Learning in Persuasion with Multiple Advisors*

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Abstract

This paper studies a persuasion game with multiple advisors revealing information to a decision maker in order to change his behavior. The advisors share perfectly aligned incentives, and they persuade the decision maker by conducting (possibly biased) investigations of noisy signals that suggest the true state of the world. When the decision maker consults multiple advisors, on the one hand he is potentially exposed to a richer set of information, but on the other hand the advisors counteract this effect by choosing more biased investigations. This paper shows that in equilibrium the latter effort never offsets the former effect. In particular, when there are multiple advisors, advisors are worse off compared to the case in which the decision maker only consults a single advisor, but the decision maker is not necessarily better off. In fact, having fewer advisors may be a Pareto improvement.

Keywords: Bayesian persuasion, multiple advisors, endogenous information acquisition, learning

1 Introduction

Consider a decision maker who consults possibly many advisors to choose an action that generates payoff dependent on an unknown state of nature. No one learns the true state, but each advisor

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can investigate an i.i.d. signal that is correlated to the true state. This is commonly true if, for example, the true state is the outcome of a future project. Even though the future outcome is never observable, the decision maker can hire advisors to investigate the outcomes of similar existing projects for a prediction. Moreover, consider the following case of misaligned incentives between the advisors and the decision maker: while the decision maker wishes to choose the action that matches the true state, the advisors prefers that he takes a single action independent of the state. For example, the advisors may always prefer that the decision maker invests in the project. This misalignment creates incentives for the advisors to initiate biased investigations that are more likely to deliver favorable results instead of the truth. For example, an advisor may adopt a lower standard in the evaluation of a project, so that even a low-quality project passes with positive probability.

Since each advisor only has access to a noisy signal instead of the true state, the employment of multiple advisors potentially exposes both the advisors and the decision maker to a richer pool of information that may enable better learning. This paper asks two questions: Does the employment of multiple independent advisors benefit each advisor? Does it benefit the decision maker? The answers: “no”, and “sometimes”.

From the perspective of the advisors, when there are many of them independently sending information to the decision maker, two forces are at work. On the one hand, fixing the investigation choices of the advisors, when the decision maker hires increasing numbers of advisors he eventually learns the true state, hence making himself better off and the advisors worse off. On the other hand, however, the advisors counteract this effect by choosing more biased investigations when they recognize the presence of other advisors. This paper shows that the latter effort never offsets the former effect. In fact, the advisors can at best replicate the single-advisor equilibrium payoff if one of them conducts an informative investigation, and the rest conduct meaningless investigations that deliver no information. If in an equilibrium there is more than one informative advisor conducting meaningful investigation, then the decision maker chooses their desired action less often, and the advisors are strictly worse off. In particular, this is true if the advisors adopt symmetric strategies in their choice of investigations.
From the perspective of the decision maker, the fact that the advisors are strictly worse off when he consults multiple informative advisors doesn’t necessarily imply that he is strictly better off himself. While it’s true that the decision maker is never worse off, for a large class of equilibria he gets the exact same payoff as in the single-advisor case when he consults more advisors. This implies that the employment of more advisors may hurt the advisors without benefiting the decision maker, even though they are potentially exposed to a richer pool of information. To see why, note that in any “intolerant equilibrium”, in which the decision maker only chooses advisors’ favorable action when all outcomes of the investigations are favorable, the decision maker must be exactly indifferent whenever he chooses that favorable action. Hence he is getting the same payoff as he would by always choosing advisors’ unfavorable action. Such intolerant equilibria exist regardless of the total number of advisors, and in particular, all equilibria are intolerant when there are two or less advisors. This implies that the decision maker is never strictly better off when he increases the number of advisors from one to two, even though the advisors are worse off. It is only in a “tolerant equilibrium”, one in which there are three or more advisors and the decision maker chooses advisors’ favorable action even when some investigation outcomes are unfavorable, that the decision maker can be strictly better off than the single-advisor scenario, because there now exists cases in which he strictly prefers to choose that favorable action.

Most results of this paper are based on the assumption that advisors choose their investigations independently. When advisors can choose correlated investigations, the game is analogous to one with a single advisor and a more accurate signal. In this case, the advisors are strictly better off when they are more numerous, and the decision maker is indifferent.

This paper is closely related to the Bayesian persuasion literature with endogenous and observable investigation such as Kamenica and Gentzkow (2011), Gentzkow and Kamenica (2012). It extends Kamenica and Gentzkow (2011) by introducing multiple advisors and by limiting their ability to acquire information. Advisors in this paper can learn a noisy signal at best, instead of the true state, which creates room for social learning when multiple advisors are introduced. While Gentzkow and Kamenica (2012) also study a model of multiple advisors, they focus on advisors
with misaligned incentives. Their paper shows that when advisors’ preferences differ, a larger number of advisors makes each of them worse off, and the decision maker better off, as a result of competition. This paper instead focuses on learning when advisors’ information is imperfect, and abstracts away from effects associated with competition by assuming advisors to have identical preferences. I show that even if all advisors share a common goal, they are worse off when the decision maker consults many of them, not as a result of competition but as a result of the lack of coordination. This paper does not provide an answer for a case with both imperfect information and competition, but based on the results in Gentzkow and Kamenica (2012), an educated guess is that the advisors are hurt even more when they differ in their preferences, which implies that what this paper describes is the best case scenario for advisors with imperfect information.

While there are papers on cheap talk persuasion games with multiple advisors (Battaglini 2002, Ambrus and Takahashi 2008, Ambrus and Lu 2010), it should be pointed out that for any game in which the advisors’ preferences are state-independent, if the decision maker only observes the outcome of the investigation but not the choice of the investigation itself, the only equilibrium is a trivial one in which the advisors always conduct completely biased investigations that only generate favorable outcomes, and the decision maker is never persuaded. (Sobel 2011). Therefore it is in fact crucial that in this paper the decision maker observes the choice of the investigation as well as its outcome. Other papers explore persuasion games with state-independent advisors under the restriction that they can only conduct unbiased investigations, but an investigation can be hidden from the decision maker if the outcome is unfavorable. Bhattacharya and Mukherjee (2013) study a setting in which an advisor learns the true state with some probability, and can only choose to report the state truthfully or stay silent. In this setting, an equilibrium is completely characterized by the default action of the decision maker. Felgenhauer and Schulte (2014) focus on a game in which in order to persuade the decision maker, an advisor performs a sequence of costly, unbiased, and noisy experiments to learn the true state. They show that in equilibrium, if the state is unfavorable, the advisor is more likely to stop experimenting, because it takes too many costly experiments to collect enough favorable evidence. In their setting the higher the decision
maker’s standard is, the better off he is. On the contrary, in this paper a more intolerant decision maker is in fact worse off.

In the remainder of the paper, a formal model setup is introduced (Section 2), followed by proofs of the results in a general setting (Section 3). A brief discussion of correlated investigations is included in the end (Section 4).

2 The Model

There are two states of the world: \( H \) or \( L \). There are \( n \) advisors and a decision maker. The decision maker’s preference is described by a utility function \( u \) that depends on his action \( a \in \{a_H, a_L\} \) and the true state. He prefers to choose the action that matches the state, i.e. \( u(a_H, H) > u(a_L, H) \) and \( u(a_L, L) > u(a_H, L) \). The advisors share a common utility function \( v \) which only depends on the action of the decision maker. WLOG assume \( v(a_H) = 1 \), and \( v(a_L) = 0 \), i.e. all advisors prefer action \( a_H \).

The advisors and the decision maker share a common prior: \( \Pr(H) = \Pr(L) = 0.5 \)\(^1\). While the true state is never observable, each advisor \( i \) can investigate an i.i.d. signal \( s_i \in \{s_H, s_L\} \) that is correlated with the true state: \( \Pr(s_H|H) = \Pr(s_L|L) = p \) for some \( p \in (0.5, 1) \). An arbitrary investigation generates an outcome \( m_i \in \{m_H, m_L\} \) with some probabilities conditional on the signal \( s_i \). The strategy of each advisor is to choose an investigation, i.e. a pair of conditional probabilities \( (x_i, y_i) \), where \( x_i \equiv \Pr(m_H|s_L) \) and \( y_i \equiv \Pr(m_H|s_H) \). The choice of investigation \( (x_i, y_i) \) and its outcome \( m_i \) are both observable by the decision maker. After observing all advisors’ choices of investigation, as well as all outcomes of the investigations, the decision maker Bayesian updates his belief about the true state, and chooses the action that maximizes his expected utility. When indifferent, assume that he chooses \( a_H \).

Let \( A \equiv \frac{u(a_L, L) - u(a_H, L)}{u(a_H, H) - u(a_L, H)} \), then \( A \) represents the minimum likelihood ratio \( \frac{\Pr(H)}{\Pr(L)} \) that induces the decision maker to choose action \( a_H \). If \( A < 1 \), the only equilibrium outcome is one in

\(^1\)The assumption of equal prior is not crucial for the results in this paper.
which the decision maker always chooses $a_H$, since his prior belief alone is strong enough to induce the advisors’ favorable action. On the other hand, if $A > \frac{p}{1-p}$, if the decision maker only consults one advisor, then he always picks $a_L$ even if he is certain that the signal investigated by his advisor is $s_H$. The rest of the paper focuses on the non-trivial cases with $A \in \left(1, \frac{p}{1-p}\right)$.

Call the case with one advisor, i.e. $n = 1$, the benchmark case. The unique equilibrium in the benchmark case is such that the advisor chooses investigation $(x, y) = (x^*, 1)$ for some $x^* \in (0, 1)$. That is, the investigation always generates outcome $m_H$ when signal is $s_H$, and generates $m_H$ with probability $x^*$ even when the signal is $s_L$. $x^*$ is chosen such that the decision maker strictly prefers, and hence chooses, $a_L$ when the outcome is $m_L$; when the outcome is $m_H$, the decision maker is exactly indifferent between the two actions, and hence chooses $a_H$ under the assumed tie-breaking rule. To see why the optimal investigation must feature $y = 1$, note that the advisor’s objective is to maximize $\Pr(m_H)$ such that when the decision maker sees $m_H$, his posterior belief $\Pr(H|m_H) = \frac{\Pr(H|m_H)}{\Pr(L|m_H)}$ is strong enough to induce action $a_H$. Since $\Pr(H|m_H) = \frac{p \cdot y + (1-p) \cdot x}{(1-p) \cdot y + p \cdot x}$ is increasing in $y$, the optimal strategy is to maximize $y$ as it not only increases $\Pr(m_H)$, but also makes the decision maker choose the desired $a_H$ more firmly when the outcome is $m_H$. For the exact same reason, even when there are multiple advisors, truthful outcome given the favorable signal (i.e. $y = 1$) remains a feature of each advisor’s optimal investigation. Therefore, each advisor $i$’s equilibrium investigation choice can be fully described by a single parameter

$$x_i \equiv \Pr(m_H|s_L).$$

### 3 Multiple advisors

#### 3.1 Advisors’ payoff

In this section, I discuss a setting in which multiple advisors choose independent investigations. I compare this setting with the benchmark case in which there is only one advisor. On the one hand,
if each advisor is choosing the same investigation as in the benchmark, when the decision maker consults increasing numbers of advisors, he eventually learns the true state, and chooses \( a_H \) less often on average. On the other hand, in equilibrium each advisor strategically counteracts such effect by choosing more biased investigation that generates outcome \( m_H \) more often. I show that in equilibrium the latter effort never offsets the former effect.

This result is proved in two steps. I first show that it is true for the case of two advisors \((n = 2)\), then extend the result to an arbitrary \( n \). The reason why \( n = 2 \) needs to be treated differently from \( n > 2 \) is that there are multiple types of equilibria when the decision maker consults three or more advisors. For example, when \( n = 3 \), there exists one type of equilibrium in which the decision maker chooses \( a_H \) if and only if all three outcomes are \( m_H \), and another type of equilibrium in which two \( m_H \) out of three outcomes is enough to induce \( a_H \). I show that from the advisors’ perspective, the former type of equilibrium dominates the latter, and therefore it is sufficient to establish the target result by showing that even the former type of equilibrium is inferior to the benchmark equilibrium.

Start with \( n = 2 \). When advisors are aware of the existence of each other, the best they can do is to let one of them choose the benchmark investigation, and the other choose uninformative investigation that always generates outcome \( m_H \) unconditionally. In other words, whenever both advisors are conducting informative investigation, they are strictly worse-off than the benchmark. To see why this is true, first note that in the equilibrium for \( n = 2 \), the decision maker is always indifferent when he chooses \( a_H \), and fairly confident that the state is \( L \) when he chooses \( a_L \). Moreover, advisors can induce the decision maker to choose \( a_H \) more often in general if they can make him more certain about the state being \( L \) when he chooses \( a_L \). Recall that in the benchmark case, the decision maker chooses \( a_L \) if and only if the outcome was \( m_L \). Now with two independent advisors, there are three outcome realizations that induce \( a_L \): \((m_H, m_L), (m_L, m_H)\), and \((m_L, m_L)\). Compared to the benchmark, when the decision maker chooses \( a_L \) in the two-advisor case, he is less certain about his action if outcomes were \((m_H, m_L)\) or \((m_L, m_H)\), and is only more certain when they were \((m_L, m_L)\). Since outcomes were generated independently, the first two cases occur more
ofter than the last, implying that the decision maker is over all less certain when he chooses \( a_L \). As a result, advisors have to choose investigations that induce a lower probability of \( a_H \) than in the benchmark. Lemma 1 elaborates this result.

**Definition 1.** When \( n = 1 \) (benchmark), let \( x^B \) denote the advisor’s equilibrium investigation\(^2\), and let \( v^B \) denote the her ex ante equilibrium payoff.

**Lemma 1.** When \( n = 2 \), let \( x_1, x_2 \) denote the equilibrium investigations of the advisors, and \( v \) denote their expected equilibrium payoffs. Then \( v \leq v^B \). Moreover, the equality holds if and only if \((x_1, x_2) = (1, x^B)\) or \((x^B, 1)\). In particular, the advisors are strictly worse off compared to the benchmark if they choose symmetric investigations \((x_1 = x_2)\).

**Proof.** Since \( A > 1 \), i.e. the posterior belief for state \( H \) has to be strictly higher than \( L \) for the decision maker to choose \( a_H \), when \( n = 2 \) the decision maker chooses \( a_H \) only if both investigations generate outcome \( m_H \). Therefore to maximize payoff, the advisors’ optimal strategy is to choose \((x_1, x_2)\) such that the decision maker is exactly indifferent when he sees two \( m_H \) outcomes. In other words,

\[
\frac{\Pr(H|m_H, m_H)}{\Pr(L|m_H, m_H)} = \frac{\Pr(m_H, m_H|H)}{\Pr(m_H, m_H|L)} = \frac{[p + (1-p)x_1][p + (1-p)x_2]}{(1-p + px_1)(1-p + px_2)} = A \tag{1}
\]

\[
x_2^*(x_1) = \frac{p^2 - (A - 1)p(1-p)x_1 - A(1-p)^2}{(A - 1)p(1-p) + [Ap^2 - (1-p)^2]x_1} \tag{2}
\]

Therefore, even when \( n = 2 \), there are infinitely many equilibria, as long as \( x_1 \) and \( x_2 \) satisfy equation (2). The goal is to show that from the advisors’ perspective, all of those equilibria are inferior to the benchmark. For convenience of analysis later in the section, I focus on the conditional payoffs

\[
v|H = \Pr(a_H|H) = \Pr(m_H, m_H|H),
\]

\[
v|L = \Pr(a_H|L) = \Pr(m_H, m_H|L),
\]

\(^2\)Specifically, \( x^B = \frac{p - A + Ap}{Ap - 1 + p} \).
and I show that both conditional payoffs are maximized exactly when \( x_1 = 1, x_2 = x^B \), or
\( x_1 = x^B, x_2 = 1 \).

First of all, notice that the expected posterior likelihood when the decision makers chooses \( a_L \),
\( l_{a_L} \equiv \frac{1 - \Pr(m_H, m_H | H)}{1 - \Pr(m_H, m_H | L)} = \frac{1 - A \cdot \Pr(m_H, m_H | L)}{1 - \Pr(m_H, m_H | L)} \), strictly decreases with \( \Pr(m_H, m_H | L) \), as well as
\( \Pr(m_H, m_H | H) = A \cdot \Pr(m_H, m_H | L) \), as a result of equation (1). In other words, \( v|H \) and \( v|L \) are
maximized exactly when \( l_{a_L} \) is minimized. For arbitrary \((x_1, x_2)\),
\[
\text{\( l_{a_L} = \frac{\Pr(m_H, m_H \text{ or } m_H, m_L \text{ or } m_L, m_H | H)}{\Pr(m_H, m_H \text{ or } m_H, m_L \text{ or } m_L, m_H | L)} \)}
\[
= \frac{(1 - x_1 x_2)(1 - p) + (2 - x_1 - x_2) p(1 - p)}{(1 - x_1 x_2) p^2 + (2 - x_1 - x_2) p(1 - p)},
\]
which decreases in both \( x_1 \) and \( x_2 \) because \((1 - p)^2 < p^2\). Moreover,
\[
\lim_{x_1 \to 1} l_{a_L} = \lim_{x_2 \to 1} l_{a_L} = \frac{1 - p}{p}. \tag{3}
\]

Since \( \frac{1 - p}{p} \) is precisely the posterior likelihood when the decision maker chooses \( a_L \) (i.e. sees
\( m_L \)) in the benchmark case, (3) implies that both \( v|H \) and \( v|L \) are maximized exactly when \( x_1 = 1 \)
or \( x_2 = 1 \); that is, the equilibria associated with the highest payoff for the advisors must have
one of the advisors choosing uninformative investigation and the other choosing the benchmark
investigation. In either case, the maximized conditional payoff is equal to the advisor’s conditional
payoff in the benchmark case, and hence \( v = v^B \). Moreover, the unique equilibrium with \( x_1 = x_2 \) in
fact generates the lowest payoff for the advisors.\(^3\)

Is there a natural extension from Lemma 1 to a general case with \( n \geq 2 \)? The answer is “yes”
only when the equilibrium is one in which the decision maker chooses \( a_L \) unless all outcomes are

\(^3\) \( v|H^x(x_1) = -\frac{2A^2 (p-1) p (2p-1)^3}{[(A-1) p^2 (x_1-1) - x_1 + p(A-1 + 2x_1)]^3} > 0; \quad v|L^x(x_1) = -\frac{2A (p-1) p (2p-1)^3}{[(A-1) p^2 (x_1-1) - x_1 + p(A-1 + 2x_1)]^3} > 0. \quad \) Moreover, \( v|H^x(x_1) = v|L^x(x_1) = 0 \) if and only if
\( x_2 = x_1 = \frac{(p-1)\sqrt{A} + p}{p-1 + p\sqrt{A}}. \)
I call it an “intolerant equilibrium”, as defined below. Lemma 2 shows that the best $n$-advisor intolerant equilibrium for the advisors features a silent advisor that chooses an uninformative investigation. Therefore by deleting silent advisors and by induction, a $n$-advisor game can be reduced to a 1-advisor game as in the benchmark.

**Definition 2.** An equilibrium is called “intolerant” if the decision maker only chooses $a_H$ when all $n$ outcomes are $m_H$. An equilibrium is called “tolerant” if there exist an integer $k < n$ such that the decision maker chooses $a_H$ when exactly $k$ outcomes are $m_H$.

**Lemma 2.** Fix $n > 2$. Let $v$ be an advisor’s highest expected payoff in an intolerant equilibrium. Then $v = v^B$.

**Proof.** Start with $n = 3$. Fix the investigation choice of the third player, $x_3$. Let $l_i$ denote the posterior likelihood ratio $\frac{\Pr(H|m_H)}{\Pr(L|m_H)} = \frac{p + (1-p)x_i}{1 - p + px_i}$ after a single outcome $m_H$ from advisor $i$’s investigation. Given $x_3$, if a pair of $(x_1, x_2)$ solves

$$\max_{x_1, x_2} v(x_1|H) \text{ and } v(x_1|L)$$

$$s.t. \ l_1 \cdot l_2 \cdot l_3 = A$$

then $(x_1, x_2)$ maximizes the payoff of the advisors. But re-arranging the constraint into $l_1 \cdot l_2 = \frac{A}{l_3}$, the optimization problem is identical to the problem in Lemma 1 with $n = 2$ and $\hat{A} = \frac{A}{l_3}$. Therefore applying Lemma 1, in the best equilibrium for the advisors, one of them must be choosing an uninformative investigation. WLOG say $x_1 = 1$ and $x_2 > 0$. Now knowing the best-response of the first two players, relax the restriction on $x_3$ to find the global optimum for all three advisors. Since advisor 1 is silent, deleting her from the group does not change the equilibrium outcome. An iterated application of Lemma 1 implies that either $(x_2, x_3) = (1, x^B)$ or $(x^B, 1)$. Although the decision maker consults 3 advisors, only one of them is conducting effective investigation $x^B$. The same logic applies to any $n \geq 3$; simply apply Lemma 1 iteratively to delete silent advisors until there is only one advisor left who follows the benchmark strategy $x^B$. Hence,
Lemma 2 partially extends Lemma 1 to the case of \( n > 2 \). But as discussed earlier, for a full extension needs to include the tolerant equilibria. Lemma 3 shows that compared to an intolerant equilibrium in which the decision maker chooses \( a_H \) only when all outcomes are \( m_H \), advisors are in fact worse off in a tolerant equilibrium in which the decision maker only requires some of the outcomes to be \( m_H \). The intuition is that when the decision maker needs more \( m_H \) outcomes to be persuaded in an equilibrium, the required informativeness of each \( m_H \) is lower, hence the advisors can get away with a more biased investigation. This effect turns out to be dominant, leading towards an overall higher ex ante probability of \( a_H \).

**Lemma 3.** Fix \( n > 2 \). Let \( v \) be an advisor’s highest expected payoff in an intolerant equilibrium. Let \( \hat{v} \) be an advisor’s highest expected payoff in a tolerant equilibrium. Then \( \hat{v} < v = v^B \).

*Proof.* Again, it is sufficient to prove the case of \( n = 3 \), as the same logic naturally extends to all \( n > 3 \). When there are 3 advisors, there are two cases: i) the decision maker chooses \( a_H \) if and only if all three outcomes are \( m_H \), or ii) the decision maker chooses \( a_H \) if at least two outcomes are \( m_H \). Start with case i). Lemma 2 implies that one of the advisors (in fact two of them) chooses uninformative investigation. Deleting that advisor doesn’t change the equilibrium outcome. Therefore \( v \) is simply the maximized unconditional probability of two \( m_H \) outcomes such that \( l_1 \cdot l_2 = A \), where \( l_i \) is the posterior likelihood ratio after one \( m_H \) as defined in the proof of Lemma 2. Now move on to case ii). WLOG assume that the decision maker still chooses \( a_H \) if the third outcome is \( m_L \), as long as the first two outcomes are \( m_H \). Let the investigation choice of the third advisor be \( x_3 < 1 \). Then \( \hat{v} \) is the maximized unconditional probability of two \( m_H \) outcomes from the first two advisors such that \( l_1 \cdot l_2 = \frac{A}{l_3} > A \). Since in this case the outcomes from the first two advisors are required to be more informative, which implies lower \( x_1 \) and \( x_2 \), \( m_H \) occurs less often, and hence \( \hat{v} < v \). The same argument holds for all \( n \geq 3 \). The best equilibrium payoff for the advisors is always higher when the decision maker require all \( m_H \) outcomes to choose \( a_H \). \( \square \)

Lemma 1, 2 and 3 combined lead to the main theorem of the section.
Theorem 1. When \( n \geq 2 \), let \( v \) be the advisors’ ex ante payoff. Then \( v \leq v^B \). Moreover, the equality holds if and only if there exists \( i \) s.t. \( x_i = x^B \) and \( x_j = 1 \) for all \( j \neq i \). Specifically, the advisors’ payoff must be strictly lower than \( v^B \) in any equilibrium featuring symmetric investigation \( x_i = x \) for all \( i \).

Proof. Theorem 1 is a direct corollary of Lemma 1, 2, and 3: with an arbitrary number of advisors, the best equilibrium for them features the decision maker choosing \( a_H \) only when all outcomes are \( m_H \), and the maximum payoff for the advisors is achieved when only one of them is choosing informative investigation with \( x^B \).

Remark 1. Since advisors are strictly worse off with symmetric investigations when \( n \geq 2 \), one may be tempted to seek for a monotonicity result: the larger \( n \) is, the worse off the advisors are. This, however, is not generally true. For example, focus on the case of \( p = 0.81 \), \( A = 3 \). When \( n = 2 \) and \( x_1 = x_2 \), the unique equilibrium payoff for the advisors is 0.5225. When \( n = 3 \) and \( x_1 = x_2 = x_3 \), there are two symmetric equilibria. In the tolerant equilibrium in which two \( m_H \) outcomes are sufficient for the decision maker to choose \( a_H \), the equilibrium payoff for the advisors is 0.5235 > 0.5225. Moreover, when three \( m_H \) outcomes are necessary for the decision maker to choose \( a_H \), the intolerant equilibrium payoff for the advisors is 0.5092 < 0.5235. Not only does this example disprove the monotonicity conjecture, but also it shows that Lemma 3 fails when the attention is restricted to equilibria with symmetric investigation only. Compared with the example of non-monotonicity given in Li and Norman (2014), this example here is not a result of unaligned preferences among advisors. Here the non-monotonicity is due to the lack of coordination among the advisors.

3.2 Decision maker’s payoff

Given that advisors are worse off when there are many of them, and strictly so when there are more than one advisor choosing informative investigation, it is worth asking if the decision maker is better off in the mean time. In other words, does a lower payoff for the advisors necessarily
imply a higher payoff for the decision maker? The answer is, in fact, “no”. I show that the decision maker simply gets a constant payoff regardless of \( n \), unless there are at least three advisors, and the equilibrium is tolerant. But even among tolerant equilibria with three or more players, both the decision maker and the advisors may be strictly worse off at the same time.

Let’s first start the analysis with an extremely simple observation, which is exaggerated into a lemma to emphasize its importance to the later results.

**Lemma 4.** If in an equilibrium the decision maker is always indifferent when he chooses \( a_H \), he gets expected payoff \( u = \frac{1}{2} [u(a_L, H) + u(a_L, L)] \).

*Proof.* There are only two actions that the decision maker can take. If he is indifferent between \( a_L \) and \( a_H \) whenever he chooses \( a_H \), his payoff is equivalent to the case in which he always chooses \( a_L \) unconditionally, that is, the default constant payoff \( u = \frac{1}{2} [u(a_L, H) + u(a_L, L)] \).

Note that the case in Lemma 4 applies to the benchmark case with a single advisor. In the benchmark case, the advisor optimally chooses an investigation that leaves the decision maker exactly indifferent and (hence chooses \( a_H \)) when the outcome is \( m_H \). In other words, whenever the decision maker picks \( a_H \) he’s just indifferent. Therefore, the decision maker’s payoff is simply the default \( u \) as if he chooses \( a_L \) all the time. To compare a case of multiple advisors with this benchmark case, simply ask whether in equilibrium the decision maker is indifferent whenever action \( a_H \) is chosen. This turns out to be true in all intolerant equilibria, and false in all tolerant equilibria, which leads to the following theorem.

**Theorem 2.** The decision maker gets expected payoff \( u \) in any intolerant equilibrium, and \( u > u \) in any tolerant equilibrium.

*Proof.* Let \( k \leq n \) be the minimum number of \( m_H \) outcomes that can induce action \( a_H \) in an equilibrium, then the optimal investigations chosen by the advisors must leave the decision maker exactly indifferent (and hence chooses \( a_H \)) when precisely \( k \) outcomes are \( m_H \). Since \( k = n \) in any intolerant equilibrium, the decision maker is indifferent whenever \( a_H \) is chosen. Hence by Lemma 4

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he always gets the default payoff $u$, independent of the number of advisors or the informativeness of the signals. On the other hand, in a tolerant equilibrium we have $k < n$, which implies that an action $a_H$ can be a result of $k + 1$ or more $m_H$, in which case the decision maker strictly prefers action $a_H$ over action $a_L$. Therefore, in a tolerant equilibrium, the decision maker gets a strictly higher expected payoff than the amount $u$ if $a_L$ is always played.

It’s particularly worth mentioning that when $n = 2$, only intolerant equilibria exist, because one $m_H$ paired with one $m_L$ are never convincing enough for the decision maker to choose $a_H$ given the assumed parameter range. This leads to an immediate corollary of Theorem 2.

**Corollary 1.** When $n = 2$, the decision maker always gets payoff $u$.

Recall that if both advisors conduct informative investigation, they are strictly worse off than the benchmark. This fact combined with Corollary 1 implies that it is in fact Pareto inferior to have two advisors instead of one, even though the players have access to more information of the true state.

## 4 Correlated investigations

All the results so far are based on the assumption that the advisors choose independent investigations. Now consider instead that the advisors may choose correlated investigations. That is, the probability of outcome $m_i$ is not only conditional on advisor $i$’s signal $s_i$, but also on all the other signals as well. In this case the advisors are better off compared to the benchmark. To see why, note that a game with $n$ advisors choosing correlated investigations is analogous to one in which there is one advisor conducting an investigation of $n$ i.i.d. signals, or a single signal with better accuracy (a higher $p$). In the latter case, the unique equilibrium features a more biased investigation (a higher $x^*$) and hence a higher payoff for the advisor. This higher $x^*$ is the result of the increased informativeness of an unfavorable outcome.
To be more specific, in equilibrium the advisors’ optimal correlated investigations truthfully reveal the signals when a large enough number of signals are \( s_L \), but otherwise generate outcomes \( m_H \). The decision maker chooses \( a_L \) in the former case, and \( a_H \) in the latter. In the latter case when the investigations generate \( m_H \), the decision maker must be exactly indifferent, because otherwise the advisors would deviate to more biased investigations that generate \( m_H \) more often. This implies that Theorem 2 can be applied here, and the receiver gets exactly \( u \) regardless of how many advisors he consults.

Theorem 3 formalizes the above intuition.

**Theorem 3.** Suppose advisors can choose correlated investigations. When \( n \geq 2 \), let \( \tilde{v} \) and \( \tilde{u} \) be the advisors’ and the decision maker’s equilibrium payoffs, respectively. Then \( \tilde{v} > v^B \) and \( \tilde{u} = u \).

**Proof.** In equilibrium advisors can always achieve \( v^B \) by choosing correlated investigations that are all based on one, instead of \( n \), signals. This way the equilibrium outcome is equivalent to the one in the benchmark. However incorporating additional signals strictly increases \( \tilde{v} \), since with better knowledge of the true state, the advisors are able to conduct more biased investigations, keeping the decision maker still indifferent given \( m_H \), while strictly more inclined towards \( a_L \) given \( m_L \). Since the decision maker is indifferent whenever he chooses \( a_H \), Theorem 2 implies that his payoff is \( u \).

5 Discussion

The results of this paper suggest that if the advisors choose investigations independently, they have incentive to prevent the decision maker from consulting more than one advisors. The implication for the decision maker is more subtle. Whether he benefits from consulting multiple advisors depends on the type of equilibrium played. Although lacking a formal proof at the moment, numerical results suggest that given a fixed number of advisors, a more “tolerant” decision maker is in fact better off than a less tolerant one. For example, when there are 5 advisors conducting symmetric investigations, the decision maker obtains the highest equilibrium payoff if he is willing
to choose $a_H$ given 3 or more favorable outcomes out of 5 (instead of 4 or more, or 5). This implies that if the decision maker can commit to a strategy, he will commit to be the most tolerant. If so, he strictly benefits from consulting more advisors in general.

References


