Information Revelation in the Property Right Theory of the Firms

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Preliminary

Abstract

I incorporate revelation of asymmetric information through shared ownership (partnership) into the Property Right Theory of the firms. Shared ownership is optimal as a joint result of mitigating hold-up and inducing information revelation. Due to the incomplete contracting nature, partnership is incentive compatible if it induces a positive probability of truthful revelation within the relationship as well as when the relationship breaks. This off-the-equilibrium-path incentive compatibility results in the optimality of partnership even for the most efficient type of the informed party. Incentive to invest in the relationship-specific asset is then distorted downward as the hold-up concern is not efficiently mitigated. The level of shared ownership reflects the relative magnitude of the information rent effect and the hold-up effect.

Keywords: Information revelation, Hold-up, Property Right Theory, Shared ownership
JEL: D23, D82
1 Introduction

I incorporate truthful revelation of asymmetric information through the allocation of ownership structure into the Property Right Theory of the firms, in which investment in a relationship-specific asset as well as terms of trade between the buyer and the investor are not ex-ante contractible. The optimal ownership structure, in addition to its conventional role to mitigate the hold-up problem, also serve as an information transmission device.

Consider a seller investing in an asset that can be purchased and used by the buyer to produce a higher valued output. Neither the ex-ante investment and the ex-post surplus is contractible ex-ante. The players can only contract on the allocation of asset ownership and an upfront fixed payment, before investment and negotiation on terms of trade occur. If information is symmetric, traditional theories on ownership allocation of the asset or firm boundary are developed from two roots of the theories of the firms: the Property Right Theory and the Transaction Cost Economics. The former emphasize on how non-integration (investor-ownership) mitigates the holdup concern and stimulates ex-ante incentive to invest in the relationship-specific asset, assuming efficient ex-post bargaining on terms of trade, whereas the latter focus on how integration (buyer-ownership) governs costly ex-post opportunism with various asset specificity.

Suppose that the marginal value of asset is the buyer’s private information as she has better access to the output market. I recognize the allocation of asset ownership as an information transmission device in the incomplete contracting relationship with asymmetric information. To facilitate ex-post negotiation on the terms of trade, the allocation of asset ownership is designed as a menu of shared ownership allocation and upfront transfer such that the buyer truthfully reveals her information on the marginal value of asset. Without the ability to contract on the investment or the terms of trade, this truthful revelation is induced if the buyer is deterred from mis-reporting when trade occurs as well as when she turns down the trade. That is, there is an additional set of incentive compatibility constraints off the equilibrium path.

Non-integration, seller-investor being the sole owner of the asset predicted in the conventional Property Right Theory, is suboptimal for any marginal value of asset

\(^{1}\)For the Property Right Theory, please refer to the pioneer works of Grossman and Hart (1986) and Hart and Moore (1990), or a survey by Segal and Whinston (2013) for more information. For the Transaction Cost Economics, please refer to the seminal works of Williamson (1975, 2002), or a survey by Tadelis and Williamson (2013) for more information. Also refer to Holmström and Roberts (1998) for a revisit and Gibbons (2005) for a neat comparison of the two and their extended theories.
due to this incentive compatibility off the equilibrium path. Shared ownership (partnership) is optimal, with the share reflecting the tradeoff of mitigating hold-up and inducing information revelation. If the information rent effect is stronger than the hold-up effect in the sense that the marginal information rent relative to the marginal value of asset is diminishing in the marginal value of asset, it is optimal to have a lower level of integration (seller having a larger share) with a buyer who has a higher marginal value of asset, vice versa.

Optimality of shared ownership captures the share-holding behavior of the modern firm mergers and acquisitions. For instance, Facebook in 2014 bought Oculus VR, a virtual reality gaming company, for $400 million in cash and 23.1 million shares of Facebook stock; MediaTek Inc., a semi-conductor company, acquired 69% stake in NuCORE Technology Inc. via share swapping in 2007. In this paper, I study one possible explanation for the optimality of shared ownership: the joint effect of information transmission and hold-up.²

The paper is organized as the following. The model is delivered in Section 2, and a benchmark with symmetric information at the trade negotiation stage is discussed in Section 3. I characterize, with binary marginal value of asset, the fully information revealing ownership allocation (level of integration) as a result of the joint effects of information rent and hold-up in Section 4. Robustness to semi-truthful information revelation is shown in Section 5. Conclusion is made in Section 6.

1.1 Related Literature

The current paper is built upon two lenses of literature. First, it contributes to the recent works in the theories of the firms with incomplete contracting and asymmetric information, e.g. Baldenius (2006), Malcomson (1997), Matouschek (2004), and Schmitz (2006, 2008). The main departure of the current paper to this lens of literature is on the role of allocating asset ownership to induce truthful revelation. They assume independency between ownership structure and information at the ex-ante investment stage as well as at the ex-post negotiation stage. The informed party never reveals his information that facilitates ex-post negotiation, i.e. ex-post bargaining is inefficient. Their focus is thus on the optimal ownership structure to motivate ex-ante investment and to reduce the probability of ex-post disagreement. In this paper, by modeling the transaction cost associated with coordination as information trans-

²Aghion and Tirole (1994), Cao (2003), Dasgupta and Tao (2000), and Schmitz (2008, 2013) also recognize the optimality of joint/split asset ownership, with various explanations to that analyzed here, none of them on information revelation.
mission between the seller and the buyer, the optimal shared ownership reflects the tradeoff between mitigation of hold-up (the focus of the Property Right Theory of the firms) and reducing transaction cost from information asymmetry (the focus of the Transaction Cost Economics).

Bester and Krähmer (2012) also shows interest in truthful revelation in an incomplete contracting environment. Their contracting environment, however, is less incomplete than that in this paper. They emphasizes on how the ex-ante contractible truth revealing terms of trade induces efficient investment, which relies on the fact that the seller is able to design the optimal transfer payment subject to trade and that subject to failure of trade independently. Without ex-ante contractible terms of trade, the players’ payoffs when trade occurs and that when trade fails both depends on the ex-ante contractible ownership structure. I then show that truthful revelation is accompanied with inefficient investment through shared ownership. Goldlücke and Schmitz (2014) also studies information transmission in an incomplete contracting environment. They focus on, however, informed seller’s signaling incentive through the observable investment, which mitigates the hold-up problem. Allocation of asset ownership is assumed exogenous. I characterize truth revealing allocation of shared ownership prior to investment and ex-post negotiation, which eliminates information asymmetry in the bargaining stage at the expense of ex-ante investment due to inefficient mitigation of hold-up.

Truth revealing ownership or control is also studied in the recent literature of complete contract (Kuribko and Lewis (2010)), that of foreign direct investment (Raff, Ryan, and Stähler (2009), and Stähler (2005)), and that of financial contracting (Dessein (2005)). The difference between information revelation under an incomplete contracting framework as in the current paper and that under the above literature is on the contractibility of ex-post return or that of ex-ante action. With such contractibility in this lens of literature, participation can be induced by the contract, as well as incentive to take a certain action. Thus, it is sufficient to induce truthful information revelation within the relationship, and there is no concern of hold-up. In comparison, I study the information transmitting role of asset ownership in an incomplete contracting environment. Efficient mitigation of hold-up is traded off to induce truthful information revelation. This tradeoff is attributed to incentive compatibility both on and off the equilibrium path.

The optimal ownership structure as an information transmission device analyzed in this paper can be regarded as a positive response to the argument by Holmström and Roberts (1998, p.91) that the theory of the firm has been “too narrowly focused
on the hold-up problem and the role of asset specificity.” It can also be regarded as an endogenous solution to the observation by Riordan (1990) and Chou (2007) that the boundary of the firm is the boundary of information, as the boundary of the firm itself is an equilibrium result of truthful information revelation.

2 Model

A seller invest $I$ in an asset, which generates non-contractible but observable quality of asset $q(I) = I$, at a cost $c(I) = \frac{1}{2}I^2$. This asset can be a physical capital of production, the right and access to contract with subordinates, or any form of intermediate output used to produce a final output. The production of the final output requires not only the asset, but also the buyer’s human capital$^3$, along with the seller’s human capital to maintain the quality of the asset during production. The value of final output produced with the asset of quality $I$ is $\mu(\beta, I) = \beta I$ within the relationship, or $\nu(\theta, \beta, I) = \theta \beta I$ outside of the relationship, absent of either the seller or the buyer. The marginal value of asset, $\beta \in \mathcal{B}$ and $\beta > 0$, follows the prior probability distribution $F(\beta)$. $F(\beta)$ is common knowledge. The buyer observes the realization of $\beta$, and there is a delay for the seller to observe $\beta$, specified at the end of this section. The buyer has information advantage due to his access to the final output market. The seller has full bargaining power. Both players are risk neutral.

Assumption 1. The asset is relationship-specific, in the sense that $\theta < 1$.

For instance, as an initial investor, the seller’s human capital is more efficient than others in maintaining the quality of the asset, and the buyer’s human capital is more efficient than others in turning the asset into the final output. $\theta$ is common knowledge, with a lower $\theta$ indicative of a more relationship-specific asset. Relationship-specificity implies that trade is efficient under perfect information.

At the first stage of the game, before investment and trade, the buyer has private information in the realization of marginal value of asset. The seller and the buyer can contract on the allocation of ownership and the upfront transfer payment. Instead of polar ownership structures, I assume that the ownership can be shared between the players, with the seller owning $s \in [0, 1]$ of the asset, and the buyer owning $1 - s$ of the asset, or that it can belong to no one, $s = \phi$. Smaller $s$ implies a higher level of integration, with $s = 0$ corresponding to the case where the seller-investor is employed

$^3$We can consider the buyer endowed with resources to transform the intermediate output into the final output.
by the buyer, and $s = 1$ corresponding to the case where the seller-investor and the buyer are non-integrated players in the asset market. The share of ownership not only gives the shareholder the right to claim a share of residual payoff when the relationship breaks, but also the power to veto usage of the asset by the other owner, with higher share of asset indicating a larger veto power. This shared ownership is assumed based on the observation that modern firm acquisition is in the form of increasing the holding of the corporate stocks, and a larger share holder tends to have a larger power in the board of directors.\footnote{Other applications include and are not limited to partnerships in law and consulting firms, and joint child custody between the parents.}

The seller offers a menu of contract on ownership share and upfront transfer payment, $\{s(\beta), t(\beta)\}$, and the buyer accepts one of the options in the menu or reject all. If the buyer rejects the ownership contract, the asset ownership is undefined, in which case the reservation payoff of each player is normalized to zero. This is the only contract they can write ex-ante.

At the second stage, after the allocation of ownership, the seller invests in the asset at a cost $c(I) = \frac{1}{2}I^2$, anticipating the negotiation outcome at the next stage. Quality of the asset is observed by both players after investment.

At the third stage, the players negotiate the share of the final output value. If negotiation is successful, trade occurs and the seller receives $p$ while the buyer receives $\mu(\beta, I) - p$. In the case where the trade negotiation fails, the seller collects $s(\beta)$ of the outside value, i.e. $s(\beta)\nu(\theta, \beta, I)$, and the buyer collects $1 - s(\beta)$ of the outside value, i.e. $(1 - s(\beta))\nu(\theta, \beta, I)$.

Some of the literature on joint ownership have a different assumption on the payoff the share owners earn when they fail to reach a trade agreement: they earn zero payoff as each of the joint owners of the asset can veto the other owners from using the asset outside of the relationship. I do not follow this lens of literature for two reasons. First, as a partial residual payoff claimant (e.g. a shareholder), even if the asset is used outside of the relationship, the player still collects a share of the outside value. She then be at least weakly better off not to veto the use of the asset. Second, each player’s veto power is, intuitively, increasing in her share of ownership. One can thus regard this share of outside value as the probability that the other player fails to veto the use of asset outside of the relationship. In this sense, the payoff each player receives outside of the relationship is her expected payoff before the use of asset is vetoed or is failed to be vetoed. Another interpretation for the share of ownership as share of outside value is in the spirit of Cao (2003). Suppose that there is an active asset market. On break of relationship, the players sell the asset in the asset market and
share the revenue $\nu(\theta, \beta, I)$ according to the share of ownership. Even for assets with indivisible control right, the share of ownership can be seen as a mixed strategy, in which the seller owns the asset with probability $s$.

I will investigate two scenarios regarding the delay of information to the seller. If the information on the realization of $\beta$ is delayed until investment is made and before negotiation starts, e.g. the seller is able to learn the profitability of the asset from the investment activity, inducing information revelation at the stage of contracting is purely for investment. If the information on the realization of $\beta$ is delayed until ex-post negotiation and trade are completed, e.g. the seller is able to learn the profitability of the asset from negotiating with the buyer, inducing information revelation at the stage of contracting is for both investment and negotiation. I characterize how and under what conditions can truthful information revelation be induced by shared ownership under each scenario.

3 Symmetric Information at Negotiation

As a benchmark, suppose that at the trade negotiation stage, $\beta$ becomes common knowledge, yet the buyer’s message $\hat{\beta}$ is already announced and the ownership structure $s(\hat{\beta})$ is contracted. Given ownership structure $s(\hat{\beta})$, and the level of investment $I$ made in the previous stage, the seller makes a take-it-or-leave-it offer $p(\beta, s(\hat{\beta}), I)$ to be paid from the buyer. The asset being relationship-specific, it is ex-post efficient to trade if

$$p(\beta, s(\hat{\beta}), I) \in \arg \max_p p$$

subject to

$$\beta I - p \geq (1 - s(\hat{\beta}))\theta \beta I$$

That is, $p(\beta, s(\hat{\beta}), I) = (1 - \theta + s(\hat{\beta})\theta)\beta I$.

At the investment stage, anticipating trade negotiation, if the seller believes that the message $\hat{\beta}$ sent by the buyer is truthful, given the ownership structure $s(\hat{\beta})$, the seller invests

$$I(\hat{\beta}, s(\hat{\beta})) \in \arg \max_I p(\hat{\beta}, s(\hat{\beta}), I) - c(I)$$

$p(\hat{\beta}, s(\hat{\beta}), I)$ being linear in $I$ and $c(I)$ satisfying Inada condition, $I(\hat{\beta}, s(\hat{\beta}))$ solves the first order condition $(1 - \theta + s(\hat{\beta})\theta)\hat{\beta} = c'(I)$. With quadratic cost of investment, $I(\hat{\beta}, s(\hat{\beta})) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}$.

At the stage of contracting on ownership, the seller proposes a take-it-or-leave-
it menu of ownership structures and upfront transfer payments, \( \{s(\beta), t(\beta)\} \), such that the buyer is willing to accept the ownership allocation and truthfully reveal his information. \( \{s(\beta), t(\beta)\} \) satisfies the following individual rationality constraint given truthful revelation.

\[
\beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \geq 0 \quad \forall \beta \quad (IR^S)
\]

For information is symmetric in the trade negotiation stage, the buyer anticipates that trade would occur regardless of his report. \( \{s(\beta), t(\beta)\} \) is designed such that the buyer does not reveal \( \hat{\beta} \neq \beta \), anticipating trade ex-post.

\[
\beta \in \arg\max_{\hat{\beta}} \beta I(\hat{\beta}, s(\hat{\beta})) - p(\beta, s(\hat{\beta}), I(\hat{\beta}, r(\hat{\beta}))) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC^S)
\]

**Proposition 1.** If information is symmetric at negotiation, the optimal allocation of ownership \( s^S(\beta) \) has \( s^S(\beta) = 1 \) for all \( \beta \), with the buyer voluntarily revealing information.

**Proof.** Appendix A.1.

In equilibrium, the parties share the surplus after investment is made. They thus have congruent interest with regards to the information on the marginal value of asset at the stage of investment. Knowing that information would be symmetric in the trade negotiation stage, the buyer voluntarily reveal the true information to facilitate the seller’s investment decision. Therefore, optimal allocation of ownership is designed to mitigate the hold-up concern alone. Inducing information revelation does not distort the efficient allocation of ownership. Non-integration is optimal regardless of the buyer’s message. Conflicton of interests arises only when information is asymmetric at the trade negotiation stage, where the buyer has incentive to opportunistically reveal incorrect information to extract a higher share of surplus.

## 4 Truthful Information Revealing Ownership

If information is asymmetric at the trade negotiation stage, given the buyer’s reported message \( \hat{\beta} \), ownership structure \( s(\hat{\beta}) \), and level of investment \( I \), the seller makes a take-it-or-leave-it offer \( p(\hat{\beta}, s(\hat{\beta}), I) \) to be paid from the buyer. The asset being relationship-specific, it is ex-post efficient to trade if the buyer has truthfully revealed his private
information. $p(\hat{\beta}, s(\hat{\beta}), I)$ is then offered such that the buyer is willing to accept.

$$p(\hat{\beta}, s(\hat{\beta}), I) \in \arg \max_p$$

subject to

$$\hat{\beta}I - p \geq (1 - s(\hat{\beta})\theta)\hat{\beta}I$$

That is, $p(\hat{\beta}, s(\hat{\beta}), I) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}I$.

At the investment stage, anticipating trade negotiation, given the buyer’s reported message $\hat{\beta}$ and ownership structure $s(\hat{\beta})$, the seller invests

$$I(\hat{\beta}, s(\hat{\beta})) \in \arg \max_I p(\hat{\beta}, s(\hat{\beta}), I) - c(I)$$

$p(\hat{\beta}, s(\hat{\beta}), I)$ being linear in $I$ and $c(I)$ satisfying Inada condition, $I(\hat{\beta}, s(\hat{\beta}))$ solves the first order condition $(1 - \theta + s(\hat{\beta})\theta)\hat{\beta} = c'(I)$. With quadratic cost of investment $c(I) = \frac{1}{2}I^2$, $I(\hat{\beta}, s(\hat{\beta})) = (1 - \theta + s(\hat{\beta})\theta)\hat{\beta}$.

At the stage of contracting on ownership, the seller proposes a take-it-or-leave-it menu of ownership structures and upfront transfer payments, $\{s(\beta), t(\beta)\}$, such that the buyer is willing to accept the ownership allocation and truthfully reveal his information. That is, $\{s(\beta), t(\beta)\}$ satisfies the following individual rationality constraint given truthful revelation.

$$\beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \geq 0 \forall \beta \quad (IR^N)$$

The difference between contracting on shared ownership here and complete contracting on profit share is the commitment to trade. In the complete contracting framework, the contracting parties commit ex-ante to a sharing rule as well as to trade ex-post; thus, it is sufficient to induce truthful information revelation within the relationship. Under the current framework, asset ownership and its corresponding upfront payment are the only elements that can be contracted on ex-ante, and therefore, truthful information revelation must be induced within as well as outside of the relationship. That is, $\{s(\beta), t(\beta)\}$ is designed to deter the buyer from revealing $\hat{\beta} \neq \beta$ and accepting the seller’s trade offer, as well as from revealing $\hat{\beta} \neq \beta$ and seeking an outside trading partner.\(^5\)

\(^5\)Deterring this sort of “off-schedule deviation” has drawn attention recently in the mechanism design theory. Athey and Segal (2013, 2475-2477) considers a general dynamic mechanism design problem when the decision rule is non-enforceable in the sense that each agent can freely choose a non-participation decision, resulting in a reservation payoff independent of the decision rule; efficiency
\[
\beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \\
\geq \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC'_I)
\]

and

\[
\beta I(\beta, s(\beta)) - p(\beta, s(\beta), I(\beta, s(\beta))) - t(\beta) \\
\geq (1 - s(\hat{\beta}))\theta \beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) \quad \forall \hat{\beta} \neq \beta \quad (IC'_O)
\]

With interim individually rational terms of trade, \((IC'_I)\) and \((IC'_O)\) can be expressed by

\[
\beta \in \arg \max_{\beta} \beta I(\hat{\beta}, s(\hat{\beta})) - p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) - t(\hat{\beta}) \quad (IC_I)
\]

and

\[
\beta \in \arg \max_{\beta} (1 - s(\hat{\beta}))\theta \beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) \quad (IC_O)
\]

Take a preliminary look at \((IC'_I)\) and \((IC'_O)\). \((IC'_I)\) captures a similar effect to the ratchet effect found in the literature of sequential screening with only short-run commitment.\(^6\) A type-\(\beta\) buyer is susceptible to lie downward on the marginal value of asset at the stage of ownership contracting, so that he is able to extract rent later at the trade negotiation stage. \((IC'_O)\) is indicative of the take-the-money-and-run effect found in the same literature. A type-\(\beta\) buyer is prone to lie upward on the marginal value of asset at the stage of ownership contracting, and later walk away from trade with a high transfer. This latter effect stemmed from the consideration of the off-the-equilibrium-path incentive compatibility is attributed to lack of long-run commitment on subsequent actions posterior to information revelation. In the literature of sequential screening with short-run commitment, however, decision rule (here, the investment) is contractible, so that the hold-up problem is absent.

With a binary marginal value of asset, \(B = \{\underline{\beta}, \overline{\beta}\}, \underline{\beta} < \overline{\beta}, (f(\overline{\beta}), f(\underline{\beta})) = (\sigma, 1-\sigma)\), can be achieved with this modification to the dynamic mechanism design problem. Compte and Jehiel (2009), on the other hand, draw an inefficiency result in a static trade mechanism with correlated types in which each player’s outside payoff is his private information. In the former paper, the role of the optimal ownership structure to mitigate the hold-up problem is absent, and in the latter paper, the hold-up problem is not studied. That is, in both papers, information revelation is the only incentive problem that the contract is designed to solve. My focus here is to see the tradeoff of investment incentive due to inefficient mitigation of hold-up when inducing truthful information revelation.

\(^6\)Please refer to Bester and Strausz (2001) and Shin and Strausz (2014).
and $C = \{(s, \overline{s}), (s, \overline{t})\}$, the incentive compatibility constraints are expressed as

$$u = \overline{\beta}I(\overline{\beta}, s) - p(\overline{\beta}, s, I(\overline{\beta}, s)) - \overline{t} \geq \overline{\beta}I(\overline{\beta}, s) - p(\overline{\beta}, s, I(\overline{\beta}, s)) - \overline{t} \quad (IC^H_I)$$

$$u = \beta I(\beta, s) - p(\beta, s, I(\beta, s)) - t \geq \beta I(\beta, s) - p(\beta, s, I(\beta, s)) - t \quad (IC^L_I)$$

Or equivalently,

$$u - u \geq (\overline{\beta} - \beta)I(\overline{\beta}, s) \quad (IC^H_I)$$

$$u - u \leq (\overline{\beta} - \beta)I(\overline{\beta}, s) \quad (IC^L_I)$$

As shown by Lemma 1 below, $(IC^H_I)$ and $(IC^L_I)$ are the relevant binding incentive compatibility constraints, and it is without loss of generality to neglect the sufficient monotonicity constraints for incentive compatibility with binary marginal value of asset.

**Lemma 1.** *Truthful information revelation is induced if*

$$(\overline{\beta} - \beta)I(\overline{\beta}, s) \leq u - u \leq (\overline{\beta} - \beta)\theta(1 - s)I(\overline{\beta}, \overline{s}) \quad (IC^B)$$

**The sufficient monotonicity constraints for incentive compatibility**

$$I(\beta, s) \leq I(\overline{\beta}, s) \quad (M^B_I)$$

and

$$(1 - s)I(\overline{\beta}, s) \leq (1 - \overline{s})I(\overline{\beta}, \overline{s}) \quad (M^B_O)$$

*automatically holds given $(IC^B)$.*

**Proof.** Appendix A.2.

Non-integration with the efficient buyer is not incentive compatible as the inefficient buyer would have incentive to report upward and reject the take-it-or-leave-it offer at
the stage of trade to walk off with a higher rent and hold up the seller, i.e. \( \overline{s} = 1 \) violates \((IC_B')\), summarized and proven in Lemma 2.

**Lemma 2.** \( \overline{s} = 1 \) is not incentive compatible.

**Proof.** If \( \overline{s} = 1 \), for any \( 0 \leq s \leq 1 \), \( (\beta - \beta)I(\beta, s) > (\beta - \beta)\theta(1 - \overline{s})I(\beta, \overline{s}) = 0 \). The set of \( \overline{u} - u \) satisfying \((IC^B)\) is empty.

Incentive compatibility off the equilibrium path results in this distortion at the top, which differs from the prediction in the traditional complete contracting literature. Exploiting the terms, the seller implements the information rent at the stage where ownership is allocated to regulate the ratchet incentive, whereas a shared ownership is contracted with even the most efficient buyer to cope with the take-the-money-and-run incentive. In addition, there is no incentive compatible allocation of ownership if the following assumption is violated such that the set of \( \overline{u} - u \) satisfying \((IC^B)\) is empty.

**Assumption 2.** \( \sigma \beta < \beta \leq \theta \beta \).

Assumption 2 places condition on the value of inducing truthful information revelation relative to asset specificity such that an incentive compatible allocation of ownership exists. By rearrangement, it is equivalent to \( \frac{\sigma}{1 - \sigma} < \frac{\beta}{\beta - \beta} \leq \frac{\theta}{1 - \theta} \). That is, an incentive compatible ownership structure exists if the marginal information rent is relatively small (the first inequality) and the asset is not too relationship-specific (the second inequality).

With binding \((IC^B)\) and \( u = 0 \), the seller’s reduced optimization problem at the ownership allocation stage is then

\[
\mathcal{P}^D: \max_{\overline{u}, \overline{s}} \sigma (\beta I(\beta, \overline{s}) - (\beta - \beta)I(\beta, s) - c(I(\beta, \overline{s}))) + (1 - \sigma) (\beta I(\beta, \overline{s}) - c(I(\beta, s)))
\]

subject to

\[
I(\beta, \overline{s}) = \theta(1 - \overline{s})I(\beta, \overline{s})
\]

\[
I(\beta, s) = (1 - \theta + \theta s(\beta))\hat{\beta} \quad \forall \hat{\beta} \in \{\beta, \overline{\beta}\}
\]

**Proposition 2.** If Assumption 2 holds, the optimal truth revealing contract on ownership structure exhibits \( 1 > \overline{s}, \overline{s} \geq 0 \), and thus \( I(\beta, s(\beta)) < I(\beta, s^S(\beta)) \) for both \( \beta \) and \( \overline{\beta} \). \( \overline{s} \geq s \) if \( \sigma \beta \geq (1 - \sigma)\beta \).

**Proof.** Appendix A.3.  

\[\square\]
With the concern of truthful information revelation to facilitate ex-post bargaining efficiency, non-integration predicted in the conventional Property Right Theory of the firms no longer holds. Equilibrium level of investment is traded off as the hold-up concern is not efficiently mitigated to induce truthful information revelation. The optimal ownership structure and the level of integration reflects the tradeoff of efficient mitigation of hold-up for truthful information revelation. Specifically, shared ownership with the buyer of low marginal value is contracted to implement a lower information rent to the buyer of high marginal value, which is at the expense of investment incentive from inefficient mitigation of hold-up. Meanwhile, shared ownership with the buyer of high marginal value is contracted to deter the buyer of low marginal value from lying and walking away from the relationship. If the marginal information rent \((\frac{\sigma_1}{\sigma_2})\) is larger than the relative marginal value of asset \((\frac{\beta}{\bar{\beta}})\), the incentive cost from the hold-up effect is relatively mild to the effect of information rent. The optimal contract thus has \(s \leq \bar{s}\) to restrict the information rent given to the buyer of high marginal value, at the expense of investment efficiency with the buyer of low marginal value. It is optimal to have a higher level of integration with the less efficient buyer than that with the more efficient buyer.

This provides an explanation and characterization for shared ownership and partnerships to be optimal under the framework of Property Right Theory of the firm with asymmetric information, even with the presence of ex-post efficient negotiation.\(^7\) The optimal ownership structure to induce truthful revelation within as well as outside of the relationship has at least some degree of integration/partnership, \(s(\beta) < 1\). This is at the expense of ex-ante investment incentive, \(I(\beta, s(\beta)) < I(\beta, s^S(\beta))\), as the hold-up concern is not efficiently mitigated to induce truthful information revelation.\(^8\)

**Application: Length of Project and Firm Boundary**

The marginal value of asset, \(\beta\), can also be interpreted as the discount factor in a dynamic production process of the final output. Suppose that it takes a time sequence of efforts for the buyer to finalize the output with the asset and yield value \(I\). The marginal value of asset in the previous section is denoted as \(\beta(t)\) instead, with \(t\) being the length of time sequence of efforts that is the buyer’s private information. \(\beta(t) \in (0, 1)\) then discount the value

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\(^7\)Matouschek (2004) and Schmitz (2008) explained optimal joint asset ownership with asymmetric information by the presence of ex-post inefficient bargaining, without truthful information revelation ex-ante.

\(^8\)Goldläcke and Schmitz (2014) predicts an opposite trade-off. When the seller, who has private information on his outside option, has incentive to signal a higher outside option through higher investment, the equilibrium would exhibit higher level of investment, at the expense of ex-post inefficiencies when the buyer mistaken a true signal as a bluff.
of final output back to its present value prior to the negotiation stage.

\( \beta(t) \) measuring the discounted value of final product which takes \( t \) sequence of efforts for the buyer to produce. Let \( \beta(t) < \beta(\bar{t}) \) when \( t < \bar{t} \). A higher marginal value of asset corresponds to a shorter sequence of efforts required for production of final output. The model then predicts a testable implication that a longer-term (shorter-term) project is optimally accompanied with a higher (lower) level of integration, if the marginal information rent is larger than the relative discount rate.\(^9\)

**Corollary 1.** \( s(\beta(\bar{t})) > s(\beta(t)) \) if \( \sigma \beta(\bar{t}) \geq (1 - \sigma) \beta(t) \).

### 5 Discussion

The analysis can be criticized for its restriction to fully truthful information revelation. I address the concern in this section.

#### 5.1 Semi-Truthful Information Revealing Ownership

Bester and Strausz (2001) have shown the validity of the revelation principle in a general contracting problems with imperfect (including short-run) commitment, under which the agent (here, the buyer) tells both truth and lie with a positive probability. I claim that the optimality of partnership derived in Proposition 2 is qualitatively robust to semi-truthful information revelation.

Suppose that, at the ownership contracting stage, the seller induces the buyer to mis-report (report truthfully) his marginal value of asset with probability \( 1 - \alpha(\beta) \) \((\alpha(\beta))\), where \( \beta \in \{\underline{\beta}, \bar{\beta}\} \) is his true marginal value. To avoid cluster, define \((\underline{\alpha}, \bar{\alpha}) \equiv (\alpha(\underline{\beta}), \alpha(\bar{\beta}))\). If the contract is individually rational, the seller updates her belief on the buyer's marginal value of asset after the buyer's acceptance, consistent to the Bayes' rule. Let \( \mu(\underline{\alpha}, \bar{\alpha}) \) and \( \mu(\alpha, \bar{\alpha}) \) be the seller's belief that the buyer is the efficient type given his message \( \beta \) and \( \underline{\beta} \), respectively.

\[
\mu(\underline{\alpha}, \bar{\alpha}) = \frac{\sigma \underline{\alpha}}{\sigma \underline{\alpha} + (1 - \sigma)(1 - \underline{\alpha})}
\]

\[
\mu(\alpha, \bar{\alpha}) = \frac{\sigma(1 - \underline{\alpha})}{\sigma(1 - \underline{\alpha}) + (1 - \sigma)\underline{\alpha}}
\]

\(^9\)On the contrary, if the final output is more valuable in the future \( (\beta(t) > \beta(\bar{t})) \), the longer time sequence it takes for the buyer to finalize the output, the lower level of integration is optimal. A longer-term project is accompanied with a lower level of integration.
The seller’s sequentially rational strategy given the belief system is then a choice between making a trade offer and an investment decision based on the buyer’s message or not. That is, the seller’s sequentially rational trade offer and investment take one of the three forms: a separating trade offer contingent on the buyer’s message, a pooling trade offer neglecting the buyer’s message, or a hybrid trade offer as a mixed strategy of the above two.

**Lemma 3.** The seller’s sequentially rational strategy following the buyer’s acceptance of the ownership contract with message \( \hat{\beta} \in \{\_, \beta\} \) is

\[
\Sigma(\hat{\beta}) = \begin{cases} 
    p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) & \text{if } \overline{\mu}(\alpha, \overline{\alpha}) \in \Omega_S \\
    \gamma p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) + (1 - \gamma) p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) & \text{if } \overline{\mu}(\alpha, \overline{\alpha}) \in \Omega_M \\
    p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) & \text{if } \overline{\mu}(\alpha, \overline{\alpha}) \in \Omega_P 
\end{cases}
\]

where \( p(\hat{\beta}, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) = (1 - \theta + \theta s(\hat{\beta})) \hat{\beta} I(\hat{\beta}, s(\hat{\beta})), I(\hat{\beta}, s(\hat{\beta})) = (1 - \theta + \theta s(\hat{\beta})) \hat{\beta}, \gamma \in (0, 1), \) and \( \Omega_S, \Omega_M, \) and \( \Omega_P \) defined in Appendix A.4.

**Proof.** Appendix A.4. \( \square \)

We now discuss the strategies at the ownership contracting stage. We first suppose that the belief system satisfies \( \overline{\mu}(\alpha, \overline{\alpha}) \in \Omega_S \cup \Omega_M, \) such that it is sequentially rational for the seller to make a separating trade offer with a positive probability. We later argue that it is payoff-equivalent to restrict our attention to a separating trade offer with a positive probability.

Upon offered the ownership contract \( C = \{(\overline{\xi}, \overline{t}), (\xi, \xi)\}, \) the efficient buyer accepts \((\overline{\xi}, \overline{t})\) truthfully with probability \( \overline{\alpha}, \) and the inefficient buyer accepts \((\xi, \xi)\) truthfully with probability \( \alpha, \) which are sequentially rational if and only if

\[
\overline{\alpha} \in \arg \max_{\alpha} U \equiv \alpha \left[ \left( \beta I(\beta, s) - p(\beta, s, I(\beta, s)) \right) + (1 - \gamma) \left( \beta I(\beta, s) - p(\beta, s, I(\beta, s)) \right) - \overline{t} \right]
\]

and

\[
\alpha \in \arg \max_{\alpha} U \equiv \alpha \left[ \left( \beta I(\beta, s) - p(\beta, s, I(\beta, s)) \right) - \overline{t} \right]
\]

and

\[
+ (1 - \alpha) \left[ \left( \beta I(\beta, s) - p(\beta, s, I(\beta, s)) \right) - \overline{t} \right].
\]
If the optimal reporting strategies \((\alpha, \alpha)\) are not deterministic, it must be that

\[
\bar{u} - u = (\beta - \beta)I(\beta, s) + (1 - \gamma)\theta(1 - \bar{s})\beta \left( I(\beta, s) - I(\beta, \bar{s}) \right) \quad (IC^H)
\]

and

\[
\bar{u} - u = (\beta - \beta)\theta(1 - \bar{s})I(\beta, \bar{s}) + (1 - \gamma)\theta(1 - \bar{s})\beta \left( I(\beta, \bar{s}) - I(\beta, s) \right) \quad (IC^L).
\]

Comparing to the case in Section 4, with semi-truthful information revelation, there is a positive probability, \((1 - \alpha)\gamma\), that the seller is actually held-up by the buyer of low marginal value. In addition, \((IC^H)\) implements the difference of a truth-telling buyer’s rent when he accepts to trade, \(\bar{u} - u\), that accounts for the rent difference of a truth-revealing buyer with high marginal value of asset between a separating and a pooling trade offer, \(\theta(1 - \bar{s})\beta \left( I(\beta, s) - I(\beta, \bar{s}) \right)\). \((IC^L)\), on the other hand, implements \(\bar{u} - u\) that accounts for the rent difference of a lying buyer with low marginal value of asset between a separating and a pooling trade offer, \(\theta(1 - \bar{s})\beta \left( I(\beta, \bar{s}) - I(\beta, s) \right)\).

Combining \((IC^H)\) and \((IC^L)\), incentive compatible ownership contract has the property that

\[
(\beta - \beta)I(\beta, s) + (1 - \gamma)\theta(1 - \bar{s})\beta \left( I(\beta, s) - I(\beta, \bar{s}) \right) = (\beta - \beta)\theta(1 - \bar{s})I(\beta, \bar{s}) + (1 - \gamma)\theta(1 - \bar{s})\beta \left( I(\beta, \bar{s}) - I(\beta, s) \right) \quad (IC^M),
\]

which holds only if \(I(\beta, s) \geq I(\beta, \bar{s})\), i.e. the sequentially rational level of investment is monotonic.

**Lemma 4.** \(s = 1\) is not incentive compatible if a separating trade offer \(p(\hat{\beta}, I(\hat{\beta}, s(\hat{\beta})))\) with a positive probability \(\gamma > 0\) is sequentially rational.

**Proof.** From \((IC^M)\), \(s = 1\) is incentive compatible and implements \(0 < \alpha, \alpha < 1\) only if \(\alpha = u\) and \(I(\beta, s) = 0\). A sequentially rational \(I(\beta, s) = (1 - \theta + \theta s)\beta\) is strictly positive for all \(0 \leq \theta < 1\). \(\square\)

Partnership is optimal under semi-truthful information revelation. Non-integration (seller-own) can implement semi-truthful information revelation provided that the buyer accepts the trade offer, yet it introduces incentive for the buyer with low marginal value of asset to lie and walk away from the trade offer with probability one, given sequentially rational investment and trade offer.
The buyer is individually rational to accept the ownership contract if
\[
U = \alpha [u - (1 - \gamma) \theta (1 - \bar{s}) \overline{I(\overline{\beta}, \overline{s})} - I(\overline{\beta}, \overline{s})] + (1 - \alpha) [u + (\overline{\beta} - \beta) I(\overline{\beta}, \overline{s})] \geq 0 \quad (IR^H)
\]
and
\[
\overline{U} = \alpha u + (1 - \alpha) [u - (\overline{\beta} - \beta) \bar{\theta} (1 - \bar{s}) I(\overline{\beta}, \overline{s}) - (1 - \gamma) \theta (1 - \bar{s}) \overline{I(\overline{\beta}, \overline{s})}] \geq 0 \quad (IR^L).
\]

Given \((IC^H)\) and \((IC^L)\), \(\overline{U} > U\). It is then straightforward that \((IR^L)\) is binding in equilibrium; otherwise, increasing both \(t\) and \(\bar{t}\) by an arbitrarily small fixed amount improves the seller’s expected payoff without violation to any of the incentive compatibility and individual rationality constraints. If \(0 < \alpha, \alpha < 1\) is implemented, \(U = 0\), and \(\overline{U} = u + (\overline{\beta} - \beta) I(\overline{\beta}, \overline{s}) = (\overline{\beta} - \beta) I(\overline{\beta}, \overline{s})\). The seller’s optimization program at the ownership contracting stage is expressed as
\[
\mathcal{P}^M : \max_{\pi, \delta, \alpha_0} \left[ \pi \left( \gamma \left( \overline{\beta} I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \right) + (1 - \gamma) \left( I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \right) \right) + (1 - \alpha) \left( \overline{\beta} I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \right) - (\overline{\beta} - \beta) I(\overline{\beta}, \overline{s}) \\
+ (1 - \sigma) \left[ \alpha \left( \overline{\beta} I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \right) \right] + (1 - \alpha) \left( \gamma \left( \theta \beta I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \right) + (1 - \gamma) \beta I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \right) \right]
\]
subject to \((IC^M)\) and \(I(\hat{\beta}, s(\hat{\beta})) = (1 - \theta + \theta s(\hat{\beta})) \hat{\beta} \) for \(\hat{\beta} \in \{\beta, \overline{\beta}\}\).

We have restricted our attention to a separating trade offer with a positive probability so far. The following lemma indicates that there is a payoff-equivalent belief system to support this sequentially rational separating trade offer with a positive probability.

**Lemma 5.** For any pair of \((0, 0) < (\alpha^*, \alpha^*) < (1, 1)\) under the ownership contract \(C = \{(\pi, \overline{\pi}), (s, \overline{s})\}\) that solve \(\mathcal{P}^M\) such that \(\pi(\alpha^*, \alpha^*) \in \Omega_P\), there exists a pair of payoff-equivalent \((\overline{\alpha}^0, \alpha^0)\) such that \(\overline{\pi}(\overline{\alpha}^0, \alpha^0) \in \Omega_S \cup \Omega_M\).

**Proof.** Appendix A.5. □

Proposition 3 concludes the qualitative robustness of partnership implied in Pro-
Proposition 3. Shared ownership is optimal, i.e. \( s, \bar{s} < 1 \), when the buyer randomizes between revealing the truth and the false information.

Proof. This is implied by Lemma 4 and Lemma 5.

6 Conclusion

Allocation of asset ownership or the level of integration as an instrument to mitigate hold-up and incentivize non-contractible investment is well recognized in the Property Right Theory of the firm with incomplete contract. This paper is complementary to the literature by studying the information revealing role of the asset ownership. I recognize the optimality of shared ownership as a joint result of truthful information revelation and mitigation of hold-up. Non-integration even with the most efficient buyer is not incentive compatible under the incomplete contracting environment, for the privately informed buyer would have incentive to lie and seek outside opportunities. Equilibrium investment is traded off for truthful information revelation, as the hold-up concern is not efficiently mitigated. The optimal level of integration reflects the relative magnitude of the information rent effect and the hold-up effect. Lower level of integration with the efficient buyer is optimal if the information rent effect relative to the hold-up effect is stronger.
A Proof of Propositions

A.1 Proof of Proposition 1

With \( p(\beta, s(\hat{\beta}), I(\hat{\beta}, s(\hat{\beta}))) = (1 - \theta + s(\hat{\beta})\theta)\beta I(\hat{\beta}, s(\hat{\beta}), (IC^S) \) is equivalent to

\[
\beta \in \arg \max_{\hat{\beta}} u^S(\beta, \hat{\beta}) \equiv (1 - s(\hat{\beta}))\theta \beta I(\hat{\beta}, s(\hat{\beta})) - t(\hat{\beta}) \forall \beta \quad (IC^S)
\]

\( (IC^S) \) is satisfied only if \( \hat{\beta} = \beta \) solves the first order condition, \( \frac{\partial u^S(\beta, \hat{\beta})}{\partial \beta} = 0 \), and if the second order condition is satisfied, \( \frac{\partial^2 u^S(\beta, \hat{\beta})}{\partial \beta^2} \leq 0 \). Taking derivative of \( \frac{\partial u^S(\beta, \hat{\beta})}{\partial \hat{\beta}} \) with respect to \( \beta \) yields

\[
\frac{d}{\partial \beta} \left[ (1 - s(\hat{\beta}))I(\hat{\beta}, s(\hat{\beta})) \right] + \theta \frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, r(\hat{\beta}))]}{d\hat{\beta}} \bigg|_{\hat{\beta} = \beta} = 0. \quad \text{If} \quad \frac{d[(1-s(\hat{\beta}))I(\hat{\beta}, r(\hat{\beta}))]}{d\hat{\beta}} \bigg|_{\hat{\beta} = \beta} \geq 0,
\]

the second order condition holds. Thus, \( (IC^S) \) can be replaced by the local incentive compatibility constraint

\[
\frac{d}{d\beta} \left[ (1 - s(\hat{\beta}))I(\hat{\beta}, s(\hat{\beta})) \right] \bigg|_{\hat{\beta} = \beta} = 0 \forall \beta \quad (LIC^S)
\]

and the monotonicity constraint

\[
\frac{d}{d\beta} \left[ (1 - s(\hat{\beta}))I(\hat{\beta}, s(\hat{\beta})) \right] \bigg|_{\hat{\beta} = \beta} \geq 0 \forall \beta \quad (M^S)
\]

It is then straightforward that \( s(\beta) = 1 \) and \( t(\beta) = 0 \) for all \( \beta \) satisfy both \( (LIC^S) \) and \( (M^S) \). With the producer having all bargaining power, \( s(\beta) = 1 \) implements first-best investment with the producer claiming the entire surplus from trade. \( \{s(\beta) = 1, t(\beta) = 0\} \) is thus the optimal contract to induce information revelation.

\( \Box \)

A.2 Proof of Lemma 1

By construction, \( (\bar{\beta} - \beta)I(\bar{\beta}, s) \geq (\bar{\beta} - \beta)\theta(1 - s)I(\bar{\beta}, s) \) and \( (\bar{\beta} - \beta)I(\bar{\beta}, \bar{s}) \geq (\bar{\beta} - \beta)\theta(1 - \bar{s})I(\bar{\beta}, \bar{s}) \). Thus, \( C = \{(\bar{s}, \bar{t}), (s, t)\} \) is incentive compatible if \( (\bar{\beta} - \beta)I(\bar{\beta}, s) \leq \bar{u} - u \leq (\bar{\beta} - \beta)\theta(1 - \bar{s})I(\bar{\beta}, \bar{s}) \). It satisfies both \( (IC^H) \) and \( (IC^L) \) if \( (\bar{\beta} - \beta)I(\bar{\beta}, s) \leq (\bar{\beta} - \beta)I(\bar{\beta}, \bar{s}) \), which holds if \( (IC^B) \) holds as \( (\bar{\beta} - \beta)I(\bar{\beta}, \bar{s}) \geq (\bar{\beta} - \beta)\theta(1 - \bar{s})I(\bar{\beta}, \bar{s}) \).
It satisfies both \((IC^H_β)\) and \((IC^L_β)\) if \((β - β)θ(1 - s)I(β, s) ≤ (β - β)θ(1 - s)I(β, s)\), which holds if \((IC^B)\) holds as \((β - β)I(β, s) ≥ (β - β)θ(1 - s)I(β, s)\).

\[\square\]

A.3 Proof of Proposition 2

The set of \(\{s, s\}\) satisfying \((IC^B)\) is empty if \(\min s I(β, s) > \max_θ (1 - s)I(β, s)\) for any \(θ\), i.e. if \(β > \theta β\). With the second inequality of Assumption 2, the set of \(\{s, s\}\) satisfying \((IC^B)\) is not empty. The seller

\[
\max_{s, s} \sigma \left(βI(β, s) - (β - β)I(β, s) - c(I(β, s))\right) + (1 - σ) \left(βI(β, s) - c(I(β, s))\right)
\]

subject to

\[I(β, s) = θ(1 - s)I(β, s)\]

Let the Lagrange function be \(L = \sigma \left(βI(β, s) - (β - β)I(β, s) - c(I(β, s))\right) + (1 - σ) \left(βI(β, s) - c(I(β, s))\right) + λ \left(θ(1 - s)I(β, s) - I(β, s)\right)\), with \(λ\) being the Lagrange multiplier of the constraint. The objective function is concave in \(I(β, s)\) and in \(I(β, s)\), which are linear in \(s\) and \(s\) respectively, and thus the objective function is concave in \(s\) and in \(s\). The set of \(s\) and that of \(s\) satisfying \((IC^B)\) are convex as the left-hand-side of \((IC^B)\) is linear in \(s\) and the right-hand-side is concave in \(s\). Hence, if \(s\) and \(s\) have interior solutions, \((s, s, λ)\) solve the following optimality conditions and the binding constraint.

\[
\frac{∂L}{∂s} = σ \left(β - (1 - θ(1 - s))β\right) + λ \left(2θ(1 - s) - 1\right) = 0
\]

\[
\frac{∂L}{∂s} = (1 - σ) \left(β - (1 - θ(1 - s))β\right) - σ \left(β - β\right) - λ = 0
\]

\[1 - θ(1 - s)β = θ(1 - s)(1 - θ(1 - s))β\]

Plugging the latter two equations into the first yields \(σθ(1 - s)β - (1 - 2θ(1 - s))(β - σβ - (1 - σ)θβ(1 - s)(1 - θ(1 - s))) = 0\). \(s < 1\) solves this equation if \(β - σβ > 0\), i.e. if the first inequality of Assumption 2 holds. If \(s = 0\) is optimal, from the binding constraint, \((1 - θ)β = θ(1 - s)(1 - θ(1 - s))β\). At \(s = 1\), \((1 - θ)β > 0\), at \(s = 0\), \(β ≤ θ β\), and the right-hand-side is strictly concave in \((1 - s)\); thus, there exist a unique \(s ∈ (0, 1)\) such that the constraint is binding at \(s = 0\). If \(s\) and \(s\) have interior solutions, from the optimality conditions, \(s = 1 - \frac{λ}{σθβ + 2θλ}\), \(s = 1 - \frac{σ(β - β) + λ}{(1 - σ)θβ}\), and by construction \(λ ≥ 0\), \(s ≥ s\) if \(s = s\) if \(σθβ ≥ (1 - σ)θβ\), i.e.
if \( \frac{\sigma}{1-\sigma} \geq \frac{\beta}{\beta} \).

\[ \square \]

### A.4 Proof of Lemma 3

Following \( \hat{\beta} = \overline{\beta} \), by offering \( p(\overline{\beta}, I(\overline{\beta})) \) and investing \( I(\overline{\beta}, \overline{s}) \), with probability \( \overline{\mu}(\alpha, \overline{\sigma}) \) the buyer has a high marginal value of asset and accept the trade offer, and with probability \( 1 - \overline{\mu}(\alpha, \overline{\sigma}) \) he has a low marginal value of asset and walk away from the trade offer. In the former situation, the seller receives \( p(\overline{\beta}, I(\overline{\beta}, \overline{s})) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \), while in the latter situation, the seller receives \( \overline{\theta} \beta I(\overline{\beta}, \overline{s}) \overline{\theta} \beta I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2 \). By offering \( p(\beta, I(\beta, s)) \) and investing \( I(\beta, s) \), the buyer, regardless of his true marginal value of asset, accepts the seller’s trade offer. The seller receives \( p(\beta, I(\beta, s)) - \frac{1}{2} I(\beta, s)^2 \). It is sequentially rational for the seller to randomize between these two trade offers and investing \( I(\beta, s) \) for \( \hat{\beta} \in \{\beta, \overline{\beta}\} \) following the buyer’s acceptance of \( \overline{s} \) if \( \overline{\mu}(\alpha, \overline{\sigma}) (p(\overline{\beta}, I(\overline{\beta}, \overline{s})) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2) + (1 - \overline{\mu}(\alpha, \overline{\sigma})) (\overline{\theta} \beta I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2) > p(\beta, I(\beta, s)) - \frac{1}{2} I(\beta, s)^2 \). Define the set of belief \( \overline{\mu}(\alpha, \overline{\sigma}) \) that satisfy the above inequality as \( \Omega_s \). It is sequentially rational for the seller to make the pooling offer \( p(\beta, I(\beta, s)) \) and investing \( I(\beta, s) \) for \( \hat{\beta} \in \{\beta, \overline{\beta}\} \) following the buyer’s acceptance of \( \overline{s} \) if \( \overline{\mu}(\alpha, \overline{\sigma}) (p(\overline{\beta}, I(\overline{\beta}, \overline{s})) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2) + (1 - \overline{\mu}(\alpha, \overline{\sigma})) (\overline{\theta} \beta I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2) < p(\beta, I(\beta, s)) - \frac{1}{2} I(\beta, s)^2 \). Define the set of belief that satisfy the above inequality as \( \Omega_P \). It is sequentially rational for the seller to randomize between these two trade offers and investments following the buyer’s acceptance of \( \overline{s} \) if \( \overline{\mu}(\alpha, \overline{\sigma}) (p(\overline{\beta}, I(\overline{\beta}, \overline{s})) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2) + (1 - \overline{\mu}(\alpha, \overline{\sigma})) (\overline{\theta} \beta I(\overline{\beta}, \overline{s}) - \frac{1}{2} I(\overline{\beta}, \overline{s})^2) = p(\beta, I(\beta, s)) - \frac{1}{2} I(\beta, s)^2 \). Define the set of belief that satisfy the above equality as \( \Omega_M \).

Following the buyer’s acceptance of \( \overline{s} \), separating trade offer and pooling trade offer yields the same payoff to the seller, \( p(\beta, I(\beta, \overline{s})) - \frac{1}{2} I(\beta, \overline{s})^2 \), any randomization between the two is sequentially rational regardless of belief.

\[ \square \]

### A.5 Proof of Lemma 5

If \( C \) is such that \( I(\beta, \overline{s})^2 - \theta \overline{s} \beta I(\beta, \overline{s}) \leq 0 \), any belief \( 0 \leq \overline{\mu}(\alpha, \overline{\sigma}) \leq 1 \) is such that \( \overline{\mu}(\alpha, \overline{\sigma}) \in \Omega_s \cup \Omega_M \). If \( C \) is such that \( I(\beta, \overline{s})^2 - \theta \overline{s} \beta I(\beta, \overline{s}) > 0 \), \( \frac{I(\beta, \overline{s})^2 - \theta \overline{s} \beta I(\beta, \overline{s})}{I(\beta, \overline{s})^2 - \theta \overline{s} \beta I(\beta, \overline{s}) - \frac{1}{2} (I(\beta, \overline{s})^2 - I(\beta, \overline{s})^2)} < 1 \), there exists \( (\alpha^0, \alpha^0) \) such that \( \overline{\mu}(\alpha^0, \alpha^0) = \frac{\theta \overline{s} \beta I(\beta, \overline{s})}{\sigma \alpha^0 + (1-\sigma)(1-\alpha^0)^2} \geq \frac{I(\beta, \overline{s})^2 - \theta \overline{s} \beta I(\beta, \overline{s})}{I(\beta, \overline{s})^2 - \theta \overline{s} \beta I(\beta, \overline{s}) - \frac{1}{2} (I(\beta, \overline{s})^2 - I(\beta, \overline{s})^2)} \). As \( (\overline{\alpha}, \overline{\alpha}) \) solves \( \mathcal{M} \) and that \( \mathcal{M} \) is linear in \( (\overline{\alpha}, \overline{\alpha}) \), the seller is indifferent between
implementing $(\pi^*, \alpha^*)$ and $(\pi^0, \alpha^0)$. The buyer is indifferent between the two as well with $(IC^H)$ and $(IC^L)$ satisfied.

□

References


