Information and Dynamic Trade

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Abstract

This paper investigates the possibility of information-based trading in a dynamic world. Information-based trading becomes possible in the dynamic world even when all assumptions in Milgrom and Stocky (1982) still hold. This result arises because the monotonic increasing feature of the information filtration implies that the current payoffs of securities can’t be made conditional on future events, constraining agents from smoothing their consumption paths through ex ante trades. Information-based trading is desirable because news about future consumption enables agents to readjust their asset portfolios to smooth consumptions over time. The no-trade theorem still holds in the dynamic environment when agents have concordant beliefs in each period and their concerns for risk aversion dominate the needs for inter-temporal substitution.

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1 Introduction

The No-trade theorem in the fundamental paper by Milgrom and Stokey (1982) shows that the arrival of private information cannot generate trade among rational agents in a complete market (complete across payoff relevant states, but not necessarily across signal states) if the initial allocation is Pareto-optimal.\footnote{Besides Milgrom and Stokey (1982), there’s a whole series of No-trade theorems that overrule the commonsense intuition of information based trade (Blume, Coury, and Easley (2006); Rubinstein (1975); Tirole (1982). And Rubinstein and Wolinsky (1990) provides an unified analysis for ”agreeing to disagree” type results, including no-trade theorems.} However, in the real world people observe significant volumes of trade everyday in markets and this level of trade is difficult to explain solely in terms of liquidity motives. This unexplained trading has been a ongoing puzzle in both finance and economic research. Other than irrational explanations like noisy traders (Grossman and Stiglitz, 1980) and Heterogenous beliefs (Harrison and Kreps, 1978), the main rational explanation is that the market might be incomplete across payoff relevant states. In an interesting paper by Blume, Coury, and Easley (2006), trade is possible when the market is state-contingent incomplete because the arrival of new information provides new insurance opportunities and makes trade Pareto-improving. Another noticeable work is Bond and Eraslan (2010), which introduces state-contingent incomplete market by considering a production economy that agents’ actions may affect asset value. Those models rely on some specific assumptions about the structure of market incompleteness to generate trade. However, since there are so many different markets with fairly different structures in the real world, these types of market incompleteness may not be able to explain the significant amount of trades happens in so many markets everyday. Also a natural prediction from those market incompleteness models is that if we introduce more payoff relevant state securities
to complete the market, we would finally reach a no-trade world.

In this paper I exam the possibility of information-based trade by re-
considering the static No-trade models in a dynamic setup. Common sense
suggests that the dynamic extension would not change the no-trade result
when the market is complete across payoff relevant states and consumption
at different periods can be treated as different goods in agents’ consump-
tion bundles, I show that information-based trade emerges in the dynamic
world even when the market is complete across payoff relevant states and
the belief condition in Milgrom and Stokey (1982) holds. The key reason
for this trade result is that the information filtration in the dynamic world
imposes a natural constraint on the market structure. The nature of time
suggests there would be no security that pays current consumption based
on the realization of future payoff relevant states. And this feature limits
agents’ abilities to reallocate their consumption paths by trading ex ante
in the market. Information-based trade becomes attractive because it gives
agents opportunities to alter their consumption paths based on future pay-
off relevant states. In other word, information-based trade can enrich the
possible consumption reallocation space for agents and thus maybe mutual
acceptable.

To illustrate this, consider a simple endowment economy with two con-
sumption goods apple and orange, and the only uncertainty is whether it
would be sunny or rainy tomorrow. In the static case when the market is
complete across payoff relevant states, there exist contingent claims on both
apples and oranges in both sunny and rainy states (that is to say, 4 different
securities), agents can reach any feasible consumption allocations by trading
ex ante. With this market structure if there exists a mutually acceptable
information-based trade, agents will prefer to do it ex ante. Now consider
the economy in a dynamic set up, following the idea that consumption at different periods can be viewed as different goods in the consumption bundle, agents consume apples today and oranges tomorrow. When the market is complete across payoff relevant states, there would be 3 different securities: security that pays apples today, and securities that pays oranges conditional on the weather in the next period. In this case, the information filtration suggests that there would be no security pays apples given what would happen tomorrow. With ex ante trade agents can not have different consumption of apples today based on the realization of states tomorrow. If some information about future states arrives today, then agents may be able to relate their consumption of apples to future states, and the desire to smooth consumption makes it potentially mutually acceptable. In contrast to market incompleteness models, information filtration is a fairly general feature for any dynamic markets and the information based trade result doesn’t rely on any specific market incompleteness assumptions. Dynamic trade is still possible here even when the market is complete across payoff relevant states.

My paper is not the first attempt to analyze no-trade theorems in a dynamic world (Harrison and Kreps, 1978; Tirole, 1982). There are many studies that generate similar No-trade results and it is widely believed that Milgrom and Stokey (1982) can be trivially extended to dynamic environments as long as the market is complete across pay-off relevant states. In this paper I show that previous dynamic No-trade results should implicitly rely on more restrictive assumptions. To understand this, I discuss a dynamic version of the No-trade theorem. Compared to Milgrom and Stokey (1982), which assumes concordant beliefs and concave utility functions, this new version of No-trade theorem requires more restrictive conditions on both beliefs and preferences. Though the new beliefs assumption can be interpreted as a nat-
ural extension of original concordant beliefs assumption, the new preference condition assumes that for agents risk aversion for one period consumption always dominates the need for intertemporal substitution. Even though the condition is very strong and questionable, I show that almost all standard utility functions used in economics and finance, including Epstein-Zin utility, satisfy it. Previous works find No-trade results in different dynamic settings because they all assume some standard utility functions. This also shows how standard utility descriptions fail to capture all the potential effects of intertemporal substitution.

The paper is organized as follows. Section 2 presents the model and discuss the difference between market structure in the static and dynamic setup. Section 3 shows that information-based trade is possible in dynamic environment and discusses under what conditions the no-trade theorem would hold in a dynamic world. Section 4 investigates whether information-based trade would lead to fully-revealing equilibrium. The final section concludes.

2 The Model

Consider a finite horizon discrete time economy with \( N \) traders and \( N_T \) periods. Let \( \Omega \) be the set of finite possible states of the world, with generic element \( \omega \). It is convenient to rewrite them as \( \Omega = \Theta \times S \) and \( \omega = (\theta, s) \). Endowments and utility functions may depend on \( \theta \), but are not affected by \( s \). Denote \( \Theta \) as the set of payoff-relevant events, and \( S \) the set of payoff-irrelevant events. Although \( s \) does not affect endowments or utility functions directly, it may be statistically correlated with \( \theta \). Let \( p^i(\omega) \) be agent \( i \)'s subjective belief about the probability of state \( \omega \) and assume \( p^i(\omega) > 0 \) for every \( i \) and every \( \omega \).
At each period $t$, agent $i$ receives some private information, denoted as an event $f^i_t$ of a partition $\mathcal{F}^i_t$ of set $\Omega$. By the construction of $\Omega$, one can also rewrite the information structure as $\mathcal{F}^i_t = \Theta_t \times S^i_t$ and $f^i_t = (\theta_t, s^i_t)$, where $\Theta_t$ is a partition of set $\Theta$ and $S^i_t$ is a partition of set $S$.

It is natural to assume that partition $\mathcal{F}^i_t$ becomes finer over time, which implies $\mathcal{F}^i_t \subseteq \mathcal{F}^i_{t+1}$, $\Theta_t \subseteq \Theta_{t+1}$ and $S^i_t \subseteq S^i_{t+1}$. The filtration $\mathcal{F}^i_t = \{\mathcal{F}^i_0, \mathcal{F}^i_1, \ldots, \mathcal{F}^i_{N_T}\}$ describes how information is revealed through time, and $\Theta^i_t = \{\Theta^i_0, \Theta^i_1, \ldots, \Theta^i_{N_T}\}$, $S^i_t = \{S^i_0, S^i_1, \ldots, S^i_{N_T}\}$ represent the dynamic payoff-relevant and payoff-irrelevant information process, respectively. Let $R^i_t$ describes the meet of $\mathcal{F}^i_0, \ldots, \mathcal{F}^i_{N_T}$, which is the set of common knowledge at period $t$, and the filtration $R = \{R_0, R_1, \ldots, R_{N_T}\}$ represents how common knowledge is revealed through time.

There are $l$ commodities in each state of world. At time $t$, each trader $i$ receives his endowment $e^i_t : \Theta_t \rightarrow R^i_{+}$. Let $C^i_t = \{c^i_t, c^i_{t+1}, \ldots, c^i_{N_T}\}$ describes agent $i$'s consumption process starts at time $t$. Agent’s taste, $U^i(C^i_t) : \Theta \rightarrow R$, may depend on $\theta$. For convenience, we denote the set of all possible consumption paths starting at time $t$ as $R^i_{+}$. Traders’ utility functions $U^i(\theta, .)$ are assumed to be increasing and concave in $C^i_t$ for all $i$. Traders update their subjective beliefs about $\omega$ according to $\{\mathcal{F}^i_0, \mathcal{F}^i_1, \ldots\}$. Following Milgrom and Stokey (1982), traders’ beliefs are concordant, defined as $p^1(s^i|\theta) = p^2(s^i|\theta) = \cdots = p^N(s^i|\theta)$ for any $i$, any $s^i \in S^i$ and any $\theta \in \Theta$.

A trade starts at period $t$ is a function from $\Omega$ to $R^i_{+}$, denoted as $T = (T^1, \ldots, T^N)$, where $T^i(\omega)$ is agent $i$’s net trade path of $l$ commodities in state $\omega$, and $T^i_t(\omega)$ is his net trade of $l$ commodities in state $\omega$ at period $t$. Since agents can only trade based on their information at that time, and

\[\text{In a more general case, agents may have different partitions } \Theta^i_t. \text{ This paper focuses on the case with a complete market for all payoff-relevant states, which implies a common partition } \Theta_t \text{ for all traders.}\]
trades are common knowledge, it is natural to assume that $T^i_t$ is a function measurable with respect to $\mathcal{R}_t$. $T$ is feasible if it satisfies $e^i_t + T^i_t \geq 0$ and $\sum_{j=1}^{N} T^i_t \leq 0$ for any $t$, $i$ and $\omega$. The proof of the original no-trade theorem builds on the concept of $\theta$-trade, which is defined as:

**Definition 1.** A trade is called a $\theta$-trade at time $t$ if it is a function from $\Theta$ to $R_{+}^L$.

Trade in the dynamic world is not necessarily mutually-acceptable all the time. Agents should find the trade contract mutually-acceptable when they trade, but the realized trade process may not be mutually-acceptable afterwards. For example, traders may find trading a European call option contract mutually-acceptable but it may not mutually-acceptable when the buyer actually exercises the option.

To understand why information-based trade may emerge in the dynamic environment, I first present a dynamic version of Milgrom and Stokey’s No-Trade Theorem.

**Proposition 1.** Suppose that all traders are weakly risk-averse, that the initial allocation $e = (e_1, \ldots, e_N)$ is Pareto-optimal relative to $\theta$-trades at time 0, that their beliefs are concordant, and that each agent $i$ observes the private information conveyed by the filtration $\{\mathcal{F}^i_t\}$. If it is common knowledge at $\omega$ at time $t$ that $T$ is a feasible $\theta$-trade and that each trader weakly prefers $T$ to the zero trade, then every agent is indifferent between $T$ and zero trade. If all agents are strictly risk-averse then $T$ is the zero trade.

**Proof.** See the Appendix. \qed

The proof is essentially the same as Milgrom and Stokey’s original one. This theorem simply states that any Pareto-superior information-based trade
can be done ex ante by $\theta$-trade. It is natural to think that any $\theta$-trade can be done ex-ante if we have a complete market for all payoff-relevant events. This intuition, though holds in the static environment, is not true in the case of dynamic model. To understand this, I introduce a new concept $\theta_t$-trade.

**Definition 2.** A trade is called a $\theta_t$-trade at time $t$ if it is a function from $\Theta$ to $R^{t,l}$, such that at each time $t$, $T_i^t$ is a function measurable with respect to $\Theta_t$.

If a trade can be done ex-ante through a complete market for all payoff-relevant events, then it must be a $\theta_t$-trade. Since $\Theta_t \subseteq \Theta$, any $\theta_t$-trade is a $\theta$-trade, but a $\theta$-trade is not necessarily a $\theta_t$-trade. In other words, we are not able to execute any $\theta$-trade even when the market is complete for all payoff-relevant events.

**Example 1.** Consider an economy with three dates $\{0, 1, 2\}$ and two potential states $\{A, B\}$ at date 2. Agents would consume at date 1 and date 2 and an agent’s consumption process can be described by a consumption bundle $(C_1, C_2A, C_2B)$. There are three securities trade in the market $\{K_1, K_2, K_3\}$. $K_1 = (1, 0, 0)$ gives one unit of consumption at period 1, $K_2 = (0, 1, 0)$ gives one unit of consumption at period 2 under state $A$, and $K_2 = (0, 0, 1)$ gives one unit of consumption at period 2 under state $B$. In other words, the market is complete for all payoff-relevant events. Now consider a one time trade at period 1 for one agent that $T = (1, 0, 0)$ when $\theta = \{1, A\}$ and $T = (0, 0, 0)$ when $\theta = \{1, B\}$. It is a $\theta$-trade but not a $\theta_t$-trade. The trade can not be done ex ante since in the market there’s no security that pays consumption at period 1 given states realized in the next period.
3 No-Trade Theorem with $\theta_t$-Trade

As argued in section 2, in a dynamic world, in general $\theta$-trade is not available ex-ante even when the market is complete for all payoff-relevant events. It would be natural to ask under what conditions does the No-trade result still hold. To answer this question, I introduce several important concepts for analysis.

**Definition 3.** Traders’ beliefs are dynamically concordant if $p_1(s_i^t|\theta_t) = \cdots = p_N(s_i^t|\theta_t)$ for any $i$, any time $t$ and any $s_i^t \in S_i^t$ and any $\theta_t \in \Theta_t$.

Of course a common prior is a special case for dynamically concordant beliefs condition. Notice that since $\theta_{Nt} = \Theta$, dynamically concordant beliefs condition implies concordant beliefs. Intuitively, when traders have dynamically concordant beliefs, then at each period $t$ they agree on how to interpret pay-off irrelevant events. The following example shows that, compared to concordant beliefs condition, dynamically concordant beliefs is a more restrictive condition.

**Example 2.** Consider an economy with three dates $\{0, 1, 2\}$ and two potential states $\{A, B\}$ at date 2. Between period 0 and period 1, there would be a public information $s \in \{S_A, S_B\}$ that reveals the true state at period 2. Suppose there are two agents $\{m, n\}$ who have heterogenous beliefs on the possibility of potential states. It is straightforward that agents have concordant beliefs since $p^m(S_A|\theta = \{1, A\}) = p^n(S_A|\theta = \{1, A\}) = 1$, but their beliefs are not dynamically concordant because $p^m(S_A|\theta_1) \neq p^n(S_A|\theta_1)$.

In static models, agents only care about risks. However, in a dynamic setup, intertemporal substitution also kicks in. The next concept imposes some strong condition on how agents trade off those two effects.
Definition 4. A trader’s preference is risk-aversion-dominating if for any random consumption \( C_i^t \in \mathbb{R}^{t+1}_+ \), \( \theta_t \in \Theta_t \), and any period \( k \geq t \), \( \theta_k \subseteq \theta_t \), we have \( E_t\{U(C_0^t)|\theta_t\} \leq E_t\{U(\{c_0^t, \ldots, E_k^t(\{c_k^t|\theta_k\}), \ldots\})|\theta_t\} \).

The concavity of utility function reflects both risk-aversion and intertemporal consumption smoothing. In Milgrom and Stokey’s static no-trade model, only risk-aversion plays a role in preferences, which suggests that agents would prefer ex ante trade to any possible information-based trade. However, in the case of a dynamic world, information about future consumption shocks enables agents to readjust their consumption path, and information-based trade becomes attractive due to the desire to smooth consumption over time. To make sure that ex ante trade is preferred to dynamic information-based trade, one needs the risk-aversion effect to dominate the concern for intertemporal consumption smoothing, which suggests the risk-aversion-dominating preference condition. The assumption of risk-aversion-dominating preferences is quite strong. There is no reason or to expect that single period risk-aversion concern should dominate incentives to smooth consumption over the entire consumption path. However, the following lemma shows that most utility functions used in finance and economics models do satisfy this property.

Lemma 1. Any utility function \( U(C_i^t) \) satisfying the following two properties is risk-aversion-dominating:

1. \( U(C_i^t) \) only depends on \( \{c_i^t, c_{i+1}^t, \ldots\} \);

2. For any \( k \geq t \), \( U(C_i^t) \) can be written as \( F(\{c_i^t, c_{i+1}^t, \ldots, c_{k-1}^t\}, U(C_k^t)) \) and is increasing in \( U(C_k^t) \) and concave in \( c_{k-1}^i \).

Proof. For any \( k \geq t \), consider \( U(C_k^t) = F(c_k^t, U(C_{k+1}^t)) \). From property 2
one obtains:

\[
E_t\{U(C_0^t)|\theta_t\} = E_t\{F(\{c_0^i, c_1^i, \ldots, c_k^i\}, U(C_{k+1}^i))|\theta_t\} \\
= E_t\{E_k[F(\{c_0^i, c_1^i, \ldots, c_k^i\}, U(C_{k+1}^i))|\theta_k]|\theta_t\} \\
\leq E_t\{E_k[F(\{c_0^i, c_1^i, \ldots, E_k(c_k^i|\theta_k)\}, U(C_{k+1}^i))|\theta_k]|\theta_t\} \\
= E_t\{F(\{c_0^i, c_1^i, \ldots, E_k(c_k^i|\theta_k)\}, U(C_{k+1}^i))|\theta_t\} \\
= E_t\{U(\{c_0^i, \ldots, E_k(c_k^i|\theta_k), \ldots\})|\theta_t\}
\] (1)

\[\Box\]

**Result 1.** All time additive utility functions \(U(C) = \sum_{t=0}^{\infty} U_t(c_t)\) are risk-aversion-dominating.

Surprisingly, Epstein-Zin utility function, though it has separable parameters to measure risk aversion and intertemporal substitution, still possesses the risk-aversion-dominating property.

**Result 2.** Epstein-Zin utility function also satisfies those two properties and thus is risk-aversion-dominating.

While this is not the first paper to investigate no-trade theorem in a dynamic setup, most previous papers assumed risk-aversion-dominating preferences. As shown in the next proposition, risk-aversion-dominating preference has played an important role in generating the dynamic version of no-trade theorem. This explains why the no-trade result in dynamic environment hasn’t been challenged for a long time.

The following proposition shows that when there is a complete market for all payoff-revelant states, the no-trade result is valid if agents have dynamically concordant belief and their preferences are risk-aversion-dominating.
Proposition 2. Suppose that all traders are weakly risk-averse, that their preferences are risk-aversion-dominating, that their beliefs are dynamically concordant, and that each agent $i$ observes the private information conveyed by the partition $\{F^i_t\}$, that the initial allocation $e = (e_1, \ldots, e_N)$ is Pareto-optimal relative to payoff-relevant contingent trade at time $0$, and it is a common knowledge at $\omega$ at time $t$ that a feasible trade $T$ and that trader weakly prefer $T$ to the zero trade. Then every agent is indifferent between $T$ and zero trade. If all agents are strictly risk-averse then $T$ is the zero trade.

Proof. See the Appendix.

Though the theorem confirms that dynamically concordant belief and risk-aversion-dominating preference are sufficient conditions for no-trade result to hold in dynamic world. The following two examples show that in some sense they are also necessary for the theorem.

Example 3 (Trade without Dynamically Concordant Beliefs). Consider an exchange economy with three consumption periods (time 0, 1 and 2) and two potential states $\{A, B\}$ at period 2. The market is the same as in example 1. There are two agents, $m$ and $n$, that both have log utility and the discount rate is normalized to 1. The initial endowment for both agents are $(1, 1, 1)$. Agents have different priors. Agent $m$ thinks state $A$ would happen with probability of $0.5 < p < 0.75$ while agent $n$ believes that state $B$ will happen with probability $p$. It is common knowledge that there would be new public information between time 0 and time 1, revealing the true state at time 2. Agents can trade at time 0 and time 1, and then consume.

The above example satisfies all conditions required for static No-trade theorem in Milgrom and Stokey (1982), but doesn’t satisfy the dynamically concordant beliefs condition. In the equilibrium, at time 0, agents reach
symmetric asset portfolios \((1, 4p − 1, 3 − 4p)\) for agent \(m\) and \((1, 3 − 4p, 4p − 1)\) for agent \(n\). This result is intuitive since the two traders are symmetric except the subjective prior on states in the future, so they hold a larger position in the states they believe are more likely to realize. When the public news arrives, two agents find they have the same position for consumption at time 1 but different allocations for future consumption. Thus the richer one (the "winner") would like to have more consumption today while the poorer one (the "loser") wants to increase his future consumption. The desire to smooth consumption over time makes it mutually acceptable for them to trade, reaching the new positions \((2p, 2p)\) for the winner and \((2 − 2p, 2 − 2p)\) for the loser.

In the first example it may not be surprising to see trade because the example involves heterogenous beliefs. In the following example, I construct a standard environment in the sense that traders have common prior and state-independent utility function, information-based trade emerges because one of the agents has preference that doesn’t satisfy the risk-aversion-dominating condition. A family of utility functions that doesn’t satisfy the risk-aversion-dominating condition are those with habit formation preferences. I will discuss some dynamic trade examples with habit formation preferences.

**Example 4** (Trade without Risk-aversion-dominating Preference). *Consider an exchange economy with three consumption periods (time 0, 1 and 2) and two potential states \(\{A, B\}\) at period 2. The market is the same as in the example 1. There are two agents, \(m\) and \(n\), and both have the same initial endowment \((2, 0.5, 2)\). Agent \(m\) has a habit-formation preference \(c_1 + \log(c_2 + 2 − c_1)\), and agent \(n\) is risk neutral. It is common knowledge that there would be new public information between time 0 and time 1, revealing the true state at time 2. Agents can trade at time 0 and time 1, and then consume.*
Because agents have common priors, agents’ beliefs are dynamically concordant. And it is easy to verify that both agents’ utility functions are weakly concave and strictly increasing in their domains. In the equilibrium, before the information is revealed, the post-trade endowment for agent $m$ is $(1, 1, 2.5)$. There’s no trade if state $A$ would realize, but agent $m$ would trade 0.75 unit of asset $K_3$ when information reveals that state $B$ will occur. Her consumption would be $(1, 1)$ in state $A$ and $(1.75, 1.75)$ in state $B$.

In this example, trade occurs because the utility of agent $m$ is not risk-aversion-dominating. Her utility from time 2 consumption $c_2$ depends on both $c_2$ and the reference point $c_1$. Intuitively, this example describes a world with a potential serious recession in the future. Because of the possible low consumption in the future, the agent with habit-formation preference chooses to consume little currently, but when the information reveals that there will be no recession in the future, the agent will want to increase her current consumption. All of this makes information-based trade desirable.

In example 3 and 4, information-based trade emerges since new information about future consumption motivates agents to reallocate their consumption portfolios, which is mutually acceptable. These adjustments can not be done ex-ante since even when we have a complete market for all pay-off relevant states, since there’s no security that pays consumption goods today given the realization of some pay-off relevant states in the future, which is exactly the difference between $\theta_t$-trade and $\theta$-trade. Information-based trade in fact expands the possible asset allocation space generated by ex-ante trade based on complete market for all pay-off relevant states, thus is Pareto-improving and is mutual acceptable.
4 Information, Market and Trade

Since informational-based trade is possible in dynamic consumption world. One may wonder whether information would be fully revealed in the equilibrium? A modified version of example 6 shows that when traders are strategic investors, there may exist an unique equilibrium in which trader’s private information is not revealed.

Example 5 (Trade with Private Information). Consider a pure exchange economy which is the same as in example 6 except that the new information is now a private information for agent $m$.

As in standard REE models, we first assume the existence of a fully revealing equilibrium, that is to say, both agents know the new information, then solve the model in the same way as in example 6. However, this fully revealing equilibrium can not hold since agent $m$ will always prefers to report the $B$ signal since it gives her more current consumption, which generates higher utility even in state $A$. The only equilibrium is a pooling equilibrium, where market belief is always $Pr(A) = Pr(B) = 0.5$, and agent $m$ consumes $(c_1, c_{2A}, c_{2B}) \approx (1.4208, 1, 1.6583)$, and any other trade will resulting in a new market belief $Pr(A) = 1$, and no trade would occur. It is interesting to see that though agent $m$ may gain by reporting false information, it in fact introduces a commitment problem: agent $m$ can not commit to report the true type, which in turn prohibits all possible further trading. The pooling equilibrium is ex-ante Pareto-inferior to the case when agent $m$ can commit to report true type.
5 Conclusion

This paper extends the celebrated No-Trade theorem to the dynamic environment. A dynamic version of no-trade theorem shows that the $\theta$-trade, the core concept in Milgrom and Stokey's proof, though is valid in static world with a complete market for all payoff-relevant states, can not be done ex-ante in a dynamic environment. In the dynamic world, the no-trade result requires agents to have risk-aversion-dominating preferences, which is a fairly strong condition.

With information-based trading, traders may have incentive to reveal their private information strategically, and fully revealing equilibrium may not exist. It maybe ex ante Pareto-superior if traders could commit to reveal their private information.
A Appendix

Proof of Proposition 1

Proof. Suppose there’s a feasible and mutually acceptable trade $T$ at $\omega'$ starts at time $t$, then for every $i$ and every $\omega \in R_t(\omega')$

$$E_i(U^i(C^i_t + T^i)|\mathcal{F}_t^i(\omega)) \geq E_i(U^i(C^i_t)|\mathcal{F}_t^i(\omega)).$$  \hspace{1cm} (2)

Suppose it is strict for trader $j$ at $\omega_t$, then consider the trade $T^* = 1_{R_t(\omega')}T$, obviously it is a feasible trade since $T$ is feasible. Viewing $T^*$ at time $t$ when the trade would occur:

$$E_i(U^i(C^i_t + T^{*^i,j}) = E_i(E_i(U^i(C^i_t + T_t1_{R_t(\omega')})|\mathcal{F}^i))$$

$$= E_i(1_{R_t(\omega')}E_i(U^i(C^i_t + T^i)|\mathcal{F}^i)) + E_i(1_{R_t(\omega')}E_i(U^i(C^i_t)|\mathcal{F}^i))$$

$$\geq E_i(U^i(C^i_t))$$

(3)

This inequality is strict for agent $j$. Now consider the ex-ante trade $T^{**} = E(T^*|\theta)$. Because agents’ beliefs are concordant, it is a feasible trade and is ex ante strictly Pareto-superior at time $t$. Contrary to our assumption of initial allocation.

If traders are strictly risk averse, and $T$ is not null, then $0.5T^{**}$ is a Pareto-improving payoff-relevant contingent trade, a contradiction. \hfill \Box

Proof of Proposition 2

Proof. Suppose there’s a feasible and mutually acceptable trade $T$ at $\omega'$ starts at time $t$, then for every $i$ and every $\omega \in R_t(\omega')$
\[ E_i(U^i(C^i_t + T^i)|\mathcal{F}^i_t) \geq E_i(U^i(C^i_t)|\mathcal{F}^i_t). \]  

(4)

Suppose it is strict for trader \( j \) at \( \omega_t \), then consider the trade \( T^* = 1_{R_t(\omega')}T \), obviously it is a feasible trade since \( T \) is feasible. Viewing \( T^* \) at time \( t \) when the trade would occur:

\[
E_i(U^i(C^i_t + T^*|\omega')) = E_i(E_i(U^i(C^i_t + T^*1_{R_t(\omega')})|\mathcal{F}^i)) \\
= E_i(1_{R_t(\omega')}E_i(U^i(C^i_t + T^i)|\mathcal{F}^i)) + E_i(1_{R_t(\omega')}E_i(U^i(C^i_t)|\mathcal{F}^i)) \\
\geq E_i(U^i(C^i_t))
\]

(5)

This inequality is strict for agent \( k \). Now consider an one time trade \( T^{**} = \sum_{t=1}^{N_T} E(T^*_k|\theta_k) \). Because agents' beliefs are dynamically concordant, it is feasible trade. It is ex-ante strictly Pareto-superior due to the risk-aversion-dominating assumption, which contradicts to the fact that the initial allocation is Pareto-optimal relative to payoff-relevant contingent trade. If traders are strictly risk averse, and \( T \) is not null, then \( 0.5T^{**} \) is a Pareto-improving payoff-relevant contingent trade, a contradiction. \( \square \)
References


