COMMON AGENCY WITH INFORMED PRINCIPALS: REVELATION PRINCIPLE
EXTENDED ABSTRACT
(THE IN A PRELIMINARY VERSION OF AN ONGOING WORK)

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ABSTRACT. This paper studies games where a group of privately informed principals design mechanisms to make a common agent to choose among allocations with each principal. The agent at the moment of taking a decision has observed his private information and may have information (endogenous) about all principals feasible allocations and types. Thus, principals may be interested in screening all this information. In this paper, we provide sufficient conditions on the agent’s payoff such that any equilibrium in this general setup will have an output equivalent equilibrium only using direct mechanisms. Depending on the conditions, we propose two different notions of a direct mechanism and discuss its applicability with some examples.

1. INTRODUCTION

Common agency problems suffer the lack of a revelation principle type of result that allows simplifying their analysis. The problem is originated in the information the agent has at the moment to communicate with each principal: Besides agent’s exogenous information (type), the agent observes each principal’s offer, which may give him valuable information that other’s principal might try to screen. This known in the literature as endogenous information, or market information. In the context of privately informed principals, this last kind of information has two components: First, the agent observes which allocations each principal offers to him. Second, the way each principal provides a set of allocation may signal his type. Thus, from one principal’s perspective, a way to screen this information is not an easy task. In this paper, we propose sufficient conditions on the agent’s payoffs such that we can restrict without loss of generality to simple mechanisms. We propose two different
notions of simple mechanisms. First one, principals just need the agent’s type space as a message space. These mechanisms are called direct mechanism in the mechanism design literature and are the usual restriction when the agent has no endogenous information. The sufficient conditions that allow us to restrict to direct mechanisms in this context prevent that the agent’s preference with one principal’s allocation to change with other’s principal’s allocations and types. If we relax the conditions related to other’s principal’s type, we have our second notion of revelation principle: Principals can use as a message space the product between agent’s type space and other’s principal’s type space. The last mechanisms are the natural generalization of direct mechanisms to the present context in the sense that the message space encapsulate all the exogenous information of the model from the perspective of one principal.

The lack of a revelation principle in common agency problems has been addressed in two different ways. The first one, initiated by Peters (2003, 2007), gives sufficient conditions on the agent and principal’s payoffs such that in a common agency model without private information the revelation principle applies. In that context, a direct mechanism corresponds to a take it or leave it offer from each principal to the agent. Later, Attar et al. (2008) obtain sufficient conditions for a setting where the agent has private information. A direct mechanism there corresponds to a map from the agent’s type space to the allocations. This project contribution is to give sufficient conditions for the most general case when also each principal has private information. The second way to address this problem is to focus on what the literature knows as menus. The idea here is instead of an emphasis on the message space of the mechanism, we can equivalently look the range of possible outcomes that a mechanism allow the agent to pick. This idea was proposed by Peters (2001) and Martimort and Stole (2002), and it is well known as a menu theorem. In this context, principals offer menus (subsets of the allocation space) from which the agent pick his most preferred allocation. Since principals strategies are now subsets, this approach is not straightforward to apply in real problems. If we assume principals have private information, Galperti (2015) shows

\[^{1}\text{Attar et al. (2011) is an excellent example of how to use this method to analyze multi-principal competition.}\]
that the same approach is useful to characterize the equilibrium in the mechanism design game, if besides the menus, the principals are allowed to send cheap-talk signals.

2. Model and preliminary results

Consider $M$ principals. Each principal privately observes $t_i \in T_i$ and decides the allocation $y_i \in Y_i$. We allow the agent to communicate with each principal. For that purpose, we assume there is a message space $M_i$ for each principal. Each principal offers to the agent a mechanism $\pi_i : M_i \rightarrow Y_i$, $\pi_i \in \Pi_i$.

The agent privately observes $\theta \in \Theta$ and send a message to each principal $m \in M = \times M_i$. Those messages will induce an allocation $\pi(m) \in Y = \times Y_i$. Payoffs for the principals and the agent are $V_i(\theta, t, y)$ and $U(\theta, t, y)$ respectively. We consider pure strategies: For the principals $\sigma_i : T_i \rightarrow \Pi_i$, $\sigma_i \in \Sigma_i$, and for the agent $\sigma_0 : \Theta \times \Pi \rightarrow M$. All these together defines the game $\Gamma_M = [(T_i)_{i \in N}, \Theta, \Pi, M, U(\ldots), (V_i(\ldots))_{i \in N}, \mu_0]$.

Consider the following reduced game. Instead of the message space $M_i$, each principal uses the space $\Theta$. Thus, each principal offers to the agent a mechanism $\tilde{\pi}_i : \Theta \rightarrow Y_i$, $\tilde{\pi}_i \in \tilde{\Pi}_i$. Pure strategies will be $\tilde{\sigma}_i : T_i \rightarrow \tilde{\Pi}_i$, $\tilde{\sigma}_i \in \tilde{\Sigma}_i$ for the principals, and $\tilde{\sigma}_0 : \Theta \times \tilde{\Pi} \rightarrow \Theta^n$ for the agent. Analogously, these defines a new game $\Gamma_\Theta = [(T_i)_{i \in N}, \Theta, \tilde{\Pi}, \Theta^n, U(\ldots), (V_i(\ldots))_{i \in N}, \mu_0]$.

We also consider the case when principals message space is $\Theta \times T_{-i}$. In this case we denote the game $\Gamma_\Theta = [(T_i)_{i \in N}, \Theta, \tilde{\Pi}, \Theta^n, U(\ldots), (V_i(\ldots))_{i \in N}, \mu_0]$.

The concept we use is Perfect Bayesian Equilibrium. $(\sigma, \mu)$ is a Perfect Bayesian Equilibrium (PBE) if $\sigma$ is sequentially rational given $\mu$ and $\mu$ is consistent with $\sigma$.

We impose the following assumptions:

**Assumption 2.1.** Weak outcome Separability. The agent’s utility is weakly outcome separable if for all $y_i, y'_i \in Y_i$, for all $y_{-i}, y'_{-i} \in Y_{-i}$, for all $\theta \in \Theta$ and for all $t \in T$

$$U(y_i, y_{-i}, \theta, t) > U(y'_i, y_{-i}, \theta, t) \Rightarrow U(y_i, y'_{-i}, \theta, t) > U(y'_i, y'_{-i}, \theta, t)$$

Agent’s preference with a particular principal does not depend on the other principals’ allocations.

**Assumption 2.2.** Weak type Separability. The agent’s utility is weakly type separable if for all $y_i, y'_i \in Y_i$, for all $y_{-i} \in Y_{-i}$, for all $\theta \in \Theta$, for all $t_i \in T_i$ and for all $t_{-i}, t'_{-i} \in T_{-i}$

$$U(y_i, y_{-i}, \theta, t_i, t_{-i}) > U(y'_i, y_{-i}, \theta, t_i, t_{-i})$$
Agent’s preference with a particular principal does not depend on the other principals’ types.

**Assumption 2.3.** Ratio Condition. The agent’s utility satisfies ratio condition for the pair \( t, t' \in T \) if for all \( y_i, y'_i \in Y_i \), for all \( y_{-i}, y'_{-i} \in Y_{-i} \):

\[
U(y_i, y_{-i}, \theta, t) > U(y'_i, y_{-i}, \theta, t) \quad \text{and} \quad U(y_i, y_{-i}, \theta, t') < U(y'_i, y_{-i}, \theta, t')
\]

\[
\Rightarrow
\]

\[
\frac{U(y_i, y_{-i}, \theta, t') - U(y'_i, y_{-i}, \theta, t')}{U(y_i, y_{-i}, \theta, t) - U(y'_i, y_{-i}, \theta, t)} = \frac{U(y_i, y'_{-i}, \theta, t') - U(y'_i, y'_{-i}, \theta, t')}{U(y_i, y'_{-i}, \theta, t) - U(y'_i, y'_{-i}, \theta, t)}
\]

**Assumption 2.4.** No Indifference. The agent’s utility satisfies no indifference if for all \( y_i \in Y_i \), for all \( \theta \in \Theta \), for all \( t \in T \) and for all \( y_{-i} \neq y'_{-i} \in Y_{-i} \):

\[
U(y_i, y_{-i}, \theta, t) \neq U(y'_i, y_{-i}, \theta, t)
\]

The agent is never indifferent between two outcomes

**Assumption 2.5.** Richness. For all \( i \in N, |M_i| \geq |\Theta \times T_{-i}| \)

Message space is rich enough compared to agent’s type space

The main two theorems of the paper are the following:

**Theorem 2.6.** If \( U \) is weakly outcome separable, satisfies ratio condition, no indifference, and richness, then for every outcome supported as a PBE of \( \Gamma_M \), there is a PBE of \( \Gamma_\Theta \) that implements the same outcome.

**Theorem 2.7.** If \( U \) is weakly outcome separable, weakly type separable, satisfies no indifference and richness, then for every outcome supported as a PBE of \( \Gamma_M \), there is a PBE of \( \Gamma'_\Theta \) that implements the same outcome.
Consider the problem of two manufacturers that compete to sell their products through the same retailer\textsuperscript{2}. Manufacturer $i$ wants to set a price $p_i$ for a quantity $x_i$, and has private information $q_i$ related with the quality of his product. Thus, manufacturer payoff is $p_i - q_i x_i$. In the other side, the retailer sells these products to consumers that value the quality $v(q_i)$. Thus, retailer’s payoff is $v(q_1)x_1 + v(q_2)x_2 - p_1 - p_2$. For the sake of simplicity, we assume the retailer does not have private information. Suppose there is a message space $M_i$ that the retailer can use to communicate with each manufacturer. Thus, each manufacturer could possibly set mechanism to try to screen retailer’s information $\pi_i : M_i \rightarrow X \times \mathbb{R}_+$. It is not difficult to see that retailer’s payoff satisfies weak outcome and type separability. Then, our result says that we can restrict without loss of generality to direct mechanism which in this context will correspond to a take it or leave it offer $(x_i, P_i) \in X \times \mathbb{R}_+$ for each type of the manufacturer. Suppose instead that the retailer’s payoff depends on the average quality he is offering to the customers. Thus, his payoff would be $v(\frac{q_1+q_2}{2})(x_1 + x_2) - p_1 - p_2$. In this case, this payoff satisfies weak outcome separability but no weak type separability. Since this function is separable in quantity and price, it satisfies the ratio condition. Thus, our result says that we can restrict to simpler mechanisms $\pi_i : Q_i \rightarrow X \times \mathbb{R}_+$.

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\textsuperscript{2}This is a modified version of the exclusive dealing model of Bernheim and Whinston (1998)

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