Regulation and the Structure of Information: The Effects of Peer Monitoring on Capital Adequacy Regulation

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1 Introduction

This paper analyzes games of incomplete information in a regulatory context. We utilize game theoretic tools to highlight a cost effective framework that allows a regulator, e.g. a central bank, to achieve a second best outcome (with respect to the complete information optimum) implemented only through cheap talk communication. More importantly, such outcomes have the features that i.) the regulator learns the private information of the agents and ii.) the regulator extracts this information without engaging in costly monitoring nor having to threaten misbehavior with harsh punishments. We provide a simple example of how such a framework can be utilized to induce banks to hold higher levels of capital in the case when the regulators monitoring of the banks’ portfolio risk is imperfect. While game theoretic tools from mechanism design have been previously introduced in the capital adequacy regulation literature (see e.g. Kupiec and O’Brien (1997)) this is, to our knowledge, the first paper to depart from the mechanism design approach and explicitly make use of the communication equilibrium solution concept to achieve a second best outcome without the use of contracting, costly monitoring by the regulator, nor punishments for misbehavior.

Next, we show that while such a framework provides the potential for the central bank to implement a welfare improving outcome via cheap talk, the set of such achievable outcomes is highly dependent on the knowledge that each agent has about the other agents’ private information. This is particularly relevant when considering the network of interbank lending relationships. It is commonly assumed that interbank lending results in peer monitoring that helps to achieve market discipline (see Rochet and Tirole (1996)). Further, there is evidence that banks maintain long lasting

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lending relationships with each other in order to make use of economies of scale in monitoring and thus to obtain lower premiums on liquidity (see e.g. Cocco, Gomes, and Martins (2009) and Afonso, Kovner, and Schoar (2011)). Additional evidence illustrates that more centralized banks pay lower premiums in times of crisis (see Craig, Fecht, and Türmer-Alkan (2012)) and that banks are more likely to lend to each other in OTC markets when they have prior relationships (see Ashcraft and Duffie (2007)). One interpretation is that interbank lending and the peer monitoring that results as a consequence creates a transmission of information between banks which leads to less uncertainty regarding the solvency or riskiness of banks that you have prior relationships with.

We analyze how such information transmission affects the set of achievable equilibrium outcomes available to the regulator in the aforementioned framework. We show, for general games of incomplete information, that more information transmission between players leads to a smaller set of equilibrium outcomes and illustrate this with our example on capital adequacy regulation. This translates to the fact that the more connected the interbank lending market is, the harder it is for the central bank to achieve its most preferred outcome without engaging in costly monitoring or enforcement through punishment. Therefore, while interbank lending and peer monitoring can help to achieve market discipline, the consequence of private information transmission can have adverse effects on welfare via the second best outcome achievable by the central bank in this framework. One insight to be gained from this result is that banks with more interbank relationships have higher incentives to use their private information advantageously and thereby shirk from risk adjusted capital adequacy requirements.

2 Framework

We suppose that the policy of each bank (e.g. their capital adequacy ratio), their private information (e.g. the risk exposure of their portfolio), and the effect of these policies on other banks (e.g. through the increased probability of default or liquidity crisis) is encompassed in the Bayesian game \( G = (I, A, \Theta, P, (u_i)_{i \in I}) \) where \( I \) represents the set of banks, \( A \) their pure actions, \( \Theta \) the possible type of each bank, \( P \) the prior beliefs of banks regarding the types of the other banks, and \( u_i : A \rightarrow \mathbb{R} \) a mapping from action profiles to payoffs for bank \( i \in I \). For example, a banks’ type could consist of solvent or insolvent in which case \( \Theta_i = \{\text{solvent}, \text{insolvent}\} \) for each \( i \in I \). It is further assumed that the type of each bank \( i \in I \) is private information to the bank \( i \) unless it is monitored by some bank \( j \), in which case the bank \( j \) is also aware of bank \( i \)’s type. We suppose that the network of monitoring is given by \( \mathcal{N} = (I, E(\mathcal{N})) \) with vertex set equal to the set of players \( I \) and edge set \( E(\mathcal{N}) \subset \{ij : i \in I, j \in I\} \). The network \( \mathcal{N} \) the provides an implicit information structure with regards to the true profile of types \( \theta \) in that player \( i \) knows player \( j \)’s type \( \theta_j \) whenever \( ij \in E(\mathcal{N}) \). In the context of peer monitoring, \( ij \in E(\mathcal{N}) \) is used to represent the fact that player \( i \) monitors player \( j \)’s type in the network \( \mathcal{N} \).
We are interested in the game $G_c$ consisting of the game $G$ augmented by some arbitrary cheap talk preplay communication phase $c$ between players and the mediator (we will use the terms mediator and regulator interchangeably in what follows). We further denote by $(G_c, \mathcal{N})$ the game $G_c$ with the information structure given by the network of monitoring $\mathcal{N}$ and $B(G_c, \mathcal{N})$ the set of Bayesian Nash equilibrium of the game $(G_c, \mathcal{N})$. In what follows we will provide a relationship between $B(G_c, \mathcal{N})$ and $B(G_c, \mathcal{N}')$ for any networks $\mathcal{N}$ and $\mathcal{N}'$ such that $\mathcal{N} \subset \mathcal{N}'$. Namely, we will investigate the effects on the set of achievable equilibrium outcomes if more or less information is shared between players before playing the game $G_c$. First, we will show that the set $B(G_c, \mathcal{N})$ can be characterized as the set of communication equilibrium $\text{CO}(G, \mathcal{N})$ of the game $(G, \mathcal{N})$.\footnote{We drop the subscript $c$ given that this holds for all arbitrary communication schemes} This is the set of equilibrium characterized as state dependent distributions over outcomes such that players report their types to a trustworthy mediator (who we think of as the regulator) who then reports back to each player a suggested strategy that is optimal for them to play (this is the revelation principle for games of incomplete information introduced by Forges (1986) and Myerson (1986)). The set of communication equilibrium, as opposed to the set of Bayesian Nash equilibrium\footnote{Here we are referring to the set of Bayesian Nash equilibrium of the game $G$ without preplay communication.}, has a very tractable structure characterized by a set of linear inequalities which makes it an excellent candidate for applied problems such as the regulation of financial markets.\footnote{For a discussion on the computational tractability of communication equilibrium in comparison to the less tractable Nash equilibrium see Papadimitriou and Roughgarden (2008).} We then follow Myerson (1982) who shows that any generalized principle agent problem incorporating both adverse selection and moral hazard can be written as a Bayesian game with preplay communication (the outcomes of which are the set of communication equilibrium). In this context, it is assumed that the principle acts as a coordinator who chooses an equilibrium from the set of communication equilibrium outcomes, to maximize their preferences over outcomes, and the players optimally play their equilibrium strategies. We consider a similar situation except we assume that the principle does not take any actions (such as writing contracts to induce outcomes) other than choosing the equilibrium and assisting in the coordination by communicating with the players. While this framework could be extended to this case where the principle does write contracts, we would like to illustrate that the regulator can achieve welfare improving outcomes without having to write contracts in order to motivate the use of these game theoretic tools in regulatory settings.

Our main theoretical result states that for any two monitoring networks $\mathcal{N} \subset \mathcal{N}'$ the set of achievable outcomes in the game $(G_c, \mathcal{N}')$ is weakly smaller than the set of achievable outcomes in the game $(G_c, \mathcal{N})$. This translates to the fact that the set of communication equilibrium is weakly monotone decreasing in the network of monitoring. Our example on capital adequacy regulation will illustrate that this relationship can be strict. It is worth noting that this result bears resemblance to Morris and Bergemann (2014) who compare the set of achievable outcomes when players may have more or less information about a payoff relevant state. The main difference between these two results is
that we consider an environment where players communicate before playing the game with the regulator who aids in coordinating their actions. In Morris and Bergemann (2014) it is shown that the set of Bayesian Nash equilibrium of any game of incomplete information (without preplay communication) where players receive signals about a payoff relevant state is equivalent to the set of Bayes correlated equilibrium and provide conditions on different information structures that cause the set of Bayes correlated equilibrium to shrink. While these results are similar to what we obtain here, the environment and therefore applications of such results are fundamentally different.

We will now highlight these results and their application to interbank relationships and capital adequacy regulation with an example.

3 Capital Adequacy Regulation and Peer Monitoring

We suppose that there are two banks \( i = 1, 2 \) who hold ex-ante identical portfolios at time \( t = 0 \). We assume that these banks receive some additional information regarding the riskiness of their projects at time \( t = 1 \). For simplicity we assume that portfolios can only become riskier conditional on this information with regards to their ex ante risk. Given that we aim at modeling incentives to hold insufficient levels of capital in an imperfect information setting this assumption allows us to restrict attention to the most relevant cases (i.e. when banks receive either no news or bad news). We model this as the private information \( \theta_i \) of player \( i = 1, 2 \) lying within the set \( \Theta_i = \{ S, R \} \) where a bank of type \( S \) has a time \( t = 1 \) portfolio that has the same level of expected risk as the time \( t = 0 \) portfolio conditional on their time \( t = 1 \) information. A bank of type \( R \) has a riskier portfolio conditional on its new information when compared to its original expectation.\(^4\) We further assume that the probability of receiving bad information (i.e. that your portfolio is more risky) is \( q_R \) and independent between banks. In the context of the game theoretic model outlined above, this implies that \( P(R) = q_R \) and therefore \( P(S) = 1 - q_R \). We assume that \( q_R \) is small to capture the fact that bad news received by the banks is unexpected by the central bank or equivalently that it is too costly for the regulator to monitor for such unlikely events.

We assume here that this ex-interim information is private to each bank. The justification for such a structure of private information comes from the fact that regulators imperfectly monitor banks.\(^5\) Namely, once this information arrives there is a natural delay between the time that the bank learns this information and the time that the regulator does. Similarly, the current standards for measurement of a banks risk exposure, the Standardized Approach, the Internal Models Approach, come with inherent weaknesses that facilitate such information asymmetries (see Bliss (1995) for a detailed discussion). Further, the Pre-Commitment Approach of Kupiec and O’Brien (1997) allows the mediator to extract the private information of the banks (although in some cases this may fail; see Kobayakawa

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\(^4\)In this preliminary draft we assume there is only one level of risk that banks can update their beliefs to.

\(^5\)This could also be justified by the fact that banks in general specialize in the types of investments that they undertake and therefore have an informational advantage over the central bank.
(1998)) using a mechanism design approach, but such a mechanism may require imposing high penalties on banks (see Prescott (1997)). The approach that we take here will avoid the necessity to impose such penalties.

Now, we assume that at time $t = 0$ the banks expected portfolio risk is characterized by the state $\theta = (S, S)$ and that each bank maintains the central banks preferred capital adequacy ratio $L$ for portfolios of type $S$. At time $t = 1$ the banks, conditional on their new information, must make a decision as whether to maintain the same ratio of $L$ or to increase the ratio to the level $H$, which we assume is the required ratio for bank $i$ with portfolio risk characterized by the state $\theta_i = R$. Each bank $i = 1, 2$ therefore takes an action in the set $A_i = \{L, H\}$ upon learning of their type $\theta_i$. We assume that these actions are taken simultaneously and independently and give payoffs characterized by figure 1. We assume that $A_L > A_H$, $B_L > B_H$, and $C_L > C_H$ to encapsulate the fact that holding a lower level of capital is preferred when projects succeed, but not preferred when projects fail. Further, we assume that $B_{j} > A_{j}$ for $j = H, L$ to capture the fact that when banks fail together there are negative externalities as opposed to when banks fail individually. Here we wish to model the case where both banks fail together as a financial crisis scenario but could equivalently assume that their simultaneous failure creates a liquidity crisis or bank run that is not present when only a single bank fails. We assume that a portfolio of type $S$ fails with probability $p_S$ and a portfolio of type $R$ fails with probability $p_R > p_S$. Therefore, the utility of player $i$ given a type profile $\theta = (j, k)$ with $(j, k) \in \{S, R\} \times \{S, R\}$ can be written as follows.

$$u_i((H, H)|(j, k)) = u_i((H, L)|(j, k)) = (1 - p_j)p_kC_H - p_j(1 - p_k)A_H - p_jp_kB_H$$

$$u_i((L, H)|(j, k)) = u_i((L, L)|(j, k)) = (1 - p_j)p_kC_L - p_j(1 - p_k)A_L - p_jp_kB_L$$

Here, while the decisions of one bank does not effect the others payoffs, the type of the other bank does. In order for our model to be of interest we will make the following assumptions:

(A1) $S$ types strictly prefer the action $L$:

$$u_i(L|(S, k)) > u_i(H|(S, k)) \text{ for } k = S, R$$

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In future work we look to extend this model to the case where banks types are continuous and can choose any possible capital ratio.
This assumption simply states that when banks are safe they prefer to maintain the same capital adequacy ratio. Given that the central bank does not know each banks private information the regulator can only require them to hold a capital ratio no higher than $L$. Therefore, $S$ type banks will optimally hold this level as it is the lowest possible capital ratio they can hold within this regulatory framework.

\textbf{(A2)} $R$ types strictly prefer the action $L$ when the other banks’ type is $S$:

$$u_i(L|(R,S)) \geq u_i(H|(R,S))$$

This assumption is motivated by the idea that banks can free ride off of deposit insurance (or believe that they are too big to fail alone) unless there is a financial crisis in which case they may be sold off given their failure. This is also an important assumption that delineates the preferences of the banks and the regulator.

\textbf{(A3)} $R$ types strictly prefer the action $H$ when the other banks’ type is $R$:

$$u_i(H|(R,R)) \geq u_i(L|(R,R))$$

This last assumption is motivated by the idea that in times of crisis banks prefer to be insured from potential losses which are higher in expectation when both types are risky. This is justified by the fact that if both banks fail then there may be a liquidity crisis requiring a fire sale of assets or the closing of a solvent bank (the possibility of which is encapsulated in the parameter $B_L$).

Let us denote by $v_i(a_i|\theta_i) = E_{\theta_{-i}}[u_i(a_i|\theta_i, \theta_{-i})]$ the expected utility of bank $i$ who takes action $a_i$ and is of type $\theta_i$. Then we are interested in the case where $v_i(L|R) > v_i(H|R)$. Namely, given that you are a risky type, the expected gain from holding low capital and succeeding outweighs the expected loss of holding low capital and failing. Based on the previous assumptions, a sufficient condition for this case is when banks receive bad news with low probability \textit{\(i.e.\)} when $q_R$ is small). This is due to the fact that when the state is $\theta = (R, R)$ then both banks prefer to hold higher levels of capital. Thus, if they expect this situation to occur with low probability then they still prefer to maintain the low capital ratio to take advantage of the gains from when their projects succeed. Now, our assumption that safe types always prefer to take action $L$ implies that $v_i(L|S) \geq v_i(H|S)$ and therefore whenever $v_i(L|R) > v_i(H|R)$ there is a unique pure Bayesian Nash equilibrium where each player maintains the low capital ratio $L$ regardless of their type (hence our interest in this case). What we will now show is that there exists a mechanism that allows the regulator to induce banks of type $R$ to take action $H$ with strictly positive probability (dependent on the parameters of the game) as
opposed to the Bayesian Nash equilibrium where action $H$ is dominated by $L$ in expectation. Further, this mechanism only requires cheap talk messages to be sent between the central bank and the individual banks.

First let us describe the first best outcome for the central planner. Given that the ratios $H$ and $L$ are results of the maximization problem solved by the central bank prior to the start of this game, we know that the central bank’s first best outcome is when $S$ type banks choose a capital adequacy ratio of $L$ and $R$ type banks choose a capital adequacy ratio of $H$. In terms of a state dependent distribution over outcomes $(a_1, a_2)$ the first best is represented by the matrices of probabilities in table 1.

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<thead>
<tr>
<th></th>
<th>$H$</th>
<th>$L$</th>
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<tbody>
<tr>
<td>$H$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$L$</td>
<td>0</td>
<td>1</td>
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</tbody>
</table>

$(\theta_1, \theta_2) = (S, S)$

Table 1: The First Best Outcome

Namely, the central banks’ most preferred outcome is when $R$ types have a capital ratio $H$ with probability 1 and $S$ types a capital ratio $L$ with probability 1. What we will show is that there is a cheap talk mechanism that allows the mediator to achieve the distribution over outcomes illustrated in table 2

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<tr>
<td>$H$</td>
<td>0</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>$L$</td>
<td>$0$</td>
<td>$1-\alpha$</td>
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$(\theta_1, \theta_2) = (R, S)$

Table 2: The Second Best Outcome Without Peer Monitoring

for all $\alpha \leq \bar{\alpha}$ where

$$\bar{\alpha} = \frac{u_i(H|\theta_1=R,\theta_2=R) - u_i(L|\theta_1=R,\theta_2=R)}{u_i(L|\theta_1=R,\theta_2=S) - u_i(H|\theta_1=R,\theta_2=S)} > 0$$

is a threshold that determines the largest value $\alpha$ such that the above distribution can be implemented as a Bayesian Nash equilibrium of the cheap talk mechanism we will describe below. What this result states is that, using only cheap talk, the central bank can i.) induce each bank to reveal their private information, ii.) induce the risky bank to maintain the capital adequacy ratio $H$ with probability $\alpha \leq \bar{\alpha}$ when one bank is risky and the other safe, and iii.) induce both banks to hold the high level of capital with probability 1 in state $(R, R)$. Given that without intervention all players
maintain a capital ratio of $L$ implies that the second best outcome (without introducing regulator monitoring, contracts, or punishments) is when the mediator induces players to choose the capital level $H$ with probability $\bar{\alpha}$. Further, we can see that given the information asymmetry, this is a large improvement over the case where the central bank simply tells banks to maintain the appropriate capital adequacy ratio with respect to the riskiness of their portfolio. Again, this comes from the fact that in our case banks will never increase their capital conditional on learning privately the news of an increase in portfolio risk.

We will now illustrate the simple canonical communication mechanism that implements the above distribution. In the canonical mechanism players privately report their types to the regulator who then draws an action profile $a \in \{H, L\} \times \{H, L\}$ according to the second best distribution above and privately tells bank $i$ the component $a_i$ for $i = 1, 2$. For example, if the profile of the banks reports is $\hat{\theta} = (S, R)$, then the mediator will tell bank 1 to play $L$ with probability 1 and bank 2 to play $H$ with probability $\alpha$ and $L$ with probability $1 - \alpha$. What we will now show is that whenever $\alpha \leq \bar{\alpha}$ then reporting truthfully and playing the strategy suggested to them by the regulator is optimal for each bank. In other words, the second best distribution is a communication equilibrium of this game.

To see why following this mechanism is an equilibrium, suppose that both players report truthfully in the first stage of the canonical mechanism. If bank 1 reports $S$ he will always be told to play $L$ which is optimal for him whenever he is of type $S$. If bank 1 reports $R$ and is told to play $L$ then he knows for sure that the state of the world is $\theta = (R, S)$ (as he is never told to play $L$ when the state is $(R, R)$) and therefore playing $L$ is again optimal. Now, if bank 1 reports $R$ and is told to play $H$ then he believes that the state of the world is $\theta = (R, R)$ with probability $\frac{\alpha}{1 + \alpha}$ and that the state of the world is $\theta = (R, S)$ with probability $\frac{1}{1 + \alpha}$. Therefore, playing $H$ conditional on reporting $R$ is optimal whenever

$$\frac{\alpha}{1 + \alpha} u_1(H|(R, S)) + \frac{1}{1 + \alpha} u_1(H|(R, R)) \geq \frac{\alpha}{1 + \alpha} u_1(L|(R, S)) + \frac{1}{1 + \alpha} u_1(L|(R, R))$$

which is precisely the case whenever $\alpha \leq \bar{\alpha}$. Further, given that players are symmetric in this game, playing the strategy suggested to her by the mediator is always optimal whenever bank 2 reports optimally in the first stage of the mechanism.

Now, we only need to check that reporting truthfully is optimal for both banks. First, note that truthfully reporting that you are type $S$ is always optimal given that in this case the bank is always told to take their preferred action whenever this report is made. Therefore, we only need to check that no bank of type $R$ wishes to report that they are of type $S$. First, note that if bank 1 reports that it is of type $S$ when it is actually of type $R$, then it is always suggested to play strategy $L$ and therefore learns no additional information about the private information of bank 2. In this case

7Note that it is crucial that the regulator be able to commit to this mechanism.
if bank 1 plays $L$ it gets a payoff of $v_1(L|R)$ and if it plays $H$ it gets a payoff of $v_1(H|R)$. Therefore, given that we are concerned with the case where $v_1(L|R) > v_1(H|R)$ it is always optimal for bank 1 to choose $L$ upon reporting $S$ when $\theta_1 = R$. Now, if bank 1 reports truthfully, then we know that it is always optimal for him to play the strategy suggested to him, therefore the expected payoff of reporting truthfully when his type is $R$ is

$$(1 - q_R)[\alpha u_1(H|(R,S)) + (1 - \alpha)u_1(L|(R,S))] + q_R u_1(H|(R,R))$$

and therefore, reporting truthfully is optimal whenever

$$(1 - q_R)[\alpha u_1(H|(R,S)) + (1 - \alpha)u_1(L|(R,S))] + q_R u_1(H|(R,R)) \geq v_1(L|R).$$

If the central bank is interested in implementing the second best distribution where $\alpha = \bar{\alpha}$ then we can see that the above inequality is satisfied whenever $q_R \leq \frac{1}{2}$. Therefore, maintaining the aforementioned assumption that $q_R$ is small, we see that the second best outcome is achievable as a Bayesian Nash equilibrium of the canonical communication mechanism.

### 3.1 Network Monitoring

Now, we will turn to the case where, due to prior peer monitoring relationships, bank 1 knows the portfolio of bank 2 and therefore its private information. 8 Such a scenario could be one where bank 1 makes a sizable loan to bank 2 and therefore bank 2 gives bank 1 the right to monitor their portfolio. 9 Then we assume that banks are similar in that the information learned by bank 2 at time $t = 1$ could also be learned by bank 1, through direct monitoring. This scenario could be interpreted as the case where the information received by the banks at time $t = 1$ is public information, but that their expertise in investment allows them to properly interpret the effect of this information on their portfolio risk as opposed to the central bank. Further, in reality it should be the case that bank $i$ of equal skill to bank $j$ can correctly interpret the effect of the time $t = 1$ public information on another banks portfolio risk, but unless bank $i$ directly monitors bank $j$ there is an asymmetry of information between these banks with regard to the exact portfolio of bank $j$.

Now, if bank 1 directly monitors bank 2 we assume that they learn of their private information at time $t = 1$. In this case, the central bank can no longer induce bank 1 to take action $H$ with probability $\bar{\alpha}$ as whenever bank 1 is perfectly informed of the state he has a dominant strategy of taking action $H$ in state $(R, R)$ and action $L$ otherwise.

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8 We assume that the portfolios are ex-ante identical in their expected return and risk but not that they are per se identical.

9 Note here that no strategic considerations need to be made about bank 2 giving bank 1 the right to monitor them as the actions of bank 1 do not affect the payoffs of bank 2 in this model.
Hence, the second best outcome for the central planner in this case is described by the following distribution.

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<tbody>
<tr>
<td>H</td>
<td>0</td>
<td>0</td>
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<tr>
<td>L</td>
<td>0</td>
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There are a few comments that can be made with regards to this situation. First, the central planner is made strictly worse off when there is peer monitoring between banks. This is due to the fact that he can no longer induce bank 1 to hold higher levels of capital with positive probability when the state is $\theta = (R, S)$. Second, in the case without intervention, monitoring makes bank 1 and the central planner strictly better off. Namely, without intervention both banks choose action $L$ no matter the state. But, once bank 1 monitors bank 2 he will always choose action $H$ in the state $\theta = (R, R)$. Thus, peer monitoring improves welfare when there is no central bank intervention but harms welfare when there is central bank intervention. Finally, one insight that can be gained here is that when the central bank intervenes in financial markets, more centralized banks (with regards to banking network relationships) are more likely to use their private information to shirk from regulatory compliance. Therefore, if the regulator must allocate resources towards costly monitoring of banks, they should focus more of these resources on banks who are more centralized in the inter bank lending relationship network.

4 General Results

We will first note that the revelation principle for games of incomplete information (see Forges (1986) and Myerson (1986)) can easily be extended to the framework of network monitoring. Namely, any outcome in the set $S(G_c, \mathcal{N})$ where $c$ is some arbitrary communication scheme can be achieved by a canonical mechanism where players report their types and the types of the players they monitor to the mediator who then makes strategy suggestions to the players. To illustrate this extension, let us denote $C(\mathcal{N}, i) = \{j : ji \in \mathcal{N}\}$ the set of predecessors to player $i$ in the network $\mathcal{N}$, and by $D(\mathcal{N}, i) := \{j : ij \in \mathcal{N}\}$ the set of successors to player $i$ in the network $\mathcal{N}$. Then we can show that the Bayesian game $(G_c, \mathcal{N})$ can be rewritten into an equivalent Bayesian game $\tilde{G}_c$ (to which the revelation principle applies). Namely, we can think of a players type in the game $(G_c, \mathcal{N})$ to be both their true type and the types of the players they monitor.

Now, given the game $\tilde{G}_c$ has the same set of strategies for each player and the same payoffs, we see that $\tilde{G}_c$ and $(G_c, \mathcal{N})$ are equivalent games in terms of information and payoffs. Further, we know that revelation principle for games of incomplete information applies to the game $\tilde{G}_c$. Namely, any Bayesian Nash equilibrium of $\tilde{G}_c$ can be implemented as a Bayesian Nash equilibrium of $\tilde{G}_c$ where $\tilde{c}$ is the canonical protocol where players first report their
types to the mediator and then the mediator reports suggested actions to the players. What this implies is that the set of Bayesian Nash equilibrium of the game $\tilde{G}_c$ is equal to the set of communication equilibrium $CO(\tilde{G})$ which is characterized by a set of linear constraints. Namely $q : \tilde{\Theta} \to \Delta(A)$ is a communication equilibrium of the game $\tilde{G}$ if and only if

$$\sum_{\theta_i \in \tilde{\Theta}_i} \sum_{a \in A} P(\theta_i | \tilde{\theta}_i) q(a | \theta) u_i(a | \theta) \geq \sum_{\theta_i \in \tilde{\Theta}_i} \sum_{a \in A} P(\theta_i | \tilde{\theta}_i) q(a | \theta', \theta_i) u_i(\delta_i(a_i), a_{-i}, \theta)$$

for all $i \in I$, $\theta_i, \theta'_i \in \tilde{\Theta}_i$ and $\delta_i : A_i \to A_i$. Let us denote by $\Theta_{D(\mathcal{N}, i)} = (\Theta_j)_{j \in D(\mathcal{N}, i)}$ the set of monitor types of player $i$ with generic element $\theta_{D(\mathcal{N}, i)}$ and by $\Theta_{D(\mathcal{N})} = (\Theta_{D(\mathcal{N}, i)})_{i \in I}$ the monitoring type profile of the players $i \in I$ with generic element $\theta_{D(\mathcal{N})}$. Further, denote by $J(D(\mathcal{N})) = \cup_{i=1}^n D(\mathcal{N}, i)$ the set of monitored players under the network $\mathcal{N}$ and let $\theta_{J(D(\mathcal{N}))} = (\theta_i)_{i \in J(D(\mathcal{N}))}$. When the network $\mathcal{N}$ is clear from context we will simply write $D(i)$ and $D$ instead of $D(\mathcal{N}, i)$ and $D(\mathcal{N})$ respectively. To simplify the notation we will now introduce a reporting operator $R^D : \Theta_{J(D)} \to \prod_{i \in I} \Theta_{J(D)} \setminus D(i)$ such that $R^D(\theta_{J(D)}) = (\theta_{J(D)} \setminus D(i))_{i \in I}$.

We can see from the above conditions on the set of communication equilibrium of the game $\tilde{G}$ that any outcome that can be achieved as a Bayesian Nash equilibrium in the game $(G, \mathcal{N})$ can also be achieved by the canonical communication protocol $\tilde{c}$ where each player $i \in I$ reports both their type $\theta_i$ and their monitor type $R^D(\theta_{D(i)}) = \theta_{D(i)}$ and then the mediator sends suggestions to the players. Finally, denoting by $\theta_{D(i)} = (\theta_j)_{j \notin D(i)}$ as the set of types not monitored by player $i \in I$ we can see that the above inequalities translate to the fact that $q : \Theta \times \Theta_D \to \Delta(A)$ is a communication equilibrium of the game $(G, \mathcal{N})$ if and only if

$$\sum_{\theta \in \Theta} \sum_{a \in A} P(\theta | \theta) q(a | \theta, R^D(\theta_{J(D)})) u_i(a, \theta) \geq \sum_{\theta \in \Theta} \sum_{a \in A} P(\theta | \theta) q(a | (\hat{\theta}_i, \theta_{-i}), (\hat{\theta}_{D(i)}, R^D_{\setminus i}((\theta_{J(D)} \setminus D(i)))) u_i(\delta_i(a_i), a_{-i}, \theta)$$

for all $i \in I$, $\theta_i, \hat{\theta}_i \in \Theta_i, \hat{\theta}_{D(i)} \in \Theta_{D(i)}$ and $\delta_i : A_i \to A_i$. Here it is worth noting that player $i \in I$ computes his expected payoff under $q$ with respect to $\theta_{J(D)}$ due to the fact that he must take into account the types reported under $R^D_{\setminus i}((\theta_{J(D)} \setminus D(i)))$, but given that the other players report truthfully, then the only monitored information reported by the other players that he does not know is $\theta_{J(D)} \setminus D(i)$. Thus, noting that $\theta_{J(D)} = (\theta_{J(D)} \setminus D(i), \theta_{I \setminus J(D)})$ we can see that the above equation sufficiently computes player $i$’s expected payoff under $q$ given the monitored information that he does not know $\theta_{J(D)} \setminus D(i)$ and the reported information (i.e. the information of players types that they report of themselves) that he does not know $\theta_{I \setminus D(i)}$. 
What we will show is that if a network \( N \subset N' \) then the set of communication equilibrium outcomes \( CO(G,N) \) of the game \( (G,N) \) is strictly larger than the set of communication equilibrium outcomes \( CO(G,N') \) of the game \( (G,N') \) in a payoff equivalent manner which we will make precise below. Note that by the revelation principle this states that the players can achieve a larger set of Bayesian Nash equilibrium outcomes in the game \( (G,N) \) in comparison to the game \( (G,N') \) whenever \( N' \subset N \).

In what follows when we write the game \( (G,N) \) we assume that this is the game \( G \) with monitoring structure \( N \) augmented by the canonical communication protocol \( \overline{c} \) introduced above. Now, let us note that the sets \( CO(G,N) \) and \( CO(G,N') \) are fundamentally different objects in the sense that if \( N \) and \( N' \) are such that \( D(N,i) \subset D(N',i) \), then in any communication equilibrium \( q^{N'} \), player \( i \) must make \( |D(N',i)\cap D(N,i)| \) more reports to the mediator than in a communication equilibrium \( q^N \). Hence, in order to directly compare the sets \( CO(G,N) \) and \( CO(G,N') \) we must introduce a way to do so. In light of this, first note that any communication equilibrium \( q^N \) of the game \( (G,N) \) is implicitly coupled with the truthful reporting strategy where players report to the mediator their own type \( \theta_1 \) and their monitor type \( \theta_{D(i)} \). Now, we will define the operator \( \pi_N : A(G,N') \to A(G,N) \) for any two networks \( N \) and \( N' \) such that \( N \subset N' \) to be such that \( \pi_N \theta_{N'} \) is the projection of the strategy profile \( q^{N'} \) onto the strategy space of the game \( (G,N) \) which we denote by \( A(G,N) \). Namely, for any communication equilibrium \( q^{N'} \) of the game \( (G,N') \) and for any network \( N \subset N' \) we denote the projected reporting strategy as \( \pi_N(\theta, \theta_{D(i)}) = (\theta_i, \theta_{D(i)}) \). Namely, if \( D'(i) = D(N',i) \), then by definition player \( i \) truthfully reports \( (\theta_i, \theta_{D'(i)}) \) in the communication equilibrium \( q^{N'} \) and therefore reports \( (\theta_i, \theta_{D(i)}) \) under the projected strategy \( \pi_N \theta_{N'} \) (Note that whenever \( D(i) \subset D'(i) \) the projection \( \pi_N \) is well defined). Further, we can define the projected action strategy as

\[
\pi_N \theta_{N'}(\cdot|\theta, R^{D'}(\theta_{J(D')})) := \sum_{\theta_{J(D\setminus D')}} P(\theta_{J(D\setminus D')}|\theta_{J(D')}q_{N'}(\cdot|\theta, R^{D'}(\theta_{J(D')}), \theta_{J(D)}))
\]

for all \( (\theta, \theta_{J(D')} \in \Theta \times \Theta_{D'} \).

In what follows when we refer to a communication equilibrium \( q^{N'} \) we implicitly assume that it is coupled with its truthful reporting strategy \( (\theta, R^{D'}(\theta_{J(D')})) \) further when we refer to \( \pi_N \theta_{N'} \) we implicitly assume that this is the projected action strategy coupled with it’s projected communication strategy \( \pi_N(\theta, R^{D'}(\theta_{J(D')})) = (\theta_i, (\theta_{D(i)})_{i \in I}) \).

**Definition** (The operator \( \sqsubseteq \)): Let \( G \) be a Bayesian game with player set \( I \) and let \( N = (I, E(N)) \) and \( N' = (I, E(N')) \) be networks over \( I \) such that \( E(N) \subset E(N') \). Then,

1.) \( CO(G,N') \sqsubseteq CO(G,N) \) if and only if for all \( q^{N'} \in CO(G,N') \) we have \( \pi_N q^{N'} \in CO(G,N) \).

2.) \( CO(G,N) \sqsubset CO(G,N') \) if and only if for all \( q^N \in CO(G,N) \) there exists a \( q^{N'} \in CO(G,N') \) such that
3.) $CO(G, N) \equiv CO(G, N')$ if and only if $CO(G, N) \subseteq CO(G, N')$ and $CO(G, N') \subseteq CO(G, N)$.

These definitions allow us to describe when a set of (payoff equivalent) equilibrium distributions grows, shrinks, or remains equal with respect to the information of the players. Intuitively, if $CO(N', G) \subseteq CO(N, G)$, then the information structure induced by $N'$ creates more incentives and thereby weakly shrinks the set of achievable equilibrium distributions when compared to $N$. We can now proceed to our main result. Our main result states that this is the case whenever $N \subset N'$.

**Theorem 1** Let $G$ be a Bayesian game with common independent prior $P$ and let $N = (I, A(N))$ and $N' = (I, A(N'))$ be mutual information networks over the set of players $I$. If $N \subset N'$, then $CO(G, N') \subseteq CO(G, N)$.

This theorem states that if the network $N'$ is more informative that the network $N$ then the set of achievable outcomes decreases. In our example on capital adequacy regulation the set of communication equilibrium was described by the set of probability distributions of the following table

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<th>$H$</th>
<th>$L$</th>
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<tbody>
<tr>
<td>$H$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$L$</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

$(\theta_1, \theta_2) = (S, S)$

for all $\alpha \geq \bar{\alpha}$. Yet, as soon as bank 1 monitors bank 2, then the set of communication equilibrium shrinks to the set of distributions characterized by:

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>$H$</td>
<td>0</td>
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<tr>
<td>$L$</td>
<td>0</td>
<td>1</td>
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</tbody>
</table>

$(\theta_1, \theta_2) = (S, S)$

for all $\alpha \geq \bar{\alpha}$.

**Sketch of proof.** The proof proceeds in three steps. First we use the fact that for any network $N$, the set of communication equilibrium $CO(G, N)$ can be written as $\bigcap_{i \in I} CO_i(G, N)$ where $CO_i(G, N)$ are the set of distributions that constitute communication equilibrium for player $i \in I$. Then, we show that whenever $N \subset N'$ and $D(N, i) = D(N', i)$,
then $CO_i(G,N) \cong CO_i(G,N')$. This is illustrated in our example, where the incentives of bank 2 do not change conditional on bank 1 learning her information. Namely, the central bank can still induce bank 2 to take action $H$ with probability $\bar{\alpha}$ even after bank 1 learns of her information. This result states that this is always the case.

The second step is to show that whenever $N \subset N'$ and $D(N,i) \subset D(N',i)$, then $CO_i(N',i) \sqsubseteq CO_i(N,i)$. Namely, whenever player $i$ learns more information this creates more incentives to deviate and therefore the the set of correlated equilibrium distributions for player $i \in I$ becomes weakly smaller. Again, our example illustrates this case where upon learning bank 2’s type, bank 1 can no longer be induced to take a high level of capital in the state $\theta = (R, S)$. Denoting by $G$ the game from our example, let $q^N \in CO_1(G, ([1, 2], \emptyset))$ be the equilibrium that induces bank 1 to take action $H$ with probability $\bar{\alpha}$ in the state $\theta = (R, S)$ as described above when the network is $N = ([1, 2], \emptyset))$. Then we can illustrate our result by noting that any equilibrium $q^{N'} \in CO_1(G, ([1, 2], \{12\}))$ is such that bank 1 plays $L$ with probability 1 in the state $\theta = (R, S)$. Therefore for every equilibrium $q^{N'} \in CO_1(G, ([1, 2], \{12\}))$ is such that $\pi_N q^{N'}((H, L)|(R, S), S) = 0$ and therefore given that $q^N((H, L)|(R, S)) = \bar{\alpha}$ implies that for all $q^{N'} \in CO_1(G, ([1, 2], \{12\}))$ we have that $\pi_N q^{N'} \neq q^N$. In the proof we show that any equilibrium distribution from the set $CO_i(N',i)$ can be implemented as and equilibrium when the network is $N$ due to the fact that there are only more incentive constraints for player $i$ when he receives more information. Our example shows that the reverse inclusion does not hold.

Finally, we use the above results on how the set of equilibrium distributions for player $i \in I$ changes with respect to the networks $N$ and $N'$ coupled with the fact that $\sqsubseteq$ is a transitive operator to show that in general $CO(G,N') \sqsubseteq CO(G,N)$ whenever $N \subset N'$.

5 Conclusion and Future Work

The aim of this paper is two-fold: First, to highlight the use of game theoretic tools (other than mechanism design) as cost efficient methods to induce second best outcomes when there are information asymmetries in a regulatory framework. Second, we show that the second best outcome is sensitive to the information structure of the players with regard to the other players’ private information. We introduce a simple model of capital adequacy regulation when banks receive updated private information about the risk exposure of their portfolios. We show that, using only cheap talk, the regulator can induce banks to hold higher levels of capital when they privately learn that their portfolios are more risky than anticipated and otherwise would prefer to hold low levels of capital. Then, we show how interbank lending relationships and the transmission of private information from one bank to another can eliminate the regulators’ ability to induce the aforementioned second best outcome. One major insight to be gained here is that this theory indicates that banks with more interbank relationships (which can be proxied by interbank lending relationships) face
more incentives to use their private information advantageously to shirk from capital adequacy regulation and therefore should be more closely monitored by the central bank.

While these results lend insights into how network relationships effect the outcomes that a regulator such as a central bank can achieve, they are primarily preliminary and can be generalized in many directions. First, we would like to generalize our model of capital adequacy regulation by allowing banks to take choose any capital adequacy ratio and to endogenize the decision of the central bank when choosing the optimal levels of required capital ratios given different profiles of risk exposure. We further would like to include the potential for interbank relationships and monitoring to not just induce perfect information revelation of types but rather more informative signals about the banks risk type that you are monitoring. Finally, we would like to explore the case where the risk types of banks are correlated and to generalize this along with the aforementioned imperfect signaling from monitoring within our theoretical framework.

References


