Expert Advice for Multiple Audiences with Conflicting Interests∗

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Abstract

This paper examines a simple (repeated) cheap talk game between a single expert and two audiences with conflicting interests. The expert, who is informed about a payoff relevant parameter, sends an unverifiable message to the receivers. Conditional on the message they observe, the receivers simultaneously choose their actions, which collectively determine the payoffs of all three. The paper answers the following questions: How valuable and informative are the expert’s advice? Under what conditions is deception consistent with equilibrium? Furthermore, if the expert is a long-lived agent who also cares about the reliability of her messages in the long-term, then what makes the expert more or less deceptive?

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1. Introduction

Following the seminal work of Crawford and Sobel (1982), an extensive literature on cheap talk games has been attacking variations of an interesting problem where a sender (the expert) is better informed about the value of a payoff relevant parameter and a receiver’s choice, which is potentially a function of the expert’s unverifiable claims, determines the payoffs of both. A common theme in cheap talk models (whether or not there are multiple receivers) is that each receiver is facing a single person decision problem, and thus, the receiver’s payoff depends only on his own action. In that regard, the literature overlooks some eminent examples where the expert sends her messages to multiple audiences and the receivers’ choices collectively determine the payoff of all parties.

For example, a central bank may affect the exchange rate or inflation rate through (cheap talk) public messages, but the supply and demand sides of the market collectively determine these macroeconomic parameters. During their campaigns, political parties would give empty promises to their audiences that have possibly conflicting interests, but the final outcome (i.e., the winning party) is the result of the voters’ collective decisions. During a political conflict, international organizations such as United Nations and NATO may change the course of a conflict through their cheap talk messages/threats, but the outcomes of the conflicts depend on the actions of the conflicting states.

Clearly the rules and the structures of the games that the receivers play in the above examples, and in many others, vary. Nevertheless, this paper focuses on a particular canonical model that is fruitful enough to draw valuable conclusions: consider a simple strategic environment in which two agents have to make a choice between approving or rejecting a proposal (or a project) that will be implemented only if both agents approve it. Suppose that there is a conflict of interest between the agents: one of the agents (receiver 2) is uncertain whether the approval of the proposal will boost or sabotage his welfare, so he prefers (ex ante) to reject it, whereas the other agent (receiver 1) prefers ex ante to approve the project. The agents are not able to resolve their uncertainties before undertaking the project. However, they can acquire advice from an expert who is better informed about the consequences of the project.

Therefore, the expert has the power to manipulate the agents’ decisions through her

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1 Please see Sobel (2011) for a detailed literature review.
2 We may alternatively consider a case where there is an ex post conflict of interest between the agents. Although it would be an interesting case per se, it automatically introduces additional scenarios, requiring detailed investigations that do not arise in the current setup. Please see Section 2 for a more detailed discussion for this point.
advice, and she would use her authority to deceive the agents for short-term gains. On the other hand, the expert would be a long-lived player who also cares about the credibility of her advice in the future; thus, her long-term incentives would restrain the expert from deceptive communication. Given that the expert has conflicting short-term and long-term interests, possibly due to the conflicting preferences of her audiences, and her advice are unverifiable (i.e., cheap talk messages), then how valuable and informative (i.e., influential) would the expert’s advice be? Under what conditions is deception consistent with equilibrium? Furthermore, what makes the expert more or less deceptive in the long-term? To answer these questions, I first examine a three-player cheap talk (stage) game between an expert and two receivers. Then I study particular cases where the stage game is repeated indefinitely.

Answers to the questions that I aim to answer in this paper depend on two critical factors: the expert’s preferences and the life span of the receivers. The first preliminary question that we should answer is “Whose side is the expert on?” If the expert has incentives to reveal the truth (i.e., the expert’s preferences are more aligned with the preferences of the second receiver), then cheap talk will be fully revealing and influential: the expert never deceives the receivers, and the receivers approve the project whenever it is beneficial for both. However, the interesting case is the one in which the expert’s short-term preferences are more aligned with the first receiver. That is, the expert wants the receivers to approve the project regardless of the true state. In this paper, I focus my attention to the latter case. The results in the following sections show that the talk between the expert and the receivers is not influential unless the expert is a long-lived agent who wants her advice to be credible in the long-term or has a sufficiently high reputation for honesty.

Empirical evidences that motivate the model and the particular assumption on the expert’s preferences are widespread. For example, an appraisal, mortgage broker, insurance agent, financial adviser, or sales agent (i.e., the expert) is usually paid by the seller (i.e., receiver 1) rather than the buyer (i.e., receiver 2) even though the buyer is likely to have the greatest information deficiency. During the housing bubble, for instance, mortgage issuers earned their money on issuing the mortgage, not holding it. Therefore, mortgage issuers wanted higher appraisals to justify higher mortgages. This bias quickly got passed through to appraisers because they were paid on a commission and faced no penalties in

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3These questions are equally relevant for standard (two-player) cheap talk games where there is an expert and a single receiver. Furthermore, there are very few papers in the literature that investigate a repeated version of a two-player cheap talk game. By dropping one of the receivers, the canonical model that I use in this paper can be transformed into a simple two-player cheap talk game where the receiver has only two actions to choose. It is fairly simple to show that all the formal results in the following sections would hold in this reduced two-player cheap talk game as well.
the event of foreclosure. Hence, both the mortgage issuers and appraisers had incentive to oversell. In some cases, appraisers may have been deliberately misleading the borrowers. Consistent with this observation, the results in the following sections show that the expert may deceive the receivers even though the expert has incentives to be truthful because she values the reliability of her messages in the long run.

The second preliminary question that we should answer is “What is the nature of the relationship between the expert and her audiences?” As in the case of the previous example, the receivers would be a series of short-lived agents who play the cheap talk game with the expert once and for all. There are other examples, such as some workplace/political disputes and elections, where the receivers are long-lived agents just like the expert. In this paper, I examine both of these cases. The results indicate that the expert is more deceptive when the expert and the receivers are engaged in a long-term relationship. The expert would deceive the receivers for a long time because the receivers are willing to disregard the expert’s deceptions if the expert promises that she will be truthful in the future. However, the expert cannot tempt the short-lived agents with future rewards, so they are less vulnerable to the expert’s deception.

Briefly, the results in the following sections indicate that cheap talk between the expert and the receivers is influential if (1) the expert is a long-lived agent who also cares about the reliability of her messages in the long-term or (2) the receivers believe, with a sufficiently high probability, that the expert is the honest type who always tells the truth. Otherwise, the talk is not influential. Given that the expert is a long-lived agent, the expert is more deceptive when (1) receivers’ monitoring ability, concerning the expert’s past play, is lower, (2) the expected return from the project is higher, (3) the receivers are also long-lived agents, and (4) the expert’s initial reputation for honesty is higher.

Sections 2 and 3 explain the details of the cheap talk game and provide the equilibrium predictions of this stage game. Sections 4, 5, and 6 study the repeated cheap talk game and present the main results of the paper. In particular, Section 4 investigates the case where the receivers are short-lived agents. In this section, I characterize the expert’s highest equilibrium payoffs under different monitoring technologies. Section 5 provides important comparative statics results concerning the case where the receivers are short-lived players. Section 6 considers the repeated cheap talk game where all three players are long-lived agents. Finally, Section 7 concludes and discusses related literature.
A simplified and outcome equivalent version of the cheap talk game that I study in this paper is the following: Two receivers, 1 and 2, simultaneously decide to approve (a) or reject (r) a project. The first receiver’s payoff in case of the approval (i.e., \( w \)) is positive whereas the second receiver’s payoff is positive in the good state and negative in the bad state. Receivers do not know the true state but believe that it is bad with probability \( \pi \in (0, 1) \). I call \( \pi \) as the likelihood of the dispute. Suppose that \( \pi \) is high enough so that the second receiver’s expected return from the project is negative (i.e., \( (1 - \pi) v_2 - \pi v_1 < 0 \)). Therefore, second receiver’s ex ante choice is to reject the project.

In the cheap talk game, the expert either lies about the state or tells the truth. After observing the expert’s unverifiable message, the receivers update their beliefs and choose their actions. Regardless of the true state, the expert gets a payoff of \( v_e > 0 \) if both receivers approve the project. Otherwise, the expert gets the payoff of 0.

Equilibrium is called influential if both receivers approve the project with a positive probability. Cheap talk is influential if there exists an influential equilibrium of the cheap talk game. In an influential equilibrium, deception may occur if the expert lies when the true state is bad and both receivers approve the project.

The results in the following sections indicate that if the expert and the receivers play the cheap talk game only once, then there is no influential equilibrium. However, if the receivers believe that the expert is the honest type, who always tells the truth, with probability \( \mu \) and opportunistic (i.e., rational in the usual sense) with probability \( 1 - \mu \), then influential equilibrium exists whenever \( \mu \) is higher than

\[
\mu^* = 1 - \frac{(1 - \pi) v_2}{\pi v_1},
\]

a function of the primitives. However, this influential equilibrium is not fully revealing. That is, the expert deceives the receivers. Therefore, the threshold \( \mu^* \) indicates the level of trust the expert needs to possess in order to deceive the receivers.
In the infinitely repeated version of the cheap talk game, I first consider the case where receiver 2 is a series of short-lived agents who play the cheap talk game once and for all. Under the assumption that the receivers fully observe the past history of the repeated cheap talk game (i.e., perfect public monitoring), fully revealing equilibrium exists. That is, the expert is truthful at all times, and the receivers approve the project only when it is the good state. If the expert is known to be the opportunistic type, then the best equilibrium for the expert is in fact the fully revealing equilibrium. This suggests that deception is not consistent with equilibrium if the expert is known to be rational. However, if the receivers believe that the expert may be the honest type, then the expert can deceive the receivers in equilibrium.

When $\mu$ is positive, then short-lived receiver 2’s monitoring technology is the important determinant of the expert’s ability to deceive. Suppose, for example, that the short-lived receiver 2 cannot observe the expert’s past play. If the expert’s initial reputation $\mu$ is high enough (i.e., $\mu \geq \mu^*$), then in equilibrium, the expert can deceive the receivers at all stages and achieve the highest possible payoff in the repeated cheap talk game. On the other hand, if the expert’s initial reputation $\mu$ is lower than $\mu^*$, then there is no influential equilibrium and no deception in the repeated cheap talk game.

I also consider the case where the expert’s past play is observed partially by the short-lived agents with a rate $\beta \in (0, 1]$, interpreted as the accuracy of the receivers’ monitoring technology. The term $\beta$ can be thought of the probability that a short-lived agent truthfully and publicly reports his experience with the expert. When the expert’s initial reputation $\mu$ is lower than $\mu^*$, then the expert builds up her reputation to a level above $\mu^*$ in order to be able to deceive the receivers. Once she builds her reputation for honesty, she deceives the receivers until she gets caught lying. The shortest time required for the expert to build up her reputation is the smallest natural number $N_G$ satisfying

$$(\mu^*)^{N_G+1} \leq \mu.$$  

The time $N_G$ decreases with the expert’s initial reputation and with the expected return of the project. However, the expected number of stages that the expert should be truthful to build up her reputation is $\frac{N_G}{\pi \beta}$. Therefore, stronger monitoring decreases the expected time spent for reputation building.

Assuming that the monitoring technology $\beta$ is in $(0, 1]$ and the expert’s initial reputation is low (i.e., $\mu < \mu^*$), then for sufficiently high values of the discount rate $\delta < 1$, the
expert’s expected payoff is maximized when

$$\beta = \frac{N_G(1 - \delta)}{\delta \pi}.$$  

That is, (1) more patient expert prefers a weaker monitoring, (2) the expert with higher reputation prefers a weaker monitoring technology, and (3) the expert with a lower reputation prefers a stronger monitoring.

However, if the expert’s initial reputation $\mu$ is higher than $\mu^*$, such that the expert does not have to build up her reputation for honesty, then the expert can deceive the receivers until she gets caught lying. The expected number of incidents wherein the expert deceives the receivers in any equilibrium is at most $1/\beta$, a fairly small number especially when the monitoring accuracy is high.

All these results hold under the assumption that the receivers are short-lived agents. However, when the receivers and the expert are long-lived agents, with the same discount factor $\delta < 1$, then the expert can deceive the receivers for a very long time, even if they can perfectly observe the expert’s past play. I construct an equilibrium in which the number of incidents that the expert deceives the receivers is the smallest natural number $M$, satisfying

$$\mu^* \leq \left( \frac{\delta \pi}{1 - \delta(1 - \pi)} \right)^M.$$  

$M$ takes large values as the long-lived players are sufficiently patient. Hence, the expert would be more deceptive when the receivers are also long-lived agents.

Finally, when the receivers are short-lived, an influential talk must be “valuable.” To put it differently, the short-lived receivers are willing to pay the expert for her advice. However, influential talk does not have to be valuable when the receivers are long-lived agents. That is, the long-lived receivers may prefer to make no payment to the expert for her advice. A long time of deception is sustainable in equilibrium with long-lived receivers because the expert can promise them that she will be truthful for the rest of the game. However, the expert exploits the receivers so long that she pushes the receivers’ expected gain from the repeated cheap talk game down to 0. On the other hand, the expert cannot give long-term promises to the short-lived agents, and so cannot exploit them as much as she exploits the long-lived receivers. Therefore, an influential talk between the expert and the short-lived receivers must be valuable.
2. The Cheap Talk Game

In this section, I will introduce and examine the cheap talk game where the expert and her audiences interact only once. Sections 3 and 4 investigate the infinite horizon version of this cheap talk game.

The timing of the cheap talk game is as follows: At the beginning, the nature determines the state $s \in \Theta = \{\theta_1, \theta_2\}$. It is the bad state (i.e., $\theta_1$) with probability $\pi \in (0, 1)$ and the good state (i.e., $\theta_2$) with probability $1 - \pi$. The expert (she) observes the true state and then sends an unverifiable public message $m \in \Theta$. That is, the expert either tells the truth or lies. After observing the expert’s message, two receivers, 1 and 2, who do not know the true state, play the one-shot, simultaneous move game where each receiver either approves ($a$) or rejects ($r$) the project. The receivers’ payoffs are as follows:

<table>
<thead>
<tr>
<th>Player 1</th>
<th>Player 2</th>
<th>Player 1</th>
<th>Player 2</th>
</tr>
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<tbody>
<tr>
<td>$a$</td>
<td>$a$</td>
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<td>$a$</td>
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<tr>
<td>$w_1, -v_1$</td>
<td>$0, 0$</td>
<td>$w_2, v_2$</td>
<td>$0, 0$</td>
</tr>
<tr>
<td>$r$</td>
<td>$r$</td>
<td>$r$</td>
<td>$r$</td>
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<tr>
<td>$0, 0$</td>
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<td>$0, 0$</td>
<td>$0, 0$</td>
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</tbody>
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The real numbers $w_2, v_1,$ and $v_2$ are all strictly positive, while $w_1 \in \mathbb{R}$ can take any values. I will suppose that the parameters satisfy

\[
-\pi v_1 + (1 - \pi)v_2 < 0 \\
\pi w_1 + (1 - \pi)w_2 \geq 0
\]

so that receiver 2’s expected return from the project is negative, but receiver 1’s ex ante return is nonnegative. Thus, ex ante, receiver 2 prefers to reject the project whereas receiver 1 weakly prefers to accept it. On the other hand, the expert gets a payoff $v_e > 0$ if both receivers play $a$ and 0 otherwise. Therefore, the expert wants to implement the project regardless of the true state. Call this cheap talk game where all parameters are common knowledge $G$. The game tree of the game $G$ is given in Figure 1.

The current setup models the receivers’ conflicting interests via their ex ante preferences. Alternatively, the source of the conflict would be the receivers’ ex post preferences. Consider, for example, the case where there are three states in which the expected benefit of approving the project is negative for both receivers. For receiver 1, the project yields positive payoffs in the first two states and a negative payoff in the third state. However, for receiver 2, the project yields a negative payoff in the first state and positive payoffs in the second and the third states. This extension would introduce additional incentives for
the expert—whether or not the expert should send her messages publicly or privately—which are not the focus of this paper.

Figure 1: Game tree of the cheap talk game G.

Let $\sigma_e(\theta_i) \in [0, 1]$ denote the probability that the expert sends the message $\theta_1$ when she observes $\theta_i$. Therefore, $1 - \sigma_e(\theta_i)$ is the probability that the expert sends the message $\theta_2$ in case she observes state $\theta_i$. Given the expert’s strategies $\sigma_e(\theta_1)$ and $\sigma_e(\theta_2)$, receivers update their beliefs about the true state according to the Bayes’ rule. For $i, j \in \{1, 2\}$, let $P(s = \theta_i | m = \theta_j)$ indicate the receivers’ posterior probability that the true state is $\theta_i$ conditional on the event that the expert sends the message $\theta_j$. Thus,

$$P(s = \theta_1 | m = \theta_1) = \frac{\pi \sigma_e(\theta_1)}{\pi \sigma_e(\theta_1) + (1 - \pi) \sigma_e(\theta_2)}$$

$$P(s = \theta_2 | m = \theta_2) = \frac{(1 - \pi)(1 - \sigma_e(\theta_2))}{(1 - \pi)(1 - \sigma_e(\theta_2)) + \pi (1 - \sigma_e(\theta_1))}.$$

Receiver $i$’s mixed strategy $\sigma_i(m)$ is possibly a function of the message $m \in \Theta$ he receives. Therefore, $\sigma_i(m) \in [0, 1]$ indicates the probability that receiver $i$ plays $a$ when he observes message $m$. A strategy profile and beliefs constitute a perfect Bayesian equilibrium (or simply equilibrium) if (1) each player’s strategy specifies optimal actions, given his/her beliefs and the strategies of the other players, and (2) given the strategy profile, the beliefs are consistent with Bayes’ rule whenever possible.

**Definitions.** An equilibrium is fully revealing if the expert is truthful, that is, she sends the message $\theta_i$ if and only if the true state is $\theta_i$. An equilibrium is influential if both receivers approve the project with a positive probability. Communication is influential if there exists an influential equilibrium of the cheap talk game G.
A fully revealing equilibrium is not necessarily be influential. That is, in equilibrium, the expert would be truthful, but the receivers would reject the project regardless of the message they observe. Likewise, an influential equilibrium does not have to be fully revealing. In fact, deception occurs at influential but not fully revealing equilibrium. In the cheap talk game G, babbling equilibrium always exists. For example, the strategy profile where the expert sends message $\theta_2$ regardless of the true state and both receivers reject the project regardless of the message forms a babbling equilibrium. The next result proves that there is no influential equilibrium.

**Proposition 1.** There is no influential equilibrium of the cheap talk game $G$.

I defer the proofs of all the results to Appendix. In any equilibrium of the cheap talk game, all three players receive 0 payoff. That is, receivers always reject the project, and so deception never occurs. It is rather easy to see why there is no fully revealing and influential equilibrium. Suppose for a contradiction that there is an equilibrium in which the expert sends the message $\theta_i$ if and only if the true state is $\theta_i$, and the receivers approve the project only when they observe message $\theta_2$. Knowing the receivers’ strategies, the best response for the expert is to lie and send the message $\theta_2$ when the true state is in fact $\theta_1$. However, unlike the standard cheap talk games with a single receiver, a fully revealing equilibrium always exists in the cheap talk game $G$, but it is not influential. That is, in a fully revealing equilibrium, the receivers will always reject the project even though they know that the expert tells the truth. The reason for this bizarre result should be clear because rejecting the project is always a best response for a receiver given that his opponent also rejects.

**Remark 1.** There exists a fully revealing equilibrium of the cheap talk game $G$. However, the fully revealing equilibrium is not influential.
3. The Cheap Talk Game with Behavioral Types

In addition to all the details of the cheap talk game $G$, I now suppose that there are two possible behavioral types of the expert: *honest* and *opportunist*. The opportunist expert is a rational player in the usual sense. That is, she chooses her message, given her beliefs about the receivers’ play, to maximize her expected payoff. However, the honest expert always tells the truth: she sends the message $\theta_i$ if and only if the true state is $\theta_i$. Let $\mu \in [0, 1)$, the initial reputation of the expert, denote the probability that the expert is honest. Call this cheap talk game where all parameters are common knowledge $G_\mu$. Note that the games $G_\mu$ and $G$ are identical when $\mu = 0$. The game tree of $G_\mu$ (not including the receivers’ moves) is given in Figure 2.

![Figure 2: The modified cheap talk game $G_\mu$, not including the receivers’ moves.](image)

The next result proves that if communication between the expert and the receivers is influential, then it will certainly be deceitful.

**Proposition 2.** There does not exist a fully revealing and influential equilibrium of the modified cheap talk game $G_\mu$.

**Proof.** Given that the opportunist expert (or simply the expert) sends the message $\theta_1$ with probability $\sigma_e(\theta_1)$ when the true state is $\theta_i$, the receivers update their beliefs about the true state by the Bayes’ rule. Therefore, conditional on the event that the receivers observe the message $\theta_1$, the receivers’ posterior beliefs that the state is $\theta_i$ (i.e., $P(s = \theta_i|m = \theta_1)$) is

$$P(s = \theta_1|m = \theta_1) = \frac{\pi\mu + \pi(1-\mu)\sigma_e(\theta_1)(1-\pi)}{\pi(1-\mu)\sigma_e(\theta_1)(1-\pi) + \pi\mu + \pi(1-\mu)\sigma_e(\theta_2)}.$$

When the receivers observe $\theta_1$, they know that there are three possible scenarios: (1) True state is $\theta_1$, and the expert is honest. The probability of this event is $\pi\mu$. (2) True
state is $\theta_1$, and the expert is opportunistic but truthful. The probability of this event is $\pi(1 - \mu)\sigma_e(\theta_1)$. (3) True state is $\theta_2$, and the expert is opportunistic and lying by sending the message $\theta_1$. The probability of this event is $(1 - \pi)(1 - \mu)\sigma_e(\theta_2)$.

On the other hand, when the receivers observe the message $\theta_2$, then the possible scenarios are as follows: (1) True state is $\theta_2$, and the expert is honest. The probability of this event is $(1 - \pi)(1 - \mu)[1 - \sigma_e(\theta_2)]$. (2) True state is $\theta_2$, and the expert is opportunistic but truthful. The probability of this event is $(1 - \pi)(1 - \mu)[1 - \sigma_e(\theta_2)]$. (3) True state is $\theta_1$, and the expert is opportunistic and lying by sending the message $\theta_2$. The probability of this event is $\pi(1 - \mu)[1 - \sigma_e(\theta_1)]$. Therefore, the receivers’ posterior beliefs that the state is $\theta_i$ is

\[
P(s = \theta_2 | m = \theta_2) = \frac{(1 - \pi)\mu + (1 - \pi)(1 - \mu)[1 - \sigma_e(\theta_2)]}{(1 - \pi)\mu + (1 - \pi)(1 - \mu)[1 - \sigma_e(\theta_2)] + \pi(1 - \mu)[1 - \sigma_e(\theta_1)]},
\]

\[
P(s = \theta_1 | m = \theta_2) = \frac{\pi(1 - \mu)[1 - \sigma_e(\theta_1)]}{(1 - \pi)\mu + (1 - \pi)(1 - \mu)[1 - \sigma_e(\theta_2)] + \pi(1 - \mu)[1 - \sigma_e(\theta_1)]}.
\]

Now, suppose for a contradiction that there exists a fully revealing and influential equilibrium of the modified cheap talk game $G_\mu$. Because the expert is truthful, then $P(s = \theta_i | m = \theta_i) = 1$, so both receivers play $a$ only if the expert sends the message $\theta_2$. Thus, the expert’s payoff is 0 if the true state is $\theta_1$ and is $v_e$ if the true state is $\theta_2$. However, the opportunistic expert deviates and sends the message $\theta_2$ whenever the true state is $\theta_1$, contradicting with the optimality of the equilibrium.

The next result and its proof show that the opportunistic expert cannot deceive the receivers unless her initial reputation is sufficiently high. For high values of $\mu$, the influential equilibrium is not fully revealing but not babbling either. Although the opportunistic expert’s strategy is independent of the true state, the receivers’ strategy is a function of the signal they receive. That is, communication has some informational value. Note that regardless of the value of $\mu$, a babbling equilibrium always exists.

**Proposition 3 (influential communication and reputation for honesty).** Suppose that the expert’s initial reputation is high enough, that is, $\mu \geq \mu^* := 1 - \frac{(1 - \pi)v_2}{\pi v_1}$, then the unique influential equilibrium of the cheap talk game $G_\mu$ is deceitful (i.e., it is not fully revealing). More formally, the opportunistic expert always sends the message $\theta_2$; the receivers approve (and reject) the project when they observe $\theta_2$ (respectively $\theta_1$).

**Proof.** It is easy to show that the above strategies form a PBE equilibrium. Suppose that the opportunistic expert’s strategies are $\sigma_e(\theta_1) = \sigma_e(\theta_2) = 0$, then $P(s = \theta_1 | m = \theta_1) = 1$,
\[ P(s = \theta_2|m = \theta_2) = \frac{1 - \pi}{1 - \pi \mu}, \text{ and } P(s = \theta_1|m = \theta_2) = \frac{\pi(1 - \mu)}{1 - \pi \mu}. \] Therefore, when the expert sends the message \( \theta_1 \), the second receiver will choose \( r \) for sure. However, when the message is \( \theta_2 \), the second receiver chooses \( a \) if and only if \( \frac{\pi(1 - \mu)}{1 - \pi \mu} v_1 + \frac{1 - \pi}{1 - \pi \mu} v_2 \geq 0 \), which is true whenever \( \mu \geq 1 - \frac{(1 - \pi)}{\pi \mu} \). Likewise, given the second receiver’s strategies, the first receiver rejects when he observes \( m = \theta_1 \). Moreover, because \( \frac{1 - \pi}{1 - \pi \mu} > 1 - \pi \) is true, \( \frac{\pi(1 - \mu)}{1 - \pi \mu} w_1 + \frac{1 - \pi}{1 - \pi \mu} w_2 > 0 \) as well. Thus, approving the project is also a best response for receiver 2 when \( m = \theta_2 \). Thus, for high values of \( \mu \), receivers play their best response strategies. Moreover, regardless of the true state, the opportunistic expert receives a payoff of \( v_e \) under these strategies. Hence, her strategies are also optimal.

For the uniqueness of this equilibrium, as Proposition 2 indicates, there is no influential equilibrium in which the expert is truthful. It is easy to see that there is no influential equilibrium in which the expert lies and sends the message \( \theta_1 \) regardless of the true state. Therefore, all we need to show is that there is no influential equilibrium in which the expert randomizes over the messages \( \theta_1 \) and \( \theta_2 \) at some state realization. The proof of the last argument is almost identical to the proof of Proposition 1, and thus I omit.

The threshold \( \mu^* \) indicates the level of trust the expert needs to possess in order to deceive the receivers. It is important to note that \( \mu^* \) depends solely on the second receiver’s expected return from the project. The reason for this is simple. Even in a babbling equilibrium, where the communication between the expert and the receivers has no informational value, the first receiver prefers to approve the project. Therefore, if communication can convince the second receiver that it is highly likely that the state is good, and thus he will be better off by approving the project, then receiver 1 is already willing to approve the project. Thus, the expert’s main concern during the communication should be convincing receiver 2.
4. Repeated Cheap Talk Game with Short-Lived Receivers

This section studies the repeated cheap talk game with short-lived receivers. The timing of the repeated cheap talk game is as follows. The nature moves first and determines the expert’s type. The expert is either honest (with probability $\mu$) or opportunistic. The type of the expert is fixed throughout the repeated cheap talk game. Only the expert knows her type. At each stage $t \in \{0, 1, \ldots\}$ the expert and receivers 1 and 2 play the stage game $G$ that was given in Section 2. At the beginning of each stage, the nature determines the true state $s \in \Theta$, where $\pi$ is the probability that the state is $\theta_1$. The expert observes the state and sends her message $m \in M = \Theta$. After observing the expert’s message, both receivers simultaneously decide whether to approve or reject the project. At the end of the stage, the players obtain their stage game payoffs that are functions of the receivers’ actions and the true state of that stage. The payoff structure of the stage game $G$ was already given in Section 2.

The expert and receiver 1 are long-lived agents with discount factors $\delta < 1$. Thus, both the opportunistic expert and receiver 1 aim to maximize their discounted lifetime payoffs. The honest expert always tells the truth. Receiver 2, on the other hand, is an infinite sequence of different short-lived agents who play the stage game with the expert and receiver 1 only once (Fudenberg and Levine 1989). Therefore, each short-lived receiver 2 aims to maximize his expected payoff in the stage game he plays. The life span of receiver 1 has no impact on our results. That is, we can prove all the results in this section under the assumption that receiver 1 consists of an infinite sequence of different short-lived agents. Call this repeated cheap talk game where all parameters are common knowledge $G_\mu^\infty$.

I will suppose, for the rest of the paper, that both the expert and receiver 1 can perfectly observe the entire history of the repeated cheap talk game. In what follows, I will fix the long-lived agents’ monitoring technology and vary the short-lived receiver 2’s monitoring ability in order to investigate the relationship between the expert’s deception behavior and the detectability of her deceits. Note that the results in this section also hold when the expert perfectly observes the history while the receivers’ monitoring technologies are identical.

Let $\mathcal{H} = \bigcup_{t=0}^{\infty}(A^2 \times M \times \Theta)^t$ denote the set of histories for the first receiver and the expert. The long-lived agents learn (at the end of each stage) the actions of receiver 1 and 2, the message of the expert, and the true state. Therefore, a behavioral strategy of the opportunistic expert and receiver 1 are $\sigma_e : \mathcal{H} \times \Theta \rightarrow \Delta(M)$ and $\sigma_1 : \mathcal{H} \times M \rightarrow \Delta(A)$, respectively. Given any history $h^t$ (possibly a null history $h^0$), $\sigma_e(h^t, \theta_i)(\theta_j)$ denotes the
probability that the opportunistic expert sends the message \( \theta_j \) given that she observes state \( \theta_i \) after history \( h^t \). Similarly, \( \sigma_1(h^t, \theta_i)(a) \) denotes the probability that the expert approves the project given that he observes the message \( \theta_i \) after history \( h^t \). The strategy of the honest expert is simple: she reports the true state at any stage.

There are infinitely many short-lived agents (i.e., potential receiver 2), and they all observe the same public signal \( y_t \in Y \) at stage \( t \). \( Y \) is a finite subset of a metric space, and more structures will be imposed on \( Y \) later. All short-lived agents observe all the public signals sent before they enter the game. Therefore, a behavioral strategy for receiver 2 is \( \sigma_2 : \mathcal{H}_2 \times M \to \Delta(A) \) where \( \mathcal{H}_2 = \bigcup_{t=0}^{\infty} Y^t \).

If \( \{(s_1(t), s_2(t))\}_{t=0}^{\infty} \) is the sequence of actions played by receivers 1 and 2 and if \( \{\theta(t)\}_{t=0}^{\infty} \) is the sequence of state realizations throughout the game, then the expert’s payoff is \( \sum_{t=0}^{\infty} \delta^t u_2(s_1(t), s_2(t)|\theta(t)) \) and the first receiver’s payoff is \( \sum_{t=0}^{\infty} \delta^t u_1(s_1(t), s_2(t)|\theta(t)) \). Finally, the payoff of receiver 2 who enters the game at stage \( t \) is simply \( u_2(s_1(t), s_2(t)|\theta(t)) \).

A. Absence of a monitoring system for receiver 2

Suppose that receiver 2 (i.e., the short-lived agents) cannot observe the history of the repeated cheap talk game \( G_\mu^\infty \). This case would resemble situations where the short-lived agents learn the true state after a significant delay. Therefore, let \( Y = \{\emptyset\} \), and so receiver 2 always receives a null signal \( \emptyset \). In any PBE of the repeated cheap talk game, receiver 2 will always reject the project if he observes the message \( \theta_1 \). However, if receiver 2 observes the message \( \theta_2 \), we know from the discussions in the proof of Proposition 3 that he will reject the project when \( \mu \) is small enough (i.e., \( \mu < \mu^* \)), and prefer to approve the project if \( \mu \geq \mu^* \).

The last observation suggests that the expert’s payoff in any equilibrium of the repeated cheap talk game \( G_\mu^\infty \) is 0 if \( \mu < \mu^* \). However, when \( \mu \) is higher than \( \mu^* \), then the expert’s highest equilibrium payoff in the repeated cheap talk game is \( \frac{E}{1-\delta} \). The expert would achieve this payoff in an equilibrium where she always deceives the receivers and sends the message \( \theta_2 \). Deception forever is sustainable in equilibrium because (1) receiver 1 ex ante prefers approving the project even if he knows that the expert is opportunistic, and communication has no informational value (as in the case of a babbling equilibrium), and (2) receiver 2 does not observe the past play, and so cannot update his belief about the expert’s type.
B. The perfect monitoring system for receiver 2

Suppose now that the short-lived agents also observe the history perfectly. That is, they observe both receiver 1 and 2’s previous actions, the expert’s message and the state realizations at all stages. Therefore, $Y = A^2 \times \Theta^2$. Note that a babbling equilibrium, where the expert sends the same message regardless of the true state she observes and the receivers reject the project independent of the message they observe, always exists. In this equilibrium, the expert’s payoff is 0. However, there are other equilibria where the expert can do better than this.

The next result, for example, proves that a fully revealing and influential equilibrium always exists. Independent of the value of $\mu \in [0,1)$, there exists an equilibrium in which the opportunistic expert sends message $\theta_i$ if and only if the true state is $\theta_i$, and the receivers approve the project only when they observe the message $\theta_2$. A punishment strategy that is sufficient to support this fully revealing and influential equilibrium is simple: if any player deviates, then all three agents play their babbling equilibrium strategies for the rest of the game.

**Proposition 4 (fully revealing and influential equilibrium).** For any $\mu \in [0,1)$ and sufficiently large values of $\delta < 1$, there exists a fully revealing and influential equilibrium of $G^\infty$, in which the expert tells the truth at all stages and the receivers approve the project whenever they observe the message $\theta_2$.

In the unique fully revealing and influential equilibrium, the expert’s discounted expected payoff is

$$v^f = \frac{(1 - \pi)v_e}{1 - \delta}.$$  \hfill (2)

In any stage, the expert receives payoff $v_e$ when the true state is good and payoff 0 when state is bad. Intuition suggests that the expert can achieve payoffs higher than this by telling the truth when the true state is good and by deceiving the receivers when the true state is bad. However, this intuition is not entirely correct. It depends whether the expert fully or partially deceives the receivers. The distinction between full and partial deceptions are directly associated with the expert’s strategy. Conditional on observing the bad state, the expert partially deceives the receivers if she mixes the two messages and fully deceives the receivers if she sends the message $\theta_2$ with certainty.

Given this distinction, it makes more sense when and why the previous intuition would be correct. For example, the expert can achieve payoffs higher than $v^f$ if she can fully

---

4More formally, after any history $h^{t-1}$, I call that the expert fully deceives the receivers in stage $t$ if $\sigma_e(h^{t-1}, \theta_1)(\theta_2) = 1$ and partially deceives the receivers in stage $t$ if $\sigma_e(h^{t-1}, \theta_1)(\theta_2) \in (0,1)$.
deceive the receivers. However, it is not so clear if the same conclusion would hold if the expert partially deceives the receivers. The (proof of) next result shows that the expert cannot achieve payoffs higher than $v^f$ when she partially deceives the receivers.

For the rest of the paper, I will refer to full deception when I simply say deception. The next result shows that deception is not consistent with equilibrium when the opportunistic expert’s type is public information. Put it differently, telling the truth is the “best” strategy the opportunistic expert can follow in the repeated cheap talk game when the receivers know her type.

**Proposition 5 (highest payoff under full rationality).** Consider a history $h^t$ of the repeated cheap talk game $G^\infty_\mu$ in which the expert is known to be the opportunistic type. The expert’s payoff in any equilibrium of the continuation game following the history $h^t$ is no more than $v^f$.

The next two results prove that the expert can do better than being truthful and deceive the receivers in equilibrium when $\mu$ is positive. However, the critical level of reputation (i.e., $\mu^* = 1 - \frac{(1-\pi)v_0}{\pi v_1}$) plays a significant role. If $\mu \geq \mu^*$, then the expert can deceive the receivers without building further reputation for honesty. However, if $\mu < \mu^*$, then she first has to build up her reputation to be able to deceive the receivers.

I will first consider the case where the expert’s initial reputation is sufficiently high, that is, $\mu \geq \mu^*$. An equilibrium that yields the highest expected payoff to the expert in this case is as follows: The expert starts the game with deception. That is, she sends the message $\theta_2$ regardless of the true state until she gets caught lying. Because the receivers perfectly monitor the history, the expert can deceive the receivers in only one period. Once the expert gets caught lying, she tells the truth for the rest of the game. Receivers always approve the project when they observe the message $\theta_2$ and reject the project if the message is $\theta_1$. In the case of deviation, the players move to the punishment phase, where regardless of the message they observe, both receivers reject the project forever.

**Proposition 6 (deception w/o reputation building).** Suppose that $\mu \geq \mu^*$ holds. The expert’s expected payoff in any equilibrium of the repeated cheap talk game $G^\infty_\mu$ is no more than $v^H = v_\epsilon \left( \frac{1+\frac{\delta(1-\pi)}{1-(1-\pi)\delta}}{1-(1-\pi)\delta} \right)$.

Next, I will find the upper boundary for the expert’s equilibrium payoffs in the repeated cheap talk game when the expert’s initial reputation $\mu$ is not high enough, (i.e., $\mu < \mu^*$.) We know from the results in the previous section that the expert cannot deceive the receivers when her reputation for honesty is low. Therefore, in order to achieve payoffs
higher than \( v' \), the expert must deceive the receivers, for which the expert must build up her reputation first.

In equilibrium, the expert’s short-term gains from lying should be transferred into higher long-term gains. Reputation for honesty is the token that materializes this payoff transfer. If the receivers’ conjecture is such that the expert strictly prefers to tell the truth at some stage, then observing the expert telling the truth at that stage does not change the receivers’ belief about the expert’s actual type (i.e., the expert’s reputation); she simply does what she was expected to do. However, observing the expert telling the truth even though she strictly prefers to lie changes the receivers’ belief about the type of the expert. But this observation also proves that the receivers’ conjecture was wrong, and “equilibrium” dictates that the receivers must have right conjectures to begin with. Therefore, in equilibrium, the expert can build her reputation for honesty if the receivers have the right conjecture where the expert has incentives to lie and to tell the truth (i.e., she is indifferent between lying and telling the truth). Thus, if she lies, then she chooses to materialize her short-term incentives as expected. If instead she tells the truth, then she chooses to postpone her short-term gains for something higher in return, which is a higher reputation for honesty.

Because we are looking for an equilibrium strategy in which the expert’s payoff is the highest, the expert’s strategy should dictate her to tell the truth (1) when the true state is \( \theta_2 \), (2) with a sufficiently low probability when the true state is \( \theta_1 \) so that her reputation is updated quickly, and (3) with a sufficiently high probability when the true state is \( \theta_1 \) so that the receivers approve the project when they observe the message \( \theta_2 \). The reason why the first condition should hold is obvious. In the next proposition, I prove that the expert prefers to play a strategy in which she can build her reputation as fast as she can. Given that the expert tells the truth at the first stage with probability \( \sigma_e(\emptyset, \theta_1)(\theta_1) \), her reputation following a history \( h^1 \), where the state is \( \theta_1 \) in the first stage and the expert tells the truth and sends the message \( \theta_1 \), is \( \mu_1 = \frac{\mu + (1 - \mu)\sigma_e}{\mu + (1 - \mu)\sigma_e} \) according to the Bayes’ rule. The expert can deceive the receivers in the second stage only if her updated reputation is higher than \( \mu^* \) (i.e., \( \mu_1 \geq \mu^* \)). Thus, the expert can update her reputation to the required level \( \mu^* \) in only one stage if the following holds:

\[
\sigma_e \leq \frac{(1 - \mu^*)\mu}{\mu^*(1 - \mu)}
\]

As for the third condition, if the expert’s strategy \( \sigma_e \) is very low (so the expert lies with a very high probability when the true state is \( \theta_1 \)), then she can build up her reputation at the very first stage, where the true state is \( \theta_1 \), by telling the truth. But if the expert’s
initial reputation \( \mu \) is low, then the message \( \theta_2 \) in the first stage is very likely to be the opportunistic expert’s deceit, and thus the receivers may prefer to reject the project even though they observe the message \( \theta_2 \). Thus, the expert should be telling the truth with a sufficiently high probability if she wants to receive positive stage game payoffs while building up her reputation. More formally, receiver 2’s expected payoff of approving the project conditional on the event that he observes the message \( \theta_2 \) is

\[
EU_2(a|m = \theta_2) = -v_1 \frac{\pi(1 - \mu)(1 - \sigma_e)}{\pi(1 - \mu)(1 - \sigma_e) + (1 - \pi)} + v_2 \frac{(1 - \pi)}{\pi(1 - \mu)(1 - \sigma_e) + (1 - \pi)}.
\]

whereas receiver 2’s expected payoff of rejecting the project when he receives the message \( \theta_2 \) is simply

\[
EU_2(r|m = \theta_2) = 0.
\]

Hence, receiver 2 prefers to approve the project if

\[
\sigma_e \geq 1 - \frac{v_2(1 - \pi)}{v_1 \pi(1 - \mu)} = \frac{\mu^* - \mu}{1 - \mu}.
\]

holds. Recall that, if receiver 2 prefers to approve the project, then receiver 1 also prefers to approve the project. Thus, inequality (4) guarantees that approving the project is an optimal for the receivers when they observe the message \( \theta_2 \).

For some values of the primitives, in particular when \((\mu^*)^2 \leq \mu\) holds, the inequalities (3) and (4) can hold simultaneously. In this case, the expert’s best equilibrium dictates that she must tell the truth when the state is \( \theta_1 \) only once with probability \( \sigma_e = \frac{\mu^* - \mu}{1 - \mu} \).

Naturally, for some values of the primitives, in particular when \((\mu^*)^2 > \mu\), the inequalities (3) and (4) do not hold simultaneously. In these circumstances, the next proposition proves that in the expert’s best equilibrium, the sufficiently patient expert will build up her reputation gradually (i.e., in more than one stage).

Next, I will calculate the shortest time that is required for the expert to build her reputation gradually up to the critical level \( \mu^* \). Recall that we consider cases where \( \mu < \mu^* \). Let \( \mu_0 = \mu \) and for all \( t \geq 0 \) define \( \sigma_e^t = \frac{\mu^* - \mu}{1 - \mu} \) and \( \mu_{t+1} = \frac{\mu}{\mu_t + (1 - \mu_t)\sigma_e^t} \) recursively. The term \( \sigma_e^t \) represents the probability that the expert tells the truth conditional on observing the state \( \theta_1 \) for the \( (t + 1) \)th time, and \( \mu_t \) represents the expert’s updated reputation given that the receivers observe the true state \( \theta_1 \) for the \( t \)th time.

Therefore, if the expert observes the bad state \( n \) times and tells the truth at all times according to \( \sigma_e^t \)'s as given above, then her reputation reaches \( \mu_n = \frac{\mu}{\mu + (1 - \mu)\Pi_{t=1}^{n-1}\sigma_e^t} \). The expert will stop building up her reputation whenever \( \mu_n \geq \mu^* \) holds, which is equivalent to \( \Pi_{t=1}^{n-1}\sigma_e^t \leq \frac{\mu^* - \mu}{(1 - \mu)\mu^*} \). Hence, the shortest time (i.e., the smallest number of stages) required for the expert to build up her reputation to \( \mu^* \)—while the receivers prefer to approve the
project conditional on observing the message $\theta_2$—is defined by

$$
N_G = \min \left\{ n \in \mathbb{N}_+ \mid \prod_{t=0}^{n-1} \sigma_e^t \leq \frac{\mu(1 - \mu^*)}{(1 - \mu)^2} \right\}.
$$

(5)

**Lemma 1.** $N_G$ is the smallest natural number satisfying $(\mu^*)^{N_G+1} \leq \mu$. Or, equivalently

$$
N_G = \min \left\{ n \in \mathbb{N}_+ \mid \ln \frac{\mu}{\ln \mu^*} - 1 \leq n \right\}.
$$

Note that $N_G$ increases with $\mu^*$ but decreases with $\mu$. Therefore, if the expert’s initial reputation (i.e., $\mu$) is higher, then she needs less time to build up her reputation. If the level of trust the expert needs to possess in order to deceive the receivers (i.e., $\mu^*$) is higher, then the expert needs more time to build up her reputation. Recall that $\mu^*$ is positively correlated with $v_1$ and $\pi$, but negatively related with $v_2$. Therefore, if the expected return of the project is lower (i.e., farther from 0), then $\mu^*$ is higher, and thus, the expert needs more time to build up her reputation.

**Proposition 7 (deception with reputation building).** Suppose that $\mu < \mu^*$ holds. The expert’s expected payoff in any equilibrium of the repeated cheap talk game $G^\infty_\mu$ is no more than $v^L = (1 - \alpha_G)v^f + \alpha_G v^H$, where $\alpha_G = \left[ \frac{\pi \delta}{1 - (1 - \pi) \delta} \right]^{N_G}$ and $N_G$ is the shortest time required for the expert to build up her reputation to $\mu^*$.

Note that $\alpha_G$ is in $(0, 1)$, and so the expert’s expected payoff is a convex combination of two payoffs. The term $v^f$ is the payoff of truthful (opportunistic) expert and the term $v^H$ is the payoff of deceitful expert who does not need to build her reputation. The expert can do better than telling the truth even if building reputation takes so much time. But, for higher values of $N_G$ (i.e., the expert requires longer times to build her reputation), the parameter $\alpha_G$ takes smaller values, and so the expert’s highest equilibrium payoff gets closer to the truthful expert’s payoff.

**C. An imperfect monitoring system for Receiver 2**

All the results in subsections A and B are true even if receiver 2 (i.e., a short-lived agent) observes only his predecessors’ payoff realizations throughout the game. In equilibrium, given the expert’s and receiver 1’s strategies, which are used to prove the results in the previous subsections, receiver 2 can perfectly deduce the expert’s previous messages and the true states only by observing receiver 2’s previous payoffs. I will not prove the
previous results with the imperfect monitoring technology that I will introduce in this
section. But, I will prove the rather general versions of the previous results under this
monitoring system.

In this section, I will consider an imperfect monitoring system for receiver 2. Suppose
that the public information at the start of stage $t$ is $h^t = (y^0, ..., y^{t-1})$. For any $t \geq 0$, $y^t \in Y = \{-v_1, v_2, 0, \emptyset\}$. By $y^t = \emptyset$, I mean that receiver 2 observes no information about
the stage $t$ play. One can interpret that the short-lived agent who plays the game with the
expert and receiver 1 in stage $t$ does not share his experience with the other short-lived
agents. For any $t$, $y^t = \emptyset$ with probability $1 - \beta$ and

$$ y^t = \begin{cases} 
- v_1, & \text{if } (a^t_1, a^t_2, s^t) = (a, a, \theta^1) \\
v_2, & \text{if } (a^t_1, a^t_2, s^t) = (a, a, \theta^2) \\
0, & \text{otherwise.} 
\end{cases} $$

with probability $\beta$, where $a^t_i$ and $s^t$ denote receiver $i$’s action and the true state in stage
$t$, respectively.

The term $\beta$ measures how strong or weak the short-lived agents’ memory about the
expert’s past behavior. Higher values of $\beta$ ensures that receiver 2 will be better informed
about the expert’s past play. The two extreme cases where $\beta \in \{0, 1\}$ are already con-
sidered in subsections A and B. Therefore, for the rest of this section, I will assume that
$\beta \in (0, 1)$.

For any values of $\beta \in (0, 1)$, and for sufficiently large values of the discount factor $\delta$, there exists a fully revealing and influential equilibrium. By being truthful, the expert
can achieve the payoff of $v^f$. In fact, it is the highest payoff the expert can achieve in
the repeated cheap talk game if $\mu = 0$ (Lemma 3 in Appendix). However, the expert can
achieve higher payoffs if $\mu > 0$. To achieve higher payoffs, the expert must deceive the
receivers and send message $\theta_2$ even when the true state is $\theta_1$.

**Proposition 8 (imperfect monitoring and deception w/o reputation building).**
Suppose that $\mu \geq \mu^*$ holds and receiver 2’s monitoring technology is $\beta \in (0, 1)$. The
expert’s expected payoff in any equilibrium of the repeated cheap talk game $G^\infty_\mu$ is no more
than $v^H_\beta = v_e \left[ \frac{1 + \frac{\delta \beta (1 - \pi)}{1 - \pi \beta \delta}}{1 - (1 - \pi \beta) \delta} \right]$.

Note that the expert’s payoff $v^H_\beta$ decreases with the monitoring technology $\beta$. That
is, higher $\beta$ results in lower values for $v^H_\beta$. Moreover, if $\beta = 1$, then $v^H_\beta = v^H$, where $v^H$ is
the upper boundary for the expert’s equilibrium payoffs under perfect monitoring. The
strategy profile yielding the payoff \( v_H^H \) is almost identical with the strategy that yields the payoff \( v_H \). The expert starts the game with deception. That is, she sends the message \( \theta_2 \) regardless of the true state until she gets caught lying. Once the expert gets caught lying, she tells the truth for the rest of the game. Receivers always approve the project when they observe the message \( \theta_2 \) and reject the project if the message is \( \theta_1 \). In case of a deviation, the players move to the punishment phase if receiver 2 observes this deviation, and they stay in the punishment phase forever. In the punishment phase, both receivers reject the project regardless of the message they observe.

**Proposition 9 (imperfect monitoring and deception with reputation building).**

Suppose that \( \mu < \mu^* \) holds and receiver 2’s monitoring technology is \( \beta \in (0, 1) \). The expert’s expected payoff in any equilibrium of the repeated cheap talk game \( C_\mu^\infty \) is no more than \( v_L^\beta = (1 - \alpha_\beta) v_I + \alpha_\beta v_H^\beta \), where \( \alpha_\beta = \left[ \frac{\pi_\beta \delta}{1 - (1 - \pi_\beta) \delta} \right]^{N_G} \) and \( N_G \) is the shortest time required for the expert to build up her reputation to \( \mu^* \).

The expert’s equilibrium strategies in which the expert achieves the payoff of \( v_L^\beta \) have three layers: The expert starts the game in reputation building phase. She sends the message \( \theta_2 \) with certainty when the true state is \( \theta_2 \), and with some positive probability that is less than 1 when the true state is \( \theta_1 \). When the expert’s reputation reaches a level above \( \mu^* \) (which happens in \( N_G \) stages in which receiver 2 observes the expert telling the truth and sending the message \( \theta_1 \)), the expert moves to the deception phase. In that phase, the expert sends the message \( \theta_2 \) regardless of the true state. The deception phase ends when receiver 2 observes the expert’s deceit. Once the expert gets caught lying, she moves to the honesty phase where she tells the truth for the rest of the game.

### 5. The Implications of The Equilibrium Strategies

In this section, I will provide some comparative statics results. First, I want to understand how \( v_L^\beta \) changes with \( \beta \). That is, what level of monitoring technology (i.e., \( \beta \)) maximizes the expert’s (highest) equilibrium payoff. The answer depends on the expert’s initial level of reputation \( \mu \) and the threshold level \( \mu^* \). If the expert’s initial reputation is high enough, in particular \( \mu^* \leq \mu \), then no monitoring (i.e., \( \beta = 0 \)) would yield the highest payoff to the expert. However, when the expert needs to build up her reputation to deceive the receivers (i.e., \( \mu < \mu^* \)), then no monitoring is not the best scenario for her. The expert prefers to be monitored perfectly while she builds up her reputation for
honesty. However, once the expert reaches the threshold level of reputation, she prefers no monitoring, so she can deceive the receivers as long as she can. Overall, there is an inverted U-shaped relationship between the expert’s maximal equilibrium payoff and the strength of the monitoring technology.

The next observation indicates that for carefully selected values of the discount rate \( \delta \), the payoff maximizing value of \( \beta \) is uniquely determined. Given the value of \( v_L^\beta \) in Proposition 9, we have

\[
\frac{\partial v_L^\beta}{\partial \beta} = \frac{\partial \alpha}{\partial \beta} \left( \frac{v_e \pi}{1 - (1 - \pi \beta) \delta} \right) + \frac{\partial v_H^\beta}{\partial \beta} \alpha. 
\]

Since \( \frac{\partial v_H^\beta}{\partial \beta} = -\frac{\delta \pi v_e}{(1 - (1 - \pi \beta) \delta)^2} \) and \( \frac{\partial \alpha}{\partial \beta} = \frac{\alpha \beta N_G (1 - \delta)}{\beta (1 - (1 - \pi \beta) \delta)} \), we have

\[
\frac{\partial v_L^\beta}{\partial \beta} = \frac{\alpha \beta v_e \pi}{(1 - (1 - \pi \beta) \delta)^2} \left[ N_G (1 - \delta) \frac{1 - \beta}{\beta} - \delta \pi \right].
\]

Equating the last equation to 0 gives the value of \( \beta \) that maximizes \( v_L^\beta \).

**Observation 1.** The function \( v_L^\beta \), the expert’s highest equilibrium payoff, is maximized when

\[
\beta = \frac{N_G (1 - \delta)}{\delta \pi}.
\]

First note that \( \beta \) decreases with \( \delta \). That is, patient expert prefers weaker monitoring. Second, if \( \mu \) increases to \( \mu^* \), then \( \beta \) decreases because higher \( \mu \) reduces \( N_G \). Therefore, the expert with a higher initial reputation reaches the threshold level of reputation faster and prefers a weaker monitoring technology. Equivalently, the expert with a low initial reputation prefers stronger monitoring system to reach the threshold level of reputation earlier. Third, if \( \pi \) increases, then \( \beta \) decreases. Therefore, if the likelihood of the bad state increases, then the expert prefers a weaker monitoring technology because she wants to reduce the likelihood of getting caught. Finally, if receiver 2’s expected return from the project increases (i.e., \( \mu^* \) is lower, and thus \( N_G \) is lower), then the expert prefers a weaker monitoring technology.

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5Proposition 9 indicates that for any \( \beta_0 \in (0, 1) \), there exists some \( \delta_3 < 1 \) high enough such that for all \( \beta \in [\beta_0, 1] \) and \( \delta \in [\delta_3, 1) \) there exist an equilibrium in which the expert’s expected payoff is \( v_L^\beta \). Therefore, we must read the next observation as follows: Fix the value of \( \delta \). Let \( \beta_0 \) be small enough, and so, \( \delta_3 \) be high enough so that \( \frac{N_G (1 - \delta)}{\delta \pi} \in [\beta_0, 1] \) and \( \delta \in [\delta_3, 1) \). Then, the expert’s expected payoff \( v_L^\beta \) takes its maximum value over \([\beta_0, 1]\) at \( \beta = \frac{N_G (1 - \delta)}{\delta \pi} \).
**Expected Length of Reputation Building:** The shortest (expected) amount of time required for the expert to build up her reputation decreases with the expert’s initial reputation $\mu$, with the monitoring strength $\beta$, and with receiver 2’s expected return for the project.

**Observation 2.** In equilibrium, the expected number of stages that the expert should be truthful to build her reputation up to $\mu^*$ is no less than $\frac{N_G}{\pi \beta}$.

For $N_G = 1$, it is easy to calculate the expected number of stages that the expert should be truthful to build her reputation up to $\mu^*$. Given the equilibrium strategies that are used to prove Proposition 9, if $N_G = 1$, then it requires only one stage, where the true state is $\theta_1$ and receiver 2 observes the payoffs, to build reputation. Therefore, if $N_G = 1$, then the expected number of stages that the expert should be truthful is

$$\sum_{i=0}^{\infty} (i + 1)(1 - \pi \beta)^i \pi \beta = \frac{1}{\pi \beta},$$

where the $(i + 1)$th stage is the first stage in which the true state is $\theta_1$ and receiver 2 observes the realized payoffs, and $(1 - \pi \beta)^i \pi \beta$ is the probability of this event. For arbitrarily large but finite $N_G$, we inductively calculate the expected number of stages that the expert should be truthful.

**Expected Length of Deception:** Recall that the expert can deceive the receivers in equilibrium only if the expert’s reputation is higher than $\mu^*$. According to the equilibrium strategies where the expert’s payoff is the highest, the expert can deceive the receivers at only one stage when $\beta = 1$. However, for smaller values of $\beta$, the expert can deceive the receivers as long as receiver 2 does not observe the previous deceptions. According to these equilibrium strategies, the probability that the expert deceives the receivers during the entire repeated cheap talk game only once is

$$\sum_{i=0}^{\infty} \pi (1 - \pi)^i \beta = \beta.$$

Similarly, the probability that the expert deceives the receivers during the entire repeated cheap talk game only twice is

$$\sum_{i=0}^{\infty} (i + 1)\pi^2 (1 - \pi)^i (1 - \beta) \beta = (1 - \beta)\beta.$$
Inductively, we can find that the probability that the expert deceives only \( n \) times is

\[
\sum_{i=n}^{\infty} \binom{i-1}{n-1} \pi^n (1-\pi)^{i-n+1} (1-\beta)^{n-1} \beta = (1-\beta)^{n-1} \beta.
\]

Hence, in equilibrium, the expected number stages that the expert deceives the receivers is at most

\[
\sum_{i=0}^{\infty} (i+1)(1-\beta)^i \beta = \frac{1}{\beta}.
\]

**Observation 3.** In equilibrium, expected number of stages that the expert deceives the receivers is at most \( \frac{1}{\beta} \).

The expected length of deception depends only on the monitoring technology. Consistent with the intuition, it decreases with the monitoring technology. However, the relationship has degree of \(-1\). The length of deception is short especially when the monitoring is strong (i.e., \( \beta \) is farther from 0). In the next section, I will show that the length of deception would be significantly longer when both receivers are long-lived agents.

**Growth Rate of Reputation:** In equilibrium where the expert’s payoff is the highest, the growth rate of the expert’s reputation is

\[
\lambda = \frac{v_2 (1-\pi)}{v_1 \pi - v_2 (1-\pi)}.
\]

Therefore, the expert’s reputation grows at a constant rate. If receiver 2’s expected return from the project increases (i.e., \( v_2 \) increases or \( v_1 \) or \( \pi \) decreases), then the expert builds its reputation at a faster rate. The growth rate is calculated by \( \frac{\mu^{i+1} - \mu}{\mu (1-\mu) \Pi_{k=0}^{i} \sigma_e^k} \). Recall that we have \( \mu_t = \frac{\mu}{\mu + (1-\mu) \Pi_{k=0}^{i-1} \sigma_e^k} \) and \( \Pi_{k=0}^{i-1} \sigma_e^k = \frac{(\mu^*)^{i-1} - \mu}{1-\mu} \). Thus, \( \lambda = \frac{1-\mu^*}{\mu^*} = \frac{v_2 (1-\pi)}{v_1 \pi - v_2 (1-\pi)} \).

6. **Repeated Cheap Talk Game with Long-Lived Receivers**

Suppose now that all three players are long-lived agents with the common discount factor \( \delta < 1 \). I will consider the case where all three players can perfectly monitor the history of the game. That is, at all stages, all three players’ actions and the true state are observable by all three players. Moreover, I will suppose that the expert is known to be the opportunistic type, so \( \mu = 0 \). The next result proves that the expert can deceive the receivers for a very long time even though she is known to be the opportunistic type,
and her deception is perfectly observable by all players. Thus, when the receivers are short-lived agents, their incentives will protect them against recurrent deceptions as long as the monitoring technology is transparent enough.

In equilibrium, the expert deceives the receivers so long that receiver 2’s expected payoff in the game is simply 0 (the receivers’ minimax value in the game). That is, by deceiving the receivers, the expert can extract all the surplus receiver 2 would achieve in the repeated cheap talk game. The reason for long periods of deception relies solely on the length of the relationship between the expert and the receivers. In equilibrium, receivers may not punish deception (even they perfectly observe it) once the expert commits herself to reward the receivers after deception by being honest for sufficiently long periods of time. Clearly, such promises do not incentivize the short-lived agents to disregard deception.

**Proposition 10.** Let $M$ be the largest natural number satisfying 
\[
M \leq \frac{\ln \mu^*}{\ln \left(\delta \pi / (1 - \delta + \delta \pi)\right)}.
\]
For sufficiently large values of $\delta < 1$, there exists an equilibrium of the repeated cheap talk game in which the expert deceives the receivers $M$ stages.

Note that $M$ is not necessarily the highest duration of deception that can be supported in equilibrium. However, $M$ takes extremely large values when $\delta$ is close to 1. The essence of the last result is to show that the expert can deceive the receivers for a very long time when the relationship between the expert and the receivers is long lasting.

Note that $M$ decreases with $\pi$. That is, if the likelihood of dispute increases, then the length of deception tends to decrease. Moreover, $M$ decreases with $\mu^*$. Therefore, if receiver 2’s expected return from the project is lower, then the length of deception is likely to decrease. The reason for this is simple: since the receiver 2’s expected return from the project is lower, receiver 2’s payoff in the repeated game will immediately fall down to 0 in fewer stages of deception. Thus, the expert cannot deceive the receivers for a very long time. With this observation, it would be fair to conjecture that deception occurs more often in “less risky” disputes (disputes where the expected loss is smaller) and less often in “risky” disputes. Equivalently, the expert would deceive risk averse (loving) agents less (more) often.

One way to prevent the expert’s deception in a long-lasting relationship is the existence of an outside option for the receivers. If the receivers have the ability to break their relationship with the expert and access to an outside option, then we should expect that the length of deception would decrease with the value of the receivers’ outside options. Competition among multiple experts, for example, would provide a valuable outside option.
for the receivers.

7. Related Literature

Our approach in this paper is distinct from the cheap talk literature in two respects. First, I consider a multi-receiver cheap talk game where the receivers play a simple voting game after observing the sender’s message. The literature mainly focuses on cheap talk games with a unique sender and receiver. Battaglini (2002), Krishna and Morgan (2001), and Ambrus and Takahashi (2008) are few exceptions where the unique receiver gets advice from multiple senders. Farrell and Gibbons (1989) and Goltsman and Pavlov (2011) consider cheap talk games with multiple receivers, where the payoff of a receiver is independent of the other receiver’s actions.

Second, unlike the usual treatment in the literature, I consider an infinite horizon repeated cheap talk game to study the trade-off between the expert’s conflicting short-term and long-term interests. The expert wants her advice to be credible in the long-term, but also wants to deceive the receivers to achieve a higher payoff in the short-term. Aumann and Hart (2003) and Krishna and Morgan (2003), for example, consider dynamic cheap talk games. However, only the talk is repeated in their settings. Golosov et al. (2014) study strategic information transmission game in a finite-horizon, dynamic Crawford and Sobel setup. The main result of their paper is that fully revealing equilibrium exists when both the expert and the receiver are long-lived and fully patient players.

Sobel (1985) considers a reputational cheap talk game where the cheap talk between the expert and the receiver is repeated finitely many stages, and both players are fully patient and long-lived. His paper assumes that the receiver is uncertain about the bias of the expert: she is either the “friendly” type, whose preferences are perfectly aligned with the receiver; or the “enemy” type, who has completely opposed preferences to the receiver. The main result of his paper is that deception is sustainable in equilibrium only if the expert has sufficiently high reputation of being the “friendly” type, and deception would occur only once. Ottaviani and Sorensen (2006, 2006b) and Morris (2001) also investigate reputational cheap talk games where the expert’s bias is unknown to the receiver. The main message of these papers is that truth telling is incompatible with equilibrium when the expert is sufficiently concerned about her reputation.

Benabou and Laroque (1992) is the most related paper to the current one. They also study an infinitely repeated cheap talk game between an expert and multiple audiences (i.e., the public). However, the players in their model have significantly different incentives. The expert receives an informative signal about the true state of the world. Unlike
the current model, the expert in Benabou and Laroque (1992) plays a trading game with her audiences right after sending her public message and directly affects her own payoff. In particular, the expert is an insider who manipulates her audiences’ opinion through her cheap talk messages and trades with them in a purely speculative market. Benabou and Laroque (1992) show that the expert will deceive her audiences (who are short-lived agents) during an unbounded length of time. They conclude, contrary to my results, that an expert with very low reputation for honesty will make no significant attempt to build her reputation, and the issue of whether intermediate reputations are worth improving by “investing in truth” remains unresolved.

**Appendix**

**Proof of Proposition 1.** Suppose that there exists a PBE strategy profile $\sigma$ in which after some message realization, both receivers play $a$ with a positive probability. Given the expert’s and the first receiver’s strategies $\sigma_e(\theta_1)$, $\sigma_e(\theta_2)$, $\sigma_1(\theta_1)$ and $\sigma_1(\theta_2)$, second receiver’s best response correspondence is calculated as follows: Suppose first that the receivers observe message $\theta_1$. Then, receiver 2’s expected payoff of playing $a$ and $r$ are as follows.

$$EU_2(a \mid m = \theta_1) = \sigma_1(\theta_1) \left[ - v_1 P(s = \theta_1 \mid m = \theta_1) + v_2 \left( 1 - P(s = \theta_1 \mid m = \theta_1) \right) \right]$$

$$EU_2(r \mid m = \theta_1) = 0.$$

Therefore, if $\sigma_1(\theta_1) > 0$, then

$$BR_2(\sigma_1 \mid m = \theta_1) = \begin{cases} 1, & \text{if } \sigma_e(\theta_1) < \frac{\sigma_e(\theta_2) v_2 (1 - \pi)}{v_1 \pi} \\ [0,1], & \text{if } \sigma_e(\theta_1) = A \\ 0, & \text{otherwise.} \end{cases}$$

Now, suppose that the receivers observe the message $\theta_2$. Then,

$$EU_2(a \mid m = \theta_2) = \sigma_1(\theta_2) \left[ - v_1 \left( 1 - P(s = \theta_2 \mid m = \theta_2) \right) + v_2 P(s = \theta_2 \mid m = \theta_2) \right]$$

$$EU_2(r \mid m = \theta_1) = 0.$$
Therefore, if $\sigma_1(\theta_2) > 0$, then

$$BR_2(\sigma_1|m = \theta_2) = \begin{cases} 
1, & \text{if } \sigma_e(\theta_1) > 1 - (1 - \sigma_e(\theta_2)) \frac{v_2(1 - \pi)}{v_1 \pi} \\
[0,1], & \text{if } \sigma_e(\theta_1) = B \\
0, & \text{otherwise.}
\end{cases}$$

Finally, we have $BR_2(\sigma_1|m) = [0,1]$ whenever $\sigma_1(m) = 0$. Since $\sigma$ is an equilibrium profile in which both receivers play $a$ with positive probabilities, we can ignore the case where $\sigma_1(\theta_1) = \sigma_1(\theta_2) = 0$. Therefore, there are three cases that we need to consider: (1) $\sigma_1(\theta_1) = 0$ but $\sigma_1(\theta_2) > 0$, (2) $\sigma_1(\theta_2) = 0$ but $\sigma_1(\theta_1) > 0$, or (3) both $\sigma_1(\theta_1)$ and $\sigma_1(\theta_2)$ are strictly positive. Next, I will prove that neither one of these cases would hold in equilibrium.

Suppose for a contradiction that the first case holds (i.e., $\sigma_1(\theta_1) = 0$). That is, the first player chooses the action $r$ with certainty whenever he receives the message $\theta_1$. However, since $\sigma$ is an equilibrium profile where both receivers choose $a$ with positive probabilities, then they must be choosing the action $a$ when they receive the message $\theta_2$, that is

$$\sigma_i(\theta_2) > 0 \text{ for } i = 1, 2. \quad (6)$$

If the last statement is true, then the expert prefers to send the message $\theta_2$ regardless of the true state. Thus, optimality of equilibrium implies that $\sigma_e(\theta_1) = \sigma_e(\theta_2) = 0$ must hold. However, if $\sigma_e(\theta_i) = 0$ for $i = 1, 2$, then the second receiver’s best response to $\sigma_1(\theta_2)$ and message $\theta_2$ is to choose $r$ with certainty. This contradicts with the optimality of $\sigma$ because the expert would want to deviate and announce $\theta_2$.

Suppose for a contradiction that the third case holds. That is, both $\sigma_1(\theta_1)$ and $\sigma_1(\theta_2)$ are strictly positive. Note that we have $A < B$ since the inequality (1) holds. Therefore, given the second receiver’s best response correspondences, there are 5 sub-cases that we need to consider. Before going through these five cases, I need to show that the expert must be indifferent between sending the message $\theta_1$ and $\theta_2$ whenever the true state is $\theta_1$ and $\sigma_e(\theta_1) \in (0,1)$ must be true. First, suppose that $\sigma_e(\theta_1) = 1$. According to the second receiver’s best response correspondences, he will play $r$ for sure when he receives the message $\theta_1$. Since both receivers play $a$ with a positive probability in $\sigma$, it must be true that the second receiver chooses $a$ with a positive probability when he observes the message $\theta_2$ (i.e., $\sigma_2(\theta_2) > 0$). However, having both $\sigma_2(\theta_2)$ and $\sigma_1(\theta_2)$ strictly positive contradicts with the optimality of $\sigma$ because the expert would want to deviate and announce $\theta_2$.
when the true state is $\theta_1$. Similar arguments show that $\sigma_e(\theta_1) = 0$ is inconsistent with equilibrium. Hence, we must have $\sigma_e(\theta_1) \in (0, 1)$. Now, I will prove that we reach a contradiction in every one of the following five cases. Suppose that the true state is $\theta_1$, then

(i) if $\sigma_e(\theta_1) < A < B$, then the second receiver plays $a$ when $m = \theta_1$ and plays $r$ when $m = \theta_2$. Therefore, the expert makes positive payoff only when she sends the message $\theta_1$. This contradicts with the fact that the expert is not indifferent on $\theta_i$'s when the true state is $\theta_1$.

(ii) if $A = \sigma_e(\theta_1) < B$, then the second receiver is indifferent between $a$ and $r$ when the expert sends the message $\theta_1$. However, when she sends the message $\theta_2$, the second receiver chooses $r$. Hence, the expert can be indifferent between the messages $\theta_1$ and $\theta_2$ if and only if the second receiver chooses $r$ regardless of the message she receives. This contradicts with the presumption that both receivers choose $a$ with a positive probability in $\sigma$.

(iii) if $A < \sigma_e(\theta_1) < B$, then the second receiver plays $r$ regardless of the expert’s message, contradicting that both receivers play $a$ with a positive probability in $\sigma$.

(iv) if $A < \sigma_e(\theta_1) = B$, then the second receiver plays $r$ when the expert sends the message $\theta_1$, and she is indifferent between $a$ and $r$ when the expert sends the message $\theta_2$. In this case, the expert is indifferent between the messages $\theta_1$ and $\theta_2$ if and only if the second receiver plays $r$ for sure when the expert’s message is $\theta_2$. This contradicts that both receivers play $a$ with a positive probability in $\sigma$.

(v) if $A < B < \sigma_e(\theta_1)$, then the second receiver plays $r$ when the expert sends the message $\theta_1$ and $a$ otherwise. Hence, the expert will strictly prefer to send the message $\theta_2$ when the true state is $\theta_1$. This contradicts with the fact that $\sigma_e(\theta_1) \in (0, 1)$ must hold.

Hence, there is no PBE strategy profile $\sigma$ in which both receivers play $a$ with a positive probability.

Q.E.D

Proof of Proposition 4. Consider the following strategy profile: The opportunistic expert starts the repeated cheap talk game by telling the truth and continues to tell the truth as long as no player deviates from his/her strategy. Receivers begin to $G^\infty_\mu$ by approving (or rejecting) the project when they observe the message $\theta_2$ (or $\theta_1$), and
continue this way unless one of the three players deviate. In case of deviation, the expert sends the message $\theta_2$ and both receivers reject the project for the rest of the game.

Next, I will discuss that this strategy profile is a PBE of the repeated cheap talk game. There are two phases of the strategy profile: the punishment and the coordination phases. In the punishment phase, both receivers reject the project, and so all three players receive 0 payoff forever. If a player deviates in the punishment phase, s/he cannot improve his/her payoff. Therefore, the punishment phase strategies are optimal for all three players.

Given the receivers’ strategies in the collusion phase, the expert’s expected payoff of playing her prescribed strategies is

$$\frac{(1 - \pi)v_e \delta}{1 - \delta} = 0 + (0\pi + (1 - \pi)v_e)\delta + (0\pi + (1 - \pi)v_e)\delta^2 + ...$$

conditional on the event that the expert observes the bad state, and is

$$v_e + \frac{(1 - \pi)v_e \delta}{1 - \delta} = v_e + (0\pi + (1 - \pi)v_e)\delta + (0\pi + (1 - \pi)v_e)\delta^2 + ...$$

conditional on the event that the expert observes the good state. According to one-shot deviation property, the expert may have an incentive to deviate only when the true state is $\theta_1$. However, her deviation will lead to the payoff of $v_e$. Therefore, $\frac{(1 - \pi)v_e \delta}{1 - \delta} \geq v_e$ holds if $\delta \geq \frac{1}{2 - \pi}$ is true. That is, the expert has no incentive to deviate if $\delta$ is high enough.

Given the other players’ strategies, receiver 1’s expected payoff of following his strategy is $\frac{(1 - \pi)w_1 \delta}{1 - \delta}$ and $w_2 + \frac{(1 - \pi)w_2 \delta}{1 - \delta}$ conditional on the events that he observes messages $\theta_1$ and $\theta_2$, respectively. However, if receiver 1 deviates and rejects (or approves) the project when the expert sends the message $\theta_2$ (or $\theta_1$), his continuation payoff will be simply 0. Hence, receiver 1 has also no incentive to deviate.

Finally, given the other players’ strategies, a short-lived agent’s expected payoff of following his strategy is 0 and $v_2$ conditional on the events that he observes the messages $\theta_1$ and $\theta_2$, respectively. However, if he deviates, his payoff will be 0. Thus, the prescribed strategies are optimal for all short-lived agents. This completes the proof.

Q.E.D

Proof of Proposition 5. Proposition 4 proves that the expert can achieve the payoff $\frac{(1 - \pi)v_e}{1 - \delta}$ in the fully-revealing and influential equilibrium. Therefore, the expert can achieve higher payoffs by making the receivers approve the project (with a positive probability) also when the true state is $\theta_1$. First, I will show that there is no equilibrium of the
continuation game following the history $h^t$ in which both receivers approve the project with a positive probability after any message they observe.

Suppose for a contradiction that there exists an equilibrium in which $\sigma_j(h^t, \theta_i)(a) \in (0, 1)$ for $i, j \in \{1, 2\}$. Recall player 2’s best response correspondences from the proof of Proposition 1. Conditional on observing the message $\theta_1$, receiver 2 approves the project if $\sigma_e(\theta_1) \leq \sigma_e(\theta_2) \frac{v_2(1-\pi)}{v_1\pi} := A$. Note that the strategy $\sigma_e(\theta_i)$ in the one stage game corresponds to $\sigma_e(h^t, \theta_i)(\theta_i)$ in the repeated cheap talk game. However, when receiver 2 observes the message $\theta_2$, he approves the project if $\sigma_e(\theta_1) \geq 1 - (1 - \sigma_e(\theta_2)) \frac{v_2(1-\pi)}{v_1\pi} := B$. Therefore, receiver 2 approves the project with a positive probability regardless of the message he observes if and only if $B \leq \sigma_e(\theta_1) \leq A$ holds. However, since the inequality (1) holds, we have $A < B$ for all values of $\sigma_e(\theta_2)$, that yields the desired contradiction.

Therefore, if the expert wants the receivers approve the project even when the true state is $\theta_1$, she must send the message $\theta_2$ regardless of the true state. However, full deception is not consistent with the equilibrium of the continuation game following the history $h^t$. If receiver 2 believes that the expert will deceive the receivers with certainty at some stage after $h^t$, then the short-lived receiver 2 prefers to reject the project at that stage. On the other hand, partial deception would be consistent with equilibrium. In particular, we know from receiver 2’s best response correspondences that if the expert sends the message $\theta_2$ with certainty when the true state is $\theta_2$ and with a probability $\sigma_e(h^t, \theta_1)(\theta_2) \leq \frac{v_2(1-\pi)}{v_1\pi}$ when the true state is $\theta_1$, then receiver 2 prefers to approve the project if he observes the message $\theta_2$. Partial deception implies that the expert plays a mixed strategy when the true state is bad. Next, I will show that partial deception does not improve the expert’s payoff.

Let $V$ be the highest equilibrium payoff of the continuation game following the history $h^t$. In any PBE following the history $h^t$, the expert’s continuation payoff is no more than $v_e + \delta V$ when the expert observes the state $\theta_2$. However, if the expert observes the state $\theta_1$ and she tells the truth, then her continuation payoff in any equilibrium is no more than $0 + \delta V$. Therefore, the expert’s highest continuation payoff is $\delta V$ if the true state is $\theta_1$. This is true because (1) if the expert tells the truth, then her continuation payoff is at most $\delta V$, or (2) if the expert partially deceives the receivers (as we know that full deception is out of consideration), then the expert’s continuation payoff of lying and telling the truth must be the same, and thus, her continuation payoff is also $\delta V$, at most. Recall that $\theta_1$ occurs with probability $\pi$. Thus, the expert’s expected payoff in the continuation game following the history $h^t$ is no more than $(1-\pi)(v_e + \delta V) + \pi\delta V$. Therefore, we must have $V \leq (1-\pi)(v_e + \delta V) + \pi\delta V$, implying that $V \leq \frac{(1-\pi)v_e + \pi\delta V}{1-\pi}$. This completes the proof.
Proof of Proposition 6. First I will show that the payoff $v^H$ can be supported in equilibrium. Then I will argue that it is the highest expected payoff that the expert can achieve in any PBE of the repeated cheap talk game. Consider the following strategy profile $\sigma$. Players start in the deception phase: The expert sends the message $\theta_2$ regardless of the true state until she gets caught lying. Once she gets caught lying, players move to the honesty phase, where the expert tells the truth as long as no player deviates. In the deception and honesty phases, receivers always approve the project when they observe the message $\theta_2$ and reject the project if the message is $\theta_1$. If (one or more of the three) players deviate, then the players move to the punishment phase, where the receivers always reject the project and the expert always sends the message $\theta_2$ for the rest of the game.

The expert’s payoff under the strategy profile $\sigma$ can be calculated by solving the following recursive equation

$$v^H = \pi \left[ v_e + \frac{\delta(1-\pi)v_e}{1-\delta} \right] + (1-\pi) \left[ v_e + \delta v^H \right]$$

The first term in the parentheses is the expert’s continuation payoff when the true state is $\theta_1$: The expert can deceive the receivers only 1 stage. Following the deception stage, the expert will be truthful forever. The second term is the expert’s expected payoff if the true state is $\theta_2$. The expert will not deceive the receivers in this stage, and so the continuation game will be identical to the game itself. The solution of this equation yields the value for $v^H$.

As we already argued before, punishment phase strategies form an equilibrium: Regardless of the message the expert sends, rejecting the project is a best response for a receiver given that the other receiver rejects as well. In the honesty phase, the receivers will know the experts type. Hence, as we argued in the proof of Proposition 4, all three players’ strategies in the honesty phase are optimal as well. Finally, in the deception phase, if the expert deviates and sends the message $\theta_1$, then her payoff will simply be 0. From Proposition 3 we know that both receiver 1 and 2’s expected payoff in the game are positive because $\mu \geq \mu^*$. Therefore, if they deviate in the deception phase and reject the project when they observe $\theta_2$, then they will get 0 payoff. Hence, deception phase strategies are also optimal for all three players.

Finally, I will argue that $v^H$ is the highest payoff the expert can achieve in an equilibrium of the repeated cheap talk game $G^\infty_\mu$. Suppose not. Recall the strategy $\sigma$. Receiver 2 approves the project if the true state is $\theta_2$. Moreover, he approves it once when the state is $\theta_1$. Deception occurs only 1 time because the expert will certainly get caught
when she lies. Therefore, a higher payoff for the expert is possible only if (1) receiver 2 approves the project at state $\theta_1$ more than once, or (2) in the deception phase, the expert sends the message $\theta_2$ with probability less than 1 when the true state is $\theta_1$, and thus reduces the chances that she gets caught lying. The first case is not possible given the monitoring technology. The second case (i.e., partial deception) would not increase the expert’s payoff (with the same reasoning of the arguments in the proof of Proposition 5): In equilibrium, the expert randomizes over the messages in $\Theta$ at some stage where the true state is $\theta_1$ if and only if she is indifferent between these messages at that stage. However, when the true state is $\theta_1$, the expert’s continuation payoff is at most $v_e + \delta (1 - \pi) v_e$ when she lies (i.e., sends the message $\theta_2$), and thus her continuation payoff should also be at most this much when she tells the truth. Therefore, if $V$ is the highest continuation payoff of the expert in any equilibrium following a history where the expert’s reputation $\mu$ is higher than or equal to $\mu^*$, then $V \leq \pi [v_e + \delta (1 - \pi v_e)] + (1 - \pi) [v_e + \delta V]$, implying that the expert cannot improve her payoff.

Q.E.D

Proof of Lemma 1. We know that $\mu^* = 1 - \frac{v_2(1 - \pi)}{v_1 \pi}$, and $\sigma^t_e = 1 - \frac{v_2(1 - \pi)}{v_1 \pi (1 - \mu)}$. Therefore, we can write

$$\sigma^t_e = \frac{\mu^* - \mu_t}{1 - \mu_t}. \quad (7)$$

Moreover, we know that $\mu_{t+1} = \frac{\mu}{\mu + (1 - \mu) \sigma^t_e}$ with $\mu_0 = \mu$, and thus, the recursive structure implies that

$$\mu_t = \frac{\mu}{\mu + (1 - \mu)(\sigma^0_e \sigma^1_e \ldots \sigma^{t-1}_e)}. \quad (8)$$

The equations in (7) and (8) imply that

$$\sigma^t_e = \mu^* - \frac{\mu(1 - \mu^*)}{(1 - \mu)(\sigma^0_e \sigma^1_e \ldots \sigma^{t-1}_e)}. \quad (9)$$

Thus, given the starting point $\sigma^0_e = \frac{\mu^* - \mu}{1 - \mu}$ and the equation (9) we can recursively calculate $\sigma^0_e \sigma^1_e \ldots \sigma^{n-1}_e$ for any $n \geq 1$ as follows: First $\sigma^0_e \sigma^1_e = \frac{(\mu^*)^2 - \mu}{1 - \mu}$. Using the last equation and (9) we find that $\sigma^0_e \sigma^1_e \sigma^2_e = \frac{(\mu^*)^3 - \mu}{1 - \mu}$. Repeating this process yields

$$\Pi_{t=0}^{n-1} \sigma^t_e = \frac{(\mu^*)^n - \mu}{1 - \mu}$$

By using the definition of $N_G$ in (5), it is rather easier to show that it is the smallest of the natural numbers $n$ satisfying $(\mu^*)^{n+1} \leq \mu$, which is equivalent to $n \geq \frac{\ln \mu}{\ln \mu^*} - 1$. 

Q.E.D
Proof of Proposition 7. First I will show that the payoff $v^L$ can be supported in equilibrium. Then I will argue that it is the highest expected payoff that the expert can achieve in any PBE of the repeated cheap talk game. Let $N_G$ be the smallest natural number satisfying $N_G \geq \frac{\ln \mu}{\ln \mu^*} - 1$. Consider the following strategy profile $\sigma$:

(i) The expert always sends the message $\theta_2$ whenever the true state is $\theta_2$.

(ii) Let $h^t$ be a history where (1) the expert’s reputation following this history (i.e., $\mu(h^t)$) is strictly positive, but strictly less than $\mu^*$, and (2) none of the three players has ever deviated from his/her prescribed strategies before. In the continuation game following the history $h^t$, (1) the expert sends message $\theta_1$ with probability $\sigma_e(h^t, \theta_1)(\theta_1) = 1 - \frac{e_2(1-\pi)}{\pi(1-\mu(h^t))}$ when the true state is $\theta_1$, and (2) the receivers reject (approve) the project if they observe the message $\theta_1$ ($\theta_2$).

(iii) Let $h^t$ be a history where (1) none of the three players has ever deviated from his/her prescribed strategies before, (2) the true state in stage $t$ was $\theta_1$, but (3) the expert has lied in stage $t$ so that her type has revealed (i.e., $\mu(h^t) = 0$). In the continuation game following the history $h^t$, (1) the expert tells the truth for the rest of the game, (2) the receivers always reject the project if the message is $\theta_1$. Finally, when the receivers observe the message $\theta_2$, they also observe the outcome of a public randomization device that has two possible outcomes A and B: At any stage (following the history $h^t$), the outcome A occurs with probability

\[
\frac{(\delta V(n) - v_e)(1-\delta)}{\delta(1-\pi)v_e},
\]

where

\[
V(n) = \left[1 - \left(\frac{\pi \delta}{1 - (1-\pi)\delta}\right)^{N_G - n}\right] v^f + \left(\frac{\pi \delta}{1 - (1-\pi)\delta}\right)^{N_G - n} v^H
\]

and $n \leq N_G$ is the number of stages (including stage $t$) in the history $h^t$ where the true state was $\theta_1$. Therefore, in the continuation game following the history $h^t$, both receivers approve the project whenever they observe the message $\theta_2$ and outcome A. However, both receivers reject the project when they observe the message $\theta_2$ and outcome B.

(iv) Let $h^t$ be a history where (1) the expert’s reputation $\mu(h^t)$ is (weakly) higher than $\mu^*$, and (2) none of the three players has ever deviated from his/her prescribed strategies before. In the continuation game following the history $h^t$, (1) the expert always sends the message $\theta_2$ (regardless of the true state) until she gets caught, and (2) the receivers reject (approve) the project if they observe the message $\theta_1$ ($\theta_2$). Once the expert gets caught lying, all three players play the rest of the game according to the strategies prescribed in (iii).
(v) After any history $h^t$ where at least one of three players has deviated from his/her prescribed strategies, receivers will reject the project for the rest of the game, and the expert always sends the message $\theta_2$.

First, note that, for sufficiently high values of $\delta$, $\sigma_1(h^t, \theta_1)(a) = \frac{(\delta V(n) - v_e)(1 - \delta)}{\delta(1 - \pi)n_e}$ is in the interval $(0, 1)$ for all $n \leq N_G$. $\sigma_1(h^t, \theta_1)(a)$ is positive because $v_e < \delta V(n)$ for high values of $\delta$. Likewise, $\sigma_1(h^t, \theta_1)(a) < 1$ because $V(n) < v^H$ and $\delta V^H < v_e + \frac{\delta(1 - \pi)v_e}{1 - \delta}$ (next, I will prove that this inequality is true). We know that $v^H = v_e \left( \frac{1 - 1/\delta + \delta(1 - \pi)}{1 - (1 - \pi)/\delta} \right)$. Therefore, the last inequality implies that $\delta \left( \frac{1 - 1/\delta + \delta(1 - \pi)}{1 - (1 - \pi)/\delta} \right) < 1 + \delta(1 - \pi)$. First, multiply both sides of this inequality with $1 - \delta$ and divide by $\delta$, then subtract $\frac{\delta(1 - \pi)}{1 - (1 - \pi)/\delta}$ from both sides to get $(1 - \pi) > \frac{\delta n}{1 - (1 - \pi)/\delta}$. Dividing both sides by $1 - \delta$ and rearranging the terms yield $\frac{\delta n}{1 - (1 - \pi)/\delta} > \frac{\delta n}{1 - \delta} > \frac{\delta n}{1 - \delta}$. Hence, $\delta V^H < v_e + \frac{\delta(1 - \pi)v_e}{1 - \delta}$ is true as claimed.

For $0 \leq n \leq N_G$, the term $V(n)$ represents the expert’s continuation payoff following a history $h^t$ where (1) none of the three players has ever deviated from his/her prescribed strategies before, and (2) the true state was $\theta_1$ in exactly $n$ stages prior to time $t$ (including stage $t$). Equivalently, $V(n)$ is the expert’s continuation payoff following a history where the expert’s reputation is $\mu_n = \frac{\mu_{n-1}}{\mu_{n-1} + (1 - \mu_{n-1})\sigma_e(h^t, \theta_1)(\theta_1)}$, where $y \leq t$ is the latest stage in which the true state was $\theta_1$. Thus,

$$V(0) = (1 - \pi)[v_e + \delta V(0)] + \pi \delta V(1)$$

$$V(1) = (1 - \pi)[v_e + \delta V(1)] + \pi \delta V(2)$$

$$V(N_G - 1) = (1 - \pi)[v_e + \delta V(N_G - 1)] + \pi \delta V(N_G)$$

and $V(N_G) = v^H$.

Recall that following the history $h^t$ where the true state was $\theta_1$ in exactly $n$ stages, the expert’s reputation is $\mu_n$. The first part of the recursive equation of $V(n)$ (i.e., $[v_e + \delta V(n)]$) is the expert’s continuation payoff conditional on the event that the true state is $\theta_2$ in stage $t + 1$. In this case, the expert tells the truth in stage $t + 1$, and so her stage-game payoff is $v_e$. The continuation game following the stage $t + 1$ will be identical to the continuation game following the history $h^t$, and thus the expert’s reputation is still $\mu_n$. Hence, the expert’s continuation payoff following stage $t + 1$ is $V(n)$.

The second part of the recursive equation $\delta V(n + 1)$ (or $\delta v^H$ in case $n = N_G - 1$) indicates the expert’s continuation payoff if the true state is $\theta_1$ in stage $t + 1$. The expert updates her reputation by telling the truth with probability $\sigma_e(h^t, \theta_1)(\theta_1)$, in which case
the expert’s stage game payoff is 0. However, the expert’s continuation payoff will be identical to the continuation payoff following a history in which the state $\theta_1$ occurs $n + 1$ times, and thus the expert’s reputation is $\mu_{n+1}$. If $n = N_G - 1$, then the expert’s reputation will reach a level above $\mu^*$ in stage $t+1$, and so the expert’s continuation payoff, according to the strategies given in (iv), will be $v^H$ (the expert’s highest equilibrium payoff in the continuation game), as we prove in Proposition 6.

The expert’s expected payoff in the repeated cheap talk game is equal to $V(0)$. In order to find its value, we must solve these $N_G$ equations recursively. Start first from solving the last equation. We find that $V(N_G - 1) = \left[1 - \frac{\pi \delta}{1 - (1 - \pi) \delta}\right] v^f + \left(\frac{\pi \delta}{1 - (1 - \pi) \delta}\right) v^H$. Hence, at the end of the process, we have

$$V(0) = \left[1 - \left(\frac{\pi \delta}{1 - (1 - \pi) \delta}\right)^{N_G}\right] v^f + \left(\frac{\pi \delta}{1 - (1 - \pi) \delta}\right)^{N_G} v^H$$

Next, I will show that the strategy profile $\sigma$ forms a PBE of the repeated cheap talk game. But first, I will prove the following result.

**Lemma 2.** Consider a history $h^t$ of the repeated cheap talk game $G^\infty_{\mu^*}$ in which the expert is known to be the opportunistic type. For any payoff $v$ in the range $[0, v^f]$ and sufficiently large values of $\delta < 1$, there exists an equilibrium of the continuation game following the history $h^t$ in which the expert’s payoff is $v$.

**Proof.** We know from Proposition 5 that a fully-revealing and influential equilibrium exists for the continuation game following the history $h^t$, where the expert’s payoff is $v^f$. Likewise, a babbling equilibrium exists for the sub game, where receivers reject the project regardless of the message they receive, and thus the expert’s payoff is 0. We can prove that any payoff $v \in [0, v^f]$ is consistent with an equilibrium of the sub game following the history $h^t$ without using a public correlation device. But, it is much simpler and shorter to provide the equilibrium strategies yielding the payoff $v$ when we use a public randomization. Therefore, for any payoff $v \in [0, v^f]$, I will construct a PBE with public correlation as follows:

Let $v = \alpha v^f$ where $\alpha \in (0, 1)$. Players start the sub game in the partially-approval phase where the expert always tells the truth and sends the message $\theta_i$ if and only if the true state is $\theta_i$. Receivers reject the project whenever they receive the message $\theta_1$. However, when they observe the message $\theta_2$, they also observe the outcome of a public randomization device that has two possible outcomes. Outcome A occurs with probability $\alpha$ and outcome B occurs with probability $1 - \alpha$. Given that the receivers observe the
message $\theta_2$ and outcome A, then both receivers approve the project. However, when the receivers observe the message $\theta_2$ and outcome B, then both receivers reject the project. If (at least) one of the three players deviate, then the players move to the punishment phase, where the expert always sends the message $\theta_2$ and the receivers reject the project forever.

In the proof of Proposition 5, I prove that almost identical strategies form equilibrium. Therefore, it is rather straightforward to show that these strategies also form a PBE of the continuation game for sufficiently high values of $\delta$, and the expert’s expected payoff is $\alpha v_f$ as required.

We already know that the punishment strategies in part (v) and the strategies in (i) are forming a PBE. Let $h^t$ be a history that is given in part (iii), and let $n \leq N_G$ be the number of stages that the true state was $\theta_1$. Conditional on the event that the true state is $\theta_1$, the expert’s expected payoff of lying is $v_e + \frac{\delta \pi_i(1-\pi)}{1-\pi} \sigma_1(h^t, \theta_1)(a)$, where $\sigma_1(h^t, \theta_1)(a) = \frac{\delta V(n) - v_e (1-\delta)}{\delta (1-\pi) v_e}$, that is identical to her expected payoff of telling the truth (i.e., $\delta V(n)$). Note that for sufficiently large values of $\delta$, $\delta V(n) < v_f$ for all $n \leq N_G$. Therefore, as I proved in Lemma 2, Propositions 5 and 6, the strategies defined in part (iii) and (iv) form a PBE of the respective continuation games.

Therefore, all we need to show that the strategies in (ii) form an equilibrium. Consider a history $h^t$ where (1) the expert’s reputation at stage $t$ (i.e., $\mu(h^t)$) is strictly positive and strictly lower than $\mu^*$, and (2) none of the three players has ever deviated from his/her prescribed strategies before. Given the strategies in (iii) and (iv), the expert is indifferent between lying and telling the truth when the true state is $\theta_1$ (as we proved above). Hence, sending the message $\theta_1$ with probability $\sigma_e(h^t, \theta_1)(\theta_1)$ is a best response for the expert. Moreover, given the expert’s and receiver 1’s strategies, the short lived receiver 2’s expected payoff of approving the project conditional on observing the message $\theta_2$ is

$$EU_2(a|m = \theta_2) = -v_1 P(s = \theta_1|m = \theta_2) + v_2 \left[1 - P(s = \theta_1|m = \theta_2)\right]$$

where $P(s = \theta_1|m = \theta_2) = \frac{\pi (1-\mu(h^t)) [1-\sigma_e(h^t, \theta_1)(\theta_1)]}{\pi (1-\mu(h^t)) [1-\sigma_e(h^t, \theta_1)(\theta_1)] + (1-\pi)}$. For the values of $\sigma_e(h^t, \theta_1)(\theta_1)$ that is given above, $EU_2(a|m = \theta_2) = 0$, and thus, receiver 2 is indifferent between approving and rejecting the project whenever he observes the message $\theta_2$. Thus, approving the project when $m = \theta_2$ and rejecting the project when $m = \theta_1$ is a best response strategy for receiver 2. Likewise, given the expert’s and the second receiver’s strategies, the first receiver’s strategy is also a best response. Hence, the strategies in (ii) (together with the strategies in (iii) and (iv)) form a PBE of the continuation game.
\[ V(0) = v^L \] is the highest PBE payoff that the expert can attain in the repeated cheap talk game \( G^\infty_{\mu} \). We can prove this recursively: By Proposition 6, in any equilibrium of the sub game following a history where the expert’s reputation is higher than \( \mu^* \), the expert’s payoff must be less than \( v^H \). Thus, we must have \( V_{N_G} = v^H \), where \( V_{N_G} \) denotes the expert’s highest equilibrium payoff in the continuation game following this history.

Then, the expert’s highest equilibrium payoff in the sub game following a history where the expert’s reputation is \( \mu_{N_G-1} \), call it \( V_{N_G-1} \), must be less than \( (1-\pi)[v_e + \delta V_{N_G-1}] + \pi \delta V_{N_G} \). This is true because in the next stage, either the true state will be \( \theta_2 \), and so the expert’s highest continuation payoff will be \( v_e + \delta V_{N_G-1} \), or the true state will be \( \theta_1 \) and the expert will tell the truth and receive at most \( 0 + \delta V_{N_G} \). Thus, solving all these inequalities recursively will yield that the expert’s highest equilibrium payoff in the game must be less than \( v^L \).

In the above strategy profiles, the expert builds her reputation gradually, which delays the payoff \( v^H \). However, the expert would play a strategy in which she builds her reputation in few stages (by telling the truth when the true state is \( \theta_1 \)) with a probability less than \( \sigma_e(h^t, \theta_1)(\theta_1) = \frac{\nu_2 (1-\pi)}{\nu_1 (1-\mu(h^t \theta_1))} \). However, if the expert follows a strategy in which she lies with a probability greater than \( \sigma_e(h^t, \theta_1)(\theta_1) \), then in equilibrium, receiver 2 certainly rejects the project regardless of the message (recall that \( \sigma_e(h^t, \theta_1)(\theta_1) \) is the probability of truth telling that makes receiver 2 indifferent between approving and rejecting the project). Thus, building reputation faster than \( N_G \) stages implies that the expert should give up her positive stage game payoffs until her reputation reaches \( \mu^* \). However, for such a strategy to be a part of an equilibrium strategy, the expert’s continuation payoff of telling the truth and lying (in case the true state is \( \theta_1 \)) must be the same. Suppose that the expert lies with a very high probability so that she can build up her reputation just in 1 stage by telling the truth when the true state is \( \theta_1 \). Therefore, the expert’s stage game payoff of sending message \( \theta_1 \) (when the true state is \( \theta_1 \)) is 0, and so, her continuation payoff is at most \( 0 + \delta v^H \). However, if she lies and gets caught, her continuation payoff will be at most \( 0 + \delta v^f \) (by Proposition 5), not \( v_e + \delta v^f \). The \( \delta v^f \) is lower than \( \delta v^H \) for all values of \( \delta \). Thus, the expert cannot get a continuation (and thus, game) payoff higher than \( v^f \) in an equilibrium strategy where the expert does not build her reputation gradually. Hence, the expert can attain her highest expected payoff in a strategy profile where she gradually builds her reputation. Since \( N_G \) is the shortest time that the expert needs to build up her reputation, \( v^L \) must be the highest payoff the expert can attain in any PBE of the repeated cheap talk game.

Q.E.D

Lemma 3. Suppose that \( \beta \in (0, 1) \). Consider a history \( h^t \) of the repeated cheap talk game
in which the expert is known to be the opportunistic type. The expert’s payoff in any equilibrium of the continuation game following the history \( h^t \) is no more than \( v^f \).

**Proof of Lemma 3.** Consider the following strategy profile: The opportunistic expert starts the repeated cheap talk game by telling the truth and continues to tell the truth as long as no player deviates from his/her strategy. Receivers begin to \( G^\infty \) by approving (or rejecting) the project when they observe the message \( \theta_2 \) (or \( \theta_1 \)), and continue this way unless one of the three players deviates. In case of a deviation that receiver 2 observes, all three players move to the punishment phase, where the expert sends the message \( \theta_2 \) and both receivers reject the project for the rest of the game. These strategies form an equilibrium if \( \delta \) is high enough (i.e., \( \frac{1}{1+\beta(1-\pi)} < \delta \)) and yield the expected payoff of \( v^f \) for the expert.

In order to achieve a higher payoff, the expert must lie and get the stage game payoff of \( v_e \) also when the true state is \( \theta_1 \). However, we know that there is no equilibrium where receiver 2 approves the project regardless of the message the expert sends. Moreover, in equilibrium, when the true state is \( \theta_1 \), the expert must tell the truth with a sufficiently high probability. Similar to the proof of Proposition 5, let \( V \) be the highest continuation payoff of the expert in any equilibrium following a history in which the expert is known to be the opportunistic type. Then, the expert’s expected payoff if the true state is \( \theta_2 \) in stage \( t+1 \) is at most \( v_e + \delta V \). However, if the true state is \( \theta_1 \) in stage \( t+1 \), then the expert’s continuation payoff of telling the truth is no more than \( 0 + \delta V \). In equilibrium where the expert tells the truth with a positive probability when the true state is \( \theta_1 \), the expert’s continuation payoff of lying and telling the truth must be the same. Hence, her continuation payoff when the state is \( \theta_1 \) should be no more than \( \delta V \). Thus, \( V \) must be less than or equal to \( (1-\pi)(v_e + \delta V) + \pi \delta V \), implying that \( V \leq \frac{(1-\pi)v_e}{1-\delta} \). Hence, \( v^f \) is the upper boundary for the expert’s equilibrium payoffs following the history \( h^t \).

Q.E.D

**Proof of Proposition 8.** First, I will show that the payoff \( v^H_\beta \) can be supported in equilibrium. Then I will argue that it is the highest expected payoff that the expert can achieve in any PBE of the repeated cheap talk game. Consider the following strategy profile \( \sigma \). Players start in the deception phase: The expert sends the message \( \theta_2 \) regardless of the true state until she gets caught lying. Once she gets caught lying by the receiver 2, all three players move to the honesty phase, where the expert tells the truth as long as no player deviates. In the deception and honesty phases, receivers always approve the project when they observe the message \( \theta_2 \) and reject the project if the message is \( \theta_1 \). If (one or more of the three) players deviate and receiver 2 observes the deviation, then the
players move to the punishment phase, where the receivers always reject the project and the expert always sends the message $\theta_2$ for the rest of the game.

The expert’s payoff under the strategy profile $\sigma$ can be calculated by solving the following recursive equation

$$v^H_\beta = \pi \left( v_e + \delta \left[ (1 - \beta) V + \beta v_f \right] \right) + (1 - \pi) \left( v_e + \delta v^H_\beta \right)$$

The first term in the parentheses is the expert’s continuation payoff when the true state is $\theta_1$: The expert will deceive the receivers. Following the deception stage, the expert will be truthful forever with probability $\beta$ because she will get caught lying. In this case, her continuation payoff is $v^f$. However, the expert will not get caught lying with probability $1 - \beta$, in which case the continuation game is identical with the game itself. The second term is the expert’s expected payoff if the true state is $\theta_2$. The expert will not deceive the receivers in this stage, and so the continuation game will be identical to the game itself. The solution of this equation yields the value for $v^H_\beta$.

As we already argued before, punishment phase strategies form equilibrium. In the honesty phase, the receivers will know the expert’s type. Hence, as we argued in Lemma 3, all three players’ strategies in the honesty phase are optimal as well. Finally, in the deception phase, if the expert deviates and sends the message $\theta_1$, then her payoff will simply be 0. From Proposition 3 we know that both receiver 1 and 2’s expected payoff in the game are positive because $\mu \geq \mu^*$. Therefore, if they deviate in the deception phase and reject the project when they observe $\theta_2$, then they will get 0 payoff. Hence, deception phase strategies are also optimal for all three players.

Finally, I will argue that $v^H_\beta$ is the highest payoff the expert can achieve in any equilibrium of the repeated cheap talk game $G^\infty_\mu$. Recall the strategy $\sigma$. Receiver 2 approves the project if the true state is $\theta_2$. Moreover, he approves it when the state is $\theta_1$ as long as he does not catch the expert lying. Moreover, once the expert is get caught lying, the expert’s type will be revealed, and the expert’s highest continuation payoff is $v^f$ as we proved in Lemma 3. Therefore, a higher payoff for the expert is possible only if the expert sends the message $\theta_2$ in the deception phase with a probability less than 1 when the true state is $\theta_1$, and thus, reduces the chances that she gets caught lying. This case (i.e., partial deception) would not increase the expert’s payoff as we previously argued: In equilibrium, the expert randomizes over the messages in $\Theta$ at some stage where the true state is $\theta_1$ if and only if she is indifferent between these messages at that stage. However, when the true state is $\theta_1$, the expert’s continuation payoff is at most $v_e + \delta \left[ (1 - \beta) V + \beta \frac{v_e(1 - \pi)}{1 - \delta} \right]$ when she lies (i.e., sends the message $\theta_2$), and thus her continuation payoff must also be
at most this much when she tells the truth. Therefore, if $V$ is the highest continuation payoff of the expert in any equilibrium following a history where the expert’s reputation is higher than or equal to $\mu^*$, then $V \leq (1-\pi)(v_e + \delta V) + \pi \left( v_e + \delta \left( 1 - \beta \right) V + \beta v_e \frac{(1-\pi)}{1-\delta} \right)$, implying that $V \leq v_e \left[ 1 + \pi \beta \frac{(1-\pi)}{1-\delta} \right]$. This completes the proof.

Q.E.D

Proof of Proposition 9. In order to prove Proposition 9, we need to modify the strategies in the proof of Proposition 7 slightly. Consider the strategies given in the proof of Proposition 7. In part (iii) we now have

$$V(n) = v_f \left( 1 - \left[ \frac{\pi \beta \delta}{1 - (1 - \pi) \delta} \right]^{N_G-n} \right) + v_H^B \left[ \frac{\pi \beta \delta}{1 - (1 - \pi \beta) \delta} \right]^{N_G-n}$$

for all $n \leq N_G$ and $\sigma_1(h^t, \theta_1)(a) = \frac{(\delta \beta V(n) - v_e)(1-\delta)}{\delta \beta (1-\pi) v_e}$. Note that for any $\beta \in (0,1)$, there exists some $\tilde{\delta} < 1$ such that $\sigma_1(h^t, \theta_1)(a) \in (0,1)$ for all $\delta \geq \tilde{\delta}$. Moreover, the game will proceed from one phase into the other only when receiver 2 observes the necessary steps required to move to the next phase (like catching the expert lying and so on). The rest of the equilibrium strategies are the same. The expert’s expected payoff in the game is a solution of the following recursive equations, where for all $0 \leq t \leq N_G - 1$, $V(t) = (1-\pi)[v_e + \delta V(t)] + \pi \delta[\beta V(t+1) + (1 - \beta) V(t)]$ and $V(t) = v_H^R$. Arguments similar to the proof of Proposition 7 shows that these strategies form an equilibrium and $V(0) = v_f^B$ is the highest payoff that the expert can achieve in any equilibrium of the repeated cheap talk game.

Q.E.D

Proof of Proposition 10. The following strategies form an equilibrium of the repeated cheap talk game where the expert deceives the receivers at $M$ subsequent stages where the true state is $\theta_1$. The equilibrium strategies consist of three phases: the deception phase, the rewarding phase and the punishment phase. Players start in the deception phase, where the expert sends the message $\theta_2$ regardless of the true state. Deception phase ends when the expert deceives the receivers $M$ stages (i.e., the expert sends the message $\theta_2$ when the true state is $\theta_1$ for $M$ times). Once the deception phase ends, the players move to the rewarding phase, where the expert always tells the truth. The rewarding phase lasts forever. In both deception and rewarding phases, receivers approve (reject) the project whenever they observe the message $\theta_2$ ($\theta_1$). If at least one of the players deviates from his/her strategies in any phase, then all three players move to the punishment phase and
stay there forever. During the punishment phase, the receivers always reject the project regardless of the message they observe, and the expert always sends the message $\theta_1$.

We know that the punishment and the rewarding phase strategies are optimal for sufficiently large values of $\delta$. All we need to show is that the deception phase strategies are also optimal. Given these strategies, the expert’s continuation payoffs in the deception phase are strictly positive. However, the best deviation and continuation payoff for the expert is 0. Therefore, deviation is never optimal for the expert. Similarly, deviation from his strategies is never optimal for receiver 1 if deviation from his strategies is never optimal for receiver 2. Therefore, all we need to check if receiver 2 has profitable deviations from his strategies in the deception phase. Let $W$ denote receiver 2’s expected payoff in the game if he plays according to his prescribed strategies. Receiver 2’s continuation payoff will be 0 when he deviates in the deception phase. Therefore, optimality of the equilibrium strategies imply that $W \geq 0$. Thus, all we need to check is whether or not $W \geq 0$ holds.

Let $W(n)$ be receiver 2’s continuation payoff following a history in which no player has ever deviated before, and the true state was $\theta_1$ in exactly $n$ times during this history. Hence, receiver 2’s game payoff can be found by the following system of recursive equations:

\[
W(0) = (1 - \pi)[v_2 + \delta W(0)] + \pi[-v_1 + \delta W(1)] \\
W(1) = (1 - \pi)[v_2 + \delta W(1)] + \pi[-v_1 + \delta W(2)] \\
\vdots \\
W(M - 1) = (1 - \pi)[v_2 + \delta W(M - 1)] + \pi[-v_1 + \delta W(M)] \\
W(M) = \frac{(1 - \pi)v_2}{1 - \delta}
\]

Solving this systems of equations yield

\[
W = W(0) = \frac{(1 - \pi)v_2}{1 - \delta} - \frac{v_1 \pi}{1 - \delta} \left[ 1 - \left( \frac{\delta \pi}{1 - \delta(1 - \pi)} \right)^M \right].
\]

Note that $W(0) < W(1) < \ldots < W(M)$, and $W(0) \geq 0$ if and only if

\[
\left( \frac{\delta \pi}{1 - \delta(1 - \pi)} \right)^M \geq 1 - \frac{(1 - \pi)v_2}{\pi v_1} = \mu^*.
\]

Since $M$ is the largest natural number satisfying $M \leq \frac{\ln \mu^*}{\ln(\delta \pi/(1 - \delta(1 - \pi)))}$, then $W(0) \geq 0$ as required. This completes the proof.

Q.E.D
References


