Supersizing: The Illusion of a Bargain and the Right-to-Split*

Ram Orzach, Miron Stano †

March 2017

Abstract

The supersizing phenomenon where menu prices for large fast food portions appear to be well below their marginal production costs is of considerable scholarly and policy interest. This article develops sufficient conditions, for a subset of cases where the single-crossing condition is violated, under which a firm can separate two different rational consumer types while maximizing and capturing the total surplus associated with marginal-cost pricing. Menu prices can be very easily determined for these cases unlike the complex solutions found under more general conditions. For our subset, the separating equilibrium creates an apparent supersizing discount even though the firm does not actually sell the additional quantity below marginal cost. With public health interest in reducing portion sizes, we introduce the right-to-split as a policy alternative that breaks the separating equilibrium and leads to smaller quantities.

**Keywords:** Optimal Pricing, 2nd Degree Price Discrimination, Fast Food, Menu Pricing, Supersizing, Revelation Principle, Spence-Mirrlees Condition

**JEL Classification:** D42, D82, I18, L12, L66

---

*We are grateful for the helpful suggestions and encouragement provided by Paul Dobson, Eitan Gerstner, David Malueg.

†Department of Economics, Oakland University, Rochester, MI 48309, USA; e-mail addresses: orzach@oakland.edu (corresponding author), stano@oakland.edu
1 Introduction

Scholars across a range of disciplines have focused on research and policy issues relating to junk food. Junk food is associated with calorie-dense food that has high levels of sugar, glycemic starch, and saturated fat. Much of the research deals with the relationship between junk food and obesity. Although a causal relationship is difficult to establish (Collins and Baker, 2009), junk food is commonly associated with obesity, especially childhood obesity (e.g., Harris et al., 2009).

The perception that consumers are getting great value from the significant quantity discounts associated with supersized portions is also of substantial scholarly interest. Although McDonald’s phased out its supersizing slogan in 2004, ever-larger portion sizes remain a staple of the fast food industry. By building on theories of price discrimination and temptation, Dobson and Gerstner’s (2010) seminal contribution (hereafter DG) develops a profit-maximizing supersizing strategy with two consumer types, ‘disciplined’ and ‘tempted,’ who face regular or supersized portions. DG show that the supersized portions, under extreme conditions, may even be offered free.

Our approach extends the framework introduced by DG to explain the apparent paradox of deep discounts associated with value size pricing where menu prices for the supersized addition appear to be well below marginal cost. In particular, we determine menu portions whereas DG take them as predetermined. By introducing low-demand and high-demand consumers whose preferences violate the single-crossing condition, and deriving menu prices for the respective profit-maximizing quantities, we show the conditions for: i. separating the two types, ii. maximizing the total surplus, iii. capturing the entire consumers surplus, and iv. creating the perception that supersizing quantities are sold below marginal cost. The firm does not sell below marginal cost but will actually charge the marginal cost plus the consumer surplus.

Section 2 introduces a model of menu pricing for a case that violates the single-crossing condition. The surprisingly simple solution provides insight into the illusion of the supersizing phenomenon. Section 3 develops implications for alternative ‘downsizing’ interventions including the right-to-split. Downsizing has considerable public health support and lends itself to creative policy strategies. Section 4 concludes the article.
2 The Model

Before developing a formal model, we start with a numerical example of a firm with residual monopoly power\(^1\) facing two types of consumers: low demand (L) and high demand (H). The firm cannot identify consumer types but knows only their proportions. Assume for simplicity that there is one L-consumer and one H-consumer with inverse demand functions:

\[ P_L = 5 - q_L \quad \text{and} \quad P_H = 2 - 0.1q_H \]

Also, for simplicity, assume a constant marginal cost of $1 per unit.

To find the menu prices that maximize profits, this firm can act as a perfect price discriminator or equivalently establish menu prices by first finding quantities where \( P = MC \) for each type. It is easily shown that when \( P = $1 \) then \( q_L = 4 \) and \( q_H = 10 \) with respective consumer surpluses of $8 and $5. By appropriating these surpluses, the menu prices \((TP_i, i \in \{L, H\})\) for the 4 and 10 unit menu quantities\(^2\) are \( TP_L = $12 \) and \( TP_H = $15 \). The respective profit margins, $8 and $5, correspond to the consumer surpluses.

Critically, in this case, the H-type will not pretend to be a L-type because he is not willing to pay $12 for 4 units. The maximum he is willing to pay, the area below his demand curve, is $7.2. Similarly, the L-type will not pretend to be an H-type as her benefit from the H-type quantity is $12.5 which falls short of its $15 price. The firm has successfully separated the two types and also maximized the total surplus.

Our primary interest, however, lies with the quantity discount. For an additional $3, H-types get 6 more units, paying $0.5 for each additional unit relative to the L-types. This is well below the $1 production cost.\(^3\) Through the following general linear formulation (nonlinear extensions do not add insight), we examine the sufficient conditions for such results which create the illusion that producers are selling below marginal cost.

\(^1\)The substantial markups for McDonalds and Burger King described by Thomadsen (2007, p.5) suggest considerable monopoly power for the major fast food brands. Armstrong and Vickers (2010) develop a theoretical duopoly model in which the two firms adopt non-linear pricing and bundling, and DG’s appendix shows that their results hold in a Hotelling-type model.

\(^2\)It is these quantities that determine the H and L labels for this example in which the SCC is violated.

\(^3\)Elsewhere, the rational addiction approach to pricing developed by Richards, Patterson, and Hamilton (2007) is predicated on pricing power for the fast food industry. They show that addictive foods, sugar-filled and high in fat, will be priced below marginal cost. More general work by Hitt and Chen (2005) also develops situations in which price can be less than marginal cost.
For our formal analysis, we continue with two types of consumers $L$ and $H$, where $\mu$ is the proportion of the $L$-type and $(1 - \mu)$ of the $H$-type. Their inverse demand functions are:

$$P_L = a_L - b_L q_L \quad \text{and} \quad P_H = a_H - b_H q_H$$

where $a_L > a_H > 1$ and $b_H < b_L$. This is a critical and nonstandard assumption\(^4\) that will lead to a violation of the single-crossing condition (SCC), i.e., the Spence-Mirrlees condition.

The extensive literature on menu pricing builds on Maskin and Riley’s (1984) characterization of nonlinear monopoly pricing that satisfies the SCC. However, we found only limited analyses of menu pricing when the SCC is violated. Andersson (2005) develops conditions under which relaxation of the SCC will still produce the Makin-Riley results, but he also (Andersson, 2008) provides a counterexample to the main Maskin-Riley result, namely that the quantities for both types can correspond to those under marginal-cost pricing with gains for both consumer-types that are zero. Chao and Nahata (2015) define the region in which the quantities for the two types, under various possibilities that violate the SCC, correspond to the efficient quantities (marginal-cost pricing) as well as where the quantity for one can exceed or fall short of the efficient quantity. They also provide more specific results for two crossing linear demands that build on Nahata Kokovin and Zhelobodko (2002). In a recent survey on general nonlinear pricing, Armstrong (2015, p. 15) states: "Demand curves might be linear, say, but consumers differ both in the slope and the intercept of demand. Such problems can often be easily solved using the demand profile approach, but are very difficult to solve using a "mechanism design" approach which focuses on preference parameters."\(^5\)

We respond to this challenge by circumventing the need to maximize a profit function with nonlinear and nonmonotonic constraints. A much simpler and straightforward approach for a subset of cases that violate the SCC determines the parameters that produce our desired solution where the firm’s quantities correspond to marginal-cost pricing while maximizing the economic surplus. If the firm succeeds in capturing the entire

\(^4\)The assumption is in the spirit of the literature on self-control and temptation reviewed and extended by Dhar and Wertenbroch (2012). It is also consistent with the $H$-consumer described by Haws and Winter (2013) where the "presence of supersized pricing automatically activates the focus on getting a deal" (p. 50).

surplus, the solution must also be consistent with profit maximization.

Let the production cost be linear. Therefore, without loss of generality, assume \( MC(Q) = 1 \) (\( Q = \Sigma q_i \)). Under the Revelation Principle (Myerson, 1979), we can concentrate only on the case where the buyer pays menu price \( TP_t(q_t) \) for quantity \( q_t \). As we have only two types of consumers, we will consider only two menu prices: \( TP_L(q_L) \) and \( TP_H(q_H) \).

Let \( U_t(TP_j, q_j) \) be the utility of type-\( t \) buyers from consuming \( q_j \) and paying \( TP_j(q_j) \); \( t \in \{L, H\} \), \( j \in \{L, H\} \). The firm’s objective function is to maximize its expected profit per consumer subject to individual rationality constraints and the incentive compatible constraints.

\[
\max_{TP_t(q_t)} \pi = \mu \cdot TP_L(q_L) + (1 - \mu) \cdot TP_H(q_H) - 1 \cdot [\mu \cdot q_L + (1 - \mu) \cdot q_H]
\]

\[\text{s.t.:}\]

\[
\begin{align*}
U_L(TP_L, q_L) & \geq 0 \quad \text{IR}_L \\
U_H(TP_H, q_H) & \geq 0 \quad \text{IR}_H \\
U_L(TP_L, q_L) & \geq U_L(TP_H, q_H) \quad \text{ICC}_L \\
U_H(TP_H, q_H) & \geq U_H(TP_L, q_L) \quad \text{ICC}_H
\end{align*}
\]

where \( \pi \) represents the expected profit per consumer; \( IR_L \) and \( IR_H \) are the individual rationality constraints indicating that consumers will buy the product only if their utilities from the exchange are non-negative; \( ICC_L \) and \( ICC_H \) are the incentive compatibility constraints requiring that neither type will pretend to be the other type and buy the other’s quantity.

**Theorem 1** If \( \left( \frac{a_H - 1}{a_L - 1} \right)^2 < \frac{b_H}{b_L} < \min\{ \frac{a_H - 1}{a_L - 1}, \frac{(a_H)^2 - 1}{(a_L)^2} \} \) and \( 2a_H < a_L + 1 \), then\(^6\)

A) there exists i) a separating equilibrium, ii) where the firm sets menu quantities that maximize the total economic surplus, and iii) in which the firm captures the entire surplus.

B) the difference in price between the price that the \( H \)-type pays compared to what the \( L \)-type pays divided by the difference in quantity is always less than the marginal cost. Namely, the 'marginal price' is less than the marginal cost.

\(^6\)All results are independent of the units of measurement including money because the results are determined by ratios and the normalized marginal cost of 1.
As the proof is constructive, it gives insight about quantity and price and also the relationship between menu prices and two-part tariffs. Consider the following claim:

**Claim 1** if \( q_L = \frac{a_L-1}{b_L} \) and \( TP_L(q_L) = \frac{(a_L)^2-1}{2b_L} \), the quantity and menu price are equivalent to a two-part tariff in a hypothetical market with only the \( L \)-types. Therefore price equals marginal cost \((p = 1 = MC)\), and the transfer price equals the \( L \)-type consumer surplus \((T_L = CS_L(p = 1))\), i.e., \( TP_L(q_L) = 1 \cdot q_L(p = 1) + T_L \), and \( U_L(P_L, q_L) = 0 \).

**Claim 2** if \( q_H = \frac{a_H-1}{b_H} \) and \( TP_H(q_H) = \frac{(a_H)^2-1}{2b_H} \), the quantity and menu price are equivalent to a two-part tariff in a hypothetical market with only \( H \)-types. Therefore, price equals marginal cost \((p = 1 = MC)\), and the transfer price equals the \( H \)-type consumer surplus \((T_H = CS_H(p = 1))\), i.e., \( TP_H(q_H) = 1 \cdot q_H(p = 1) + T_H \), and \( U_H(P_H, q_H) = 0 \).

Claims 1 and 2 are apparent and intuitive. The nontrivial part is to develop the conditions for which none of the types will buy the menu of the others so that the ICC\( s \) are satisfied (see Claims 4 and 5).

**Claim 3** If \( \frac{b_H}{b_L} < \frac{a_H-1}{a_L} \) then \( q_H > q_L \). Moreover, \( q_H > q_0 \), where \( q_0 \) is the quantity that the \( L \)-type will buy if the price is zero.

Note that, if the price is zero, the \( L \)-type will buy \( q_0 = \frac{a_L}{b_L} \) and \( q_L = \frac{a_H-1}{b_H} \). Claim 3 is a technical one that prevents the need to divide the proof into two cases (the other in which \( q_H < q_0 \)). Claim 3 is more applicable to fast food where consumption is usually immediate but it may not hold for other goods\(^7\).

**Claim 4** If \( \frac{b_H}{b_L} < \frac{(a_H)^2-1}{(a_L)^2} \), then the ICC\( L \) is satisfied. Namely, \( U_L(TP_L, q_L) \geq U_L(TP_H, q_H) \).

**Claim 5** if \( 2a_H < a_L + 1 \), then the ICC\( H \) is satisfied. Namely, \( U_H(TP_H, q_H) \geq U_H(TP_L, q_L) \).

Claims 4 and 5 are easier to satisfy (see Appendix) under menu pricing compared to two-part tariffs. By preventing consumers from choosing quantities under the same transfer price, menu prices are more effective for the firm than two-part tariffs (Wolfstetter, 1999), and also very applicable to fast food.

Claim 6 establishes the condition in which part B of Theorem 1 is satisfied and creates the perception to consumers that 'marginal price' is less than the marginal cost.

\(^7\)For example, the U.S. clothing chain Jos A Bank is well-known for its quantity discounts such as "buy one suit at the regular price and get two free". \( L \)-types can enjoy the benefits of the other two suits at later dates (so that we do not observe a separate \( L \)-type menu). For fast food, with its immediate consumption property, the \( L \)-type is less likely to buy and save excess amounts even at a zero price.
Claim 6 if \( (\frac{a_H - 1}{a_L - 1})^2 < \frac{b_H}{b_L} \), then \( \frac{TP_H(q_H) - TP_L(q_L)}{q_H - q_L} < 1 \), namely, the difference in menu price divided by the difference in quantity is less than the marginal cost (proof in the Appendix).

Theorem 1 easily derives from a reorganization of the conditions for Claims 3-6.

To interpret the sufficient conditions for Theorem 1, consider the inequality \( (\frac{a_H - 1}{a_L - 1})^2 < \frac{b_H}{b_L} \). Here, the ratio of the slope of the \( H \) to \( L \) demands, must be greater than the square of the ratio of their maximum willingness-to-pay for the good, considering the normalized marginal cost. This will ensure that the \( L \)-menu price will be less than the \( H \)-menu price but that the difference is limited. The inequality \( \frac{b_H}{b_L} < \min\left\{ \frac{a_H - 1}{a_L - 1}, \frac{(a_H)^2 - 1}{(a_L)^2} \right\} \) ensures that the \( L \)-type will not pretend to be an \( H \)-type as the \( H \)-type’s demand slope leads to a large quantity that the \( L \)-type will not consume. The inequality, \( 2a_H < a_L + 1 \), sets the requirement for the maximum willingness-to-pay so that the \( H \)-type will not pretend to be an \( L \)-type for a small quantity.

These conditions are illustrated in Figure 1 using the parameters of our initial example. They can be seen as representing DG’s ‘disciplined’ and ‘tempted’ consumers. The \( L \)-type (disciplined) is willing to pay a relatively high unit price and the rate of increase in her consumer surplus from a higher quantity is initially greater than that of the \( H \)-type (tempted); and vice-versa at higher quantities.

Our example and Theorem 1 violate the Spence-Mirrlees condition. Increasing the quantity benefits the \( L \)-type more than \( H \)-type initially and vice versa at the higher quantity, leading to TPs ($15 and $12 in our example) that are relatively close, and where \( q_L \) is much smaller than \( q_H \). This creates the perception that marginal price is much less than marginal cost.
A model that violates the SCC is especially relevant to the fast food industry which has targeted lower-income, minority neighborhoods (Meltzer and Shuetz, 2012; Jou, 2017). In our example, the $L$-type is willing to pay a price of $12 for the smaller 4 unit quantity, behavior that may be more representative of higher income consumers. In contrast, the $H$-type are only willing to pay $7.5 for 4 units but up to $15 for 10 units. This behavior may be more commonly associated with lower-income consumers.

3 The Right-to-Split

Various market failures associated with junk food, especially with supersized portions, have been used to justify public health efforts to reduce food portion sizes in general and promote a variety of downsizing policy strategies. These include reducing food portion and packaging sizes; restricting advertising of highly sweetened products with low nutritional value to children; and restricting both the promotion of supersized portions and the price discounts associated with them (Marteau et al., 2015). Most of the regulatory and voluntary initiatives that have been adopted, however, relate to food labelling and the posting of caloric and nutritional information on products, menus, or menu boards. In the United States, many restaurants post such information even where it is not mandated.

The most visible and contentious regulatory measure, however, was launched in 2012 by former New York City mayor, Michael Bloomberg. The regulation would have barred certain restaurants and many other businesses from selling sugar-sweetened beverages (SSBs) in cups larger than 16 ounces. The New York Supreme Court subsequently ruled against the ban.

Economists have been especially interested in the effects of taxes especially those on (SSBs). Of numerous attempts across the United States to impose a soda tax, in March 2015, Berkeley became the first city to establish one at the rate of $0.01 per ounce of SSBs. After three months, Falbe et al. (2015) estimated that the tax was partially shifted to consumers, e.g., 69 percent for sugar-sweetened sodas, a necessary step for tax policy to be effective. Falbe et al. (2016) also compared consumption of SSBs in low-income neighborhoods with high minority populations in Berkeley with consumption in similar neighborhoods in San Francisco and Oakland, California. Households in these neighborhoods are likely to consume more SSBs and to have more obesity-related problems. Adjusted for various sociodemographic characteristics, consumption of regular
soda decreased 26% in Berkeley compared to a 10% increase in the comparison cities.

Mexico, in response to its obesity epidemic, introduced a one peso per liter SSB tax in January 2014. The tax is roughly 9 percent of average retail price. Grogger’s (2015) preliminary work shows that prices of regular sodas increased by 12-14 percent relative to other beverages right after the tax. With a unitary price elasticity for SSBs in Mexico (Colchero et al., 2015), consumption of SSBs can be significantly impacted through tax policy. Also on a national scale, the United Kingdom recently announced a tax on the production of sugar-containing soft drinks. Effective January 2018, the novel two-tiered tax increases with sugar content.

In our simple example, a unit tax up to $0.22 on the large portion will not disturb the separating equilibrium in which the firm captures the entire consumer surplus (minus the tax) but it will reduce the large portion to 7.8 units with $TP_H = $12.55 which is greater than the maximum of $12.5 that $L$ is willing to pay.

Eliminating “supersizing” by requiring the store to sell only one size will lead our firm to set a $TP = $12.5 so that the $L$-type does not drop out. To get $TP = $12.5 for the $H$-type requires a quantity of 7.75 units. The $L$-type disposes 2.75 units and total profit is $9.5. Interestingly, the firm could get around such a regulation and even do better by offering a quantity of 5 units with $TP = $12.5 but with free refills even in 5-unit portions. The $L$-type will not refill; the $H$-type refills once and total profit is $10. (The total profit = $12.3 with a refill size of 2.75).

These interventions are highly impractical, difficult to enforce, and would be regarded by many as coercive. The inherent arbitrariness of defining products and businesses subject to taxes or limits can also create legal barriers as with the New York City proposal (Min 2013). Instead, we focus on adding a new intervention that we define as the “right-to-split.” Consider two variants of this right:

1. A “strong right-to-split” where a customer would be able to buy half the large portion for half the price assuming that half portions or less are on the menu. This approach corresponds to what is called proportional or linear pricing in the public health literature.\(^8\)

2. A “weak right-to-split” where a customer would be able to buy the large quantity and have it divided

\(^8\)Vermeer and colleagues (2009, 2014) conducted experimental studies in the Netherlands at different settings such as a fast food restaurant and worksite cafeteria. The authors found that proportional pricing in a fast food setting did not have substantial effects but that overweight customers were more likely to choose smaller portions.
(and packaged) into two, i.e., a form of plate-sharing. For simplicity, we assume that the marginal cost of splitting the transaction is zero.

The Bloomberg soda regulation did not prohibit establishments from posting a price for a 32 ounce soda portion if it were packaged into two of the maximum allowable 16 ounce cups. This practice would be consistent with the weak right-to-split. The strong version would enable the customer to buy a 16 ounce drink at half the large serving price.

Consider first the strong version. This will break the separating equilibrium under which the firm can extract the entire consumer surplus from both consumers. The reasoning is well-developed in Alger’s (1999) extensive analyses of multiple and joint purchases. Assume a consumer is allowed to buy any proportion at a proportional price where price per unit in the menu price exceeds the consumer’s marginal benefit. Then the consumer will prefer to buy a smaller portion. This behavior will lead the firm to set the standard monopoly solution, a result that applies even to a single consumer type. In our more restrictive intervention, if the firm intends to serve the two consumer types, it will at a minimum leave some consumer surplus for at least one type and/or reduce the portion size for at least one type.

Now consider the weak version. To be relevant, there must be more than one consumer in the transaction. DG actually discuss the sharing of supersized meals among family and friends. If supersizing is banned, DG claim the firm may circumvent the ban by offering multiples at reduced prices. If the firm offers such deals, $L$-types can take advantage by sharing with family and friends.

Within the context of our model, suppose that the $L$ and $H$-types can cooperate. At a $15 total price, the $H$-type can split the 10 unit portion with the $L$-type so that both gain because the $L$-type is willing to pay up to $12.5 for 5 units and the $H$-type is willing to pay up to $8.75. Their respective consumer surpluses would be $5 and $1.25 if they split the bill at $7.5 each. Of course, where we have more $L$-types, they could cooperate by splitting the $H$-menu and even two $H$-types could benefit by splitting the $H$-menu. If cooperation were common among a firm’s customers, this again would break the separating equilibrium where the quantities correspond to $P = MC$ for each type and the firm captures the entire surplus.
4 CONCLUSION

We introduced a simple linear model for a firm with market power facing two types of consumers where consumer type is private information. Without dealing with the mathematical complexities associated with profit maximization under nonlinear and nonmonotonic constraints, we show the sufficient conditions for separating the two types and where the firm can capture the entire consumer surplus associated with marginal-cost pricing. We then show how this leads to value size pricing and the mythical discount associated with supersizing, even though the firm does not sell below marginal cost. There is no need for any rational addiction or other strategies to generate this bargain price. Our model is constructive and shows the derivation of quantities and the total menu prices.

We consider different interventions and introduce the right-to-split as an alternative. Consistent with policy goals, either the strong or the weak versions (or both) will likely reduce the supersized portions. Currently, in the United States, there are no clear practices on such splitting and it is possible that many consumers are too embarrassed to request split portions.

From a small convenience sample of consumers in Minnesota, O’Dougherty et al. (2006) found that 57 percent of survey participants somewhat or strongly opposed legislation that would force restaurants to offer better prices on smaller portions. Regulatory interventions, including the Bloomberg soda ban or taxation of soda, have not received broad public support (Min 2013). To counter the temptation created by value size pricing, public health education and other efforts to pressure restaurants to reduce portion sizes, or to make it more acceptable for consumers to shamelessly request split portions or plate-sharing, could encourage the availability of downsized options at downsized prices.
Appendix: Proofs of Claims 4-6

Proof of claim 4

As $TP_L(q_L) = 1 * q_L(p = 1) + CS_L(p = 1)$, then $U_L(TP_L, q_L) = 0$. Therefore, it is left to show that $U_L(TP_H, q_H) < 0$. Next observe that if $\frac{bu}{bl} < \frac{a_H - 1}{a_L}$, then $W_L(q_H) = \frac{(a_L)^2}{2b_L}$ is the willingness to pay of the $L$ type for the quantity of the $H$ type (notice claim 3). Therefore, $U_L(TP_H, q_H) = \frac{(a_L)^2}{2b_L} - TP_H(q_H) = \frac{(a_L)^2}{2b_L} - \frac{(a_H)^2 - 1}{2b_H}$. As it left to show that $U_L(TP_H, q_H) < 0$, then $\frac{(a_L)^2}{2b_L} - \frac{(a_H)^2 - 1}{2b_H} < 0$, which leads to $\frac{bu}{bl} < \frac{(a_H)^2 - 1}{(a_L)^2}$. ■

Proof of claim 5

As $TP_H(q_H) = 1 * q_H(p = 1) + CS_H(p = 1)$, then $U_H(TP_H, q_H) = 0$. It is left to show that $U_H(TP_L, q_L) < 0$. Next observe that $W_H(q_L) = \frac{1}{2}(a_H + a_H - b_H((a_L + 1) \frac{a_L - 1}{b_L}))$ (notice that $p(q_L) = a_H - b_H((a_L + 1) \frac{a_L - 1}{b_L}))$. Therefore,

$$U_H(TP_L, q_L) = \frac{a_H + a_H - b_H((a_L - 1) \frac{a_L - 1}{b_L})}{2b_L} - TP_L(q_L)$$

$$= (2a_H - \frac{b_H}{b_L}(a_L - 1)) \frac{(a_L - 1)}{2b_L} - \frac{(a_H)^2 - 1}{2b_L}$$

$$= \frac{(2a_H - \frac{bu}{bl}(a_L - 1))(a_L - 1) - (a_L - 1)(a_L + 1)}{2b_L}$$

Therefore, $U_H(TP_L, q_L) < 0$ iff $(2a_H - \frac{bu}{bl}(a_L - 1))(a_L - 1) - (a_L - 1)(a_L + 1) < 0$.

As $(a_L - 1) > 0$, $U_H(TP_L, q_L) < 0$ iff $(2a_H - \frac{bu}{bl}(a_L - 1)) - (a_L + 1) < 0$.

As $\frac{bu}{bl}(a_L - 1) > 0$, then $U_H(TP_L, q_L) < 0$ if $2a_H - (a_L + 1) < 0$ or $2a_H < (a_L + 1)$. ■

Proof of claim 6

Observe that:

$$\frac{TP_H(q_H) - TP_L(q_L)}{q_H - q_L} = \frac{(a_H)^2 - 1}{2b_H} - \frac{(a_L)^2 - 1}{2b_L}$$

$$= \frac{b_L((a_H)^2 - 1) - b_H((a_L)^2 - 1)}{b_H b_L}$$

$$= \frac{1}{2} \frac{b_L((a_H)^2 - 1) - b_H((a_L)^2 - 1)}{b_L(a_H - 1) - b_H(a_L - 1)}$$

Therefore, $\frac{TP_H(q_H) - TP_L(q_L)}{q_H - q_L} < 1$ leads to $\frac{b_L((a_H)^2 - 1) - b_H((a_L)^2 - 1)}{b_L(a_H - 1) - b_H(a_L - 1)} < 2$. As $q_H > q_L$, then $b_L(a_H - 1) - b_H((a_L)^2 - 1) > 0$. 

12
Therefore, \( \frac{TP_H(q_H) - TP_L(q_L)}{q_H - q_L} < 1 \) leads to:

\[
\begin{align*}
  b_L((a_H)^2 - 1) - b_H((a_L)^2 - 1) &< 2[b_L(a_H - 1) - b_H((a_L)^2 - 1)] \text{ or} \\
  b_L((a_H)^2 - 1) - 2b_L(a_H - 1) &< b_H((a_L)^2 - 1) - 2b_H(a_L - 1) \text{ or} \\
  b_L(a_H - 1)(a_H + 1) - 2b_L(a_H - 1) &< b_H(a_L - 1)(a_L + 1) - 2b_H(a_L - 1) \text{ or} \\
  b_L(a_H - 1)((a_H + 1) - 2) &< b_H(a_L - 1)((a_L + 1) - 2) \text{ or} \\
  b_L(a_H - 1)^2 &< b_H(a_L - 1)^2
\end{align*}
\]

Therefore, \( \frac{TP_H(q_H) - TP_L(q_L)}{q_H - q_L} < 1 \) leads to:

\[
\left( \frac{a_H - 1}{a_L - 1} \right)^2 < \frac{b_H}{b_L}
\]

\[\blacksquare\]
References


Falbe, J., Thompson, H.R., Becker, C.M., et al., 2016. Impact of the Berkeley excise tax on sugar-


