Authority and motivation in situations of open conflict

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We study the interplay between the authority to select a project and the motivation to work on it in a principal-agent problem with non-transferable utility and two distinct features. First, the project’s success depends on effort by both players. Second, it is common knowledge that, conditional on success, the two players prefer different projects to be selected whereas a player’s motivation to work on the other player’s preferred project is his private information. Our main result provides a rationale for delegation when effort by both players is essential for success. (JEL D23, D82, L22, M54)
1. Introduction

In a principal-agent relationship, it is often no secret that the two parties have conflicting interests about the future direction of a common project. Yet, although it may be clear that the principal and the agent would like different projects to be successful, the extent of their conflict may be unclear. In such a context, the allocation of the authority to select a project will affect the actual project selection and both players’ motivation to work on the selected project. We are interested in the interaction between authority and motivation when the players’ effort decisions are strategic complements, effort by both players is essential for the project’s success, and the use of monetary incentives is either not feasible or not desirable. Our main results will provide a rationale for delegation in such situations.

As an example, think of a division manager and an engineer who can develop a product together. The engineer prefers a more innovative product that allows him to better signal his technical skills, whereas the manager prefers a less innovative product that is more profitable and allows him to better signal his management skills. Essential for the success of the project is a design for that both have strong incentives to work for.

In our base model, there are two projects, \( p \) and \( a \), and it is common knowledge that the principal strictly prefers project \( p \) whereas the agent strictly prefers project \( a \) conditional on that both projects will be equally successful. However, the success of the project depends on effort by both players. While it is common knowledge what the players like most, it is less well known how much they are hurt by what they like less. That is, the principal is uncertain about how much the agent likes project \( p \) and the agent is uncertain about how much the principal likes project \( a \). As a consequence, each player is uncertain about the other player’s eagerness to work on his less preferred project. After the project is selected, it becomes apparent to both players how much both of them like the selected project and they simultaneously decide on how much effort they will exert. Thus, the project selection affects both players’ effort incentives and their effort incentives interact in a non-trivial way. Ex ante, when the relationship between the principal and the agent is established and before details about later direction decisions are known, the principal decides who has the authority to make such decisions.

What are the effects implied by the authority decision? On one hand, delegation leads to a loss of control. That is, project \( a \) may be chosen too often for the principal’s taste. On the other hand, the principal wants project \( p \) to be selected unless the agent is sufficiently uneager to work on it. As this project selection behavior depends on the agent’s private information, the principal may have an incentive to delegate the project choice in order to use the agent’s information. Whether delegation will be optimal depends on the agent’s actual willingness to compromise on the project choice when he likes project \( p \).

Our main finding is that for a large class of problems that exhibit strategic complementarities between the effort decisions and where effort by both players is essential for success, delegation is optimal when it is sufficiently unlikely that the principal likes project \( a \). In a certain sense, this means that the principal delegates the decision to select the project when he has strong feelings about it. Intuitively, delegation disciplines the agent when it is likely that the principal dislikes project \( a \). This allows the principal to obtain his preferred project selection behavior. Notice that the interaction between the principal’s and the agent’s eagerness to work on the selected project is crucial for our rationale for delegation. This differentiates our model from articles that consider one-side effort decisions after project selection (e.g., Van den Steen, 2006, 2009; Landier et al., 2009) or effort decisions prior to project selection (e.g., Aghion and Tirole, 1997).
We further explain the role of essentiality of effort by both players. Delegation may still be optimal when effort decisions are strategic complements but effort by both players is not essential for success as it is the case when effort decisions are perfect substitutes. The scope for delegation is then however very limited.

On the other hand, we explain in an extension of our base model that the scope for delegation becomes even larger when the players’ information stays private prior to the effort decisions. Responsible for this is that choosing project \( a \) under delegation is then associated with an additional, discouraging effect. This is because the agent is then uncertain about whether the principal likes this project and is willing to spend effort on it. He must thus fear that the project will be unsuccessful and his own effort will be wasted. On the other hand, when he chooses project \( p \) this choice comes along with a signaling effect that takes the fear of wasting effort on this project away from the principal. This makes compromising on the project choice under delegation relatively more attractive for the agent and, in consequence, delegating the authority to select the project more attractive for the principal.

Finally, we discuss the robustness of our rationale for delegation with respect to our informational assumptions. First, the possibility of cheap talk prior to project selection has no impact on our results. Second, if the principal can affect whether he interacts with the agent in an environment where the two players learn about each others’ types prior to the effort decisions or not, an environment without learning may be optimal.

Our article is organized as follows: We introduce our base model with an exogenously given authority structure in the next section and analyze it in Section 3. Then, in Section 4, we provide our rationale for why it may be better not to be the player who selects the project. Subsequently, we discuss in Section 5 the most crucial assumptions in our model. Subsection 5.1 explains why the players have no incentives to share private information through cheap talk already at the project selection stage. Subsection 5.2 discusses which effects are added when the players’ signals remain private information at the effort provision stage. Subsection 5.3 endogenizes the information structure. Finally, Section 6 presents the literature review and Section 7 concludes.

2. The model

We introduce now the model for a given allocation of the authority to select the project. There is a team of two players, say players 1 and 2. The principal may either assume the role of player 1 or of player 2; the agent assumes the other role. Who assumes which role is not important until we discuss how the principal wants to assign these roles in Section 4.

Player 1 chooses a project \( x \in \{1, 2\} \) on which both of them will subsequently work. Working on the project corresponds to simultaneously choosing an effort level \( e_i \in E \) at quadratic effort cost \(-e_i^2/2\). The effort space \( E \) is a compact subset of \( \mathbb{R}_+ \) that includes “shirking” \((e_i = 0)\). The success of the project is measured by a project success function \( s(e_1, e_2) \). This function is non-decreasing in each player’s effort decision, symmetric, and normalized such that it assumes a value of zero if both players shirk. We say that effort by both players is essential for success if \( e_i = 0 \) implies \( s(e_1, e_2) = 0 \). The structure of player \( i \)'s payoff is

\[
    u_i(e_1, e_2; v_i(x, \theta_i)) \equiv v_i(x, \theta_i) \cdot s(e_1, e_2) - \frac{1}{2} e_i^2
\]

with \( v_i(x, \theta_i) \geq 0 \). \( v_i(x, \theta_i) \) describes player \( i \)'s (marginal) value of success and \( \theta_i \) describes a private signal that he possesses.
We are interested in problems with two distinct features. First, both players have an interest in coordinating on a project for which both of them are eager to work. This property will be reflected in our model by considering success functions for which the two effort decisions are strategic complements. We will discuss a number of examples for this in the next section. Second, the two players face a conflict because it is common knowledge that they obtain a higher marginal value of success from different projects. More specifically, we assume that project $i$ is player $i$’s favorite project in the sense that $v_i(i, \theta_i) = 1$ and $v_i(-i, \theta_i) = \theta_i < 1$. The private signal $\theta_i$ describes how much player $i$ likes the other project. With probability $q_i \in (0, 1)$, player $i$ still “likes” this project: $\theta_i = \alpha \in (0, 1)$; with probability $1 - q_i$, he “dislikes” this project: $\theta_i = 0$. Each player $i$ knows his own signal $\theta_i$ but only the distribution of the other player’s signal $\theta_{-i}$. We assume at first that both signals become observable after the project is selected and before effort decisions are made. This reflects that team members learn about each other once they start working together on a particular project.

The timing is thus as follows: (1) Each player privately learns his signal $\theta_i$. (2) Player 1 chooses a project $x$. (3) The project choice and both private signals become publicly observable. (4) Both players simultaneously take effort decisions $e_i$ and payoffs realize.

As equilibrium concept, we adopt the notion of Perfect Bayesian Equilibrium in pure strategies with a refinement that strengthens the sequential rationality requirement: If the continuation game that is played after a project is selected has multiple equilibria and if one of these equilibria Pareto dominates any other equilibrium, then this equilibrium is selected.

3. Motivation and project selection behavior

3.1. Motivation under different success functions: Some examples

Consider the continuation game that is played after the project choice is made. The players’ values $v_1$ and $v_2$ are in this game commonly known. We will discuss this effort provision game for general values $v_1, v_2 \geq 0$. This will be useful for extending our results in Section 5.

The main goal of this article is to present a rationale for delegation in a framework with an open conflict and endogenous effort. We will start by discussing the implications of different specific success functions on effort incentives. This will allow us to better explain afterwards which properties of the success function are important for our rationale and which are not. In the first three examples, each player’s effort decision is continuous but the complementarity between the two effort decisions is varied. In the last example, the effort decisions are perfectly complementary and binary.

Example PC-C: Continuous, perfectly complementary effort. Suppose $E = \mathbb{R}_+$ and $s(e_1, e_2) = \min\{e_1, e_2\}$. Because effort is perfectly complementary, both players provide in each equilibrium the same effort. For each $e \in [0, \min\{v_1, v_2\}]$, $(e, e)$ constitutes an equilibrium of the continuation game. The equilibria can be ordered according to whether both effort decisions are lower or higher. Because payoffs are weakly increasing in the other player’s effort decision and because a revealed preferences argument applies, the equilibrium with the highest effort

\footnote{The players’ values describe the only information that is payoff-relevant. Equilibrium effort provision behavior could, in principle, also depend on information that is not payoff-relevant. However, for the success functions we will be interested in, there exists a unique candidate for the equilibrium effort provision behavior that depends only on payoff-relevant information.}
decisions Pareto dominates any other equilibrium.\footnote{Compare player 1’s payoff for effort vectors \((e', e')\) and \((e'', e'')\) with \(e', e'' \in [0, \min\{v_1, v_2\}]\) and \(e' < e''\). By monotonicity of the success function in effort decisions and \(v_i \geq 0\), player 1 prefers \((e', e'')\) over \((e', e')\). By a revealed preference argument, he further prefers \((e', e'')\) over \((e'', e'')\).} Hence, in the equilibrium that is selected by our refinement, each player \(i\) provides effort \(\hat{e}_i(v_1, v_2) = \min\{v_i, v_{-i}\}\) and gets a reduced-form payoff of

\[
\hat{u}_i(v_1, v_2) = v_i \cdot \min\{v_i, v_{-i}\} - \frac{1}{2} \min\{v_i, v_{-i}\}^2 = \left\{ \begin{array}{ll} \frac{1}{2} v_i^2 & \text{if } v_i \leq v_{-i} \\ v_i v_{-i} - \frac{1}{2} v_{-i}^2 & \text{if } v_i > v_{-i} \end{array} \right. .
\]

**Example CD-C: Continuous, imperfectly complementary effort.** Cobb-Douglas. Suppose \(E = \mathbb{R}_+\) and \(s(e_1, e_2) = \sqrt{e_1 e_2}\). The effort provision game possesses two equilibria, a no effort equilibrium and a unique equilibrium with strictly positive effort decisions. The latter equilibrium Pareto dominates the former by the same argument as in Example PC-C. Player \(i\) provides effort \(\hat{e}_i(v_1, v_2) = \frac{1}{2} v_i^{3/4} v_{-i}^{1/4}\). Because the effort decisions are imperfectly complementary, each player’s optimal effort decision is responsive to both values, but the own value has a stronger effect. Player \(i\)’s reduced-form payoff is

\[
\hat{u}_i(v_1, v_2) = \frac{3}{8} v_i^{3/2} v_{-i}^{1/2} .
\]

**Example PS-C: Continuous, perfectly substitutive effort.** Suppose \(E = \mathbb{R}_+\) and \(s(e_1, e_2) = (e_1 + e_2)/2\). The effort provision game possesses a unique equilibrium in that each player \(i\) provides effort \(\hat{e}_i(v_1, v_2) = \frac{1}{2} v_i\). The consequence of perfectly substitutive effort is that each player \(i\)’s optimal effort decision depends only on his own value. However, reduced-form payoffs still depend positively on both values:

\[
\hat{u}_i(v_1, v_2) = \frac{1}{8} v_i^2 + \frac{1}{4} v_i v_{-i} .
\]

Responsible for this is that a player does still “free-ride” on the other player’s effort if \(v_i > 0\). That is, he benefits from it without having to bear the cost of it.

**Example PC-B: Binary, perfectly complementary effort.** Lastly, suppose that each player can only decide between providing effort at cost \(c\) and not providing effort. The project is successful if, and only if, both players provide effort. In case of success, player \(i\) realizes a value \(v_i \geq 0\). To make the problem interesting, we need to assume that \(c \in (0, \alpha)\). This problem fits into our framework when we define \(E = \{0, \sqrt{2c}\}\) and \(s(e_1, e_2) = 1\) if \(\min\{e_1, e_2\} = \sqrt{2c}\) and \(s(e_1, e_2) = 0\) otherwise. The effort provision game possesses then a Pareto dominant equilibrium in that player \(i\) provides effort \(\hat{e}_i(v_1, v_2) = \sqrt{2c}\) if \(v_i, v_{-i} \geq c\) and \(\hat{e}_i(v_1, v_2) = 0\) otherwise. It gives rise to a reduced-form payoff of

\[
\hat{u}_i(v_1, v_2) = \left\{ \begin{array}{ll} v_i - c & \text{if } v_i \geq c \text{ and } v_2 \geq c \\ 0 & \text{if } v_1 < c \text{ or } v_2 < c \end{array} \right. .
\]

3.2. Motivation: Common properties implied by the examples

All of our examples imply a number of common properties. We will describe these properties in this subsection. Our analysis (except for Proposition 4) will only depend on these implied properties and on whether effort by both players is essential for success or not.

The first property concerns the optimal effort decisions. After stating this property, we will discuss its different parts and explain which implications they have for payoffs.
Property 1 (Reducibility and properties of equilibrium effort)  

(a) The continuation game that is played after the project choice is made possesses a Pareto dominant Nash equilibrium that depends only on the payoff-relevant information $v_1$ and $v_2$, say $\hat{e}_1(v_1, v_2)$ and $\hat{e}_2(v_1, v_2)$.  

(b) $\hat{e}_i(v_1, v_2)$ is non-decreasing in $v_1$ and in $v_2$.  

(c) $\hat{e}_1(v_1, v_2) = \hat{e}_2(v_2, v_1)$.  

(d) $\hat{e}_1(0, v_2) = \hat{e}_2(v_1, 0) = 0$. If effort by both players is essential for success, $\hat{e}_2(0, v_2) = \hat{e}_1(v_1, 0) = 0$.

It follows from Property 1 (a) that there exists a single candidate for effort provision behavior that can be part of a Perfect Bayesian Equilibrium that satisfies our refinement criterion. This allows us to reduce the effort provision stage and to solve the game backwards. In general, player $i$’s effort provision behavior is described by a function $e_i(x, \theta_1, \theta_2)$. The property also states that this behavior depends only on the values that are implied by the project choice and the players’ signals. This allows us to express it as a function $\hat{e}_i(v_1, v_2)$ such that reduced-form payoffs are given by

$$\hat{u}_i(v_1, v_2) \equiv u_i(\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2); v_i).$$  

(3)

The intuition for Property 1 (b) is as follows: A higher marginal value of success by player $i$ directly increases his incentive to provide effort. Because the effort decisions are strategic complements in each of the four examples, this increases also (weakly) the other player’s incentive to provide effort, which in turn (weakly) increases player $i$’s incentive, and so on. As a consequence, the optimal effort decisions increase (weakly) in both players’ values. The perfect substitutes case (Example PS-C) represents the polar case where the indirect effects are mute such that a player’s effort decision depends only on his own marginal value. As a weakly higher value and a weakly higher effort level by the other agent cannot harm an agent, weak monotonicity of effort decisions in values implies weak monotonicity of payoffs in values. Moreover, by a revealed preferences argument, a player’s payoff must increase strictly in the own value once the payoff is positive.

Property 2 (Positive interdependence)  

$\hat{u}_i(v_1, v_2)$ increases weakly in $v_i$ and in $v_{-i}$ with $\hat{u}_i(0, 0) = 0$. Moreover, for any given $v_{-i}$, $\hat{u}_i(v_1, v_2)$ is either zero or strictly increasing in $v_i$.

Property 1 (c) is not very surprising as the two players are symmetric in the continuation game except for their values. The property implies that values but not identities matter for payoffs:

Property 3 (Permutation symmetry)  

$\hat{u}_1(v_1, v_2) = \hat{u}_2(v_2, v_1)$.

This property will be useful for getting less convoluted effects when we come to the question who should have the authority to select the project.

Assumptions that imply the following property are necessary to render our problem interesting.

Property 4 (Positive payoffs from coordination)  

$v_1, v_2 \geq \alpha$ implies $\hat{u}_i(v_1, v_2) > 0$.

In our model, the value of one player, say player $i$, is 1 and the value of the other player is either $\theta_{-i} = \alpha$ or $\theta_{-i} = 0$. If Property 4 did not hold, it would not be possible to induce positive effort in our main case where effort by both players is essential for success. In our example with continuous effort, the property arises naturally; in our example with binary effort it follows from assuming that effort cost are not too high.
The first statement of Property 1 (d) is obvious: If a player’s marginal value of success is zero, he will not provide positive effort and he will get a zero payoff. If effort by both players is essential for success (like in Examples PC-C, CD-C and PC-B), this implies that a player does also not provide effort when the other player’s value is zero. As a consequence, if a project is selected that one of them does not like, both players obtain a zero payoff.

**Property 5 (Need for coordination)** \( v_i = 0 \) implies \( \hat{u}_i(v_1, v_2) = 0 \). If effort by both players is essential for success, \( v_i = 0 \) implies also \( \hat{u}_{-i}(v_1, v_2) = 0 \).

Hence, when effort by both players is essential for success, the two players have an incentive to coordinate on a project that both of them like despite the open conflict regarding which project has a higher value. They only face a conflict when there exist multiple projects on that they can coordinate. This is the case when \( \theta_1 = \theta_2 = \alpha \). The project choice corresponds then to the choice between the value vectors \((1, \alpha)\) and \((\alpha, 1)\). The following property implies that each player prefers his own favorite project to be selected in this case:

**Property 6 (Single-crossing)** \( v_i > v_{-i} \) implies \( \hat{u}_i(v_1, v_2) \geq \hat{u}_{-i}(v_1, v_2) \) with a strict inequality if \( \hat{u}_{-i}(v_1, v_2) > 0 \).

Verifying that the properties hold for the four examples leads to the following lemma:

**Lemma 1 (Implications of optimal effort)** Properties 1–6 hold for each of our four examples.\(^3\)

As our results will hold for any combination of project success function and effort space that implies the six properties, we will study from now on the reduced-form problem on that we directly impose these assumptions:

**Assumption:** Suppose for the rest of this article that the success function \( s(e_1, e_2) \) and the effort space \( E \) are such that Properties 1–6 hold.

Properties 1 (a)–(c), 2 and 3 depend essentially only on the project success function reflecting strategic complementarities and on players 1 and 2 being symmetric in the continuation game except for values (see Appendix A). Property 4 follows from restricting attention to the interesting case. Properties 1 (d) and 5 are an immediate implication of the definition of essentiality of effort by both players. Even though the last property sounds quite intuitive, it relies on the most restrictive assumptions. In particular, it is not implied by strategic complementarity and symmetry alone. See the last paragraph of Appendix A for a discussion.

**3.3. Incentives to compromise on the project choice**

If player 1 chooses his own favorite project \( x = 1 \), his payoff depends on whether the other player likes this project. His payoff is \( \hat{u}_1(1, \alpha) \) if player 2 likes it and \( \hat{u}_1(1, 0) \) if player 2 does not like it. When player 1 compromises on the project choice by choosing project \( x = 2 \), he obtains a certain payoff of \( \hat{u}_1(\theta_1, 1) \). As choosing his own favorite project gives rise to a strictly positive expected payoff (by Properties 2 and 4) whereas compromising on the project choice gives a zero payoff when player 1 dislikes project \( x = 2 \) (by Property 5), only two different project

\(^3\)Note that the second part of Property 5 is meaningless in Example PS-C where effort by both players is not essential for success.
selection rules \( x(\theta_1) \) can be optimal. We will refer to the project selection rule \( x(0) = x(\alpha) = 1 \) as \textit{never compromising on the project choice} and to the rule that is specified by \( x(0) = 1 \) and \( x(\alpha) = 2 \) as \textit{compromising on the project choice whenever player 1 likes player 2’s favorite project}.

Never compromising on the project choice is strictly optimal when

\[
q_2 \hat{u}_1(1, \alpha) + (1 - q_2)\hat{u}_1(1, 0) > \hat{u}_1(\alpha, 1). \tag{4}
\]

Compromising on the project choice whenever \( \theta_1 = \alpha \) is strictly optimal when the converse inequality holds. This gives rise to the following lemma:

**Lemma 2 (Project selection)** Define

\[
q^* = \begin{cases} 
\frac{\hat{u}_1(\alpha, 1) - \hat{u}_1(1, 0)}{\hat{u}_1(1, \alpha) - \hat{u}_1(1, 0)} & \text{if } \hat{u}_1(\alpha, 1) > \hat{u}_1(1, 0) \\
0 & \text{if } \hat{u}_1(\alpha, 1) \leq \hat{u}_1(1, 0)
\end{cases}
\]

If \( q_2 > q^* \), player 1 never compromises on the project choice. If \( q_2 < q^* \), player 1 compromises on the project choice whenever he is able to compromise.

If effort by both players is essential for success, the threshold \( q^* \) that describes the optimal project selection behavior lies strictly between 0 and 1 (by Properties 5 and 6). That is, if it is sufficiently likely that player 2 likes project 1, it is optimal for player 1 to never compromise on the project choice. Yet if it is sufficiently likely that player 2 dislikes project 1, it is optimal for player 1 to compromise on the project choice whenever he likes project \( x = 2 \). In our three examples with essential effort, the threshold is given by \( q^* = \alpha/(2 - \alpha) \) (PC-C), \( q^* = \alpha \) (CD-C) and \( q^* = (\alpha - c)/(1 - c) \) (PC-B), respectively. See Figures 1a, 1b and 1c for illustrations. In each of these figures, the square displays the relevant part of the parameter space. Never compromising is optimal for player 1 in the white region; compromising whenever player 1 likes project \( x = 2 \) is optimal in the grey region.

If effort by both players is not essential for success, never compromising becomes relatively more attractive for player 1 as he may then also obtain a positive payoff from his favorite project when it turns out that player 2 dislikes it. Yet it may still be optimal to compromise on the project choice in order to induce more effort by the other player. However, such “free-riding” on the other player’s effort can only be attractive if the value from compromising is sufficiently large (i.e., when \( \alpha \) is large). In our example with perfectly substitutive effort (PS-C), this is the case for \( \alpha > \sqrt{2} - 1 \). The threshold is then \( q^* = (\alpha^2 + 2\alpha - 1)/(2\alpha) \in (0, 1) \) and it has the same interpretation as in the case where effort by both players is essential. If \( \alpha \leq \sqrt{2} - 1 \), never compromising is optimal for all \( q_2 \). See Figure 1d.
4. Who should have the authority to select the project?

It follows from our analysis so far how the payoffs of the players who assume the role of player 1 and of player 2 depend on the parameters of the model. If the probability that player 2 likes player 1’s favorite project is high \( q_2 > q^* \), player 1 will never compromise on the project choice. Player \( i \)’s expected payoff is then

\[
U_i(q_1, q_2) \equiv q_2 \hat{u}_i(1, \alpha) + (1 - q_2)\hat{u}_i(1, 0).
\]  

(5)

On the other hand, if the probability that player 2 likes player 1’s favorite project is low \( q_2 < q^* \), player 1 will compromise on the project choice whenever he likes player 2’s favorite project. Player \( i \)’s expected payoff is then

\[
U_i(q_1, q_2) \equiv q_1 \hat{u}_i(\alpha, 1) + (1 - q_1)[q_2 \hat{u}_i(1, \alpha) + (1 - q_2)\hat{u}_i(1, 0)].
\]  

(6)

We are now interested in the augmented problem between a principal \( P \) and an agent \( A \) who both can either assume the role of player 1 or of player 2. The principal decides in an initial stage before private signals realize who should have the authority to select a project; that is, he decides who assumes the role of player 1 and of player 2. We denote the principal’s (agent’s) favorite project by \( p(a) \), and the probability that the principal (agent) likes project \( a \) (project \( p \)) by \( q_P(q_A) \).保持 the authority to select the project is optimal for the principal if

\[ U_1(q_P, q_A) \geq U_2(q_A, q_P) \]

whereas giving the authority to the agent is optimal when the converse inequality holds. As the cases with essential and with non-essential effort will imply structurally different effects, we will consider them separately.

4.1. Delegation incentives with essential effort

Consider first our main case where effort by both players is essential for success. What would be the principal’s and the agent’s preferred project selection behavior? There is a need for coordination when only one of the projects is liked by both players (by Property 5 and essentiality of effort by both players). More specifically, both players prefer the principal’s favorite project to be selected if \( \theta_P = 0 \) and \( \theta_A = \alpha \); both players prefer the agent’s favorite project to be selected if \( \theta_P = \alpha \) and \( \theta_A = 0 \). If both projects are liked by both players, each of them has a strict preference for his own favorite project to be selected (by Properties 4 and 6). Finally, if none of the projects is liked by both players, the project choice does not matter as either project will imply a zero payoff for both players (by Property 5 and essentiality of effort by both players). Table 1 summarizes the principal’s and the agent’s preferred project in the four cases. Intuitively, the principal prefers a project selection behavior where the agent makes always just as many compromises as he is able to do. That is, he wants project \( p \) to be selected if the agent likes project \( p \) and he wants project \( a \) to be selected if the agent does not like project \( p \). Interestingly, the principal’s preferred project selection behavior depends only on the information of the agent.

As a consequence, the principal’s preferred project selection behavior can never be obtained when the principal chooses the project himself (\( P \)-authority). Yet it would be obtained if the principal delegated the project choice to the agent (\( A \)-authority) and the agent compromised on the project choice whenever he likes project \( p \). Whether the agent is willing to behave in this way under delegation depends on his incentives to compromise on the project choice. If

\[4\]This implies that if the principal assumes the role of player 1, then \( p = 1, q_1 = q_P, a = 2 \) and \( q_2 = q_A \). Conversely, if the principal assumes the role of player 2, then \( p = 2, q_2 = q_P, a = 1 \) and \( q_1 = q_A \).
the agent thinks that the principal likes project $a$ with a sufficiently high probability ($q_P > q^*$), he will select project $a$ irrespective of his information. As the principal can obtain this project selection behavior also when he selects the project himself, $P$-authority is at least weakly optimal. By contrast, if the agent thinks that the principal likes project $a$ with a sufficiently low probability ($q_P < q^*$), the agent compromises on the project choice whenever he is able to compromise, rendering $A$-authority strictly optimal.

**Proposition 1 (Optimal authority; essential effort)** Suppose that effort by both players is essential. If $q_P < q^*$, $A$-authority is strictly optimal. If $q_P > q^*$, $P$-authority is strictly optimal.

Notice that unlike in Aghion and Tirole (1997), the reason for delegation in this proposition is not only to induce effort by the agent, but rather to get the project that gives the principal the higher marginal value whenever the agent is willing to provide effort for this project. The crucial prerequisite for this is that the agent is uncertain and sufficiently pessimistic about whether the principal likes project $a$. Also, notice that the result does not depend on $q_A$. That is, the principal’s uncertainty about whether the agent likes project $p$ does not matter. Figure 2 displays how the optimal authority decision depends on $\alpha$ and $q_P$ for our three examples where effort is essential for success. Delegation is optimal in the grey regions.

**4.2. Delegation incentives with non-essential effort**

What changes when effort by both players is not essential for success? As the principal then does not rely on the agent’s effort to get a positive payoff, project $p$ becomes relatively more attractive for the principal in the case where the agent dislikes this project. This has two consequences. First, the principal may strictly prefer project $p$ when $\theta_P = \theta_A = 0$ (instead of being indifferent). Second, it becomes a priori unclear whether he prefers project $a$ or project $p$ when $\theta_P = \alpha$ and $\theta_A = 0$ (instead of strictly preferring project $a$). Project $p$ may in this case lead to a strictly positive payoff but project $a$ may still be better as it allows the principal to “free-ride” on the agent’s effort choice.

To be more concrete, consider our example with perfectly substitutive effort. The principal then strictly prefers project $p$ if $\theta_P = \theta_A = 0$ but he still strictly prefers project $a$ if $\theta_P = \alpha$
and \( \theta_A = 0 \). Table 2 summarizes the principal’s preferred project in the four cases. As before, \( A \)-authority may only be better for the principal than \( P \)-authority when it implies a project selection behavior that is responsive to the agent’s information. By the reasoning in Subsection 3.3, the agent is never willing to compromise for \( \alpha \leq \sqrt{2} - 1 \). The agent’s project selection behavior is only responsive to his information if \( \alpha > \sqrt{2} - 1 \) and \( q_P < q^* \) (see Figure 1d). Yet when \( q_P \) is sufficiently small, the principal prefers project \( p \) in the case where the agent does not compromise on the project choice (i.e., conditional on \( \theta_A = 0 \)). It is in this case better for the principal to retain control and to enforce the choice of project \( p \). Hence, for any \( \alpha \), delegation may only be optimal for an intermediate region of \( q_P \)-values.

**Proposition 2 (Optimal authority; non-essential effort)** Suppose that effort is perfectly substitutive as introduced in Example PS-C. Moreover, suppose that \( q_P = q_A = q \in (0,1) \). Define \( q^* \equiv (1/\alpha)^2 - \sqrt{(1/\alpha)^2 - 1/\alpha} \). If \( q \in (q_*, q^*) \), \( A \)-authority is strictly optimal. If \( q \in (0, q_*) \cup (q^*, 1) \), \( P \)-authority is strictly optimal.

See Figure 3 for an illustration. \( A \)-authority is only optimal in the grey region. Notice that this region is quite small. Without the essentiality assumption, delegation is only optimal under two additional kinds of constraints which imply that \( \alpha \) must be sufficiently large and that \( q_P \) must not be too small. This demonstrates that essentiality of effort by both players may significantly increase the scope of our rationale for delegation.

5. Discussion

5.1. Cheap talk prior to project selection: Delegation dominates communication

In our base model, the players’ signals became publicly observable once they started working together on a particular project but these signals were private information at the project selection stage. As communication prior to project selection is possible in virtually every application, we are now interested in whether the players have an interest in sharing information about their signals already at the project selection stage. Suppose the principal and the agent can communicate with cheap talk messages according to an arbitrary protocol after the principal has made his authority decision and before the project selection decision is made.

**Proposition 3 (Cheap talk prior to project selection)** Cheap talk prior to project selection does never alter the project selection behavior or the players’ motivation.
To obtain an intuition for the result, suppose player 2 can send a cheap talk message to player 1 before player 1 selects a project. Player 2 knows that he only has a chance to affect the behavior of player 1 if player 1 likes project $x = 2$. However, conditional on that this is the case, player 2 strictly benefits if it becomes more likely that player 1 selects project $x = 2$. Hence, player 2 would always send a message that implies the highest probability that project $x = 2$ is selected. This renders communication that affects the project selection behavior impossible.

Proposition 3 has two important implications: First, keeping the authority to select the project and asking the agent for his signal is not an equivalent replacement for delegation. Second, in situations where the principal wants to delegate, he has an incentive to commit to the agent’s choice as not committing corresponds to degrading his choice to a cheap talk message. In practice, reputation in a repeated framework where the principal interacts with a sequence of different agents could be an instrument for such a commitment.

5.2. Private information at the effort provision stage: Signaling through compromise

Another assumption that may be too extreme for some applications is that the players do publicly learn each others’ signals prior to the effort provision stage. We explain now what changes when we consider the other polar case where this information stays private. To distinguish the settings, we will refer to this case as the private information case and to the case that we considered so far as the public information case. As the analysis of our base model suggests that the parameter $q_A$ is not very interesting, we consider henceforth the symmetric case with $q_P = q_A = q$ and focus on the role of the informational assumption. Moreover, we restrict attention to the case where effort by both players is essential for success. Finally, we impose for the remainder of this article a somewhat more stringent version of Property 4 to exclude some less interesting special cases.

Property 4’ (Positive payoffs from coordination) $v_1, v_2 \geq \min\{\alpha, q\}$ implies $\hat{u}_i(v_1, v_2) > 0$.

Notice that this more stringent property holds for Examples PC-C and CD-C. In example PC-B, it holds when we additionally assume that $q > c$. We will refer to the version of this example with the additional assumption as PC-B’.

What impact does incomplete information have on player 1’s and player 2’s motivation at the effort provision stage? As each player $i$ knows only his own signal $\theta_i$ and the project $x$ chosen by player 1, the effort provision behavior can be described by functions $e_i(x, \theta_i)$. Even though player 1’s signal is not directly observable to player 2, his project choice may signal information about it. We denote the probability that player 2 believes that $\theta_1 = \alpha$ conditional on observing the project choice $x$ by $\mu_x$.

If player 1 chooses project $x = 1$, it is common knowledge that player 1’s value is 1 but player 1 is uncertain about player 2’s value. When player 2 dislikes project 1, he will provide no effort. As effort by both players is essential for success, player 1 “wastes” his effort with probability $1 - q$. To put this differently, player 1 always has to bear the effort cost but his value realizes only with probability $q$. Effectively, player 1’s uncertainty about whether player

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5 A justification for this assumption could be that players often have an incentive to share information before they engage in interactions that reflect strategic complementarities.

6 This represents the interesting case. In the case with perfectly substitutive effort, the other player’s signal does not affect a player’s optimal effort decision. Nothing would change if this information stayed private. That is, the results from the public information case directly carry over to the private information case.
2 likes project \( x = 1 \) results in a reduction of player 1’s value from 1 to \( q \). When we denote player 2’s effort decision conditional on that \( \theta_2 = \alpha \) by \( e_2' \), player 1’s objective function is

\[
q \cdot s(e_1, e_2') - \frac{1}{2} e_1^2
\]

and the objective function of player 2 with information \( \theta_2 = \alpha \) is

\[
\alpha \cdot s(e_1, e_2') - \frac{1}{2} e_2'^2.
\]

This corresponds just to the objective functions from Subsections 3.1 and 3.2 with \( v_1 = q \) and \( v_2 = \alpha \). Thus, the players behave here as if values are commonly known but player 1’s value is reduced from 1 to \( q \). As our analysis in Subsections 3.1 and 3.2 did not rely on the exact values of \( v_1 \) and \( v_2 \), it applies also here. It follows that player 1 provides effort \( \hat{e}_1(q, \alpha) \) (irrespective of \( \theta_1 \) and \( \theta_2 \)) and that player 2 provides effort \( \hat{e}_2(q, \alpha) \) if \( \theta_2 = \alpha \) and \( \hat{e}_2(q, 0) = 0 \) if \( \theta_2 = 0 \).

Player 1’s expected payoff from choosing his own favorite project \( x = 1 \) corresponds to \( \hat{u}_1(q, \alpha) \). As this is strictly positive by Property 4’, he may only choose project \( x = 2 \) in equilibrium if this project choice implies a strictly positive expected payoff as well. Since this is only possible if player 1 likes project \( x = 2 \), player 2 will infer in any equilibrium from \( x = 2 \) that \( \theta_1 = \alpha \). That is, \( \mu_2 = 1 \). When we denote player 1’s effort decision conditional on \( \theta_1 = \alpha \) by \( e_1' \), the objective function of player 1 with signal \( \theta_1 = \alpha \) is

\[
\alpha \cdot s(e_1', e_2) - \frac{1}{2} e_1'^2
\]

and the objective function of player 2 is

\[
[\mu_2 \cdot s(e_1', e_2) + (1 - \mu_2) 0] - \frac{1}{2} e_2^2 = 1 \cdot s(e_1', e_2) - \frac{1}{2} e_2^2.
\]

This corresponds just to the objective functions from Subsections 3.1 and 3.2 with \( v_1 = \alpha \) and \( v_2 = 1 \). Thus, due to the signaling effect that comes with the project choice, the players behave as if their values were commonly known. It follows that player 1 provides effort \( \hat{e}_1(\alpha, 1) \) and that player 2 provides effort \( \hat{e}_2(\alpha, 1) \).

The subsequent lemma summarizes what we have just discussed:

**Lemma 3 (Effort provision in the private information case)** Suppose that effort by both players is essential for success and that \( q_1 = q_2 = q \in (0, 1) \). Then, in any equilibrium, effort decisions are like in an auxiliary version of the problem with public information at the effort provision stage where the marginal values of success are modified. More specifically, \( e_1(1, \theta_1) = \hat{e}_1(q, \alpha) \), \( e_2(1, \theta_2) = \hat{e}_2(q, \theta_2) \), \( e_1(2, \theta_1) = \hat{e}_1(\theta_1, 1) \), and \( e_2(2, \theta_2) = \hat{e}_2(\alpha, 1) \).

The lemma allows us to reduce the effort provision stage again. Player 1’s reduced expected payoff from decision \( x = 1 \) is

\[
1 \cdot [q s(e_1(1, \theta_1), e_2(1, \alpha)) + (1 - q) s(e_1(1, \theta_1), e_2(1, 0))] - \frac{1}{2} e_1(1, \theta_1)^2
\]

\[
= q \cdot s(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha)) - \frac{1}{2} \hat{e}_1(q, \alpha)^2 = \hat{u}_1(q, \alpha). \tag{7}
\]

His reduced expected payoff from decision \( x = 2 \) is 0 if \( \theta_1 = 0 \) and

\[
\alpha \cdot [q s(e_1(2, \alpha), e_2(2, \alpha)) + (1 - q) s(e_1(2, \alpha), e_2(2, 0))] - \frac{1}{2} e_1(2, \alpha)^2
\]

\[
= \alpha \cdot s(\hat{e}_1(\alpha, 1), \hat{e}_2(\alpha, 1)) - \frac{1}{2} \hat{e}_1(\alpha, 1)^2 = \hat{u}_1(\alpha, 1). \tag{8}
\]
Lemma 4 (Project selection in the private information case) Suppose that effort by both players is essential for success and that \( q_1 = q_2 = q \in (0, 1) \). Then, there exists a unique value \( q^{**} \in [0, 1] \) such that the following is true: If \( q > q^{**} \), player 1 does never compromise on the project choice. If \( q < q^{**} \), player 1 compromises on the project choice whenever he is able to compromise. Moreover, \( q^{**} \geq q^* \) and \( q^{**} \geq \alpha \).

In our examples where effort is continuous and effort by both players is essential for success, the threshold is given by \( q^{**} = \alpha \) (PC-C), \( q^{**} = \alpha^{2/3} \) (CD-C) and \( q^{**} = \alpha \) (PC-B’), respectively. See Figure 4 for an illustration. The figure displays also the \( q^* \)-curve that was relevant in the public information case. In the private information case, never compromising is optimal for player 1 in the white region; compromising whenever player 1 likes project \( x = 2 \) is optimal in the two grey regions. Since \( q^{**} \geq \alpha \), this region amounts to at least half of the parameter space which is as in the preceding figures indicated by the square. The dark grey region displays the part of the parameter space where player 1 is in the private information case but not in the public information case willing to compromise on the project choice when he likes project \( x = 2 \).

We are now set to consider again the augmented problem where the principal decides in an initial stage whether he wants to assume the role of player 1 (P-authority) or of player 2 (A-authority). Like in the public information case, the optimality of delegation with essential effort is strongly related to player 1’s willingness to compromise on the project choice. More specifically, for our three examples where effort by both players is essential for success, the result from the public information case (Proposition 1) extends to the private information case: If the agent is willing to compromise on the project choice conditional on that he likes the principal’s favorite project, delegation is optimal.

Proposition 4 (Optimal authority in the private information case) Consider Example PC-C, CD-C or PC-B’ and suppose that \( q_P = q_A = q \in (0, 1) \). If \( q < q^{**} \), then A-authority is strictly optimal. If \( q > q^{**} \), then P-authority is strictly optimal.

The intuition for the result is very similar to that in the public information case but the derivation of the result is somewhat more involved. Responsible for this is that the optimal
effort provision behavior depends in the private information case not only on the project choice and the players’ private signals, but also on who selected the project (i.e., on whether the project choice was accompanied by a signaling effect or not). In the proof of Proposition 4, we show that the project selection effects that drove the result in the public information case are not overturned by the effort effects that arise additionally in the private information case.

Delegation is optimal in the grey regions of Figure 4. The dark grey region shows the part of the parameter space where delegation is optimal in the private but not in the public information case. Since \( q^{**} \geq \alpha \) (by Lemma 4), our rationale for delegation applies in the private information case for at least half of the parameter space.

5.3. Endogenous information at the effort provision stage: A rationale for secrecy

In some applications, the principal may be able to also affect the informational environment besides the authority structure. In Subsection 5.1, we have learnt that players have no incentive to communicate informatively prior to the project selection. This raises the question of whether they can have an interest in going even further by undertaking measures in order to keep information secret at the effort provision stage secret.

Consider the framework from the preceding subsection and suppose now that the principal can additionally decide at the initial stage whether he will interact with the agent in an environment where signals stay private after the project selection or whether they become observable. The most interesting case arises when the two informational environments induce different project selection behaviors. In our three examples, this is the case for \( q \in (q^*, q^{**}) \). A-authority is optimal in the private information case, whereas P-authority is optimal in the public information case.

When the principal has the possibility to decide between the private and the public information case, he faces the following trade-off: In the private information case, not compromising on the project choice leads with a certain probability to a waste of effort. This makes the agent more eager to compromise on the project choice which is good for the principal. On the other hand, the waste of effort by the agent is harmful for both players as it lowers his (and therewith—due to the strategic complementarities in the effort decisions—also the principal’s) effort incentives. The following result shows that the better project selection behavior dominates the lower effort effect.

**Proposition 5 (Secrecy after project selection)** Suppose that effort by both players is essential for success and that \( q_1 = q_2 = q \in (q^*, q^{**}) \). If P-authority is optimal in the public information case but A-authority is optimal in the private information case (like it is the case in Examples PC-C, CD-C and PC-S), then the principal prefers the public information case over the private information case. That is, A-authority in conjunction with undertaking measures to keep signals private after the project selection is optimal.

Two remarks are in order: First, it is only important that the principal is able to keep his own signal private secret. When the agent compromises on the project choice, only the agent’s private information is important for effort provision. However, this information is signaled through the project choice anyway. When the agent does not compromise, it is only the principal’s information that is important for effort provision. The proposition implies that the principal has an incentive to commit to keeping this information private. Such a commitment could come from spatial separation or, if we embed our model in a framework where the principal interacts repeatedly with different agents, from reputation effects.
Second, notice that it follows from our analysis so far that both players prefer the public information case over the private information case when either \( q \in (0, q^*) \) or \( q \in (q^{**}, 1) \). The only potential benefit of the private information case is that it may induce a project selection behavior that is better for the principal. If the project selection behavior is not affected (like it is the case for low and high \( q \)), the potential waste of effort that arises in the private information case can only be harmful as it lowers effort incentives. Hence, the public information case must be optimal.

6. Literature

Our delegation problem with two-sided effort provision after project selection is related to several strands of literature that can be classified according to who has to provide effort when.

No effort provision. There exists an extensive literature on the optimal delegation of a decision to a systematically biased agent who is in possession of information that is payoff-relevant to the principal (e.g., Holmström, 1984; Melumad and Shibano, 1991; Dessein, 2002; Martimort and Semenov, 2006; Alonso and Matouschek, 2008; Amador and Bagwell, 2013). This literature is concerned with how to restrict the agent’s discretion in order to use his information optimally. Motivational issues play no role. Although the agent in our model is also systematically biased, he is not in possession of any information that is directly payoff-relevant for the principal. The only reason for delegation is to improve the agent’s motivation which interacts with the principal’s motivation in a non-trivial way.

Effort provision before project selection. In their seminal article on formal authority (the right to decide) and real authority (the effective control over decisions), Aghion and Tirole (1997) study the interaction between authority and motivation. A principal and an agent can both provide effort to acquire information before a project is selected. The sequence of project selection and effort provision thus is reversed compared to our article and the effort decisions are strategic substitutes instead of strategic complements. As a consequence, the channels through that delegation affects motivation differ crucially from those in our article. Delegation generally has a motivating effect. Moreover, in sharp contrast to our article, communication prior to the project selection can have an impact on the project selection behavior.\(^7\)

One-sided effort provision after project selection. In a further strand of literature, the probability with that the selected project is successful depends on one-sided effort by the agent (Van den Steen, 2006, 2009; Landier et al., 2009). Effort and the “right” project selection are complementary. In Van den Steen (2006), an open conflict exists in the sense that principal and agent have different priors about the state of the world. As this implies that the agent believes that he is able to take better decisions than the principal, delegation has a motivating effect. Landier et al. (2009) take the perspective of a third party (the organization) and explore the role of dissent between a decision-maker and an implementer. Decision-maker and implementer have intrinsic and possibly differing preferences over projects but share an interest in the project’s success. The decision-maker anticipates the effect of her project choice on the implementer’s motivation. If there is dissent, this can prevent him from following her intrinsic bias and to choose the project that is more likely to be right. That is, dissent has a disciplining effect that improves the implementer’s motivation to provide effort and can render a dissenting

\(^7\)Marino et al. (2010) extend the study of formal and real authority by introducing limits to authority which arise through ineffective enforcement. Szalay (2005) introduces one-sided information acquisition by the agent in an optimal delegation problem. See also Section 3.1 in Armstrong and Vickers (2010).
organization optimal. Our model differs in two important respects. First, the nature of the projects differs. There is no objectively “right” project in our article. The difference in projects is just a matter of taste. Second, the success of the project relies on effort provision by both players. Each player internalizes the effect of project selection on the other player’s motivation. That is, delegation and non-delegation both have a disciplining effect.

Bénabou and Tirole (2003) study the interplay between intrinsic and extrinsic motivation in a setting with an agent with imperfect self-knowledge and an informed principal. The principal’s decision to delegate can signal confidence in the agent’s ability and thus have a motivating effect. By contrast, in our model, a motivating signaling effect only arises under non-delegation.

One-sided implementation decision after project selection. Aghion et al. (2004) study the allocation of decision rights for a problem where, after the project is selected, the principal can decide between implementing it and stopping it at “intermediate payoffs”. Such an implementation decision is related to the principal’s effort decision in our article. The agent’s type is payoff-relevant for the principal and the agent is already informed about his type at the time the allocation decision is made. They show that learning about the agent’s type may take place in a drastically different way when decision rights are contractible (i.e., when the allocation of decision rights can be message contingent) and when they are simply transferable. Besides the difference in focus, the authors also abstract from a motivational problem on the agent’s side.

Authority, motivation and monetary incentives. Zabojnik (2002) and Bester and Krähmer (2008) study the interaction between authority and the agent’s motivation to exert implementation effort when monetary incentives are feasible. Contracts can specify monetary transfers that condition on performance but not on project selection. In our model, we are interested in situations in that it is either not desirable or for exogenous reasons not possible to set monetary incentives. In many problems, people are motivated by other things than money, such as job characteristics, feelings of empowerment or how a specific project relates to own career objectives or personal interests. By focusing on this last aspect, our model is also related to the literature on intrinsic motivation and mission-orientation, going back to Besley and Ghatak (2005), Francois (2000) and Prendergast (2007), among others.

7. Conclusion

This article argues that in situations in that a principal can undertake a project together with an agent on that both of them need to work to make the project a success, there is scope for delegation of the project choice even when there is an open conflict regarding which project the partners prefer to be successful. A crucial role is played by the dissent between the principal and the agent as measured by the probability with that the principal also likes the project that

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8Bester and Krähmer (2016) introduce an exit option in a delegation problem with monetary incentives.

9In Zabojnik (2002), principal and agent possess independent information about what the right project is and the agent’s implementation effort and the quality of decision-making are complementary. He shows that delegation may allow the principal to save on high-powered incentives when the agent is protected by limited liability. Delegation can thus be optimal even if the principal is better informed than the agent. Melumad and Reichelstein (1987) analyze the case in that project selection is contractible. See Vidal and Möller (2016) for a mechanism design approach for a problem with two-sided effort and contractible project selection. Bester and Krähmer (2008) consider a problem without asymmetric information. Projects differ in the private benefits that they generate in case of success and the agent can increase the success probability by exerting implementation effort. They show that the need to motivate the agent makes the principal less willing to delegate. When the agent is protected by limited liability, delegation is generally suboptimal.
the agent prefers to be successful. Delegation of the project choice to the agent is optimal if the dissent is sufficiently strong. Otherwise, the agent does not feel the need to compromise on the project choice when he is able to compromise and retaining the control over the project choice is optimal for the principal.

Appendix A. Discussion of a general class of project success functions

Consider the effort provision problem from Subsections 3.1 and 3.2 where the players’ values \( v_1, v_2 \geq 0 \) are commonly known. Instead of considering specific examples, we consider now a general class of project success functions \( s(e_1, e_2) \). In addition to what we have assumed in Section 2, suppose that the project success function satisfies the following two assumptions:

- \( s(e_1, e_2) \) is upper semi-continuous;
- \( s(e_1, e_2) \) has increasing differences in \( e_1 \) and \( e_2 \) (i.e., \( e'_1 \geq e_1 \) and \( e'_2 \geq e_2 \) implies \( s(e'_1, e'_2) + s(e_1, e_2) \geq s(e'_1, e_2) + s(e_1, e'_2) \)).

Standard results from the theory of supermodular games (Topkis, 1979; Milgrom and Roberts, 1990) imply then that the effort provision game possesses always an equilibrium that is very focal.

**Lemma A1** (a) There exists a largest Nash equilibrium \((\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2))\) in pure strategies. This equilibrium dominates any other Nash equilibrium at least weakly according to the Pareto criterion. (b) \( \hat{e}_i(v_1, v_2) \) is nondecreasing in \( v_1 \) and in \( v_2 \). (c) \( \hat{e}_1(v_1, v_2) = \hat{e}_2(v_2, v_1) \).

The preceding lemma proves that Properties 1 (a)–(c) hold for the general class of success functions. The subsequent lemma shows that this implies that also Properties 2 and 3 hold for this entire class.

**Lemma A2** (a) \( \hat{u}_i(\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2); v_i) \) increases weakly in \( v_i \) and in \( v_{-i} \). Moreover, for a given value of \( v_{-i} \), \( \hat{u}_i(\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2); v_i) \) is locally for any \( v_i \) either zero or strictly increasing in \( v_i \). (b) \( u_1(\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2); v_i) = u_2(\hat{e}_1(v_1, v_2), \hat{e}_2(v_2, v_1); v_i) \).

Part (a) is driven by the fact that effort decisions are strategic complements and that effort has positive spillovers. Part (b) is essentially a consequence of imposing a symmetric structure on payoff functions.

A rough intuition for why the assumptions we imposed on the general class of success functions are not sufficient for Property 5 is the following: It follows from Lemma A2 (b) that when both players have the same value, they attain the same payoff. Property 5 does thus basically state that when player \( i \)'s value increases relative to this symmetric situation, his own payoff increases stronger than that of the other player. This property sounds quite intuitive, but it requires some additional assumptions. Intuitively, the direct effect of an increase in player \( i \)'s value is only an increase in player \( i \)'s payoff. As value and success are complementary, player \( i \) may further increase his payoff by increasing his effort. However, interestingly, the decision to increase his effort in response to an increase in his value may benefit the other player more than it benefits player \( i \). Responsible for this is that the other player does not have to bear the accompanied increase in effort cost.
Appendix B. Proofs

Proof of Lemma 1
It is straightforward to check the six properties for our four examples. q.e.d.

Proof of Lemma 2
It follows from the discussion in the text that choosing project \( x = 1 \) is strictly optimal for player 1 if \( \theta_1 = 0 \). Suppose that \( \theta_1 = \alpha \). Then \( x = 1 \) is by the reasoning in the text strictly optimal if (4) holds. We distinguish two cases.

Case 1: \( \hat{u}_1(\alpha, 1) \leq \hat{u}_1(1, 0) \). Properties 4 and 6 imply that \( \hat{u}(1, \alpha) > \hat{u}(\alpha, 1) \). By this and the supposition, the left-hand side of (4) strictly exceeds the right-hand side for any \( q_2 \in (0, 1) \). This corresponds to the statement in the lemma with the threshold \( q^* = 0 \).

Case 2: \( \hat{u}_1(\alpha, 1) > \hat{u}_1(1, 0) \). We can rewrite (B.1) as

\[
q_2(\hat{u}(1, \alpha) - \hat{u}(1, 0)) > \hat{u}(\alpha, 1) - \hat{u}(1, 0).
\]  

(B.1)

By Properties 4 and 6, \( \hat{u}(1, \alpha) > \hat{u}(\alpha, 1) \). By this and the supposition, \( \hat{u}(1, \alpha) > \hat{u}(1, 0) \). This implies that the left-hand side of (B.1) is strictly positive for any \( q_2 \in (0, 1) \). It follows that \( x = 1 \) is strictly optimal if \( q_2 \) does strictly exceed the threshold \( q^* \) as stated in the lemma (and that \( x = 2 \) is strictly optimal if \( q_2 < q^* \)). q.e.d.

Proof of Proposition 1
The principal’s expected payoff is \( U_1(q_P, q_A) \) under \( P \)-authority and \( U_2(q_A, q_P) \) under \( A \)-authority. By (5) and (6), we obtain that the principal’s expected payoff under \( P \)-authority is

\[
U_1(q_P, q_A) = \begin{cases} 
q_A \hat{u}_1(1, \alpha) + (1 - q_A) \hat{u}_1(1, 0) & \text{if } q_A > q^* \\
q_P \hat{u}_1(1, \alpha) + (1 - q_P)[q_A \hat{u}_1(1, \alpha) + (1 - q_A) \hat{u}_1(1, 0)] & \text{if } q_A < q^* 
\end{cases}
\]

(B.2)

and that his expected payoff under \( A \)-authority is

\[
U_2(q_A, q_P) = \begin{cases} 
q_P \hat{u}_2(1, \alpha) + (1 - q_P) \hat{u}_2(1, 0) & \text{if } q_P > q^* \\
q_A \hat{u}_2(1, \alpha) + (1 - q_A)[q_P \hat{u}_2(1, \alpha) + (1 - q_P) \hat{u}_2(1, 0)] & \text{if } q_P < q^* 
\end{cases}
\]

(B.3)

and that his expected payoff under \( A \)-authority is

\[
U_2(q_A, q_P) = \begin{cases} 
q_P \hat{u}_2(1, \alpha) + (1 - q_P) \hat{u}_2(1, 0) & \text{if } q_P > q^* \\
q_A \hat{u}_2(1, \alpha) + (1 - q_A)[q_P \hat{u}_2(1, \alpha) + (1 - q_P) \hat{u}_2(1, 0)] & \text{if } q_P < q^* 
\end{cases}
\]

The transformations arise as follows: In the first case, the second equality follows from essentiality of effort by both players for success and the second part of Property 5. In the second case, the second equality follows from Property 3 and from the first part of Property 5. We distinguish four cases. To prove the result, we have have to show that \( P \)-authority is strictly optimal in the first tow cases and that \( A \)-authority is strictly optimal in the last two cases.

Case 1: \( q_P > q^* \) and \( q_A > q^* \). \( P \)-authority is then strictly optimal if

\[
q_A \hat{u}_1(1, \alpha) > q_P \hat{u}_1(1, \alpha).
\]

By Lemma 2 and \( \hat{u}_1(1, 0) = 0 \), \( q^* = \hat{u}_1(\alpha, 1)/\hat{u}_1(1, \alpha) \). This allows us to write the inequality as \( q_A > q_P q^* \). Since the right-hand side exceeds \( q^* \), the inequality holds by the supposition of this case.
Case 2: $q_P > q^*$ and $q_A < q^*$. $P$-authority is then strictly optimal if
\[ q_P \hat{u}_1(\alpha, 1) + (1 - q_P)q_A \hat{u}_1(1, \alpha) > q_P \hat{u}_1(\alpha, 1). \]
The inequality holds since $\hat{u}_1(1, \alpha) > 0$ by Property 4.

Case 3: $q_P < q^*$ and $q_A < q^*$. $A$-authority is then strictly optimal if
\[ q_A \hat{u}_1(1, \alpha) + (1 - q_A)q_P \hat{u}_1(1, \alpha) > q_A \hat{u}_1(\alpha, 1) + (1 - q_A)q_P \hat{u}_1(1, \alpha). \]

It follows from simplifying that the inequality corresponds to $\hat{u}_1(\alpha, 1) < \hat{u}_1(1, \alpha)$. This holds by Properties 4 and 6.

Case 4: $q_P < q^*$ and $q_A > q^*$. $A$-authority is then strictly optimal if
\[ q_A \hat{u}_1(1, \alpha) + (1 - q_A)q_P \hat{u}_1(1, \alpha) > q_A \hat{u}_1(\alpha, 1). \]
The inequality holds since $\hat{u}_1(\alpha, 1) > 0$ by Property 4. q.e.d.

**Proof of Proposition 2**

The principal’s expected payoff under $P$-authority is given by (B.2):
\[
U_1(q, q) = \begin{cases} 
q\hat{u}_1(1, \alpha) + (1 - q)\hat{u}_1(1, 0) & \text{if } q > q^* \\
q\hat{u}_1(\alpha, 1) + (1 - q)[q\hat{u}_1(1, \alpha) + (1 - q)\hat{u}_1(1, 0)] & \text{if } q < q^*
\end{cases}.
\]

His expected payoff under $A$-authority is given by (B.3):
\[
U_2(q, q) = \begin{cases} 
q\hat{u}_1(\alpha, 1) & \text{if } q > q^* \\
q\hat{u}_1(1, \alpha) + (1 - q)q\hat{u}_1(\alpha, 1) & \text{if } q < q^*
\end{cases}.
\]

We distinguish two cases.

Case 1: $q > q^*$. $P$-authority is then strictly optimal if
\[ q\hat{u}_1(\alpha, 1) + (1 - q)\hat{u}_1(1, 0) > q\hat{u}_1(\alpha, 1). \]

This holds because $\hat{u}_1(1, 0) > 0$ in Example PS-C by (2).

Case 2: $q < q^*$. $P$-authority is then strictly optimal if
\[ q\hat{u}_1(\alpha, 1) + (1 - q)[q\hat{u}_1(1, \alpha) + (1 - q)\hat{u}_1(1, 0)] > q\hat{u}_1(1, \alpha) + (1 - q)q\hat{u}_1(\alpha, 1) \]
and $A$-authority is strictly optimal if the converse inequality holds. By simplifying (B.4) and by using then that (2) holds in Example PS-C, this becomes
\[
(1 - q)^2 \hat{u}_1(1, 0) > q^2(\hat{u}_1(1, \alpha) - \hat{u}_1(\alpha, 1))
\]
\[
\Leftrightarrow (1 - q)^2 \frac{1}{8} > q^2\left(\frac{1}{8} + \frac{1}{4}\alpha\right) - \left(\frac{1}{8}\alpha^2 + \frac{1}{4}\alpha\right)
\]
\[
\Leftrightarrow (1 - q)^2 > q^2(1 - \alpha^2)
\]

By solving this quadratic inequality, we obtain that $P$-authority is strictly optimal when
\[
q < q_* \equiv \left(\frac{1}{\alpha}\right)^2 - \frac{1}{\alpha} \sqrt{\left(\frac{1}{\alpha}\right)^2 - 1}
\]
and that $A$-authority is strictly optimal when $q > q_*$. q.e.d.
Proof of Proposition 3

Suppose that cheap talk happens according to some protocol prior to player 1’s project choice. It will not be important for the argument here how exactly the cheap talk protocol looks like. First, notice that since effort decisions in the continuation game that starts after a project is selected depend by Property 1 (a) only on payoff-relevant information, they are never affected by cheap talk. Cheap talk may only affect the belief player 1 holds about player 2’s signal at the project selection stage and thus the project selection behavior. By a reasoning like in the first paragraph of Subsection 3.3, player 1 will for any such belief choose \( x = 1 \) if \( \theta_1 = 0 \). Hence, cheap talk may only affect player 1’s project selection behavior conditional on that \( \theta_1 = \alpha \).

When player 1 selects in this case project \( x = 1 \), player 2’s payoff is \( \hat{u}_2(1, \theta_2) \) whereas player 2’s payoff is \( \hat{u}_2(\alpha, 1) \) when player 1 selects project \( x = 2 \). By Properties 2, 4 and 6, player 2 does for any realization of \( \theta_2 \) strictly prefer the selection of project \( x = 2 \) over project \( x = 1 \). This renders cheap talk that affects player 1’s project selection behavior impossible as player 1 would always send the message that leads to the highest probability with that project \( x = 2 \) will be selected. Hence, cheap talk has also no effect on player 1’s project selection behavior. \( \square \).

Proof of Lemma 3

This result follows directly from the discussion in the text. \( \square \).

Proof of Lemma 4

Part 1: Project selection behavior. Suppose that \( \theta_1 = 0 \). Player 1’s payoff from project \( x = 2 \) is then zero whereas his payoff from project \( x = 1 \), (7), is by Property 4’ strictly positive. Hence, player 1 has a strict incentive to choose project \( x = 1 \).

Suppose next that \( \theta_1 = \alpha \). Choosing project \( x = 2 \) is in this case by (7) and (8) strictly optimal if

\[
\hat{u}_1(\alpha, 1) > \hat{u}_1(q, \alpha)
\]

(B.5)

and choosing project \( x = 1 \) is strictly optimal if the converse inequality holds. By Property 4’ and the second part of Property 2, the right-hand side of (B.5) is strictly increasing in \( q \). Hence, there exists for any \( \alpha \) a threshold \( q^{**} \in [0, 1] \) such that \( x = 2 \) is strictly optimal if \( q < q^{**} \) and \( x = 1 \) is strictly optimal if \( q > q^{**} \).

Part 2: \( q^{**} \geq \alpha \). Since (B.5) holds for and \( q < \alpha \) by the first part of Property 2, \( q^{**} \geq \alpha \) must be true.

Part 3: \( q^{**} \geq q^* \). Consider again the public information case with \( q_1 = q_2 = q \) and suppose that effort by both players is essential for success. By the reasoning in the first paragraph of Subsection 3.3, choosing project \( x = 2 \) is then strictly optimal if

\[
\hat{u}_1(\alpha, 1) > q\hat{u}_1(1, \alpha).
\]

(B.6)

Sufficient for \( q^{**} \geq q^* \) is thus that for any \( q \) the right-hand side of (B.5) is smaller than the right-hand side of (B.6):

\[
\hat{u}_1(q, \alpha) \leq q\hat{u}_1(1, \alpha).
\]
I obtain this from the following sequence of transformations:

\[ \hat{u}_1(q, \alpha) = u_1(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha); q) \]
\[ = q s(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha)) - \frac{1}{2} \hat{e}_1(q, \alpha)^2 \]
\[ \leq q \left( 1 \cdot s(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha)) - \frac{1}{2} \hat{e}_1(q, \alpha)^2 \right) \]
\[ \leq q \left( 1 \cdot s(\hat{e}_1(q, \alpha), \hat{e}_2(1, \alpha)) - \frac{1}{2} \hat{e}_1(q, \alpha)^2 \right) \]
\[ \leq q \left( 1 \cdot s(\hat{e}_1(1, \alpha), \hat{e}_2(1, \alpha)) - \frac{1}{2} \hat{e}_1(1, \alpha)^2 \right) \]
\[ = q \hat{u}_1(1, \alpha). \]

The transformations arise as follows: The first equality follows from the definition of reduced-form payoffs, (3). The second equality follows from the definition of payoffs, (1). The first inequality follows from adding \((1 - q)^2 \hat{e}_1(q, \alpha)^2\). The second inequality follows since \(\hat{e}_2(q, \alpha) \leq \hat{e}_2(1, \alpha)\) by Property 1 (b) and since the success function is weakly increasing in effort decisions. The expression in brackets describes the payoff of a player with value 1 who does not respond optimally to the effort decision \(\hat{e}_2(1, \alpha)\) of player 2. The third inequality follows since the expression in brackets corresponds now to the payoff from the optimal response. The last equation follows from the definition of payoffs, (1), and of reduced-form payoffs, (3). q.e.d.

**Proof of Proposition 4**

We denote the expected payoff of player \(i\) in the public information case with \(q_1 = q_2 = q\) by \(V_i(q)\). To prove the result, we need to show that \(V_2(q) > V_1(q)\) if \(q < q^{**}\) and that \(V_1(q) > V_2(q)\) if \(q > q^{**}\).

Case 1: \(q > q^{**}\). In this case player 1 never compromises on the project choice. By (7), his expected payoff is

\[ V_1(q) = \hat{u}_1(q, \alpha). \]

Player 2’s expected payoff is

\[ V_2(q) = q \cdot [\alpha s(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha)) - \frac{1}{2} \hat{e}_2(q, \alpha)^2] + (1 - q) \cdot [0] = q \hat{u}_2(q, \alpha). \]

We obtain \(V_1(q) > V_2(q)\) from the following transformations:

\[ V_2(q) = q \hat{u}_1(q, \alpha) \leq q \hat{u}_1(q, \alpha) < V_1(q). \]

The transformations arise as follows: The second equality follows from using that essentiality of effort by both players implies \(\hat{u}_2(q, 0) = 0\) (Property 5) and permutation symmetry of reduced-form payoffs (Property 3). Since \(q > \alpha\) by our supposition and the last statement in Lemma 4, the first inequality follows from Property 6. The second inequality follows from \(q \in (0, 1)\).

Case 2: \(q < q^{**}\). In this case player 1 compromises on the project choice whenever he likes project \(x = 2\). By (7) and (8), his expected payoff is

\[ V_1(q) = q \hat{u}_1(\alpha, 1) + (1 - q) \hat{u}_1(q, \alpha) \]
\[ = q^2 \hat{u}_1(\alpha, 1) + (1 - q) q [\hat{u}_1(\alpha, 1) + \frac{1}{q} \hat{u}_1(q, \alpha)]. \]
When we use the definition of reduced-form payoffs, (3), and of payoffs, (1), this becomes

\[ V_2(q) = q\hat{u}_2(\alpha, 1) + (1 - q)[q\hat{u}_2(q, \alpha) + (1 - q)0] \]
\[ = q\hat{u}_1(1, \alpha) + (1 - q)q\hat{u}_1(\alpha, q) \]
\[ = q^2\hat{u}_1(1, \alpha) + (1 - q)q[\hat{u}_1(\alpha, q) + \hat{u}_1(1, \alpha)] \]

The second equality follows from using permutation symmetry of reduced-form payoffs (Property 3). The third equality follows from writing the expected payoff in a more complicated way.

Since \( \hat{u}_1(1, \alpha) > \hat{u}_1(\alpha, 1) \) by Properties 4 and 6, sufficient for \( V_2(q) > V_1(q) \) is

\[ \hat{u}_1(\alpha, q) + \hat{u}_1(1, \alpha) \geq \hat{u}_1(\alpha, 1) + \frac{1}{q}\hat{u}_1(q, \alpha). \]

When we use the definition of reduced-form payoffs, (3), and of payoffs, (1), this becomes

\[ \left[ \alpha s(\hat{e}_1(\alpha, q), \hat{e}_2(\alpha, q)) - \frac{1}{2}\hat{e}_1(\alpha, q)^2 \right] + \left[ 1s(\hat{e}_1(1, \alpha), \hat{e}_2(1, \alpha)) - \frac{1}{2}\hat{e}_1(1, \alpha)^2 \right] \]
\[ \geq \left[ \alpha s(\hat{e}_1(\alpha, 1), \hat{e}_2(\alpha, 1)) - \frac{1}{2}\hat{e}_1(\alpha, 1)^2 \right] + \frac{1}{q}\left[ qs(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha)) - \frac{1}{2}\hat{e}_1(q, \alpha)^2 \right]. \]

By rearranging and by using symmetry of the success function and permutation symmetry of the effort decisions (Property 1 (c)), we can rewrite this as

\[ (1 - \alpha)\left[ s(\hat{e}_1(1, \alpha), \hat{e}_2(1, \alpha)) - s(\hat{e}_1(q, \alpha), \hat{e}_2(q, \alpha)) \right] \]
\[ \geq \frac{1}{2}\left[ \hat{e}_1(1, \alpha)^2 - \hat{e}_1(\alpha, 1)^2 + \hat{e}_1(\alpha, q)^2 - \frac{1}{q}\hat{e}_1(q, \alpha)^2 \right]. \]

To conclude the proof, it suffices to show that this inequality holds for the examples PC-C, CD-C and PC-B.

Case 2.1: PC-C. Since \( q^{**} = \alpha \) holds in this case, it follows that \( q < \alpha \) under the supposition of Case 2. By using that \( \hat{e}_i(v_1, v_2) = \min\{v_1, v_2\} \) and that \( s(e_1, e_2) = \min\{e_1, e_2\} \), we obtain

\[ (1 - \alpha)(\alpha - q) \geq \frac{1}{2}q(1 - q). \]

Case 2.2: CD-C. By using that \( \hat{e}_i(v_1, v_2) = \frac{1}{2}v^{3/4}_i v^{1/4}_{-i} \) and that \( s(e_1, e_2) = \sqrt{e_1 e_2} \), we obtain

\[ \frac{1}{2}(1 - \alpha)\alpha^{1/2}(1 - q^{1/2}) \geq \frac{1}{8}(1 - \alpha)\alpha^{1/2}(1 - q^{1/2}). \]

Case 2.3: PC-B'. By using that \( v_i, v_{-i} \geq c \) implies \( \hat{e}_i(v_1, v_2) = \sqrt{2c} \) and \( s(e_1, e_2) = 1 \), we obtain

\[ 0 \geq c\left( 1 - \frac{1}{q} \right). \]

As the inequality does obviously hold in each of the three subcases, we obtain the result. q.e.d.
Proof of Proposition 5
Suppose that \( q \in (q^*, q^{**}) \). By (B.7), the principal’s expected payoff is
\[
V_2(q) = q \hat{u}_1(1, \alpha) + (1 - q)q \hat{u}_1(\alpha, q)
\]
in the public information case. By (7), his expected payoff is
\[
U_1(q, q) = q \hat{u}_1(1, \alpha) + (1 - q)\hat{u}_1(1, 0)
\]
in the private information case. Since \( \hat{u}_1(\alpha, q) > 0 \) by Property 4' and since essentiality of effort by both players for success implies \( \hat{u}_1(1, 0) = 0 \) by the second part of Property 5, we obtain that \( V_2(q) > U_1(q, q) \). q.e.d.

Proof of Lemma A1
(a) According to Topkis (1998), the effort provision game is a supermodular game if (i) \( E \) is a sublattice of \( \mathbb{R}^m \), (ii) \( u_i(e_1, e_2; v_i) \) has increasing differences in \( e_i \) and \( e_{-i} \), and (iii) \( u_i(e_1, e_2; v_i) \) is supermodular in \( e_i \). Properties (ii) and (iii) follow from \( E \) being a compact subset of the one-dimensional space \( \mathbb{R}_+ \). To prove Property (ii), we have to show that \( e_i' \geq e_i \) and \( e_i'' \geq e_i' \) imply that \( u_i(e_i, e_i'; v_i) + u_i(e_i', e_i''; v_i) \geq u_i(e_i', e_i''; v_i) + u_i(e_i, e_i''; v_i) \). By using the definition of \( u_i(e_1, e_2; v_i) \) and by simplifying, we obtain that the inequality can be rewritten as \( v_i \cdot s(e_i', e_i'') \geq v_i \cdot (s(e_i', e_i') + s(e_i'', e_i')) \). Increasing differences of the payoff function in \( e_i \) and \( e_{-i} \) follows then from increasing differences of the project success function in \( e_i \) and \( e_{-i} \). Since upper semi-continuity of \( s(e_1, e_2) \) in \( e_i \) implies that also \( u_i(e_1, e_2; v_i) \) is upper semi-continuous in \( e_i \) and since \( E \) is compact, Theorem 4.2.1 in Topkis (1998) applies. That is, the set of pure strategy Nash equilibria is non-empty and contains a largest equilibrium. We denote this equilibrium by \( (\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2)) \). It remains thus only to argue that this equilibrium is for any fixed parameters \( v_1 \) and \( v_2 \) for each player at least weakly better than any other equilibrium. This is a direct consequence of each player’s payoff function being non-decreasing in the other player’s effort decision and a revealed preferences argument. See Theorem 7 in Milgrom and Roberts (1990) for a similar argument.

(b) We need to show that \( u_i(e_1, e_2; v_i) \) has increasing differences in \( (e_i, v) \) with \( v = (v_1, v_2) \). The result follows then from Theorem 4.2.2 in Topkis (1998). Consider without loss of generality player 1 and suppose that \( e_i' \geq e_i, e_i'' \geq e_i' \) and \( v_2'' \geq v_2' \). We have to show that \( u_i(e_i', e_2; v_1') + u_i(e_i', e_2; v_2') \geq u_i(e_i', e_2; v_1') + u_i(e_i', e_2; v_2') \). By using the definition of \( u_i(e_1, e_2; v_i) \) and by simplifying, the inequality can be rewritten as \( (v_i'' - v_i')(s(e_i'', e_2) - s(e_i', e_1)) \geq 0 \). Since \( s(e_1, e_2) = 0 \) is non-decreasing in \( e_1 \), our supposition implies that this inequality holds.

(c) This follows directly from the symmetry of the game and the existence of a unique largest equilibrium. q.e.d.

Proof of Lemma A2
(a) Suppose that \( v_i'' \geq v_i' \) and that \( v_2'' \geq v_2' \). If values increase weakly from \( (v_i', v_2') \) to \( (v_i'', v_2'') \), then player \(-i\) will weakly increase his effort by Lemma A1 (b); i.e., \( \hat{e}_{-i}(v_i'', v_2'') \geq \hat{e}_{-i}(v_i', v_2') \). If player \( i \) would stick to the effort \( \hat{e}_i(v_i', v_2') \), he would be at least weakly better off by the increase in the other player’s effort decision and by the increase in his own value. Moreover, if his payoff is strictly positive for \( (v_i', v_2') \), he would be strictly better off by the increase in his own value. By a revealed preferences argument, he must be even better off if he decides to provide effort \( \hat{e}_i(v_i'', v_2'') \) instead of \( \hat{e}_i(v_i', v_2') \). Hence, it follows that
\[
u_i(\hat{e}_1(v_i'', v_2''), \hat{e}_2(v_i'', v_2''); v_i'') \geq \nu_i(\hat{e}_1(v_i', v_2''), \hat{e}_2(v_i', v_2''); v_i')
\]
and, if \( v''_i > v'_i \) and \( u_i(\hat{e}_1(v'_1, v'_2), \hat{e}_2(v'_1, v'_2); v'_i) > 0 \), that
\[
u_i(\hat{e}_1(v''_1, v''_2), \hat{e}_2(v''_1, v''_2); v''_i) > u_i(\hat{e}_1(v'_1, v'_2), \hat{e}_2(v'_1, v'_2); v'_i).
\]

(b) This follows from the following transformation:
\[
u_1(\hat{e}_1(v_1, v_2), \hat{e}_2(v_1, v_2); v_1) = u_1(\hat{e}_2(v_2, v_1), \hat{e}_1(v_2, v_1); v_1)
\]
\[= v_1s(\hat{e}_2(v_2, v_1), \hat{e}_1(v_2, v_1)) - \frac{1}{2}v_1^2
\]
\[= v_1s(\hat{e}_1(v_2, v_1), \hat{e}_2(v_2, v_1)) - \frac{1}{2}v_1^2
\]
\[= u_2(\hat{e}_1(v_2, v_1), \hat{e}_2(v_2, v_1); v_1).
\]
The transformations arise as follows: The first equality follows from Lemma A1 (c). The second equality follows from the definition of the payoff function. The third equality follows from symmetry of the project success function. The fourth equality follows again from the definition of the payoff function. q.e.d.

References


