Overconfidence, Imperfect Competition, and Evolution∗

Karen Khachatryan†
Middlesex University London
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Abstract

This study explores whether market competition between firms owned and run by managers favors overconfident managers. We study this question in a linear duopoly setting with differentiated products. The main result is that when there is complete information about the competitor’s type, evolutionary market selection forces will always favor a positive degree of managerial overconfidence. This result is robust to both the form of the strategic interaction and the nature of product differentiation. We also study the case of incomplete information about the competitor’s type under quantity competition and show that evolutionary forces may still favor overconfident managers if market selection is driven by relative rather than absolute profit performance.

Keywords: Overconfidence, imperfect competition, product differentiation, evolution, market selection.

JEL codes: C72, D01, D03, D43, D83, L10

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†Middlesex University Business School, The Burroughs, London NW4 4BT, UK; k.khachatryan@mdx.ac.uk.
1. Introduction

Humans show many psychological biases, of which overconfidence is one of the most documented. In this paper we examine the possibility that biased beliefs regarding self-confidence, and in particular overconfidence, may be an asset in competition. More specifically, that firms run by overconfident managers may earn higher profits than competing firms run by managers with correct beliefs about their own firm’s demand, as in textbook models. Will market competition select for a certain degree of overconfidence that will prevail in the long run?

We study this possibility in a simple oligopoly setting with differentiated products. In our model each of the two firms is run by a manager-cum-owner who may have incorrect beliefs about the relative merits of their own firm’s products. We interpret a tendency to overestimate one’s own firm’s product demand—or the extent of product differentiation—as an expression for overconfidence.

Our main contribution is to formalize these phenomena exactly in a few simple market settings, and to show under which conditions market competition selects overconfident managers. Indeed, we shown that a firm run by a moderately overconfident manager will earn higher profits than its competitor if the latter is run by a manager with correct beliefs about own firm’s demand if either the firms compete by setting prices and the products are complementary or the firms compete by setting quantities and the products are imperfect substitutes.

The intuition is simple. Under quantity competition the best response against an overconfident competitor is to cut down one’s own production when the goods are substitutes and increase production when the goods are complementary. Furthermore, overconfidence has an asymmetric effect on prices. When the goods are imperfect substitutes this will favor the overconfident manager relatively more. However, when the goods are imperfect complements this will favor the rational manager. Similarly, under price competition the best response against an overconfident competitor is to lower your own price when the products are imperfect substitutes and raise your own price when the products are complementary. However, the effect on demand is asymmetric because your own price has a bigger impact on your own profits than your competitor’s price. The second effect will dominate when the goods are imperfect substitutes, and the first effect will dominate when the goods are imperfect complements.

Moreover, under quantity competition two competitors run by equally overconfident managers will earn higher profits than two competitors run by managers with correct beliefs when the goods are imperfect complements. Similarly, under price competition two competitors run by equally overconfident managers will earn higher profits than two competitors run by managers with correct beliefs when the goods are substitutable. Given this tendency towards overconfidence one may ask what degree of confidence, if any, will prevail in the long run in a given market context, if there is free entry and exit of managers with different degrees of self-confidence. We identify and characterize this degree of self-confidence, which we call the evolutionarily robust degree of self-confidence.

Overconfidence has been attracting much attention recently from psychologists and economists.
The literature on judgment under uncertainty has found that people tend to be overconfident about the information they have, in that their subjective probability distributions on relevant events are too tight (Cesarini et al., 2006; Kahneman et al., 1982). Overconfidence has also been found in various professions such as entrepreneurs (Busenitz and Barney, 1997; Cooper et al., 1988), policy experts (Tetlock, 1999), and security analysts (Chen and Jiang, 2006). The implications of overconfidence for economic choices and especially for financial markets have recently been studied by numerous researchers (Gervais and Odean, 2001; Kyle and Wang, 1997; Scheinkman and Xiong, 2003). Hvide (2002) offers a theoretical argument for the endogenous emergence of overconfidence. Recently, Johnson and Fowler (2011) have developed an evolutionary model demonstrating that overconfident populations are evolutionarily stable in a wide range of environments.

In order to model the level of confidence, we allow for asymmetric beliefs about the extent of product differentiation. Recent papers that involve asymmetric beliefs (differing priors) include Admati and Pfleiderer (2004); de la Rosa (2011); Fang and Moscarini (2005); Van den Steen (2004, 2007), and Grubb (2009), among others. Admati and Pfleiderer (2004) study an information transmission game where the sender can be overconfident in his ability to observe the true state. Fang and Moscarini (2005) consider the workers’ overconfidence in their skills on wage policies, and de la Rosa (2011) studies the effects of overconfidence on incentive contracts in a moral hazard framework. Van den Steen (2004) models how overconfidence can arise from heterogeneous priors when individuals have a choice over projects, while Van den Steen (2007) examines worker incentives when a worker may disagree with the manager regarding the best course of action. Grubb (2009) develops a model of screening with consumers who overestimate the precision of their demand forecasts. Heller (2011) presents an evolutionary foundation for overconfidence based on diversification of risk.

Finally, this paper is part of a larger literature on the evolution of preferences, beginning with Güth and Yaari (1992) and Güth (1995), which is based on the indirect evolutionary approach. A number of contributions have shown that evolution may yield preferences that deviate from the underlying “fitness” function; see e.g. Heifetz et al. (2007a,b), Dekel et al. (2007) and Alger and Weibull (2012) for very general formulations.¹

The paper is organized as follows. Section 2 introduces the model setup. Section 3 analyzes the model in which firms compete by setting quantities and there is complete information about managerial types. Section 4 extends the analysis to an incomplete information scenario. Section 5 instead focuses on price competition with complete information about managerial types. Section 6 concludes with a summary and a discussion of some directions for further research. Some graphical illustrations and the proofs of Propositions 6 and 14, are given in appendixes at the end of the paper.

¹The relation to the literature on the evolution of preferences will be clarified in the next version of the paper.
2. The Model

Consider two firms that produce differentiated goods. The representative consumer’s utility, following Dixit (1979) and Singh and Vives (1984), is a function of the consumption of the two differentiated goods and the composite good \( I \) and is given by

\[
U(q_1, q_2, I) = \alpha (q_1 + q_2) - \frac{1}{2} \left( q_1^2 + q_2^2 + 2\gamma q_1 q_2 \right) + I.
\]

Thus utility is quadratic in the consumption of \( q \)-goods and linear in the consumption of other goods, \( I \). The parameter \( \gamma \in [-1, 1] \) measures the substitutability between the two products. When goods are substitutes, the degree of substitutability could be interpreted in terms of horizontal product differentiation. If \( \gamma = 0 \), the products are unrelated and each firm has a monopolistic market power, while if \( \gamma = 1 \), the products are perfect substitutes. A negative \( \gamma \) implies that the goods are complementary. Finally, \( \alpha > 0 \) measures quality in a vertical sense.²

Consumers maximize utility subject to the budget constraint \( \sum p_i q_i + I \leq m \), where \( m \) denotes income and the price of the composite good is normalized to one.

This utility function gives rise to a linear demand structure. The first-order condition determining the optimal consumption of good \( i \) is

\[
\frac{\partial U}{\partial q_i} = \alpha - q_i - \gamma q_j = p_i
\]

for \( i = 1, 2, j \neq i \). Therefore, the inverse demand is given by

\[ p_1 = \alpha - q_1 - \gamma q_2 \]

for firm 1 and

\[ p_2 = \alpha - \gamma q_1 - q_2 \]

for firm 2 in the region of quantity space where prices are positive.

Inverting, we get the following direct demand system

\[
\begin{align*}
q_1 &= \frac{1}{1 - \gamma^2} \left( \alpha (1 - \gamma) - p_1 + \gamma p_2 \right) \\
q_2 &= \frac{1}{1 - \gamma^2} \left( \alpha (1 - \gamma) + \gamma p_1 - p_2 \right)
\end{align*}
\]

in the region of price space where quantities are positive. Demand for good \( i \) is always downward sloping in its own price and increases (decreases) with increase in the price of the competitor if the goods are imperfect substitutes (complements).

²Note that an increase in the degree of product differentiation (a decline in \( \gamma \)) shifts the demand curves for both firms outwards.
Both firms have the same constant marginal cost, $u$ and there are no fixed costs of production. We consider from now on prices net of marginal cost. This is without loss of generality since if marginal costs are positive, we may replace $\alpha$ by $\alpha - u$.

Each firm $i$ is owned and run by a manager. Each manager $i$ can be overconfident or underconfident, which is modeled as follows. Each manager $i$ is allowed to have different beliefs (perceptions) about the parameter $\gamma$. Manager $i$ believes $\gamma$ to be $\gamma_i = k_i \gamma$, where $k_i \in K = (0, 1/|\gamma|)$\(^3\). We will refer to $k_i$ as manager $i$’s degree of self-confidence or simply the type of the manager. We assume that the managers correctly perceive their opponents’ beliefs about the market, but do not adjust theirs accordingly. In other words, we assume that the type of the manager is observable and the managers simply “agree to disagree” about the representative consumers’ preferences (parameter $\gamma$).\(^4\) We will also consider the case where a manager’s type is not observable.

First, suppose the goods are imperfect substitutes ($0 < \gamma < 1$). Then a manager with self-confidence $k_i < 1$ is overconfident, while a manager with $k_i > 1$ is underconfident.\(^5\) An overconfident manager perceives the products to be less substitutable (more horizontally differentiated) than they actually are, while an underconfident manager perceives the products to be more substitutable (less horizontally differentiated) than they actually are.

Second, suppose the goods are complementary ($-1 < \gamma < 0$). Then a manager with self-confidence $k_i > 1$ is overconfident, while a manager with $k_i < 1$ is underconfident. An overconfident manager believes that the products are more complementary than they actually are, while an underconfident manager believes the products to be less complementary than they actually are.

Now, when the two managers disagree about $\gamma$ and manager 1 has self-confidence $k_1$ and manager 2 has self-confidence $k_2$ then manager 1 thinks that the indirect demand system is given by

\[
\begin{align*}
    p_1 &= \alpha - q_1 - k_1 \gamma q_2 \\
    p_2 &= \alpha - k_1 \gamma q_1 - q_2
\end{align*}
\]

in the quantity space where prices are positive, while the direct demand is given by

\[
\begin{align*}
    q_1 &= \frac{1}{1 - k_1^2 \gamma^2} \left( \alpha (1 - k_1 \gamma) - p_1 + k_1 \gamma p_2 \right) \\
    q_2 &= \frac{1}{1 - k_1^2 \gamma^2} \left( \alpha (1 - k_1 \gamma) - p_2 + k_1 \gamma p_1 \right)
\end{align*}
\]

\(^3\)In other words we need to have $\gamma_i^2 < 1$ to ensure concavity of the consumer’s utility function.
\(^4\)Strictly speaking we do not have to endow our managers with point beliefs. We could alternatively assume, that there is uncertainty about demand (consumers’ preferences). One may think of a lottery over different levels of $\gamma$ in the representative consumer’s utility function, keeping $\alpha$ fixed. An overconfident manager underestimates the risk of a high realization of $\gamma$ and therefore expects a higher demand (price). Owning to risk neutrality, all the results would go through if $\gamma$ represents the true expectation. Then this could be a model of non-common priors, for example.
\(^5\)To see this note that a higher perceived $|\gamma_i|$ would imply a higher (perceived) marginal utility of $q_i$ for the representative consumer.
in the price space where quantities are positive.

Similarly, from the perspective of manager 2, the inverse demand is given by

\[
\begin{align*}
p_1 &= \alpha - q_1 - k_2 \gamma q_2 \\
p_2 &= \alpha - k_2 \gamma q_1 - q_2
\end{align*}
\] (5)

in the quantity space where prices are positive, and the direct demand is given by

\[
\begin{align*}
q_1 &= \frac{1}{1 - k_2^2 \gamma^2} (\alpha (1 - k_2 \gamma) - p_1 + k_2 \gamma p_2) \\
q_2 &= \frac{1}{1 - k_2^2 \gamma^2} (\alpha (1 - k_2 \gamma) - p_2 + k_2 \gamma p_1)
\end{align*}
\] (6)

in the price space where quantities are positive.

3. Quantity Competition

In this section, we analyze the game in which firms compete by setting quantities under complete information, i.e., when both managerial types are mutually known. For expository purposes, we start with the case of one over- or underconfident manager and then move on to the more general case of two arbitrarily confident managers.

3.1. One Overconfident Manager

Consider the case when manager 1 holds correct beliefs about \( \gamma \), \( k_1 = 1 \), while manager 2 is biased and has self-confidence \( k_2 \neq 1 \). Manager 2 is overconfident if either \( \gamma > 0 \) and \( k_2 < 1 \) or \( \gamma < 0 \) and \( k_2 > 1 \). These beliefs are common knowledge. Each manager strives to maximize his or her perceived profits.

The manager of firm 1 solves

\[
\max_{q_1 \geq 0} \{ (\alpha - q_1 - \gamma q_2) q_1 \}
\]
taking \( q_2 \) as given. The first-order condition for profit maximization is

\[
\alpha - 2q_1 - \gamma q_2 = 0,
\]

from which we get the following best-response function

\[
q_1 (q_2) = \frac{1}{2} (\alpha - \gamma q_2).
\] (7)

\footnote{Throughout the paper, in the first-order conditions for profit maximization and the associated best response functions, we suppress the nonnegativity constraint(s) but always make sure that none of them is violated.}
The manager of firm 2 solves
\[
\max_{q_2 \geq 0} \{ (\alpha - k_2 \gamma q_1 - q_2) q_2 \}
\]
taking \( q_1 \) as given. The first-order condition for profit maximization is
\[
\alpha - 2q_2 - k_2 \gamma q_1 = 0,
\]
from which we get the following best-response function
\[
q_2(q_1) = \frac{1}{2} [\alpha - k_2 \gamma q_1]. \tag{8}
\]
Solving the best response functions (7) and (8) simultaneously, we get that in the unique interior Nash equilibrium of the quantity competition game
\[
q_1^* = \frac{\alpha (2 - \gamma)}{4 - k_2 \gamma^2}
\]
and
\[
q_2^* = \frac{\alpha (2 - k_2 \gamma)}{4 - k_2 \gamma^2}.
\]
These quantity choices imply the following (true) equilibrium prices
\[
p_1^* (q_1^*, q_2^*) = \frac{\alpha (2 - \gamma)}{4 - k_2 \gamma^2},
\]
and
\[
p_2^* (q_1^*, q_2^*) = \frac{\alpha (2 - (2 - k_2) \gamma + (1 - k_2) \gamma^2)}{4 - k_2 \gamma^2}.
\]
The (true) equilibrium profits are, respectively,
\[
\pi_1^* = \frac{\alpha^2 (2 - \gamma)^2}{(4 - k_2 \gamma^2)^2},
\]
and
\[
\pi_2^* = \frac{\alpha^2 (2 - k_2 \gamma) (2 - (2 - k_2) \gamma + (1 - k_2) \gamma^2)}{(4 - k_2 \gamma^2)^2}.
\]
\footnote{Note that the associated second-order conditions for a maximum are trivially satisfied and the resulting equilibrium quantities and prices are indeed positive given the parameter ranges for \( \gamma \) and \( k \).}
Hence, we have the following differences:

\[ q_1^* - q_2^* = \frac{\alpha (k_2 - 1) \gamma}{4 - k_2 \gamma^2} \]
\[ p_1^* - p_2^* = \frac{\alpha (1 - k_2) \gamma (1 - \gamma)}{4 - k_2^2 \gamma^2} \]
\[ \pi_1^* - \pi_2^* = \frac{\alpha \gamma^2 (k_2 - 1) (1 + k_2 (1 - \gamma))}{(4 - \gamma^2 k_2)^2} \]

First, note that when both managers are rational and so have neutral self-confidence, these differences vanish. Second, note that a manager’s self-confidence affects quantities and prices in an opposite manner. Which effect dominates for profits? To answer this question, we now look at the case of imperfect substitutes and imperfect complements separately.

### 3.1.1. Substitutes

**Proposition 1** Suppose the goods are imperfect substitutes (\( \gamma > 0 \)) and manager 2 is overconfident (\( k_2 < 1 \)). In the unique interior Nash equilibrium of the quantity competition game we have the following:

(a) \( q_2^* > q_1^* \)
(b) \( p_2^* < p_1^* \)
(c) \( \pi_2^* > \pi_1^* \)

In other words, firm 2—run by an overconfident manager—produces more and earns a higher profit than its unbiased competitor, firm 1.

The diagram in Figure 1 shows how \( \pi_2^* \) (solid curve) and \( \pi_1^* \) (dashed curve) depend on \( k_2 \) (the horizontal axis), for \( \gamma = 0.6 \) and \( \alpha = 1 \). The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all \( k_2 < 1 \) the overconfident manager 2 produces more and makes a higher profit than unbiased manager 1,

(b) There is an optimal degree of overconfidence for manager 2, \( k_2 \approx 0.67 \),

(c) The industry profit is increasing in \( k_2 \),

(d) For moderate degrees of overconfidence (\( 0.3 < k_2 < 1 \))\(^8\), manager 2 earns a higher profit than had she also had correct perceptions (had \( k_2 = k_1 = 1 \)).

\(^8\)For other \( \gamma \), this interval would be \( \min \left\{ 0, \frac{4 \gamma + 2 \gamma^2 - 4}{2 \gamma^2 + 3 \gamma + 1}, 1 \right\} \).
3.1.2. Complements

Proposition 2 Suppose the goods are imperfect complements ($\gamma < 0$) and manager 2 is overconfident ($k_2 > 1$). In the unique interior Nash equilibrium of the quantity competition game, we have the following:

(a) $q_2^* > q_1^*$
(b) $p_2^* < p_1^*$
(c) $\pi_2^* < \pi_1^*$

In other words, the firm run by an overconfident manager produces more, but earns lower profit, than its unbiased competitor. In this case, the value of being overconfident is not positive. Similarly,

Proposition 3 Suppose the goods are imperfect complements ($\gamma < 0$) and manager 2 is underconfident ($k_2 < 1$). In the unique interior Nash equilibrium of the quantity competition game, we have the following:

(a) $q_2^* < q_1^*$
(b) $p_2^* > p_1^*$
(c) $\pi_2^* > \pi_1^*$
In other words, the firm run by an underconfident manager produces less and earns a higher profit, than its unbiased competitor. When the goods are imperfect complements it pays off to be underconfident.

The diagram in Figure 2 shows how $\pi_2^*$ (solid curve) and $\pi_1^*$ (dashed curve) depend on $k_2$ (the horizontal axis), for $\gamma = -0.6$ and $\alpha = 1$. The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all $k_2 > 1$ the overconfident manager 2 produces more and makes a lower profit than unbiased manager 1,

(b) However, for all $k_2 < 1$ the underconfident manager 2 makes a higher profit,

(c) The industry profit is increasing in $k_2$,

(d) For moderate degrees of overconfidence ($1 < k_2 < 1.6$)\(^9\), manager 2 earns a higher profit than had she also had correct perceptions (had $k_2 = k_1 = 1$),

(e) For all degrees of underconfidence manager 2 earns a lower profit than had she also had correct perceptions (had $k_2 = k_1 = 1$).

\(^9\)For other $\gamma$, the exact interval is $\left(1, \frac{1 + 2\gamma^2 - 4}{2\gamma^3 + 2\gamma - 4}\right)$.
3.1.3. Discussion and Intuition

First, note that manager 2, who has biased self-confidence $k_2 \neq 1$, is maximizing a payoff, which can be written in the following form:

$$\tilde{\pi}_2(q_1, q_2) = (\alpha - k_2 \gamma q_1 - q_2) q_2$$

$$= (\alpha - \gamma q_1 - q_2) q_2 + (1 - k_2) \gamma q_1 q_2,$$

a sum of two terms. The first term is the (true) payoff, $\pi_2(q_1, q_2) = (\alpha - \gamma q_1 - q_2) q_1$ and the second term is due to biased self-confidence and is positive if the manager is overconfident and negative if underconfident.

Second note that

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_1} = -\gamma q_2$$

and

$$\frac{\partial \pi_2(q_1, q_2)}{\partial q_2 \partial q_1} = -\gamma.$$

Thus when $\gamma > 0$ the managers impose negative externalities on one another (i.e., the larger is the action of manager 2, $q_2$, the lower is manager 1’s payoff, and visa versa. Moreover, the actions, i.e., quantity choices in this case, are strategic substitutes in the sense of Bulow et al. (1985) (the best response functions are decreasing in the $(q_1, q_2)$ space). In contrast, when $\gamma < 0$, the managers impose positive externalities on one another, and their actions are strategic complements.

Now, consider the full perceived payoff and note that

$$\frac{\partial \tilde{\pi}_2(q_1, q_2)}{\partial q_1} = -k_2 \gamma q_2$$

and

$$\frac{\partial \tilde{\pi}_2(q_1, q_2)}{\partial q_2 \partial q_1} = -k_2 \gamma.$$

Suppose the goods are imperfect substitutes, $\gamma > 0$. Thus, an overconfident manager 2 not only overestimates the returns from her own action for any action taken by the other manager $((1 - k_2) \gamma q_1 > 0)$, but also underestimates the returns from the opponent’s action. Thus biased-beliefs operate through two different channels that reinforce each other. An overconfident manager acts more aggressively than a rational manager as he not only exaggerates the impact of his actions on his payoffs but also because he underestimates the impact of other’s actions on his payoffs. When goods are imperfect substitutes, the players’ actions are strategic substitutes, so the aggressive behavior of an overconfident manager induces the rival to play soft. In other words, a managers’ actions impose negative externalities on their opponent, so the soft behavior of his opponents benefits the overconfident player.

Next, suppose the goods are complementary, $\gamma < 0$. An overconfident manager 2 still overestimates the returns from her own action for any action taken by the other manager $((1 - k_2) \gamma q_1 > 0)$, but she also overestimates the returns from the opponent’s action. Thus biased-beliefs operate through two different channels that work against each other. When $\gamma < 0$, actions are strategic complements, so aggressive behavior of an overconfident manager induces rivals to play aggressively as well. Since now the actions of managers impose positive externalities on each other, the
aggressive behavior of an overconfident manager benefits both managers in this case.

Of course, being overconfident is also costly, since a manager is not making an optimal decision. When \( \gamma > 0 \), the benefit of overconfidence gained through strategic advantage outweighs the cost, so in relative terms the overconfident manager is still better off. However when \( \gamma < 0 \), the benefit of overconfidence gained through strategic advantage is higher for the opponent, so in relative terms the overconfident manager is worse off.

### 3.2. Two Arbitrarily Confident Managers

We now turn to the analysis of the case of two arbitrarily confident managers. Throughout the analysis remember that \( k_1, k_2 \in K = (0, |1/\gamma|) \).

The manager of firm 1 solves

\[
\max_{q_1 \geq 0} \{ (\alpha - q_1 - k_1 \gamma q_2) q_1 \}
\]

taking \( q_2 \) as given. The first-order condition for profit maximization is

\[
\alpha - 2q_1 - k_1 \gamma q_2 = 0,
\]

from which we get the following best-response function

\[
q_1 (q_2) = \frac{1}{2} [\alpha - k_1 \gamma q_2].
\] (10)

Similarly, the manager of firm 2 solves

\[
\max_{q_2 \geq 0} \{ (\alpha - k_2 \gamma q_1 - q_2) q_2 \}
\]

taking \( q_1 \) as given. The first-order condition for profit maximization is

\[
\alpha - 2q_2 - k_2 \gamma q_1 = 0,
\]

from which we get the following best-response function

\[
q_2 (q_1) = \frac{1}{2} [\alpha - k_2 \gamma q_1].
\] (11)

Note that when the goods are imperfect substitutes (\( \gamma > 0 \)) we have downward sloping best response functions and quantities are strategic substitutes. However, when the goods are complementary (\( \gamma < 0 \)), the best response functions are upward sloping and the quantities are strategic complements.

Solving the best response functions (10) and (11) simultaneously, we get that in the unique
interior Nash equilibrium of the quantity competition game\textsuperscript{10}

\[ q^*_1 = \frac{\alpha (2 - k_1 \gamma)}{4 - k_1 k_2 \gamma^2} \]

and

\[ q^*_2 = \frac{\alpha (2 - k_2 \gamma)}{4 - k_1 k_2 \gamma^2}. \]

These equilibrium quantities are indeed positive for all \( k_1, k_2 \in K \).

These quantity choices by the managers’ imply the following (true) equilibrium prices

\[ p^*_1 (q^*_1, q^*_2) = \frac{\alpha (2 - (2 - k_1) \gamma + (1 - k_1) k_2 \gamma^2)}{4 - k_1 k_2 \gamma^2}, \]

and

\[ p^*_2 (q^*_1, q^*_2) = \frac{\alpha (2 - (2 - k_2) \gamma + (1 - k_2) k_1 \gamma^2)}{4 - k_1 k_2 \gamma^2}. \]

The (true) equilibrium profits are, respectively,

\[ \pi^*_1 (k_1, k_2) = \frac{\alpha^2 (2 - k_1 \gamma) (2 - (2 - k_1) \gamma + (1 - k_1) k_2 \gamma^2)}{(4 - k_1 k_2 \gamma^2)^2}, \]

and

\[ \pi^*_2 (k_1, k_2) = \frac{\alpha^2 (2 - k_2 \gamma) (2 - (2 - k_2) \gamma + (1 - k_2) k_1 \gamma^2)}{(4 - k_1 k_2 \gamma^2)^2}. \]

Note that these prices, and hence profits, are always positive given parameter restrictions on \( \gamma \) and \( k_i \).

Comparing equilibrium quantities we see that

\[ q^*_1 - q^*_2 = \frac{\alpha \gamma (k_2 - k_1)}{4 - k_1 k_2 \gamma^2}. \]

The denominator is always positive, therefore

\[ \text{sign} (q^*_1 - q^*_2) = \text{sign} (\gamma (k_2 - k_1)). \]

This implies that \( q^*_1 > q^*_2 \) if and only if:

(a) \( \gamma > 0 \) and \( k_1 < k_2 \), i.e. the more self-confident manager will produce more when the goods are imperfect substitutes.

(b) \( \gamma < 0 \) and \( k_1 > k_2 \), i.e. the more self-confident manager will produce more when the goods are imperfect complements.

\textsuperscript{10}Note that the first-order conditions for profit maximization are also sufficient.
In other words, irrespective of whether the goods are imperfect substitutes or imperfect complements, the more self-confident managers will produce more. In particular, we already saw in the previous section that when an overconfident and a rational manager interact, the overconfident manager will produce more.

Comparing equilibrium prices we see that

\[
p_1^* (q_1^*, q_2^*) - p_2^* (q_1^*, q_2^*) = \frac{\alpha \gamma (1 - \gamma)(k_1 - k_2)}{4 - k_1 k_2 \gamma^2},
\]

implying that

\[
\text{sign} (p_1^* - p_2^*) = \text{sign} (\gamma (k_1 - k_2)).
\]

We note that self-confidence moves prices and quantities into opposite directions. We have that \(p_1^* > p_2^*\) if and only if:

(a) \(\gamma > 0\) and \(k_1 > k_2\), i.e. the less self-confident managers will face a higher market price when the goods are imperfect substitutes.

(b) \(\gamma < 0\) and \(k_1 < k_2\), i.e. the less self-confident managers will face a higher market price when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or imperfect complements, more self-confident managers will face a lower market price than they have perceived. In particular, when an overconfident and a rational manager interact, the overconfident manager will face a lower price.

We have seen that the manager’s level of self-confidence affects prices and quantities in an opposite manner. Which effect will dominate? What is the overall effect on equilibrium profits?

The (true) equilibrium profit difference is

\[
\pi_1^* - \pi_2^* = \frac{\alpha^2 \gamma^2 (k_2 - k_1) (k_1 - k_1 k_2 \gamma + k_2)}{(4 - k_1 k_2 \gamma^2)^2}.
\]

It is easy to verify that the last factor is always positive for all \(k_1, k_2 \in K\). Therefore, \(\pi_1^* > \pi_2^*\) if and only if \(k_1 < k_2\).

We summarize the findings above in the following two propositions:

**Proposition 4** Suppose the goods are imperfect substitutes (\(\gamma > 0\)) and manager \(i\)’s degree of self-confidence is \(k_i \in K\), \(i = 1, 2\). Then in the unique interior Nash equilibrium of the quantity competition game, the following are equivalent:

(a) \(k_i < k_{-i}\)

(b) \(q_i^* > q_{-i}^*\)
(c) \( p_i^* < p_{-i}^* \)

(d) \( \pi_i^* > \pi_{-i}^* \)

**Proposition 5** Suppose the goods are imperfect complements \( (\gamma < 0) \) and manager \( i \)'s degree of self-confidence is \( k_i \in K, \ i = 1, 2 \). Then in the unique interior Nash equilibrium of the quantity competition game, the following are equivalent:

(a) \( k_i < k_{-i} \)

(b) \( q_i^* < q_{-i}^* \)

(c) \( p_i^* > p_{-i}^* \)

(d) \( \pi_i^* > \pi_{-i}^* \)

In other words, the manager with a higher degree of confidence will produce more and make higher profits when the goods are imperfect substitutes but will make lower profits when the goods are imperfect complements. In particular, when two underconfident managers interact, the one who is less underconfident will make a higher profit when the goods are imperfect substitutes but will make a lower profit if the goods are imperfect complements. When two overconfident managers interact, the more overconfident manager will earn higher profits when the goods are imperfect substitutes but will make lower profits when the goods are imperfect complements.

Next, is there an optimal degree of self-confidence for a manager to have? Before we answer that question, we make the following observation.

**Proposition 6** Suppose \( \gamma \in \left(-\frac{6}{7}, 1\right) \) and \( k_1, k_2 \in K \). As a function of own confidence, \( k_i \), the equilibrium profit \( \pi_i^* \) to manager \( i \) is a strictly concave function with a unique maximum at

\[
\hat{k}_i (k_j) = \frac{4 (1 - k_j \gamma)}{4 - 2k_j \gamma - k_j \gamma^2}.
\]

(12)

**Proof.** See appendix.

In particular if \( k_1 = 1 \) and \( \gamma = 0.6 \ (-0.6) \) as in the preceding subsection, then \( \hat{k}_2 = 0.656 \ (1.322) \), in broad agreement with the above approximate observation. More generally, (1) for \( \gamma > 0 \), equation (12) defines \( \hat{k}_i \) as a number in \((0, 1)\); and (2) for \( \gamma < 0 \), equation (12) defines \( \hat{k}_i \) as a number larger than 1. Furthermore, if \( \gamma \in \left(-\frac{2}{3}, 1\right) \), then \( \hat{k}_i < \left|\frac{1}{\gamma}\right| \). We henceforth assume that \( \gamma \in \left(-\frac{2}{3}, 1\right) \).

Next, we turn to market selection based on absolute profits and robustness analysis.
3.3. Market Selection: Evolutionarily Robust Self-Confidence

Consider a manager with confidence $k'$ who is matched against a manager with confidence $k$. The profit to the first manager is

$$
\pi^* = v(k', k) = \frac{\alpha^2 (2 - k'\gamma) ((1 - k') k\gamma^2 + (k' - 2) \gamma + 2)}{(4 - k'k\gamma^2)^2}
$$

We will call a degree of self-confidence $k$ evolutionarily robust if there for every $k' \neq k$ exists an $\bar{\varepsilon} > 0$ such that for all $\varepsilon \in (0, \bar{\varepsilon})$:

$$(1 - \varepsilon) v(k, k) + \varepsilon v(k, k') > (1 - \varepsilon) v(k', k) + \varepsilon v(k', k')$$  (13)

In words: consider a pool of managers who all have the same degree of self-confidence, $k$. Suppose that a small population fraction of these would “mutate” to some other degree of self-confidence, $k'$. Let $0 < \varepsilon < 1$ be the population share of such “mutants”. The expected profit that a manager of the “incumbent” type $k$ would make when matched against another manager of the same incumbent type would be $v(k, k)$, while the expected profit to the same manager if matched against a manager of the “mutant” type $k'$ would be $v(k', k)$. Hence, the left-hand side in (13) is the expected profit to a manager of the incumbent type when randomly matched against another manager in the pool of managers. Likewise, the right-hand side of (13) is the expected profit to a manager of the mutant type when randomly matched against another manager in the pool.\(^{11}\) The incumbent degree of self-confidence is thus defined as evolutionarily robust if there exists no other degree of self-confidence that, if appearing in a sufficiently small share of the manager pool, would earn at least the same expected profit as the managers of the incumbent type do, in the mixed population.

For a degree of self-confidence $k$ to be evolutionarily robust in this sense, it is clearly necessary that

$$v(k', k) \leq v(k, k) \text{ for all } k' > 0.$$  

In other words, $k$ should be optimal against itself.\(^{12}\) Conversely, suppose that $k$ is optimal against itself. If, moreover, no other $k'$ is also optimal against $k$, then it follows that $k$ is evolutionarily robust. Hence, it is of interest to find those $k$ that are optimal against themselves.

A robust degree of confidence $k$ would need to satisfy equation (12) for $k_i = k_j = k$. The unique degree of confidence with this robustness property is evidently\(^{13}\),

$$
k^r = \frac{2}{2 + \gamma}.
$$

\(^{11}\)We assume that the pool is so large that we do not have to adjust the matching probabilities by first subtracting the manager himself from the pool, before randomizing.

\(^{12}\)For if there would exist a manager type $k'$ that would earn more against $k$ than $k$ does, that is, $v(k', k) > v(k, k)$, then, for $\varepsilon > 0$ sufficiently small, (13) would be violated.

\(^{13}\)Note that (12) has two fixed points: $-\frac{2}{\gamma}$ and $\frac{2}{2 + \gamma}$. The first one falls out of the interval $K$. 

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For the case of substitutes, we illustrate this in Figure 3, showing the contour map of the function $v$, with $k'$ on the horizontal axis and $k$ on the vertical, when $\alpha = 1$ and $\gamma = 0.6$. Figure 4 the same illustration for the case of complements, when $\alpha = 1$ and $\gamma = -0.4$. A degree of confidence $k^r$ is robust against market selection if and only if the tangent of the isoquant through the point $(k, k)$—the thick curve—is horizontal at that point, and no other point on the horizontal line through that point has a higher $v$-value. Inspecting the graph we conclude that this degree is approximately $k = 0.77$ when the goods are imperfect substitutes and $k = 1.25$ when the goods are imperfect complements (indicated by the vertical and horizontal lines).

More formally, we have\footnote{Remember that we need to restrict $\gamma$ to $\left(-\frac{2}{3}, 1\right)$ to ensure that the evolutionarily robust self-confidence is less than $\frac{1}{k}$.}:

**Proposition 7** Suppose $\gamma \in \left(-\frac{2}{3}, 1\right)$. Under quantity competition the unique degree of confidence that is robust to market selection driven by absolute profits is

$$k^r = \frac{2}{\gamma + 2}, \quad (14)$$

**Proof.** Recalling Proposition 6, the proof is evident. \(\blacksquare\)

We illustrate the evolutionarily robust degree of self-confidence under quantity competition in Figure 5. We note that this evolutionarily robust degree of confidence is of the overconfident type and is independent of whether the goods are imperfect substitutes or imperfect complements, and
Figure 4. Evolutionarily robust degree of self-confidence when the goods are imperfect complements.

hence whether the choice variables are strategic substitutes or strategic complements.

Note that, when the goods are imperfect substitutes, the degree of overconfidence is higher the more substitutable the goods are. When the goods are independent, and each firm has a monopoly power, there is no strategic or selection value to being overconfident. This implies that overconfident managers should be more prevalent in industries where product differentiation is lower and there is more room for strategic interaction.

When the goods are imperfect complements, the degree of overconfidence is higher the more complementary the goods are. This implies that overconfident managers should be more prevalent in industries where product complementaries are higher.

3.3.1. Profit Comparison

Although in the evolutionarily robust equilibrium all the firms would be run by overconfident managers, it is instructive to compare profits with the case where all the managers were rational. The common profits when both managers have the evolutionarily robust degree of confidence is

\[
\pi^r = \frac{\alpha^2 (2 - k'\gamma)}{(4 - k'k\gamma^2)^2} \left( (1 - k') k\gamma^2 + (k' - 2) \gamma + 2 \right) \bigg|_{k=\frac{2}{2+\gamma}, k' = \frac{2}{2+\gamma}}
\]

\[
= \frac{\alpha^2 (4 - \gamma^2)}{16 (1 + \gamma)}.
\]
whereas the common profits when both managers are rational is

$$\pi^* = \left( \frac{\alpha}{2 + \gamma} \right)^2$$

Comparing these profits, we see that the difference is

$$\pi^r - \pi^* = \frac{\alpha^2 (4 - \gamma^2)}{16 (1 + \gamma)} - \left( \frac{\alpha}{2 + \gamma} \right)^2$$

$$= -\frac{1}{16} \frac{\alpha^2 \gamma^3 (4 + \gamma)}{(1 + \gamma) (2 + \gamma)^2},$$

which implies that

$$\text{sign} (\pi^r - \pi^*) = -\text{sign} (\gamma).$$

Thus the profit difference is negative if the products are imperfect substitutes ($\gamma > 0$) and positive if the products imperfect complements ($\gamma < 0$).

### 4. Quantity Competition under Incomplete Information

In the previous section we analyzed the quantity competition game under the assumption that the managers in a duopoly could observe each other’s types or levels of self-confidence. What can be said if they do know or cannot observe each other’s types? To analyze this scenario, we proceed
in two steps. In the first step, for expository purposes, we will look at the case when there is a large population of potential managers who are randomly matched and there are only two types of managers: rational, with self-confidence \( k = 1 \), and biased with self-confidence \( k \neq 1 \). Then we will allow for arbitrarily levels of self-confidence for all managers.

### 4.1. One Overconfident Manager

Suppose there is a mutually incomplete information about the manager’s type. Let \( 1 - \varepsilon \) be the population share of rational managers with self-confidence \( k = 1 \) and \( \varepsilon \) the population share of managers with self-confidence \( k \neq 1 \) (or alternatively, each manager will hold the belief that his opponent has self-confidence \( k = 1 \) with probability \( 1 - \varepsilon \) and self-confidence \( k \neq 1 \) with probability \( \varepsilon \)). Suppose there are only these types and the type of a manager is a private knowledge. We note that the average degree of confidence in the population, \( \bar{k} \), is

\[
\bar{k} = (1 - \varepsilon) \cdot 1 + \varepsilon \cdot k
\]

\[
= 1 + \varepsilon (k - 1)
\]

\[
= 1 - \varepsilon + \varepsilon k < 1 / |\gamma|,
\]

Also note that \( \bar{k} > k \) if \( k < 1 \) and \( \bar{k} < k \) if \( k > 1 \).

We are looking for a pure strategy Bayesian Nash equilibrium. At a Bayesian Nash equilibrium of the quantity competition game we must have that

\[
q^k \in \arg \max_{q^i \geq 0} \mathbb{E}[\pi | k],
\]

\[
q^1 \in \arg \max_{q^i \geq 0} \mathbb{E}[\pi | 1],
\]

where \( q^k \) is the quantity choice if a manager is of type \( k \neq 1 \) and \( q^1 \) is the quantity choice of a rational manager.

More precisely, we must have that

\[
q^k \in \arg \max_{q^i \geq 0} \left\{ \varepsilon(\alpha - q^i - k\gamma q^k)q^i + (1 - \varepsilon)(\alpha - q^i - k\gamma q^1)q^i \right\}
\]

\[
q^1 \in \arg \max_{q^i \geq 0} \left\{ \varepsilon(\alpha - q^i - \gamma q^k)q^i + (1 - \varepsilon)(\alpha - q^i - \gamma q^1)q^i \right\}
\]

or more simply

\[
q^k \in \arg \max_{q^i \geq 0} \left\{ (\alpha - q^i - \varepsilon k\gamma q^k - (1 - \varepsilon)k\gamma q^1)q^i \right\}
\]

\[
q^1 \in \arg \max_{q^i \geq 0} \left\{ (\alpha - q^i - \varepsilon \gamma q^k - (1 - \varepsilon)\gamma q^1)q^i \right\}
\]
The first-order conditions for maximization (which are also sufficient) yield:

\[
\begin{align*}
\alpha - 2q^i - \varepsilon k\gamma q^k - (1 - \varepsilon)k\gamma q^1 &= 0 \\
\alpha - 2q^i - \varepsilon \gamma q^k - (1 - \varepsilon)\gamma q^1 &= 0
\end{align*}
\]

These first-order conditions have to be satisfied on the equilibrium path, so we must have that

\[
\begin{align*}
\alpha - 2q^k - \varepsilon k\gamma q^k - (1 - \varepsilon)k\gamma q^1 &= 0 \\
\alpha - 2q^1 - \varepsilon \gamma q^k - (1 - \varepsilon)\gamma q^1 &= 0
\end{align*}
\]

or

\[
\begin{align*}
(2 + \varepsilon k\gamma) q^k + (1 - \varepsilon)k\gamma q^1 &= \alpha \\
\varepsilon \gamma q^k + (2 + (1 - \varepsilon)\gamma) q^1 &= \alpha
\end{align*}
\]

The solution to this system of equations is given by

\[
q^k = \frac{\alpha (2 + (1 - \varepsilon) (1 - k) \gamma)}{2 (2 + (1 - \varepsilon) \gamma + \varepsilon k\gamma)} = \frac{\alpha \left(2 + (\bar{k} - k) \gamma\right)}{2 \left(2 + \bar{k}\gamma\right)}
\]

and

\[
q^1 = \frac{\alpha (2 - \varepsilon (1 - k) \gamma)}{2 (2 + (1 - \varepsilon) \gamma + \varepsilon k\gamma)} = \frac{\alpha \left(2 + (\bar{k} - 1) \gamma\right)}{2 \left(2 + \bar{k}\gamma\right)}
\]

We note that both of these quantities are positive and that the difference is

\[
q^k - q^1 = \frac{\alpha (1 - k) \gamma}{2 \left(2 + \bar{k}\gamma\right)},
\]

which is positive if either \(\gamma > 0\) and \(k < 1\) or \(\gamma < 0\) and \(k > 1\). In other words, like under complete information, an overconfident manager will act more aggressively (produce more) and this is irrespective of whether the products are imperfect complements or imperfect substitutes.

True equilibrium prices, in a match between a rational manager and a manager with self-confidence \(k\) are, respectively
We note that
\[ p(k, 1) - p(1, k) = \frac{\alpha \gamma (1 - \gamma) (k - 1)}{2 (2 + \bar{k} \gamma)}, \]
which is negative if either \( \gamma > 0 \) and \( k < 1 \) or \( \gamma < 0 \) and \( k > 1 \). In other words, like under complete information, an overconfident manager will face a lower market price than its unbiased competition irrespective of whether the products are imperfect substitutes or imperfect complements.

For completeness, we also note that prices in other matches are given by\(^\text{15}\)
\[ p(k, k) = \alpha - q_k - \gamma q_k = \frac{\alpha (2 + \gamma (k - 1) (\varepsilon - \gamma \varepsilon + 1))}{2 (2 + \gamma (k \varepsilon + 1 - \varepsilon))} \]
\[ = \frac{\alpha (2 + \gamma [\bar{k} + k - 2 + (1 - \bar{k}) \gamma])}{2 (2 + \bar{k} \gamma)} > 0, \]
and
\[ p(1, 1) = \alpha - q_1 - \gamma q_1 = \frac{\alpha (2 + \gamma (k - 1) \varepsilon (1 - \gamma))}{2 (2 + (1 - \varepsilon) \gamma + \varepsilon k \gamma)} \]
\[ = \frac{\alpha (2 + (\bar{k} - 1) (1 - \gamma) \gamma)}{2 (2 + \bar{k} \gamma)} > 0. \]

We see again, that self-confidence affects equilibrium quantities and prices in an opposite manner. Which effect dominates for (true) equilibrium profits?

Let \( \pi^* (1, k) \) and \( \pi^* (k, 1) \) denote the (true) equilibrium profits to a rational and an overconfident

\(^{15}\text{We also note that both prices at the equilibrium quantities are positive from the (ex ante) perspective of any type of a manager.}\)
manager, respectively, in a match between them. Then we have that

\[
\pi^* (1, k) = \frac{\alpha^2 (2 + \gamma (k - 1) (\gamma + \varepsilon - \gamma \varepsilon)) (2 - \varepsilon (1 - k) \gamma)}{4 (2 + (1 - \varepsilon) \gamma + \varepsilon k \gamma)^2}
\]

\[
= \frac{\alpha^2 \left(2 + \gamma \left[\bar{k} - 1 + (k - \bar{k}) \gamma\right]\right) (2 + (\bar{k} - 1) \gamma)}{4 \left(2 + \bar{k} \gamma\right)^2}
\]

and

\[
\pi^* (k, 1) = \frac{\alpha^2 (2 + \gamma (k - 1) (\varepsilon - \gamma \varepsilon + 1)) (2 + (1 - \varepsilon) (1 - k) \gamma)}{4 (2 + (1 - \varepsilon) \gamma + \varepsilon k \gamma)^2}
\]

\[
= \frac{\alpha^2 \left(2 + \gamma \left[\bar{k} + k - 2 + (1 - \bar{k}) \gamma\right]\right) (2 + (\bar{k} - k) \gamma)}{4 \left(2 + \bar{k} \gamma\right)^2}
\]

The profit difference is

\[
\pi^* (k, 1) - \pi^* (1, k) = \frac{\alpha^2 \gamma^2 (1 - k^2)}{4 \left(2 + \bar{k} \gamma\right)^2},
\]

which is positive if \( k < 1 \).

In other words, when the goods are imperfect substitutes, in a match between an overconfident and a rational manager, the overconfident manager earns a higher profit. When the goods are complementary, then an overconfident manager earns less. This effect again, is qualitatively the same as under complete information. However, this does not imply that the type that earns the highest profits on average is the overconfident type when the goods are imperfect substitutes and the underconfident type when the goods are imperfect complements.

The profits when two rational managers are matched and the profits when two managers with self-confidence \( k \) are matched, are, respectively

\[
\pi^* (1, 1) = \frac{\alpha^2 (2 + \gamma (k - 1) \varepsilon (1 - \gamma)) (2 - \varepsilon (1 - k) \gamma)}{4 (2 + (1 - \varepsilon) \gamma + \varepsilon k \gamma)^2}
\]

\[
= \frac{\alpha^2 \left(2 + \gamma (1 - \gamma) \left(\bar{k} - 1\right)\right) (2 + (\bar{k} - 1) \gamma)}{4 \left(2 + \bar{k} \gamma\right)^2}
\]

and

\[
\pi^* (k, k) = \frac{\alpha^2 (2 + \gamma (k - 1) (\gamma + \varepsilon - \gamma \varepsilon + 1)) (2 + (1 - \varepsilon) (1 - k) \gamma)}{4 (2 + (1 - \varepsilon) \gamma + \varepsilon k \gamma)^2}
\]

\[
= \frac{\alpha^2 \left(2 + \gamma \left[\bar{k} + k - 2 + \gamma \left(k - \bar{k}\right)\right]\right) (2 + (\bar{k} - k) \gamma)}{4 \left(2 + \bar{k} \gamma\right)^2}
\]
Therefore, the average expected profit to a rational manager, over all possible matchings is

\[
\mathbb{E}(\pi|1) = \varepsilon \pi(1,k) + (1 - \varepsilon) \pi(1,1) \\
= \frac{\alpha^2 (k\gamma - \gamma + 2)^2}{4 (\gamma - \gamma\epsilon + k\gamma + 2)^2} \\
= \frac{\alpha^2 (2 + (\bar{k} - 1)\gamma)^2}{4 (2 + \bar{k}\gamma)^2},
\]

while the average expected profit to a manager with self-confidence \(k\), over all possible matchings is

\[
\mathbb{E}(\pi|k) = \varepsilon \pi(k,k) + (1 - \varepsilon) \pi(k,1) \\
= \frac{\alpha^2 (2 + \gamma - k\gamma - \gamma\epsilon + k\gamma\epsilon)(2 - \gamma + k\gamma - \gamma\epsilon + k\gamma\epsilon)}{4 (\gamma - \gamma\epsilon + k\gamma\epsilon + 2)^2} \\
= \frac{\alpha^2 (2 + (\bar{k} - k)\gamma)(2 + (\bar{k} + k - 2)\gamma)}{4 (2 + \bar{k}\gamma)^2}
\]

This expected profit difference is

\[
\mathbb{E}(\pi|1) - \mathbb{E}(\pi|k) = \frac{\alpha^2 \gamma^2 (k - 1)^2}{4 (2 + \bar{k}\gamma)^2} > 0. \tag{15}
\]

We see from (15), that \(\mathbb{E}(\pi|1) > \mathbb{E}(\pi|k)\) for all \(k\). In other words, the type that earns highest average expected profits is the homo oeconomicus irrespective of whether the goods are imperfect substitutes or imperfect complements.

We gather the findings above in the following proposition.

**Proposition 8** Suppose the managers could have only two possible degrees of self-confidence, one of which is the neutral one, the type of the manager is a private knowledge and the firms compete by setting prices. There exists a unique pure-strategy Bayesian Nash equilibrium. If the goods are imperfect substitutes, in equilibrium, in a pair of duopolists, the overconfident type produces more and earns higher profits. If the goods are imperfect complements, then in this equilibrium, in a pair of duopolists, the underconfident type produces more and earns higher profits. However, the type that earns highest average expected profits, over all possible matchings, is the homo oeconomicus.

We next move on to the case of arbitrarily self-confident managers.
4.2. Arbitrarily Self-Confident Managers

Consider again a large population of potential managers and let \( \mu \) be a probability measure representing the distribution of their confidence degrees \( k \) with support on \( K = (0, \lfloor 1/\gamma \rfloor) \). Denote \( \mathbb{E}_\mu [k] = \bar{k} \) and note that \( \bar{k} \in K \). Suppose that the managers are pair-wise randomly matched and that the type of one’s competitor is not observable. Assume that the population is so large that we can approximate the conditional distribution of one’s opponent’s type, given one’s own type, by \( \mu \).

We are looking for a pure strategy symmetric Bayesian Nash equilibria of the quantity competition game. At such an equilibrium we must have that

\[
q(k_i) \in \arg \max_{q_i \geq 0} \{ (\alpha - q_i - k_i \gamma \bar{q}) \cdot q \} \text{ for } \forall k_i \in K,
\]

where \( \bar{q} = \mathbb{E}[q(k_i)] \) is the average quantity choice in the population.

The first-order conditions for profit maximization yield (for each \( k \)):

\[
\alpha - 2q(k_i) - k_i \gamma \bar{q} = 0,
\]

or

\[
q(k_i) = \frac{\alpha - k_i \gamma \bar{q}}{2} \text{ for } \forall k_i \in K.
\]

Since this holds for all \( k \), we can take the expectation of both sides

\[
\mathbb{E}[q(k_i)] = \bar{q} = \frac{\alpha - \bar{k} \gamma \bar{q}}{2},
\]

which gives

\[
\bar{q} = \frac{\alpha}{2 + \bar{k} \gamma}.
\]  \hspace{1cm} (16)

Inserting (16) back into the original first-order condition, gives

\[
q^*(k_i) = \frac{\alpha \left( 2 + \left( \bar{k} - k_i \right) \gamma \right)}{2 \left( 2 + \bar{k} \gamma \right)} \text{ for } \forall k_i \in K.
\]

We note that, as expected, \( q^*(k_i) \) is decreasing in \( k \) when the goods are imperfect substitutes (having a low \( k \) is like being more self-confident) and increasing in \( k \) if the goods are imperfect complements (having a high \( k \) is like being more self-confident):

\[
q^*(k) - q^*(k') = \frac{\alpha (k' - k) \gamma}{2 \left( 2 + \bar{k} \gamma \right)}.
\]

Note that all the equilibrium quantities and both resulting equilibrium prices in any match are positive.
Hence, when a $k$ and a $k'$ type managers are randomly matched, the more self-confident type will produce more than the less confident type and this is irrespective of whether the goods are substitutes or complements.

The resulting equilibrium prices, in a match between a $k$ and a $k'$ type manager are, respectively,

$$p^* (k, k') = \frac{\alpha \left( 2 + \gamma \left( \bar{k} + k - 2 + (k' - \bar{k}) \gamma \right) \right)}{2 \left( 2 + \bar{k} \gamma \right)}$$

and

$$p^* (k', k) = \frac{\alpha \left( 2 + \gamma \left( \bar{k} + k' - 2 + (k - \bar{k}) \gamma \right) \right)}{2 \left( 2 + \bar{k} \gamma \right)}.$$

We see again, that the difference is

$$p^* (k, k') - p^* (k', k) = \frac{\alpha \gamma (1 - \gamma) (k - k')}{2 \left( 2 + \bar{k} \gamma \right)}$$

implying that the more confident manager of the pair, irrespective of the nature of the goods, will face a lower market price. Like under complete information, the degree of self-confidence affects equilibrium quantities and prices in an opposite manner. Which effect will dominate for profits?

For completeness, the prices in other matches are, respectively,

$$p (k, k) = \frac{\alpha \left( 2 + \left( \bar{k} + k - 2 + (k - \bar{k}) \gamma \right) \gamma \right)}{2 \left( 2 + \bar{k} \gamma \right)}$$

and

$$p (k', k) = \frac{\alpha \left( 2 + \left( \bar{k} + k' - 2 + (k' - \bar{k}) \gamma \right) \gamma \right)}{2 \left( 2 + \bar{k} \gamma \right)}.$$

In a match between a $k$ and $k'$ type manager, the equilibrium profits are

$$\pi^* (k, k') = \frac{\alpha^2 \left( 2 + \gamma \left( \bar{k} + k - 2 + (k' - \bar{k}) \gamma \right) \right) \left( 2 + (k - \bar{k}) \gamma \right)}{4 \left( 2 + \bar{k} \gamma \right)^2}$$

to manager $k$, and

$$\pi^* (k', k) = \frac{\alpha^2 \left( 2 + \gamma \left( \bar{k} + k' - 2 + (k' - \bar{k}) \gamma \right) \right) \left( 2 + (k' - \bar{k}) \gamma \right)}{4 \left( 2 + \bar{k} \gamma \right)}$$

to manager $k'$, respectively.
The difference is
\[
\pi^* (k, k') - \pi^* (k', k) = \frac{\alpha^2 \gamma^2 ((k')^2 - k^2)}{4 \left(2 + \bar{k}\gamma\right)^2} = \frac{\alpha^2 \gamma^2 (k' - k) (k' + k)}{4 \left(2 + \bar{k}\gamma\right)^2},
\]
which implies that
\[
\text{sign} (\pi^* (k, k') - \pi^* (k', k)) = \text{sign} (k' - k).
\]
This condition says that the manager who earns a higher profit is the more confident one when the goods are imperfect substitutes and the less confident one when the goods are imperfect complements.

However, this does not imply that the type that earns the highest profits on average is the overconfident type when the goods are imperfect substitutes and the underconfident type when the goods are incomplete complements.

The expected profit to a manager of type \( k_i \), over all possible matchings is
\[
\mathbb{E}_\mu [\pi^* (k_i)] = (\alpha - q (k_i) - \gamma \bar{q}) \cdot q (k_i)
\]
\[
= \left( \alpha - \frac{\alpha \left(2 + (\bar{k} - k_i) \gamma\right)}{2 \left(2 + \bar{k}\gamma\right)} - \gamma \frac{\alpha \left(2 + (\bar{k} - k_i) \gamma\right)}{2 \left(2 + \bar{k}\gamma\right)} \right)\frac{\alpha \left(2 + (\bar{k} - k_i) \gamma\right)}{2 \left(2 + \bar{k}\gamma\right)}
\]
\[
= \frac{\alpha^2 \left(2 + (\bar{k} - k_i) \gamma\right) \left(2 + (\bar{k} + k_i - 2) \gamma\right)}{4 \left(2 + \bar{k}\gamma\right)^2},
\]
a quantity that is maximized at \( k_i = 1 \). Hence, the type \( k \) that earns the highest expected profit is \( k = 1 \), or, in other words, homo oeconomicus.

To see this, let’s take the derivative of \( \mathbb{E}_\mu [\pi^* (k_i)] \) with respect to \( k_i \).
\[
\frac{\partial \mathbb{E}_\mu [\pi^* (k_i)]}{\partial k_i} = \frac{\alpha^2 \gamma^2 (1 - k_i)}{2 \left(2 + \bar{k}\gamma\right)^2} = 0
\]
implies that \( k_i = 1 \). Moreover,
\[
\frac{\partial^2 \mathbb{E}_\mu [\pi^* (k_i)]}{(\partial k_i)^2} = -\frac{\alpha^2 \gamma^2}{2 \left(2 + \bar{k}\gamma\right)^2} < 0
\]
so, this is indeed a maximum. In sum:

**Proposition 9** There exists a unique symmetric pure-strategy Bayesian Nash equilibrium. In this
equilibrium:

(a) All the managers expect positive profits and they all use the strategy

\[ q^* (k_i) = \frac{\alpha \left(2 + \left(\bar{k} - k_i\right) \gamma \right)}{2 \left(2 + \bar{k} \gamma \right)} \text{ for } \forall k_i \in K \]

(b) In a pair of duopolists, the more confident manager earns a higher profit if the goods are substitutes, and the less confident manager earns a higher profit if the goods are complementary.

(c) The expected average equilibrium profit to a manager \( i \) with confidence \( k_i \), in a random match with another manager from the type distribution \( \mu \), is

\[
E_\mu [\pi^* (k_i)] = \frac{\alpha^2 \left(2 + \left(\bar{k} - k_i\right) \gamma \right) \left(2 + \left(\bar{k} + k_i - 2\right) \gamma \right)}{4 \left(2 + \bar{k} \gamma \right)^2}
\]

and this profit is maximized at \( k_i = 1 \).

We conclude with a brief discussion of market selection. Suppose, first that managers are selected for according to how well their firms do in terms of their expected profit. A degree of managerial confidence \( k \) would be robust against such market selection if it were the optimal degree of confidence, in terms of average earned profit, in a random match against a type drawn from the type distribution \( \mu \). All other types would be selected against and eventually disappear from the population of active managers. Only the optimal degree of confidence would prevail.

According to Proposition 9, the unique robust degree of confidence that the market will select for, would be \( k = 1 \), granted this belongs to the type distribution. In the terminology of the previous section, \( k^r = 1 \), no matter the nature of the goods. *Homo oeconomicus* would thus prevail. This contrasts sharply with the outcome of market selection according to absolute performance under complete information, where we found that a certain degree of overconfidence was selected for, see Proposition 7.

Secondly, if selection is based on relative profits, instead, then the rational manager will be selected against. In this case it seems that the most overconfident type in the population will be selected for when the goods are imperfect substitutes and the least underconfident type when the goods are imperfect complements.

5. Price Competition

In this section instead of a quantity competition game, we imagine that firms compete by setting prices. The steps in the analysis in this section follow closely to that of the quantity competition case under complete information (mutually observable types).
5.1. One Overconfident Manager

Consider the case when manager 1 holds correct beliefs about $\gamma$, $k_1 = 1$, while manager 2 is biased and has self-confidence $k_2 \neq 1$. Manager 2 is overconfident if either $\gamma > 0$ and $k_2 < 1$ or $\gamma < 0$ and $k_2 > 1$. These beliefs are common knowledge. We also assume that $k_2 \in (0, 1/|\gamma|)$ when the goods are imperfect complements ($-1 < \gamma < 0$) and that

$$\max \left\{ 0, \frac{\gamma^2 + 2\gamma - 2}{\gamma} \right\} < k_2 < \frac{\sqrt{3 + \gamma} - 1}{\gamma}$$  \hspace{1cm} (17)

when the goods are imperfect substitutes ($0 < \gamma < 1$). The first constraint in (17) ensures that equilibrium quantities at the interior equilibrium will indeed be positive, even if $\gamma > \gamma^* = \sqrt{3} - 1$. The second constraint guarantees that even the perceived demands are both positive at the equilibrium prices from either of the managers’ perspective.

The manager of firm 1 solves

$$\max_{p_1 \geq 0} \left\{ p_1 \cdot \frac{1}{1 - \gamma^2} (\alpha (1 - \gamma) - p_1 + \gamma p_2) \right\}$$

taking $p_2$ as given. The first-order condition for profit maximization, which is also sufficient, reads

$$\alpha (1 - \gamma) - 2p_1 + \gamma p_2 = 0,$$

from which we get the following best-response function

$$p_1 (p_2) = \frac{1}{2} [\alpha (1 - \gamma) + \gamma p_2]. \hspace{1cm} (18)$$

The manager of firm 2 solves

$$\max_{p_2 \geq 0} \left\{ p_2 \cdot \frac{1}{1 - k_2^2 \gamma^2} (\alpha (1 - k_2 \gamma) + k_2 \gamma p_1 - p_2) \right\}$$

taking $p_1$ as given. The first order condition for profit maximization is

$$\alpha (1 - k_2 \gamma) + k_2 \gamma p_1 - 2p_2 = 0,$$

from which we get the following best-response function

$$p_2 (p_1) = \frac{1}{2} [\alpha (1 - k_2 \gamma) + k_2 \gamma p_1]. \hspace{1cm} (19)$$

Note that when the goods are imperfect substitutes ($\gamma > 0$) we have upward sloping best response functions and prices are strategic complements. However, when the goods are imperfect complements ($\gamma < 0$), the best response functions are downward sloping and the prices are strategic.
substitutes. This problem is the dual of the quantity competition case, with prices playing the same role as quantities in the quantity competition case and visa versa.

Solving the best response functions (18) and (19) simultaneously, we get that in the unique interior Nash equilibrium of the price competition game

\[ p^*_1 = \frac{\alpha (2 - \gamma - k_2 \gamma^2)}{4 - k_2 \gamma^2} \]

and

\[ p^*_2 = \frac{\alpha (2 - k_2 \gamma - k_2 \gamma^2)}{4 - k_2 \gamma^2}, \]

which are indeed positive.

These price choices imply that (true) equilibrium demands are

\[ q^*_1 (p^*_1, p^*_2) = \frac{\alpha (2 - \gamma - k_2 \gamma^2)}{(1 - \gamma^2) (4 - k_2 \gamma^2)} \]

and

\[ q^*_2 (p^*_1, p^*_2) = \frac{\alpha (2 - (2 - k_2) \gamma - \gamma^2)}{(1 - \gamma^2) (4 - k_2 \gamma^2)}. \]

The (true) equilibrium profits, are

\[ \pi^*_1 = \frac{\alpha^2 (2 - \gamma - k_2 \gamma^2)^2}{(1 - \gamma^2) (4 - k_2 \gamma^2)^2}, \]

and

\[ \pi^*_2 = \frac{\alpha^2 (2 - k_2 \gamma - k_2 \gamma^2) (2 - (2 - k_2) \gamma - \gamma^2)}{(1 - \gamma^2) (4 - k_2 \gamma^2)^2}. \]

Hence, we have the following differences:

\[ p^*_1 - p^*_2 = \frac{\alpha \gamma (k_2 - 1)}{4 - k_2 \gamma^2} \]

\[ q^*_1 - q^*_2 = \frac{\alpha \gamma (1 - k_2)}{(1 - \gamma) (4 - k_2 \gamma^2)} \]

\[ \pi^*_1 - \pi^*_2 = \frac{\alpha^2 \gamma^2 (1 - k_2) (3 - k_2 - k_2 \gamma - k_2 \gamma^2)}{(1 - \gamma^2) (4 - k_2 \gamma^2)^2} \]

We note that \(3 - k_2 - k_2 \gamma - k_2 \gamma^2 > 0\) for all \(k_2 < \frac{3}{1+\gamma+\gamma^2}, \gamma \in (-1, 1)\).

First, note that if both managers were rational, then these differences would vanish. Second note that, again, the level of a manager’s self-confidence affects prices and quantities in an opposite manner. Which effect dominates for profits? To answer this question, we now look at the case of imperfect substitutes and imperfect complements separately.
Proposition 10 Suppose the goods are imperfect substitutes \((\gamma > 0)\). In the unique interior Nash equilibrium of the price competition game we have:

(a) If \(\max \left\{0, \frac{\gamma^2 + 2\gamma - 2}{\gamma} \right\} < k_2 < 1\), then

(a) \(p_2^* > p_1^*\)

(b) \(q_2^* < q_1^*\)

(c) \(\pi_2^* < \pi_1^*\)

(b) If \(1 < k_2 < \min \left\{\frac{3}{1 + \gamma + \gamma^2}, \frac{\sqrt{3 + \gamma} - 1}{\gamma} \right\}\), then

(a) \(p_2^* < p_1^*\)

(b) \(q_2^* > q_1^*\)

(c) \(\pi_2^* > \pi_1^*\)

In other words, firm 2—if run by an overconfident manager—would set a higher price, but nevertheless earn lower profits than its unbiased competitor, firm 1. Under price competition it does not pay off to be overconfident when the goods are substitutable. If firm 2 is run by an underconfident manager, it will set a lower price and earn a higher profit. The diagram in Figure 6 shows how \(\pi_2^*\) (solid curve) and \(\pi_1^*\) (dashed curve) depend on \(k_2\) (the horizontal axis), for \(\gamma = 0.6\). The thin curve is the average of the two curves—half the industry profit.

We see that:
(a) For all $k_2 < 1$ the overconfident manager 2 charges a higher price but makes less profit than unbiased manager 1,

(b) For moderate degrees of overconfidence, $0.73 < k_2 < 1$, manager 2 earns a higher profit that had she also held correct perceptions (had $k_2 = k_1 = 1$). In that sense, there is an optimal degree of overconfidence for manager 2, $k_2 \approx 0.86$,

(c) The industry profit first increases and then decreases in $k_2$,

(d) For all degrees of underconfidence ($k_2 > 1$), manager 2 earns less profit than had she also had correct perceptions (had $k_2 = k_1 = 1$). However, for all admissible levels of underconfidence ($1 < k_2 < 1.5$) manager 2 earns higher profits than manager 1. The relative profit difference is maximized for $k_2 \approx 1.27$.

5.1.2. Complements

Proposition 11 Suppose the goods are complementary ($\gamma < 0$) and $1 < k_2 < \min\left\{\frac{3}{1+\gamma+\gamma^2}, \frac{1}{\gamma} \right\}$. Then, in the unique interior Nash equilibrium of the price competition game, we have the following:

(a) $p_2^* > p_1^*$

(b) $q_2^* < q_1^*$

(c) $\pi_2^* > \pi_1^*$

In other words, firm 2—run by an overconfident manager—sets a higher prices and nevertheless earns higher profits, than his or her rational competitor, firm 1. Under price competition with complementary goods an underconfident manager will always earn less profits than his or her rational competitor. The diagram in Figure 7 shows how $\pi_2^*$ (solid curve) and $\pi_1^*$ (dashed curve) depend on $k_2$ (the horizontal axis), for $\gamma = -0.6$ and $\alpha = 1$. The thin curve is the average of the two curves—half the industry profit.

We see that:

(a) For all $k_2 > 1$ the overconfident manager 2 sets a higher price and makes a higher profit than unbiased manager 2,

(b) There is an optimal degree of overconfidence for manager 2, $k_2 \approx 1.5$,

(c) The industry profit is decreasing in $k_2$,

(d) For all degrees of overconfidence, manager 2 earns a higher profit than had she also had correct perceptions (had $k_2 = k_1 = 1$).

17The more exact value would be $k_2 = \frac{8-2\gamma}{2\gamma+\gamma^2-\gamma^3+4}$.
In general, this last point is true for other values of negative $\gamma$ if $1 < k_2 < \frac{4\gamma - 2\gamma^2 - 4}{2\gamma^2 + \gamma^3 - \gamma^4 - 4}$. For example, if $\gamma = -0.4$, then this upper bound for overconfidence would be about 1.4.

### 5.2. Two Arbitrarily Confident Managers

We now turn to the analysis of the more general case of an interaction between two arbitrarily confident managers.

The manager of firm 1 solves

$$\max_{p_1 \geq 0} \left\{ p_1 \cdot \frac{1}{1 - k_1^2 \gamma^2} \left( \alpha (1 - k_1 \gamma) - p_1 + k_1 \gamma p_2 \right) \right\}$$

taking $p_2$ as given. The first-order condition for profit maximization is

$$\alpha (1 - k_1 \gamma) - 2p_1 + k_1 \gamma p_2 = 0,$$

from which we get the following best-response function

$$p_1 (p_2) = \frac{1}{2} \left[ \alpha (1 - k_1 \gamma) + k_1 \gamma p_2 \right]. \quad (21)$$

The manager of firm 2 solves

$$\max_{p_2 \geq 0} \left\{ p_2 \cdot \frac{1}{1 - k_2^2 \gamma^2} \left( \alpha (1 - k_2 \gamma) + k_2 \gamma p_1 - p_2 \right) \right\}$$
taking \( p_1 \) as given. The first-order condition for profit maximization is
\[
\alpha (1 - k_2 \gamma ) + k_2 \gamma p_1 - 2p_2 = 0,
\]
from which we get the following best response function
\[
p_2 (p_1) = \frac{1}{2} [\alpha (1 - k_2 \gamma ) + k_2 \gamma p_1]. \tag{22}
\]
Note that when the goods are imperfect substitutes \( (\gamma > 0) \) we have upward sloping best response functions and the prices are strategic complements. However, when the goods are imperfect complements \( (\gamma < 0) \), the best response functions are downward sloping and the prices are strategic substitutes.

Solving the best response functions (21) and (22) simultaneously, we get that in the unique interior Nash equilibrium
\[
p_1^* = \frac{\alpha (2 - k_1 \gamma - k_1 k_2 \gamma^2)}{4 - k_1 k_2 \gamma^2},
\]
and
\[
p_2^* = \frac{\alpha (2 - k_2 \gamma - k_1 k_2 \gamma^2)}{4 - k_1 k_2 \gamma^2}.
\]
Note that these prices are indeed positive given that \( k_i \in K = (0, |1/\gamma|) \) for \( i = 1, 2 \).

These prices choices imply that true (rather than perceived) equilibrium demands are
\[
q_1^* (p_1^*, p_2^*) = \frac{\alpha (2 - k_1 \gamma - k_1 k_2 \gamma^2)}{(1 - \gamma^2) (4 - k_1 k_2 \gamma^2)},
\]
and
\[
q_2^* (p_1^*, p_2^*) = \frac{\alpha (2 - k_2 \gamma - k_1 k_2 \gamma^2)}{(1 - \gamma^2) (4 - k_1 k_2 \gamma^2)}.
\]

The (true) equilibrium profits are, respectively,
\[
\pi_1^* = \frac{\alpha^2 (2 - k_1 \gamma - k_1 k_2 \gamma^2) (2 - (k_1) \gamma - k_2 \gamma^2)}{(1 - \gamma^2) (4 - k_1 k_2 \gamma^2)^2},
\]
and
\[
\pi_2^* = \frac{\alpha^2 (2 - k_2 \gamma - k_1 k_2 \gamma^2) (2 - (k_2) \gamma - k_1 \gamma^2)}{(1 - \gamma^2) (4 - k_1 k_2 \gamma^2)^2}.
\]

Note that for negative \( \gamma \), these quantities indeed define an interior Nash equilibrium for any degree of permissible self-confidence \( k_1, k_2 \in K \). However, when the goods are imperfect substitutes, we need additional restrictions on the admissible levels of self-confidence, namely we need to impose
Figure 8. The set, where condition (23) is satisfied, $\gamma = 0.75$

the following technical condition:

$$
\begin{cases}
2 - (2 - k_1) \gamma - k_2 \gamma^2 > 0 \\
2 - (2 - k_2) \gamma - k_1 \gamma^2 > 0 \\
2 - (2k_1 - k_2) \gamma - k_1^2 \gamma^2 > 0 \\
2 - (2k_2 - k_1) \gamma - k_2^2 \gamma^2 > 0
\end{cases}
$$

(23)

The first two inequalities ensure that the true equilibrium demand is positive for both firms. The first two inequalities are not binding unless $\frac{2}{3} < \gamma < 1$.

The last two inequalities require that even perceived demands at equilibrium prices be positive for either of the firm. We illustrate condition (23) in Figure 8 for $\gamma = 0.75$. For further illustrations, see Appendix A. In essence condition (23) requires the beliefs not to be too far apart, when both managers are underconfident, and also in case $\gamma > 2/3$, the beliefs cannot diverge too much even in case of overconfidence.

Comparing equilibrium prices we see that

$$p_1^* - p_2^* = \frac{\alpha \gamma (k_2 - k_1)}{4 - k_1 k_2 \gamma^2}.$$ 

The denominator is always positive, therefore

$$\text{sign } (p_1^* - p_2^*) = \text{sign } (\gamma (k_2 - k_1)).$$
This implies that $p_1^* > p_2^*$ if and only if

(a) $\gamma > 0$ and $k_1 < k_2$, i.e. the more confident manager will set a higher price when the goods are imperfect substitutes.

(b) $\gamma < 0$ and $k_1 > k_2$, i.e. the more confident manager will set a higher price when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or complements, more confident managers will set higher prices. We see that prices play the same role in the price competition scenario, as quantities did under quantity competition.

Comparing equilibrium quantities we see that

$$q_1^* (p_1^*, p_2^*) - q_2^* (p_1^*, p_2^*) = \frac{\alpha \gamma (k_1 - k_2)}{(1 - \gamma) (4 - k_1 k_2 \gamma^2)},$$

implying that

$$\text{sign} \left( q_1^* - q_2^* \right) = \text{sign} \left( \gamma (k_1 - k_2) \right) = -\text{sign} \left( p_1^* - p_2^* \right).$$

We note that the degree of self confidence affects equilibrium prices and demands in an opposite manner.

We have that $q_1^* > q_2^*$ if and only if

(a) $\gamma > 0$ and $k_1 < k_2$, i.e. the less self-confident manager of the pair will face a higher demand when the goods are imperfect substitutes,

(b) $\gamma < 0$ and $k_1 > k_2$, i.e. the less self-confident manager of the pair will face a higher demand when the goods are imperfect complements.

In other words, irrespective of whether the goods are imperfect substitutes or complements, more confident managers will face a lower demand than anticipated. Again, the duality with the quantity competition case is self-evident.

We have seen that, again, the managers’ level of self-confidence moves the prices and quantities in opposite directions. Which effect will dominate? What is the overall effect on equilibrium profits?

The (true) equilibrium profit difference is

$$\pi_1^* - \pi_2^* = \frac{\alpha^2 \gamma^2 (k_1 - k_2) (4 - k_1 - k_2 - k_1 k_2 \gamma - k_1 k_2 \gamma^2)}{(1 - \gamma^2) (4 - k_1 k_2 \gamma^2)^2}$$

(24)

While in general terms it seems difficult to determine the sign of

$$(k_1 - k_2) \left( 4 - k_1 - k_2 - k_1 k_2 \gamma - k_1 k_2 \gamma^2 \right),$$

we can establish the following:
Proposition 12 Suppose the goods are imperfect substitutes ($\gamma > 0$) and manager $i$’s has self-confidence $k_i \leq 1$, $i = 1, 2$, and condition (23) is satisfied. Then in the unique interior Nash equilibrium of the price competition game, the following are equivalent:

(a) $k_i < k_{-i}$
(b) $p_i^* > p_{-i}^*$
(c) $q_i^* < q_{-i}^*$
(d) $\pi_i^* < \pi_{-i}^*$

Proof. The proof follows easily from noting that in the relevant range, $4 - k_i - k_{3-i} - k_i k_{3-i} \gamma - k_i k_{3-i} \gamma^2 < 0$. ■

In other words, when the goods are imperfect substitutes, both managers are overconfident, then under price competition the less overconfident manager of the pair will set lower prices and earn higher profits. Similarly,

Proposition 13 Suppose the goods are imperfect complements ($-1 < \gamma < 0$) and manager $i$’s degree of self-confidence is $k_i \geq 1$, $i = 1, 2$. Then in the unique interior Nash equilibrium of the price competition game, the following are equivalent:

(a) $k_i < k_{-i}$
(b) $p_i^* < p_{-i}^*$
(c) $q_i^* > q_{-i}^*$
(d) $\pi_i^* < \pi_{-i}^*$

In other words, when the goods are imperfect complements, both managers are overconfident, then the more overconfident manager will set a higher price and earn more profits.

Is there an optimal level of self-confidence for a manager in this set-up, in the sense of being robust to market selection forces? Before we can answer that question, we make the following observation.

Proposition 14 Assume $\gamma \in \left(-\frac{2}{3}, 1\right)$. As a function of own confidence, $k_i$, the equilibrium profit $\pi_i^*$ to manager $i$ under price competition is a strictly concave function with a unique maximum at

$$\hat{k}_i = \frac{4}{4 + 2k_j \gamma - k_j \gamma^2 - k_j^2 \gamma^3}$$

(25)
Proof. See appendix. ■

In particular if \( k_1 = 1 \) and \( \gamma = 0.6 \ (-0.6) \) as in the preceding subsection, then \( \hat{k}_2 = 0.866 \ (1.506) \), in broad agreement with the above approximate observation.

More generally, (1) for \( \gamma > 0 \) (25) defines \( \hat{k}_i \) as a number in \((0,1)\); and (2) for \(-\frac{1}{2} < \gamma < 0\) equation (25) defines \( \hat{k}_i \) as a number in \((1,1/|\gamma|)\). We henceforth assume that \( \gamma \in \left(-\frac{1}{2},1\right) \) and turn to market selection and evolutionarily robustness analysis under price competition.

5.3. Market Selection: Evolutionarily Robust Self-Confidence

Consider again the setup from Section 3.3 but instead of quantity competition, assume the firms compete by setting prices. Also, assume that \(-\frac{1}{2} < \gamma < 1\) and that \( \gamma \neq 0, k_i \in K, i = 1, 2, \) and that condition (23) is satisfied.

Consider a manager with self-confidence \( k' \) who is matched against a manager with self-confidence \( k \). The profit to the first manager is

\[
\pi^* = v(k', k) = \alpha^2 \left(2 - k'\gamma - k'k\gamma^2\right) \left(2 - (2 - k')\gamma - k\gamma^2\right)
\]

\[
\frac{1}{(1 - \gamma^2)(4 - k'k\gamma^2)^2}
\]

Again, a degree of self-confidence \( k \) is evolutionarily robust if condition (13) holds. A robust degree of self-confidence \( k \) would need to satisfy equation (25) for \( \hat{k}_i = k_j = k \). The unique degree of confidence with this robustness property, under price competition, is\(^{18}\)

\[
k^R = \frac{1}{2\gamma^2}\left(\gamma - \sqrt{4\gamma - 7\gamma^2 + 4 + 2}\right).
\]

For the case of substitutes, we illustrate this in Figure 9, showing the contour map of the function \( v \), with \( k' \) on the horizontal axis and \( k \) on the vertical, when \( \alpha = 1 \) and \( \gamma = 0.7 \). Figure 10 illustrates the same for the case of complements, \( \alpha = 1 \) and \( \gamma = -0.3 \). A degree of confidence \( k^R \) is robust against market selection if and only if the tangent of the isoquant through the point \((k, k)\)—the thick curve—is horizontal at that point, and no other point on the horizontal line through that point has a higher \( v \)-value. Inspecting the graph we conclude that this degree is approximately \( k^R = 0.88 \) when the goods are imperfect substitutes and \( k^R = 1.26 \) when the goods are imperfect complements (indicated by the vertical and horizontal lines).

Formally, we have:

**Proposition 15** Suppose \( \gamma \in \left(-\frac{1}{2},1\right) \). The unique degree of confidence that is robust to market

\(^{18}\)Note that (25) has three fixed points: \( -\frac{2}{\gamma}, \frac{\gamma + \sqrt{4\gamma - 7\gamma^2 + 4 + 2}}{2\gamma^2} \) and \( \frac{\gamma - \sqrt{4\gamma - 7\gamma^2 + 4 + 2}}{2\gamma^2} \). The first two are outside of the interval \((0, |\frac{1}{\gamma}|)\).
Figure 9. Evolutionarily robust degree of self-confidence when the goods are imperfect substitutes.

Figure 10. Evolutionarily robust degree of self-confidence under price competition with imperfect complements.
selection driven by absolute profits under price competition is

\[ k^R = \frac{1}{2\gamma^2} \left( \gamma - \sqrt{4\gamma - 7\gamma^2 + 4 + 2} \right). \]

**Proof.** The proof follows from Proposition (14). ■

We illustrate this in the figure 11.

We note, that the evolutionarily robust self-confidence is again of the overconfident type, and this does not depend on whether the goods are imperfect substitutes or imperfect complements. Note, that when the goods are complementary (but not too complementary), then as in the case of quantity competition the degree of overconfidence is higher the more complementary the goods are.

When the goods are imperfect substitutes, the relationship between product differentiation and robust level of self-confidence is a U-shaped one.

### 5.4. Profit Comparison

When both managers in a duopoly have this evolutionarily robust degree of confidence, each firm’s profit is

\[ \pi^R = \frac{2\alpha^2\gamma \left( \gamma + \sqrt{4\gamma - 7\gamma^2 + 4 - 2} \right)}{(1 + \gamma) \left( 3\gamma + \sqrt{4\gamma - 7\gamma^2 + 4 - 2} \right)^2}. \]
When both managers in a duopoly instead are *homo oeconomicus*, then their profits are

\[
\pi^* = \frac{\alpha^2 (1 - \gamma)}{(1 + \gamma) (2 - \gamma)^2}.
\]

The profit difference is

\[
\pi^R - \pi^* = \frac{\alpha^2}{(1 + \gamma)} \left( \frac{2\gamma \left( \gamma + \sqrt{4\gamma - 7\gamma^2 + 4} - 2 \right)}{(3\gamma + \sqrt{4\gamma - 7\gamma^2 + 4} - 2)^2} - \frac{1 - \gamma}{(2 - \gamma)^2} \right),
\]

which is negative if \( \gamma < 0 \) and positive if \( \gamma > 0 \). Hence, market selection tends to select overconfident managers and thereby reduce industry profit if the goods are complementary and increase industry profits if the goods are substitutable.

6. Conclusion

In this paper we examined the role of managerial self-confidence in product market competition. We have studied a market interaction between two firms in a differentiated goods duopoly run by owner-managers who have asymmetric beliefs (“confidence”) about the degree of product differentiation. We have shown that under quantity competition, more confidence entails more aggressive play irrespective of whether the products are imperfect substitutes or imperfect complements. Subsequently, firms run by more confident managers gain a competitive edge when the goods are imperfect substitutes and the less confident managers gain a competitive edge when the goods are complementary. On the other hand, under price competition more confident managers gain a competitive edge when the goods are complementary and the less confident managers gain a competitive edge when the goods are substitutable. So, in essence overconfident managers get a competitive edge if the choice variables are strategic substitutes. However, the analysis also suggests that market selection forces will always favor some degree of managerial overconfidence, irrespective of whether the choice variables are strategic substitutes or strategic complements.

In this paper we modeled overconfidence as asymmetric beliefs about a particular parameter, the extent of horizontal product differentiation with symmetric firms. It would be interesting to see whether the conclusions reached in this paper would be robust to alternative models of self-confidence in an oligopoly setting, say about market size. It is also possible to envision a model with endogenous product differentiation, in which managers invest in horizontal product differentiation by exerting some effort, and that this could depend on both skill and luck. Then there are more dimensions to consider, and it would be interesting to look at the interaction of skill and various forms of overconfidence.
Figure 12. The set, where condition (23) is satisfied, $\gamma = 0.5$

Figure 13. The set, where condition (23) is satisfied, $\gamma = 0.9$
Appendix B

Following are the proofs of Propositions 6 and 14.

Proof of Proposition 6.  Recall that
\[ \pi_i^*(k_i, k_j) = \frac{\alpha^2 (2 - k_i \gamma)}{(4 - k_i k_j \gamma^2)^2} \left[ (1 - k_i) k_j \gamma^2 + (k_i - 2) \gamma + 2 \right] \text{ for } \forall k_i, k_j \in K, \]
where \( K = (0, |1/\gamma|), \ i = 1, 2 \) and \( j \neq i \). Differentiating \( \pi_i^*(k_i, k_j) \) with respect to \( k_i \)
\[ \frac{\partial \pi_i^*(k_i, k_j)}{\partial k_i} = \frac{\alpha^2 \gamma^2 (2 - k_j \gamma)}{(4 - k_i k_j \gamma^2)^2} \left[ k_i k_j \gamma^2 + 2 k_i k_j \gamma - 4 k_j \gamma - 4 k_i + 4 \right] \]
and equating the resulting expression to zero, implies that
\[ \hat{k}_i(k_j) = \frac{4 (1 - k_j \gamma)}{4 - 2 k_j \gamma - k_j \gamma^2}. \]
Furthermore,
\[ \frac{\partial^2 \pi_i^*(k_i, k_j)}{\partial k_i^2} = \frac{2 \alpha^2 \gamma^2 (2 - k_j \gamma)}{(4 - k_i k_j \gamma^2)^4} \left[ 4 \gamma k_j - 6 \gamma^3 k_j^2 + 8 \gamma^2 k_j - 4 \gamma^2 k_i k_j \right] \left( 2 \gamma^3 k_j^2 + \gamma^4 k_i k_j^2 - 8 \right). \]
The first factor is positive. Hence, if we can show that the last factor is negative everywhere on the relevant range, then we would have that
\[ \frac{\partial^2 \pi_i^*(k_i, k_j)}{\partial k_i^2} < 0 \]
and the proof would be complete. The derivative of the last factor with respect to \( k_i \) is
\[ \gamma^2 k_j \left( \gamma k_j (\gamma + 2) - 4 \right) < 0 \text{ for } \forall k_j \in K, \]
so that the whole expression is decreasing in \( k_i \). Therefore, it remains to show that
\[
4 \gamma k_j - 6 \gamma^3 k_j^2 + 8 \gamma^2 k_j - 4 \gamma^2 k_i k_j + 2 \gamma^3 k_i k_j^2 + \gamma^4 k_i k_j^2 - 8 \bigg|_{k_i=0}
= -6 \gamma^3 k_j^2 + 8 \gamma^2 k_j + 4 \gamma k_j - 8
= -2 \left( 3 \gamma^3 k_j^2 - 2 \gamma k_j - 4 \gamma^2 k_j + 4 \right)
< 0,
\]

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on the relevant range. First, consider the case when $0 < \gamma < 1$. Then we have that

$$3\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4 = \gamma^3k_j^2 + 2\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4$$

$$= \gamma^3k_j^2 - 2\gamma k_j \left(1 - \gamma^2k_j\right) + 4 \left(1 - \gamma^2k_j\right)$$

$$= \gamma^3k_j^2 + 2 \left(1 - \gamma^2k_j\right) \left(2 - \gamma k_j\right)$$

$$> 0.$$ 

Second, consider the case when $-\frac{6}{7} < \gamma < 0$. Then $3\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4$ is a downward “facing” parabola with

$$3\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4 \bigg|_{k_j = 0} = 4 > 0,$$

$$3\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4 \bigg|_{k_j = -\frac{1}{\gamma}} = 7\gamma + 6 > 0.$$ 

Therefore $3\gamma^3k_j^2 - 2\gamma k_j - 4\gamma^2k_j + 4 > 0$ for $\forall k_j \in K$. This concludes the proof. □

**Proof of Proposition 14.** Recall that,

$$\pi_i^* (k_i, k_j) = \frac{\alpha^2 (2 - k_i \gamma - k_i k_j \gamma^2) \left(2 - (2 - k_i) \gamma - k_j \gamma^2\right)}{(1 - \gamma^2) (4 - k_i k_j \gamma^2)^2} \text{ for } \forall k_i, k_j \in K,$$

where $K = (0, 1/\gamma)$, $i = 1, 2$ and $j \neq i$. Differentiating $\pi_i^* (k_i, k_j)$ with respect to $k_i$

$$\frac{\partial \pi_i^* (k_i, k_j)}{\partial k_i} = \frac{\alpha^2 \gamma^2 (2 + k_j \gamma)}{(1 - \gamma^2) (4 - k_i k_j \gamma^2)^3} \left(k_i k_j \gamma^3 + k_i \gamma^2 k_j - 2k_i k_j \gamma - 4k_i + 4\right)$$

and equating the resulting expression to zero, implies that

$$\hat{k}_i = \frac{4}{4 + 2k_j \gamma - k_j \gamma^2 - k_j^2 \gamma^3}.$$ 

Furthermore,

$$\frac{\partial^2 \pi_i^* (k_i, k_j)}{\partial k_i^2} = \frac{2\alpha^2 \gamma^2 (2 + k_j \gamma)}{(1 - \gamma^2) (4 - k_i k_j \gamma^2)^4} \left(2\gamma^3 k_j^2 - 4\gamma k_j + 8\gamma^2 k_j - 4\gamma^2 k_i k_j \right) \left(-2\gamma^3 k_i k_j^2 + \gamma^4 k_i k_j^2 + k_i^2 \gamma^3 - 8\right).$$

The first factor is positive. Hence, if we can show that the last factor is negative everywhere on the relevant range, then we would have that

$$\frac{\partial^2 \pi_i^* (k_i, k_j)}{\partial k_i^2} < 0$$
and the proof would be complete. The derivative of the last factor with respect to $k_i$ is (factoring out a positive multiplier)

$$
\gamma^3 k_j^2 + \gamma^2 k_j - 2\gamma k_j - 4 = -(1 + k_j \gamma) \left(2 - k_j \gamma^2 \right) - 2
$$

$$
< 0 \text{ for } \forall k_j \in K,
$$

so that the whole expression is decreasing in $k_i$. Therefore, it remains to show that

$$
\left. \left(2\gamma^3 k_j^2 - 4\gamma k_j + 8\gamma^2 k_j - 4\gamma^2 k_i k_j - 2\gamma^3 k_i k_j^2 + \gamma^4 k_i k_j^2 + \gamma^5 k_i k_j^3 - 8\right) \right|_{k_j=0} = 2 \left(\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4\right) < 0
$$

on the relevant range. First, consider the case when $0 < \gamma < 1$. Then $\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4$ is an “upward” facing parabola in $k_j$ with

$$
\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4 \bigg|_{k_j=0} = -4 < 0,
$$

$$
\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4 \bigg|_{k_j=\frac{1}{\gamma}} = 5\gamma - 6 < 0.
$$

Next consider the case when $-\frac{2}{3} < \gamma < 0$. Then $\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4$ is a “downward” facing parabola with roots

$$
k_{j1} = \frac{1}{\gamma^2} \left(1 - 2\gamma - \sqrt{4\gamma^2 + 1}\right) \text{ and } k_{j2} = \frac{1}{\gamma^2} \left(1 - 2\gamma + \sqrt{4\gamma^2 + 1}\right).
$$

Note that $k_{j1} = -\frac{1}{\gamma}$ when $\gamma = -\frac{2}{3}$ and we subsequently have that $0 < k_j < -\frac{1}{\gamma} < k_{j1} < k_{j2}$ for all $\gamma \in \left(-\frac{2}{3}, 0\right)$, which implies that $\gamma^3 k_j^2 + 4\gamma^2 k_j - 2\gamma k_j - 4 < 0$ for all $k_j \in K$. This completes the proof. ■

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References


