frac{1}{2}
$$
Consider again the example of section 1.1, and assume that \( \pi = \frac{1}{2} \). In this case, if \( c \leq \frac{1}{8} \), then \( \pi^c \geq \pi \) and the constant lie \( \lambda(s) = \frac{1}{4} \) works in that every agent consults the expert and the belief that \( \theta = F \) lasts forever. Consider, however, lie \( \hat{\lambda}(s) \) which, in this example, is such that

\[
\hat{\lambda}(s) = \min \{ s, 1 - s \}
\]

It follows that each agent \( t \)'s willingness to consult the expert is equal to \( \frac{3}{4} \) for any \( s_t \) and so it works provided \( c \leq \frac{1}{4} \). So, for any \( c \in \left( \frac{1}{8}, \frac{1}{4} \right) \), it follows that \( \pi^c > \frac{1}{2} \) but with the alternative lie \( \hat{\lambda} \), a prior \( \pi = \frac{1}{2} \) is sufficient - see Figure 4.

One problem that \( \hat{\lambda} \) has, however, is that it depends on \( \pi \) in its construction. So, if, by whatever reason, the expert underestimates \( \pi \), the lie \( \hat{\lambda} \) might not work. Going back to the example, imagine that the expert designs \( \hat{\lambda} \) believing that \( \pi = \frac{1}{2} \) but in reality \( \pi \) is very close to 1. In that case, the agent’s problem is analogous to when the expert was honest - if his signal is very close to 0 or 1, he will not want to consult the expert. Figure 5 illustrates.

3. Even if \( \pi \) is small, the expert might be able to create a permanent reputation with positive probability.

Fix some \( p \) and \( c \) such that condition (2) holds but assume that \( \pi < \pi^c \) and that the lie of the previous point is undesirable. While it may not be possible, in such circumstances, for the expert to succeed in getting every agent to consult her with certainty, it might be with some positive probability as I illustrate with the example of section 1.1.
Figure 5: Expected net benefit of consulting the expert when the lie is \( \hat{\lambda}(s) = \min\{s, 1-s\} \) and \( c \in (\frac{1}{8}, \frac{1}{4}) \) for different values of \( \pi \). If \( \pi \) is too large the agent will not consult the expert if \( s \) is sufficiently different from \( \frac{1}{2} \).

Suppose that \( c = \frac{1}{8} \) so that \( \pi^c = \frac{1}{2} \). Assume instead that \( \pi = 0.05 \). The problem that the expert faces is that the prior belief is too low. This does not imply that the expert should settle to being honest, because that will imply a long term average profit of 0 almost surely (if \( \delta \to 1 \)). Instead, it is in the best interest of the expert to "build up" the belief that \( \theta = F \) until it reaches \( \pi^c \) and then revert to the original lie - that \( \lambda^h_t(s) = \frac{1}{4} \) for all \( s, h^t \) and \( t \).

Consider the following lie:

1) For any period \( t \leq 12 \), choose \( \lambda^h_t(s) = \frac{1}{2} \) for any \( s \in [0, 1] \) and for any public history \( h^t \).

2) For any period \( t > 13 \), choose \( \lambda^h_t(s) = \frac{1}{4} \) for any \( s \in [0, 1] \) and for any public history \( h^t \).

Consider a PBE where, for every history such that every preceding agent has consulted the expert and has been successful, the expert gives as good advice as she is able to, while she reports uninformatively otherwise. As for the agents, they report truthfully. This means that, once some agent refuses to consult the expert, or receives unsuccessful advice from her, no agent will ever consult the expert again.

Consider what happens at period 1. The agent consults the expert if and only if

\[
s_1 \in [0, 0.3947] \cup [0.6053, 1]
\]

At period 2, agent 2 updates his beliefs about \( \theta \). If agent 1 has been unsuccessful, the belief
that $\theta = F$ will drop to 0, because in the false world, the agent should always be successful. If, however, agent 1 was successful, then the belief that $\theta = F$ will increase because, in the true world, the probability of success is given by

$$\int_{0.6053}^{1} \frac{sds}{0.6053} = 0.80265 < 1$$

In particular, at period 2, if agent 1 has consulted the expert and has been successful, the public belief that $\theta = F$ is equal to

$$\frac{0.05}{0.05 + 0.95 \times 0.80275} = 0.06153$$

By this logic, it is only a matter of time until the belief that $\theta = F$, conditional on all preceding agents having consulted the expert and having been successful, reaches $\frac{1}{2}$. In particular, that happens at period 13. While the probability that every agent before period 13 consults the expert and is successful is small, from that point forward, by reverting to the "sustainable" lie the expert is able to guarantee that every agent consults her. So, the long-run average profit for the expert will be larger than if he was honest (if $\delta \rightarrow 1$).

Notice that, in this example, the expert did not need to switch lies at period 13, i.e. she could have increased the belief even further and make it as close to 1 as she wanted. So, it is perfectly possible to have a sustainable very large belief in the supernatural, even though the odds of that happening are small. In Appendix C, I discuss how to generalize this algorithm of creating a lie even with a small prior, which works in a more general setting and not only for "simple" lies.

5 Endogenous price

Throughout I have assumed that the price the expert charges is constant and equal to $c$. The motivation was to display a world where competition would drive prices to the marginal cost of the expert and where experts are unable or unwilling to borrow money. Nevertheless, I believe it is worthwhile to discuss what would happen if, instead, the expert was a monopolist with no costs and was able to select what price to charge in each period, given each history.

Assume first that the expert is honest so that $\pi = 0$. In this case, the expert would prefer to charge each agent their willingness to pay. However, the expert does not observe each agent’s signal, so perfect discrimination would be impossible. Nevertheless, one can be
certain that the expert will do better by selecting a variable price. If nothing else, when some client decides not to consult the expert at some period $t$, the expert has an incentive to lower the price of period $t+1$ all the way to 0 in order to restart the chain of clients who consult her.

Now, consider the case where the expert is able to lie. Given that the cost of the expert is 0, the pareto efficient outcome would be for every agent to consult the expert in every period for any signal. So, any strategy of the expert (which now involves not only the choice of the lie but also the pricing scheme) that is successful is getting every agent to consult the expert maximizes the sum of her utility and the agents'. Consider, as an illustration, the example of section 1. Assume that $\pi = \frac{1}{2}$ and that the expert tells the "optimal" lie described in section 4:

$$\lambda^h_t(s_t) = \min\{s_t, 1 - s_t\} \text{ for any } s_t \in [0, 1] \text{ and for any } h^t \in H^t$$

Recall that this lie averages out the willingness to pay of the agent. So, for any $s_t \in [0, 1]$, the agent is willing to pay $\frac{1}{4}$ to consult the expert, conditional on every previous agent having consulted the expert. But then, consider the following pricing scheme: have the expert charge a price equal to $\frac{1}{4}$ in every period and for any history. Every agent has just enough incentives to consult the expert, provided every preceding agent did so as well. Furthermore, even though every agent consults the expert and the belief $\pi$ lasts forever, the expected surplus of each of the agents is 0, because they are paying exactly $\frac{1}{4}$ - their average willingness to pay. As a result, this is the best the expert can do.

Endogenizing the price allows the expert to do better whether he is honest or not. However, the ability to choose both the price and the lie allows him to extract all the surplus from his interactions with the agents.\(^7\) In particular, the agents are not made better off by the existence of the expert, which is in stark contrast with the case where the price was fixed as, in that case, having a lying expert was actually good for the agents.

6 Conclusion

There is a widespread belief that there are people in the world with special predicting abilities. Often times, one attributes these beliefs to some type of behavioral bias. In this paper, I put forward a theory where, despite agents having rational expectations, it is possible for an expert to perpetuate the belief that she has special predicting skills even though she does not. The lie that is told attracts paying clients to the expert, which allows her to obtain information from them and, as a result, enables her to be better prepared to make predictions.

\(^7\)In appendix D I show that this statement holds more generally.
Experts resort to this elaborate lie rather than confess that the source of their "power" is the data because it makes them less susceptible to being dismissed by their clients. If a client is aware that the expert’s ability to give advice depends on the data she has, he might prefer not to consult with her if he finds that data unimportant. But this would cause a negative externality to all other agents who are left without the information held by the client himself.

Moreover, I also dispute the notion that it is bad that experts pretend to have special predicting skills that they do not have. I have shown that, when prices are constant, it is quite possible that almost all agents benefit from having a dishonest expert because an honest expert would not be able to have a long-term profit and, so, would not be able to provide advice for too long.

7 Appendix

7.1 Appendix A - Simple Lies

Simple lies are defined to be history independent lies. Formally, the expert tells a simple lie if $\lambda_t^{h_t}$ is independent of $h_t$ and of $t$.

From condition (3), one can see that the history dependence that a lie must have in order for proposition 2 to hold comes from the fact that $g_t^{h_t}$ might be history dependent. So, for the lie to be history independent, it is sufficient that $g_t^{h_t}$ is also history independent. The reason that $g_t^{h_t}$ might depend on the history is that the beliefs of an agent regarding the previous period’s signal evolve over time as more and more evidence is realized. Hence, in order to eliminate that dependence, it is sufficient that each agent does not change his beliefs about the previous signal after observing whether the previous agent was successful - formally, if $p$ is such that $d_t^{h_t} = d_1^{h_1}$ for all $h_t \in H^t$ and $t$.

**Proposition 3** If $v$ is constant, then $d_t^{h_t}$ is independent of $h_t \in H^t$ and $t$.

**Proof.** See Appendix B. ■

If $v$ is constant, there is no information being transmitted at the end of the period, because a success is equally likely to happen for any signal. As a result, $d_t^{h_t} = d_1^{h_1}$ for any $h_t \in H^t$ and for any $t$ so that the problem of each agent is always the same on the path of play.

**Example 4**

$$p(s, x) = \begin{cases} 
\nu & \text{if } s \geq x \\
1 - \nu & \text{if } s < x
\end{cases} 
\quad \text{for some } \nu \in \left[\frac{1}{2}, 1\right]
$$

In this case,

$$v(s) = \nu \text{ for all } s \in [0, 1]$$

23
It is also possible to find \( p \) where, even though \( d_t^h \) may not be history independent, \( g_t^h \) will.

**Proposition 5** If \( p \) is such that

\[
p(s, x) \in \{\nu^s, 1 - \nu^s\} \text{ for some } \nu^s \in \left[\frac{1}{2}, 1\right]
\]

ii) 

\[\nu^s = \nu^{1-s} \text{ for all } s\]

and iii) 

\[
p(s, x) + p(s, 1-x) = \zeta^s \text{ for some } \zeta^s \in \left[\frac{1}{2}, 1\right]
\]

then \( g_t^h \) is independent of \( h^t \in H^t \) and \( t \).

**Proof.** See Appendix B. ■

### 7.2 Appendix B - Proofs

#### 7.2.1 Proof of Proposition 1

First, notice that if \( c > \frac{1}{2} \), no agent will consult the expert. So, consider the case where \( c \leq \frac{1}{2} \). Notice that if

\[
\int_0^1 1 \{s \in I^c\} \, ds > 0
\]

then there must be some \( \kappa \in (0, c) \) such that

\[
\int_0^1 1 \{s \in I^\kappa\} \, ds > 0
\]

I show that there is a probability of at least

\[
\varepsilon = \left(1 - \frac{\kappa}{c}\right) \int_0^1 1 \{s \in I^\kappa\} \, ds > 0
\]

that, for any PBE and for any public history \( h^t \), if \( h^t \) is such that \( b_r = 1 \) for all \( \tau < t \), then the probability that agent \( t + 1 \) does not consult the expert is at least \( \varepsilon \).

Take any PBE. I show the statement by induction.
Consider agent $t = 1$. The probability that he consults the expert is maximal when the 
expert truthfully reports at period $t = 1$. In that case, and by definition of $I^c$, there is a 
probability of
\[
\int_0^1 \mathbf{1}\{s \in I^c\} \, ds > \varepsilon
\]
of him not consulting the expert.

Now consider any agent $t$ after some public history $h^t$ such that $b_\tau = 1$ for all $\tau < t$. Notice that
\[
v(s_{t+1}) = \int_0^1 f(s_t|h^t) \max \{p(s_{t+1}, s_t), 1 - p(s_{t+1}, s_t)\} \, ds_t
\]
\[
= \int_0^1 \sum_{h^{t+1}} \Pr\{h^{t+1}|h^t\} f(s_t|h^{t+1}) \max \{p(s_{t+1}, s_t), 1 - p(s_{t+1}, s_t)\} \, ds_t
\]
\[
= \sum_{h^{t+1}} \Pr\{h^{t+1}|h^t\} \int_0^1 f(s_t|h^{t+1}) \max \{p(s_{t+1}, s_t), 1 - p(s_{t+1}, s_t)\} \, ds_t
\]
where $h^{t+1}|h^t$ is understood as representing a public history at period $t + 1$ consistent with 
public history $h^t$. Notice that, for any $h^{t+1},$
\[
\int_0^1 f(s_t|h^{t+1}) \max \{p(s_{t+1}, s_t), 1 - p(s_{t+1}, s_t)\} \, ds_t \geq \tilde{v}^{h^{t+1}}(s_{t+1})
\]
where $\tilde{v}^{h^{t+1}}(s_{t+1})$ denotes the expected utility of consulting the expert at public history 
$h^{t+1}$, given signal $s_{t+1}$ and the expert’s strategy. The inequality follows because the expected 
utility of consulting the expert is larger if the expert gives as good advice as possible. This 
means that
\[
v(s) \geq \sum_{h^{t+1}} \Pr\{h^{t+1}|h^t\} \tilde{v}^{h^{t+1}}(s)
\]
It is also the case that

\[
q(s_{t+1}) = \max \left\{ \int_0^1 f(s_t|h_t') p(s_{t+1},s_t) \, ds_t, \int_0^1 f(s_t|h_t) (1 - p(s_{t+1},s_t)) \, ds_t \right\}
\]

\[
= \max \left\{ \int_0^1 \sum_{h^{t+1}} \Pr\{h^{t+1}|h_t\} f(s_t|h_t') p(s_{t+1},s_t) \, ds_t, \right\}
\]

\[
\int_0^1 \sum_{h^{t+1}} \Pr\{h^{t+1}|h_t\} f(s_t|h_t') (1 - p(s_{t+1},s_t)) \, ds_t \right\}
\]

\[
\leq \sum_{h^{t+1}} \Pr\{h^{t+1}|h_t\} \max \left\{ \int_0^1 f(s_t|h_t') p(s_{t+1},s_t) \, ds_t, \int_0^1 f(s_t|h_t') (1 - p(s_{t+1},s_t)) \, ds_t \right\}
\]

Notice that, for all \(h^{t+1}\),

\[
\max \left\{ \int_0^1 f(s_t|h_t') p(s_{t+1},s_t) \, ds_t, \int_0^1 f(s_t|h_t') (1 - p(s_{t+1},s_t)) \, ds_t \right\} = \tilde{q}^{t+1}(s_{t+1})
\]

where \(\tilde{q}^{t+1}(s_{t+1})\) denotes the expected opportunity cost of consulting the expert at public history \(h^{t+1}\) and given signal \(s_{t+1}\). This means that

\[
q(s) \leq \sum_{h^{t+1}} \Pr\{h^{t+1}|h_t\} \tilde{q}^{t+1}(s)
\]

Therefore, it follows that

\[
g(s) \geq \sum_{h^{t+1}} \Pr\{h^{t+1}|h_t\} \tilde{g}^{t+1}(s)
\]

where

\[
\tilde{g}^{t+1}(s) = v^{t+1}(s) - \tilde{q}^{t+1}(s)
\]

Take any \(s \in I^t\). It follows that

\[
c > g(s) \geq \sum_{h^{t+1}} \Pr\{h^{t+1}|h_t\} \tilde{g}^{t+1}(s)
\]

Let

\[
Z^{h_t}(s) = \left\{ h^{t+1}|h_t : \tilde{g}^{t+1}(s) < c \right\}
\]

so that one can write

\[
\sum_{h^{t+1} \in Z^{h_t}(s)} \Pr\{h^{t+1}|h_t\} \tilde{g}^{t+1}(s) + \sum_{h^{t+1} \not\in Z^{h_t}(s)} \Pr\{h^{t+1}|h_t\} \tilde{g}^{t+1}(s) \leq g(s) < c
\]
Notice that

\[
\Pr \left\{ h^{t+1} \in Z^h_t(s) \mid h^t \right\} \hat{g}^h_t(s) + \left( 1 - \Pr \left\{ h^{t+1} \in Z^h_t(s) \mid h^t \right\} \right) \tilde{g}^h_t(s) \leq g(s) < c
\]

where

\[
\hat{g}^h_t(s) = \sum_{h^{t+1} \in Z^h_t(s)} \frac{\Pr \left\{ h^{t+1} \mid h^t \right\}}{\sum_{h^{t+1} \in Z^h_t(s)} \Pr \left\{ h^{t+1} \mid h^t \right\}} \tilde{g}^{h^{t+1}}(s)
\]

and

\[
\tilde{g}^h_t(s) = \sum_{h^{t+1} \notin Z^h_t(s)} \frac{\Pr \left\{ h^{t+1} \mid h^t \right\}}{\sum_{h^{t+1} \notin Z^h_t(s)} \Pr \left\{ h^{t+1} \mid h^t \right\}} \tilde{g}^{h^{t+1}}(s)
\]

Because

\[
\hat{g}^h_t(s) < c \leq \tilde{g}^h_t(s)
\]

it follows that

\[
\Pr \left\{ h^{t+1} \in Z^h_t(s) \mid h^t \right\} \geq \frac{\tilde{g}^h_t(s) - g(s)}{\hat{g}^h_t(s) - \tilde{g}^h_t(s)} \geq \frac{\tilde{g}^h_t(s) - g(s)}{\tilde{g}^h_t(s)} = 1 - \frac{g(s)}{\tilde{g}^h_t(s)} 
\geq 1 - \frac{c}{g(s)} \geq 1 - \frac{\kappa}{c}
\]

Therefore, the probability that agent \( t + 1 \) does not consult the psychic is at least

\[
\int_0^1 \Pr \left\{ h^{t+1} \in Z^h_t(s) \mid h^t \right\} \mathbf{1} \{ s \in I^c \} ds \geq \left( 1 - \frac{\kappa}{c} \right) \int_0^1 \mathbf{1} \{ s \in I^c \} ds = \varepsilon
\]

### 7.2.2 Proof of Proposition 3

Let \( \tau = v(s) = v^h_1(s) \) for all \( s \in [0, 1] \). Notice that, by definition, \( d^h_1(x) = 1 \) for any \( x \in [0, 1] \). Take any arbitrary period \( t \) and any arbitrary history \( h^t \in H^t \). Assume that \( v^h_t(s) = \tau \) for all \( s \in [0, 1] \). It follows that

\[
d^{h^{t+1}}_{t+1}(x) = \begin{cases} \frac{\tau}{1} = 1 & \text{if } u^t \mid h^{t+1} = 1 \\ \int_0^\tau da & \text{if } u^t \mid h^{t+1} = 0 \\ \frac{1-\tau}{1} = 1 & \text{if } u^t \mid h^{t+1} = 0 \\ \int_0^{1-\tau} da & \text{if } u^t \mid h^{t+1} = 1 \end{cases}
\]

and so \( d^{h^{t+1}}_{t+1}(x) = 1 \) for any \( x \in [0, 1] \) and any \( h^{t+1} \in H^{t+1} \) which follows \( h^t \).
7.2.3 Proof of Proposition 5

Notice that it is enough to show that \( g_{h^2} = g \) when \( h^2 = \{b_1 = 1, b_1u_1 = 1\} \equiv h^2 \), because this would imply that \( g_{h^2} = g \) for \( h^2 = \{b_1 = 1, b_1u_1 = 0\} \) and, recursively, that \( g_{h^t} = g \) for any \( h^t \in H^t \) and \( t \).

Notice that, by i),
\[
v(s) = \int_0^1 \max\{p(s,x), 1 - p(s,x)\} \, dx = \nu^s
\]

Let
\[
\epsilon = \int_0^1 \nu^s \, ds
\]

It follows that
\[
v_{h^2}^2 = \int_0^1 \frac{\nu^x}{\epsilon} \max\{p(s,x), 1 - p(s,x)\} \, dx = \nu^s
\]

Notice also that, by iii)
\[
\int_0^1 p(s,x) \, dx = \frac{\zeta^s}{2} \quad \text{for all } s \in [0,1]
\]

so that
\[
q(s) = \max\left\{\frac{\zeta^s}{2}, 1 - \frac{\zeta^s}{2}\right\}
\]

It is also the case that
\[
\int_0^1 \frac{\nu^x}{\epsilon} p(s,x) \, dx = \int_0^1 \frac{\nu^x}{\epsilon} (p(s,x) + p(s,1-x)) \, dx = \int_0^1 \frac{\nu^x}{\epsilon} \zeta^s \, dx = \frac{\zeta^s}{2}
\]

where the first equality follows from ii), and the second equality follows from iii). Therefore,
\[
q_{h^2}^2(s) = \max\left\{\frac{\zeta^s}{2}, 1 - \frac{\zeta^s}{2}\right\} = q(s)
\]

7.3 Appendix C - Small Prior

In this part, I generalize the procedure of section 4 to show how to create a permanent lie that has a positive probability of attracting every agent to consult the expert. After bringing
the belief up, the idea of the argument is to use Proposition 2 to argue that, going forward, it is enough to switch to the "sustainable" lie. However, this requires that the belief that the first agent who is confronted with the sustainable lie has with respect to the previous signal, when \( \theta = T \), is uniform, in order to make him equivalent to agent 1 in Proposition 2. In the example, this was the case. However, in general it may not be. Therefore, there needs to be a period in between the two sets of periods of the example to "reset" the belief about the previous signal. For simplicity, I assume that there is \( \chi \in (0,1) \) such that, for all \( (s,x) \in [0,1]^2 \), \( p(s,x) \leq \chi \).

Let \( \varphi_r(h^t) \) denote the history of the last \( r \) periods of public history \( h^t \) so that

\[
\varphi_r(h^t) = \{b_{t-r}, b_{t-r}u_{t-r}, \ldots, b_{t-1}, b_{t-1}u_{t-1}\}
\]

Consider the following lie:

1) For any period \( t < \bar{t} \), choose \( \lambda_t^h(s) = \frac{1}{2} \) for any \( s \in [0,1] \) and for any public history \( h^t \).
2) If \( t = \bar{t} \), choose \( \lambda_t^h(s) = \chi - \frac{1}{2} + \epsilon \) for some \( \epsilon \in (0,1 - \chi) \).
3) For any period \( t > \bar{t} \), choose

\[
\lambda_t^h(s) = \int_{0}^{1} \varphi_{t-\bar{t}}(h^t)(s) \, ds - \frac{1}{2}
\]

for any \( s \in [0,1] \) and for any public history \( h^t \).

As for the strategies, the PBE is such that each agent reports truthfully, while the expert reports as follows:

i) For every \( t < \bar{t} \),
   a) the expert gives as good advice as possible whenever all preceding agents have consulted the expert and have succeeded,
   b) the expert gives uninformative advice otherwise.
ii) If \( t = \bar{t} \),
   a) the expert reports as good advice as possible if \( \theta = F \) and all preceding agents have consulted the expert and have succeeded,
   b) the expert gives uninformative advice otherwise.
iii) If \( t > \bar{t} \),
   a) the expert reports as good advice as possible if all agents before \( \bar{t} \) have consulted the expert and have succeeded,
   b) the expert gives uninformative advice otherwise.
Part i) is as in the example and simply builds up the belief in the event that every preceding agent consults the expert and is successful. Let $\pi_T$ be the belief at the beginning of period $T$, when every agent has consulted the expert and was successful. Notice that

$$\pi_T \geq \hat{\pi}_T$$

where $\hat{\pi}_t$ is defined recursively as

$$\hat{\pi}_1 = \pi \in (0, 1)$$

and

$$\hat{\pi}_{t+1} = \frac{\hat{\pi}_t}{\hat{\pi}_t + (1 - \hat{\pi}_t) \chi}$$

for all $t \geq 1$. Notice that $\hat{\pi}_T$ is strictly increasing and $\hat{\pi}_T \to 1$ as $T \to \infty$, so that the belief $\pi_T$ can be made arbitrarily large, provided that the history where all preceding agents consult the expert and succeed has positive probability. Assume that is the case. Choose $T$ to be large enough so that

a) $$\hat{\pi}_T \left( \chi - \frac{1}{2} + \epsilon \right) \geq c \Leftrightarrow \hat{\pi}_T \geq \frac{c}{\chi - \frac{1}{2} + \epsilon}$$

so that, at period $T$, agent $T$ wants to consult the expert for any signal, provided every preceding agent has consulted the expert and succeeded.

b) $$\hat{\pi}_T \geq \pi^*$$

where $\pi^*$ uniquely solves

$$\pi^* \left( 1 - \chi - \epsilon \right) \pi^* \left( 1 - \chi - \epsilon \right) + (1 - \pi^*) \frac{1}{2} = \pi_c$$

The right hand side comes from Proposition 2. Recall that if $\pi \geq \bar{\pi}_c$, the lie of Proposition 2 is successful in getting every agent to consult the expert. So, this condition guarantees that, at period $T + 1$, if all agents before $T$ consulted the expert and were successful, the posterior belief that $\theta = F$ is still large enough for the sustainable lie to work, even if agent $T$ is unsuccessful.

The argument is the following. First, the expert tells the appealing lie to get the belief $\pi_T$ arbitrarily close to 1. Then, if it happens that every agent consults the expert and is successful, at period $T$, the expert only gives good advice if $\theta = F$. Given that $\pi_T$ is sufficiently large, agent $T$ consults the expert for any signal. At period $T + 1$, the agent updates the beliefs about $\theta$ given the success of agent $T$. If agent $T$ is successful the belief will increase, if not it will decrease. However, because $\pi_T$ was so large, the posterior belief will still be large enough.
at period $t + 1$ that the sustainable lie gets agent $t + 1$ to consult the expert. Furthermore, the beliefs of agent $t + 1$ will be that signal $s_T$ is uniform if $\theta = T$. So then, by Proposition 2, every agent from then on will consult the expert and the belief will remain the same forever.

Notice that the reporting strategies of the agents and the expert form a PBE. In particular, at period $t$, if all preceding agents have consulted the expert and have been successful, the expert is indifferent on what to report because, no matter what, every future agent will consult the expert. The only condition for this procedure to work is that there is a positive probability that the history for which every agent consults the expert and is successful until period $t$ occurs. In that case, telling this lie gives the expert a positive probability bounded away from 0 when $\delta \to 1$ and is, therefore, preferable to being honest.

### 7.4 Appendix D - Variable Price

Throughout I assume that there is $\chi \in (0, 1)$ such that, for all $(s, x) \in [0, 1]^2$, $p(s, x) \leq \chi$. The goal is to find a profile that is a PBE that gets every agent to consult the expert and minimizes their expected utility. The reporting strategy of the expert and of the agents is the same as in section 3.2 and the profile is also such that the public belief $\pi$ is constant. The only things that change are the lie and the pricing scheme. For the pricing scheme, consider the following: in each period and for each history choose the largest price which ensures that every agent consults the expert for any signal $s_t$ given the lie told by the expert.

Consider the following lie by the expert. Fix some $\iota \in (0, \frac{1}{2})$. For all $h^t \in H^t$ and whenever

$$\int_0^1 g_t^{h^t}(s_t) \, ds_t \geq \iota$$

let $\lambda_t^{h^t}(s_t)$ be such that

$$\pi \lambda_t^{h^t}(s_t) + (1 - \pi) g_t^{h^t}(s_t) = \pi \int_0^1 \left( \nu_t^{h^t}(s_t) - \frac{1}{2} \right) \, ds_t + (1 - \pi) \int_0^1 g_t^{h^t}(s_t) \, ds_t$$

Notice that, provided $\pi$ is sufficiently large, $\lambda_t^{h^t}(s_t) \in [0, \frac{1}{2}]$. For any other circumstances, let

$$\lambda_t^{h^t}(s_t) = \int_0^1 \nu_t^{h^t}(s_t) \, ds_t - \frac{1}{2}$$

In words, if the average willingness to pay of the expert on the path of play is larger than $\iota$, the expert, through the lie, makes it as though the agent’s willingness to pay is independent of signal $s_t$. If not, the expert simply tells the lie of section 3.2. In either case, should every agent consult the expert, the public belief remains equal to $\pi$. 

31
The lie and the pricing scheme put together imply the following. On the path of play, whenever
\[ \int_{0}^{1} g_{t}^{h_{t}^{r}}(s_{t}) \, ds_{t} \geq \tau > 0 \]
the expert is getting the agent to consult her but is able to get all the surplus, because the price would be equal to \[ \int_{0}^{1} g_{t}^{h_{t}^{r}}(s_{t}) \, ds_{t} \]. However,
\[ \int_{0}^{1} g_{t}^{h_{t}^{r}}(s_{t}) \, ds_{t} < \tau \]
even though the agent consults the expert for any signal, the expert does not get all the surplus. Nevertheless, reducing \( \tau \) reduces the probability of that happening so that the profit of the expert converges to its maximum possible value as \( \tau \) converges to 0.
References


