Authoritarian Election as an Incentive Scheme*

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July 15, 2015

Abstract
Authoritarian rule requires teamwork of political elites. However, members of the elite class may lack incentives to contribute efforts. In this paper, we develop a model to study authoritarian rulers’ decision to introduce election. In our model, election motivates the ruling class to devote more effort to public good provision. As a result, election alleviates the moral-hazard-in-teams problem within authoritarian government. While too much electoral control hinders the introduction of election, mild electoral control facilitates it. This offers a new perspective in understanding authoritarian elections and explains a number of stylized facts in authoritarian regimes.

1 Introduction

Even the most powerful dictator cannot rule alone. Authoritarian rule requires contributions from a group of elites who share political power. Nazi Germany relied on Goebbels to cook propaganda. Stalin’s Soviet Union depended upon Beria’s secret police to purge dissidents. Khomeini would not have sustained the theocratic rule in Iran without the support of other senior Shia scholars. Authoritarian elites, however, have personal agendas. They may shirk from consolidating the authoritarian regime and instead reap private interests at

*We would like to thank our advisor, Kalyan Chatterjee, for his guidance and support. This paper has benefited greatly from discussions with Sourav Bhattacharya, John Duggan, Ryan Fang, Barry Ickes, Marcin Pęski, and Joseph Wright. Needless to say, all remaining errors are our own.
the cost of the regime and other elites. For example, the USSR military used to exaggerate the need to invest in military defense, at the expense of the rest of the economy and other party elites\(^1\). Authoritarian governance does not provide effective discipline for dictators themselves. Inevitably, dictators suffer from a moral-hazard-in-teams problem.

Holmstrom (1982) has shown that breaking the budget can solve the moral hazard problem in teams. Namely, if the entire team will be penalized when any given player shirks, the moral hazard problem can be alleviated. Elections are usually a tool for voters to hold politicians accountable. When politicians rule as a team, however, they could also use elections to hold each other accountable. In this paper, we argue that elections serve as a means to break the budget if all incumbent politicians bear the cost of losing.

We developed a model to study authoritarian rulers’ decision to introduce elections. In our model, a regime is ruled by two politicians. At date 0, they decide whether to hold an election at date 1. At date 1, politicians exert efforts in a joint production of a public good. A production shock is realized after politicians have taken actions. As the politicians cannot contract on each others’ effort, they suffer from a moral-hazard-in-teams problem. If the election has not been introduced, the game ends. If the election has been introduced, a representative voter announces a satisfaction level. The politicians choose effort levels after observing the announced satisfaction level. After the shock is realized, the voter re-elects the politicians if the satisfaction level is met, and the incumbent politicians continue to hold power. Otherwise, incumbents have to conduct costly electoral fraud to remain in office.

We first show that the politicians will not introduce the election at date 0 when the cost of electoral fraud is too high. Intuitively, if the election is too costly to lose, the politicians will have to spend too much effort in producing public good. Duggan and Martinelli (2014) have a similar result in their single-agent model that voters could motivate any level of effort from politicians given the rewards of office are high enough. They interpret this result as an evidence of democracy’s responsiveness to voters’ demand. Our result,

\(^1\)Various researchers estimated that around 30%-40% of USSR’s total budget outlay was spent on defense in the 1980s (see Harrison (2003) for a review). Some scholars argued that the heavy burden of military spending finally brought down the Soviet Union (Easterly and Fischer, 1995).
however, indicates that an expectation of excessive electoral control prevents authoritarian rulers from introducing elections in the first place.

We then characterize voter’s optimal satisfaction level at date 1. A satisfaction level too low will not provide politicians enough incentive. It is also counterproductive to set a satisfaction level too high to reach. In such case, politicians will not exert additional effort but take their chance. We show that the voter chooses a satisfaction level that maximizes the marginal impact of shirking on the chance of the realized output reaching it.

Given voter’s optimal satisfaction level at date 1, we establish conditions under which politicians will introduce the election at date 0. The election serves politicians as the budget-breaking device. Given a satisfaction level, politicians exert more effort than they do without election to avoid the cost of electoral fraud. The election thus benefits a politician through motivating the other politician to exert more effort. However, in the event of an adverse production shock, it is costly for the politicians to remain in office. When the satisfaction level is set high, the chance that politicians need to commit costly electoral fraud is high. The expected cost of electoral fraud, therefore, outweighs the motivating effect of the election. In this case, both politicians prefer not to introduce the election. When the satisfaction level is moderate, the probability that politicians need to commit electoral fraud is not too high. The expected cost of electoral fraud, therefore, is compensated by the benefit from extra effort motivated by the election. In this case, both politicians prefer to introduce the election at date 0. We show that election is not always introduced even if the motivating effect is always positive. Election will only be introduced when the distribution of production shock is favorable. When an adverse production shock is more likely, election will not be introduced.

Observers sometimes view authoritarian elections as meaningless if they are unlikely to lead to democratization, or if the incumbents have an unshakable dominance. Our result challenges this opinion. We show that even when elections are manipulated, they can still provide incentives for effective governance. As long as the incumbents have some stake in the election game, election will not be inconsequential.
1.1 Literature Review

Our model belongs to the electoral control literature initiated by Barro (1973). Electoral control model studies to what extent voters can discipline politicians when politicians cannot commit to the behavior preferred by voters. Barro (1973) establishes the responsive democracy result, i.e. the level of public good provision converges to the voters’ optimal when office reward is sufficiently large. Ferejohn (1986) introduces moral hazard into the electoral control model. The incumbent politician has private information about a productivity shock, and his effort choice is unobservable to the voters. He shows that the highest equilibrium payoff for the voters is increasing in the office reward. Voters in Banks and Sundaram (1993) face both moral hazard and adverse selection problem in an infinite-period election model, i.e. politician’s abilities are unknown, and their effort levels are not directly observable. Electoral control helps voters to choose the high ability politician and to motivate politicians to put in more effort. Banks and Sundaram (1998) show the same result does not necessarily hold with a term limit, i.e. both types of politicians will shirk at the end of their terms. Fearon (1999) studies a two-period election model. He argues that elections should be mainly understood as mechanisms of selecting good politicians, rather than of sanctioning politicians’ misbehavior. Bank and Duggan (2008) consider the adverse selection problem in a multidimensional model. They show that politicians can be partially constrained by concerns of re-election when their non-policy office reward is sufficiently large. Duggan and Martinelli (2014) review the literature and establish the responsive democracy result in a series of models. They also find negative result in an infinite-horizon model with term limits. Different from these papers, we study the behavior of multiple incumbents under electoral control. We also focus on how electoral control

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2 Fearon (1999) compares three cases in his model: pure adverse selection, pure moral hazard, and the mixed case. In the case of pure adverse selection, the electorate is capable to reelect the good politician as monitoring approaches perfection. In the pure moral hazard case, the politician’s choice converges to the voter’s ideal point as monitoring keeps improving. In the mixed case, the electorate focuses less on preventing the bad politician from shirking as compared with the pure moral hazard case, even when the chance of finding a good politician is very small.

3 A multidimensional electoral control model is different from a one-dimensional model in that a majority undominated policy may not exist. Therefore, researchers cannot study whether the equilibrium policy will converge to an ideal one preferred by the median voter, but only whether politicians will deviate from their own ideal point.
might mitigate the moral-hazard-in-teams problem.

Researchers have proposed alternative views on the role of elections in authoritarian regimes. Some scholars think elections are introduced by autocracies under internal and international pressure for democratization, nothing more than authoritarian window dressing (Waterbury, 1999; Schedler, 2006). Others claim that elections are a means by which dictators hold onto power (Gandhi and Lust-Okar, 2009). Elections transmit information. Through elections, dictators could credibly commit to a constrained confiscatory behavior (Wright, 2008a), gather information about opposition strongholds (Cox, 2009; Egorov and Sonin, 2014), or demonstrate invincibility (Magaloni 2006; Geddes, 2009). Elections are also thought of as a power-sharing mechanism. Through elections, dictators form alliances (Boix and Svolik 2013; Bidner, Francois, and Trebbi, 2014; Gandhi, 2008), keep the opposition divided (Lust-Okar, 2005), or incorporate the opposition (Gandhi and Przeworski, 2007). In our model, election serves as an incentive device to solve the moral hazard problem within authoritarian government.

In broader terms, our paper contributes to the literature that studies incentive schemes in authoritarian regimes. The majority of the work in this literature studies how dictators motivate bureaucrats or low-tier officials. For example, Qian and Roland (1998) study how fiscal competition among local governments alleviates the soft budget constraint problem. Lazarev (2007) studies the optimal promotion contract within regime parties to encourage bureaucratic effort. Dictators could also use media freedom (Egorov et al., 2009) to provide incentives for low-tier officials. Our paper is different from these studies in that dictators introduce institutions not to discipline their subordinates but themselves.

The remainder of the paper is organized as follows. The next section introduces the model. Section 3 presents the main results. Section 4 discusses extensions and the relationship between our model and existing empirical facts. Section 5 concludes the paper and suggests directions for future research.
2 Model

Our model consists of three players, two politicians $i \in \{1, 2\}$ and a representative voter $v$. There are two dates $t = 0, 1$.

At date 0, the two politicians decide by unanimous vote the game to play at date 1. There are two possible date-1 games, the diarchy game and the election game. The election game is played at date 1 only if both politicians vote for it at date 0. Otherwise, the diarchy game is played.

The two politicians engage in the production of public good regardless of the game chosen, the output level $\tilde{y}$ at date 1 is given by

$$\tilde{y}(a_1, a_2) = y(a_1, a_2) + \tilde{\theta},$$

where $a_1, a_2 \in \mathbb{R}_+$ are the politicians’ effort levels and $\tilde{\theta}$ is an exogenous production shock. The production shock $\tilde{\theta}$ is distributed according to a probability distribution function $F : \mathbb{R} \to \mathbb{R}_+$. The distribution $F$ has full support on the real line $\mathbb{R}$ and admits a probability density function $f$ that has continuous and bounded derivative. We also assume the distribution is unimodal. The public good production function $y : \mathbb{R}_+^2 \to \mathbb{R}_+$ is assumed to be twice-continuously-differentiable, strictly increasing, strictly concave, and supermodular. The politicians bear costs by exerting efforts to produce the public good. Politician $i$’s cost function $c_i : \mathbb{R}_+ \to \mathbb{R}_+$ is twice-continuously-differentiable, strictly increasing and strictly convex. Moreover, $c(0) = c_i'(0) = 0$ and $\lim_{a_i \to \infty} c_i'(a_i) = \infty$.

We assume that voter’s utility $V : \mathbb{R}_+ \to \mathbb{R}_+$ is a strictly increasing function of the level of public good provision $\tilde{y}$. We also assume that a politician’s utility increases with the level of the public good and decreases with the cost of his effort level. Denote $\alpha_i \in \mathbb{R}_{++}$ the weight of public good in politician $i$’s utility. A politician with a higher $\alpha$ has more incentives to provide public good. In reality, the $\alpha$’s are endogenously determined.

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4The moral-hazard-in-teams problem that we are interested in only occurs when there are more than one politician. The main results of this paper generalize easily to the case of $N \geq 2$ politicians.

5We use the male pronoun to refer to a politician and the female pronoun to refer to the voter.

6We assume unimodality just for the convenience of expression. Our main results easily generalize to production shocks with a multimodal distribution.
by various characteristics of an authoritarian regime. For example, dictators are more inclined to invest in public goods when they have a longer time horizon (Wright, 2008b). Autocracies with larger coalition system will also have a greater incentive to provide public goods (Bueno de Mesquita et al, 2003). We treat \( \alpha_i \) as an exogenous variable to focus on the effect of election on public good provision for given preferences of regime rulers.

Finally, we assume that for all sequences \( \{(a^n_1, a^n_2)\}_{n \in \mathbb{N}} \) with either \( a^n_1 \) or \( a^n_2 \to \infty \),

\[
\min_{i \in \{1, 2\}} \left\{ \alpha_i y \left( a^n_i, a^n_j \right) - c_i(a^n_i) \right\} < 0.
\]

This assumption implies that complementarity between the politicians’ efforts cannot be too strong so that the expected payoff of at least one of the politicians must become negative as the effort levels increase. When the production function \( y \) is either bounded, decreasing return to scale or constant return to scale, this assumption is automatically satisfied.

**Diarchy game:** In the diarchy game, the politicians choose effort levels simultaneously. Then, the production shock \( \tilde{\theta} \) is realized and the game ends. The payoff of politician \( i \) is given by

\[
u^D_i (a_i, a_j) = \alpha_i \tilde{y} (a_i, a_j) - c_i(a_i).
\]

The payoff of the voter is simply \( V(\tilde{y}(a_i, a_j)) \).

**Election game:** In the election game, the voter chooses a satisfactory output level \( \tilde{y} \in \mathbb{R} \cup \{-\infty\} \cup \{+\infty\} \). The two politicians, after observing \( \tilde{y} \), choose effort levels simultaneously. Then, the production shock \( \tilde{\theta} \) is realized and the voter observes the output level of public good \( \tilde{y} \). The voter reelects the incumbent politicians if and only if the output level of public good reaches the satisfaction level, i.e., \( \tilde{y} > \bar{y} \). If the politicians fail to get the vote, they commit electoral fraud to stay in office. As the incumbent politicians always hold office, the voter is indifferent to the characteristics and behaviors of the challenger. As a result, we do not explicitly model the challenger.\(^7\) Electoral fraud is costly and the cost \( Z \) is shared between the two politicians according to the sharing rule \((r_1, r_2)\), where \( r_i > 0 \) and \( r_1 + r_2 = 1 \). Our assumption of costly electoral fraud is motivated by the fact

\(^7\)Following Duggan and Martinelli (2014), we can interpret an equilibrium of the election game as the selected equilibrium of an alternative game, in which the voter votes after observing \( \tilde{y} \) without precommitting to a cutoff voting rule. In the alternative game, the voter is always indifferent, but voting according to the equilibrium cutoff rule of the election game maximizes her ex ante payoff.
that in reality, any forms of electoral fraud, such as vote buying, intimidation, misleading ballot papers, and ballot stuffing, incur costs. Even if the regime party simply misrecords votes, such act may infuriate the public and cause instability.

In standard models of electoral accountability (Duggan and Martinelli, 2014), the cost of losing an election to a politician is the forgone benefit from office for an extra term. This assumption, while appropriate in describing democratic elections, is unsuitable for the study of authoritarian elections. Unlike the benefit from holding office, the cost of committing electoral fraud can be quite small comparing to politicians’ current payoff. This observation motivates us to consider the small cost limit in our analysis.

The timeline of the election game is presented in Figure 1.

![Timeline of the election game](image)

Figure 1. Timeline of the election game.

The payoff of politician $i$ is given by

$$u_i^E (a_i, a_j) = \alpha_i y (a_i, a_j) - c_i (a_i) - 1_{\{\bar{y} < y\}} r_i Z.$$ 

### 2.1 Equilibrium

The equilibrium of the model consists of the politicians’ vote at date 0, the politicians’ effort levels in the diarchy game, $\{a_1^d, a_2^d\}$, the voter’s satisfaction level, $\bar{y}^*$, and the two politicians’ effort levels in the election game given the voter’s satisfaction level, $\{a_1^e (\bar{y}), a_2^e (\bar{y})\}$, such that

1. $\{a_1^d, a_2^d\}$ forms a Nash equilibrium in the diarchy game, i.e.

   $$a_i^d \in \arg \max_{a_i} \alpha_i y (a_i, a_j) - c_i (a_i) \text{ for } i \in \{1, 2\},$$

2. $\{a_1^e (\bar{y}), a_2^e (\bar{y})\}$ forms a Bayesian Nash equilibrium in the election game for every $\bar{y}$.
i.e.

\[ a_i^e (\bar{y}) \in \arg \max_{a_i} \alpha_i y (a_i, a_j^e (\bar{y})) - c_i (a_i) - F \left[ \bar{y} - y (a_i, a_j^e (\bar{y})) \right] r_i Z \text{ for } i \in \{1, 2\} \]

3. given \{a_1^e(\bar{y}), a_2^e(\bar{y})\},

\[ \bar{y}^* \in \arg \max_{\bar{y}} y (a_1^e (\bar{y}), a_2^e (\bar{y})) \]

4. given \{a_1^d, a_2^d\}, \bar{y}^*, and \{a_1^e(\bar{y}), a_2^e(\bar{y})\}, politician \(i\) vote for election if and only

\[
\alpha_i y \left( a_1^d, a_2^d \right) - c_i \left( a_i^d \right) \leq \alpha_j y \left( a_1^e (\bar{y}^*), a_2^e (\bar{y}^*) \right) - c_i \left( a_i^e (\bar{y}^*) \right) - F \bar{y}^* - y \left( a_1^e (\bar{y}^*), a_2^e (\bar{y}^*) \right) r_i Z.
\]

3 Main Results

To decide whether to introduce the election, politicians compare equilibrium payoffs in the diarchy game and the election game. We begin our analysis by characterizing the equilibrium of the diarchy game:

Lemma 1 The equilibrium \((a_1^d, a_2^d)\) of the diarchy game is unique. Moreover, \(a_i^d\) uniquely satisfies

\[
\frac{\partial y (a_i^d, a_j^d)}{\partial a_i} - c_i' (a_i^d) = 0.
\]

The uniqueness of equilibrium follows from the strict concavity of the production function. In the unique equilibrium, the weighted marginal output of effort must equals the marginal cost of effort for each politician. Lemma 1 implies that the pair of equilibrium effort levels, \((a_1^d, a_2^d)\), does not maximize the sum of the two politicians’ payoff, which is defined by

\[
(\alpha_i + \alpha_j) \frac{\partial y (a_i, a_j)}{\partial a_i} - c_i' (a_i) = 0.
\]

Both politicians can be better off if they both exert more efforts. This is because each politician only consider the marginal benefits of effort to himself, while increasing the effort
level can benefit the other politician. As efforts cannot be contracted on, there is a moral-

hazard-in-teams problem. From a planner’s perspective who consider both the politicians' and the voter’s utility, \((a_1^d, a_2^d)\) is also not optimal.

To solve the election game, we work backward from the end of the game. Given \(\tilde{y}\) and politician \(j\)’s effort level \(a_j\), politician \(i\)’s problem is

\[
\max_{a_i \in \mathbb{R}_+} \alpha_i y(a_i, a_j) - c_i(a_i) - F[\tilde{y} - y(a_i, a_j)] r_i Z
\]

(2)

which has the following first order condition:

\[
(\alpha_i + r_i f[\tilde{y} - y(a_i, a_j)] Z) \frac{\partial y(a_i, a_j)}{\partial a_i} - c_i'(a_i) = 0.
\]

(3)

The first order condition (3) is different from the first order condition (1) for politician \(i\) in the diarchy game. By exerting more efforts, a politician not only benefits from producing more public good, but also from reducing the chance of paying the cost of electoral fraud.

Given a satisfaction level \(\tilde{y}\), denote the voter-preferred equilibrium effort levels (i.e. the pair of equilibrium effort levels that maximize output) by \((a_1^e, a_2^e)\). The voters’ decision problem is

\[
\max_{\tilde{y} \in \mathbb{R}} y[a_1^e(\tilde{y}), a_2^e(\tilde{y})]
\]

(4)

Clearly, the voter prefers both politicians to exert as much effort as possible. However, she does not care about the cost of politicians’ effort. Therefore, the voter might set a satisfaction level too high for the election to be beneficial for politicians. We will discuss this in detail in section 4.1.

It is well-known in the electoral control literature that the politicians’ problem (2) is in general non-concave and the characterization of the equilibria is nontrivial even in the single-politician case (Duggan and Martinelli 2014). To see this, we can rewrite politician \(i\)’s problem as
\[
\max_{a_i \in \mathbb{R}^+, s \in \mathbb{R}} \alpha_i y(a_i, a_j) - c_i(a_i) + sr_iZ \\
\text{s.t. } s + F[\tilde{y} - y(a_i, a_j)] \leq 0.
\]

The objective function is concave in \((a_i, s)\). However, if the distribution function is non-convex, the constraining set is not convex. This leads to the possibility of multiple optima. Therefore, instead of providing a complete characterization, we take an approach to analyze the limit cases. We first show the conditions under which the election is not introduced by politicians:

**Proposition 1** As the cost of electoral fraud \(Z\) increases, at least one of the politicians’ effort increases without bound. Moreover, when the cost of electoral fraud \(Z\) is large enough, the diarchy game is chosen at date 0 in equilibrium.

To see the first result, suppose neither politician’s effort increases without bound as \(Z\) increases. In that case, the expected loss from an adverse production shock increases without bound. Moreover, the marginal expected loss from withholding effort also goes to infinity and exceeds the marginal cost of effort. This contradicts optimality. When at least one of the politicians’ effort increases beyond bound, it follows from our assumption that at least one of the politician must have a negative payoff. This politician thus does not prefer to bring in the election. Intuitively, if the election is too costly to lose, the politician will have to spend too much effort in producing public good.

Proposition 1 is an analog of the *strong responsiveness of democracy* result in the pure moral hazard model of electoral accountability in Duggan and Martinelli (2014). In a single-agent model, they show that as re-election incentives increase, the equilibrium effort of the incumbent politician increases without bound. Based on this and other related results, they conclude that electoral accountability is possible under democracy. Excessive electoral control, however, is fatal to the introduction of elections in authoritarian regime. Our model suggests that authoritarian leaders simply will not introduce elections if they cannot afford to lose.
However, when the cost of losing is not too high, politicians may actually prefer the election game. To see this, we first provide the equilibrium characterization when the cost of electoral fraud $Z$ is small enough:

**Proposition 2** When the cost of electoral fraud $Z$ is small enough, the equilibrium satisfaction level $\bar{y}^*$ solves $\theta^* = \bar{y}^* - y[a_1^r(\bar{y}^*), a_2^r(\bar{y}^*)]$, where $\theta^*$ is the mode of the distribution $F$. Moreover, for both $i \in \{1, 2\}$, $a_i^r(\bar{y}^*) > a_i^d$.

When the cost of electoral fraud is small, a politician’s problem becomes strictly concave, and the equilibrium effort pair is uniquely determined by (3). We then use the strict concavity and supermodularity of a politician’s payoff function to prove Proposition 2. Fearon (1999) has a similar result in a moral hazard model, where the politician’s payoff function is quadratic in his own policy choice, and the noise is symmetrically distributed. We generalize his result to a model with any strictly concave and supermodular payoff function and any distribution with a full support. Moreover, our result is established for more than one agent.

Intuitively, the voter’s satisfaction level decides the probability for the politicians to win the vote given any level of public good provision. Proposition 2 states that the voter will set this probability as sensitive as possible to the level of public good produced. In such way, the politicians will have the highest incentive possible to exert effort. Compared to the diarchy game, politicians bear more consequences by shirking in the election game. Therefore, they will exert more effort in the election game than in the diarchy game.

Proposition 2 offers a way to solve the equilibrium when $Z$ is small. First, we find the mode, $\theta^*$. Then, we solve $(a_1^v(\bar{y}^*), a_2^v(\bar{y}^*))$ from the first order conditions (3) where we substitute $\bar{y} - y(a_i, a_j)$ with $\theta^*$. Finally, we have $\bar{y}^* = y(a_1^v(\bar{y}^*), a_2^v(\bar{y}^*)) + \theta^*$. It is clear that politicians produce more public good than the voter’s satisfaction level if $\theta^* < 0$, i.e., the production shock at mode is negative. Conversely, the politicians will produce less public good than the voter’s satisfaction if $\theta^* > 0$. The probability that the realized output of public good will reach the satisfaction level, however, depends on the cumulative probability at mode, $F(\theta^*)$. We will see how this result relates to the conditions under which politicians would like to introduce the election in Proposition 4.
The characterization in Proposition 2 also allows us to partially rank different distributions in terms of equilibrium effort levels.

**Proposition 3** Consider two distributions $F$ and $G$ which satisfy all the assumptions in Section 2 and denote their modes by $\theta^*_F$ and $\theta^*_G$, respectively. There exists a $\bar{Z}_{F,G} > 0$, such that for all $Z < \bar{Z}_{F,G}$, the efforts of the two politicians are strictly larger in the diarchy game when $\theta$ has distribution $F$ rather than distribution $G$ iff $f(\theta^*_F) > g(\theta^*_G)$.

The proof of Proposition 3 follows naturally from Proposition 2. The marginal impact of shirking at the equilibrium effort level is determined by the density at mode of the production shock. Thus, a production shock with higher density at mode enables the voter to better motivate politicians to exert effort. For example, given two normally distributed production shocks, politicians will exert more effort under the production shock with lower variance.

The production shock could also be understood as a noise added to the observation of the level of public good when the voter monitors the two politicians’ performance. In a principal-agent context, Kim (2005) shows that different information systems can be ranked in efficiency by the mean preserving spread relation between the likelihood ratio of distributions derived from the information systems. In our political agency model, two information systems can be ranked in motivating effect by comparing the density at mode alone, provided that the cost of electoral fraud $Z$ is small enough.

Now, we show the condition under which both politicians are willing to introduce the election:

**Proposition 4** When the cost of electoral fraud $Z$ is small enough, there exists a constant $K > 0$, independent of the distribution $F$, such that if the reverse hazard rate at mode, denoted by $R(\theta^*)$, is strictly larger than $K$, then the election game is chosen at date 0 in equilibrium. Moreover, if $R(\theta^*) < K$, the diarchy game is chosen at date 0 in equilibrium.

To see this result, first note that politicians’ equilibrium payoffs in the diarchy game equal the limit of the election equilibrium payoffs as the cost of electoral fraud $Z$ approaches
zero. When $Z$ is small, therefore, we could use first order expansion to decompose the consequence of introducing the election on a politician’s payoff. Denote the unique equilibrium effort level $\hat{a}_i^*(\tilde{y}^*)$ for a given $Z$ as $\hat{a}_i^*(Z)$. Then,

$$
E \left[ u_i^E \left( a_i^*(\tilde{y}^*), a_j^*(\tilde{y}^*) \right) \right] - E \left[ u_i^D \left( a_i^d, a_j^d \right) \right] 
\approx \frac{\partial E \left[ u_i^E \left( a_i^d, a_j^d \right) \right]}{\partial a_i} \frac{d\hat{a}_i^*(0)}{dZ} Z + \frac{\partial E \left[ u_i^E \left( a_i^d, a_j^d \right) \right]}{\partial a_j} \frac{d\hat{a}_j^*(0)}{dZ} Z + \frac{\partial E \left[ u_i^E \left( a_i^d, a_j^d \right) \right]}{\partial Z} Z
\approx 0 \cdot Z + \frac{\partial y \left( a_i^d, a_j^d \right)}{\partial a_j} \frac{d\hat{a}_j^*(0)}{dZ} Z - r_i F(\theta^*) Z
$$

The effect of the election has three components. The first component is a indirect effect of the election through a politician’s own optimal effort level, which is zero by the Envelope Theorem. The second component is an indirect effect of the election through other’s optimal effort level, which is always positive in the equilibrium. The third component is a direct effect of the election through potential election fraud in the equilibrium, which is always negative. When $Z$ is small, the positive indirect effect is approximately linear in the product of $Z$ and the density at mode $f(\theta^*)$, and the negative indirect effect is linear in the product of $Z$ and the probability of potential election fraud in the equilibrium $F(\theta^*)$. The relative size of these effects, therefore, is increasing linearly with the reverse hazard rate at mode, $R(\theta^*) = \frac{f(\theta^*)}{F(\theta^*)}$. When $R(\theta^*)$ is large enough, the indirect effect exceeds the direct effect, and both politicians are willing to introduce the election.

The reversed hazard rate at mode measures the conditional probability for a slightly adverse shock to happen, given the shock is adverse. Intuitively, for politicians to introduce election, the marginal impact of shirking should be high enough to motivate effort, while the overall chance of losing the vote should be small enough to avoid cost. Specifically, politicians will introduce election when the distribution of production shock is skewed enough to the left.

We have shown that electoral control can benefit politicians. This benefit is resulted from mitigating the moral-hazard-in-teams problem in public good provision. By introducing election, politicians enforce extra punishment for shirking. In Holmstrom (1982)’s
language, the voter acts like a new role that administrates an incentive scheme that does not balance the budget.

Using the first order expansion, we can also reach the following result about the effect of the sharing rule of the cost of electoral fraud on the introduction of the election. Denote $K$ as the threshold in Proposition 4.

**Proposition 5** If the two politicians have symmetric payoff functions in the diarchy game, i.e. $u_i^D(a_i, a_j) = u_j^D(a_j, a_i)$, then $K$ is linearly increasing in $\max\{\frac{r_1}{r_2}, \frac{r_2}{r_1}\}$. Moreover, $K$ reaches its minimum when $r_1 = r_2 = 1/2$.

Proposition 5 implies that the election is more likely to be introduced when the cost of election is shared fairly. To see this result, recall the first order expansion of the consequence of election on a politician’s payoff in (5). We show in the appendix that $\frac{r_i}{r_j}$ linearly reduce the relative size between the positive and the negative effect of election on politician $i$’s payoff, independently of the reverse hazard rate at mode. As the relative size linearly increases in the reverse hazard rate at mode, $R(\theta^*)$, a higher $\max\{\frac{r_1}{r_2}, \frac{r_2}{r_1}\}$ correlates to a higher threshold of $R(\theta^*)$ above which both politicians benefit from introducing the election. The intuition is as follows. The higher the $r_i$, the more politician $i$ bears the cost of election. In the meanwhile, the other politician $j$ has less incentive to provide effort as he suffers less loss from shirking. Consequently, politician $i$ benefits less from the increase in politician $j$’s effort, while paying more electoral fraud cost. Therefore, politician $i$ is more willing to introduce election when $\frac{r_j}{r_i}$ is larger. But symmetrically, politician $j$ is more willing to do so when the same ratio is smaller. As a result, election is more likely to be introduced when $\max\{\frac{r_1}{r_2}, \frac{r_2}{r_1}\}$ is lower.

### 4 Discussion

#### 4.1 The Voter’s Commitment Problem

In our model, the voter chooses the satisfaction level $\bar{y}$ only after the election is introduced at date 1. This particular sequence of events creates a commitment problem on the voter’s
side. To see this, suppose the voter now chooses the satisfaction level \( \bar{y} \) at date 0 instead. The politicians, upon observing \( \bar{y} \), vote for the date-1 game. The voter can then choose a lower satisfaction level \( \bar{y}^{**} \) to induce the election when the optimal date-1 satisfaction level \( \bar{y}^{*} \) is too high for the election to take place. Although the new satisfaction level \( \bar{y}^{**} \) Pareto-improves welfare, it is not an equilibrium choice in the original model. Because, at date 1, the voter chooses the satisfaction level \( \bar{y}^{*} \) taking the election as given. This strategic tension is best illustrated with a normally distributed production shock when the cost of electoral fraud \( Z \) is low.

**Proposition 6** Suppose the production shock \( \hat{\theta} \) is normally distributed, then for all \( Z \) small enough, there exists a \( \bar{y}^{**} \in \mathbb{R} \) such that if the voter can commit to the satisfaction level rule \( \bar{y}^{**} \) at date 0, the two politicians would agree to introduce the election. Moreover, each player’s welfare is strictly higher with election than without election.

With normally distributed shock, the reverse hazard rate at mode \( R(\theta^{*}) \) equals \( \frac{1}{\sigma \sqrt{2\pi}} \). Thus, given a normal distribution with standard deviation \( \sigma > \frac{1}{\sqrt{2\pi} K} \), by Proposition 4, there exists a \( \tilde{Z} > 0 \), such that for all \( Z < \tilde{Z} \), election is not introduced in equilibrium. By Proposition 6, if the voter can commit to the satisfaction level rule at date 0, Pareto-improvement can be made over the equilibrium outcome without commitment.

Therefore, the voter’s commitment problem may hinder the introduction of election that is Pareto improving. We suggest that this may be one of the reasons why some authoritarian countries fail to launch election, even when political reform seems to be a promising way to enhance economic performance and regime stability.

The voter’s commitment problem in this model is similar to those discussed in Acemoglu and Robinson (2005a), who argue that the elite might overthrow a democratic regime because the mass cannot commit to make policies less pro-majority. It also echoes Huntington (1991)’s reasoning that democratization could be impeded if voters could not commit to pardon rulers who are responsible for human rights abuses.
4.2 Democratization

We assume the politicians always bear the cost of electoral fraud, and do not have a choice to step down. Such feature resembles electoral authoritarian instead of democracy. Relaxing such assumptions, our framework could be used to discuss democratization in authoritarian regimes as well. In this subsession, the same game is played with the slight change in date-1 election game:

If the politicians fail to win the vote, they can choose between staying in office by committing electoral fraud and stepping down voluntarily. In such case, politician \( i \) gets a payoff of:

\[
\begin{align*}
    u_i^E(a_i, a_j) &= \alpha_i y(a_i, a_j) - c(a_i) - r_i \min \{ Z, R \}. \\
\end{align*}
\]  

(6)

where \( R \) is the cost of losing office. By introducing the option of resigning, we put an upper bound on the cost of not getting the vote, which is \( \min \{ Z, R \} \). Clearly, our results on small \( Z \) still hold, and the election is more likely to be introduced even when \( Z \) is large. Whether the election introduced by autocrats will be free and fair, and likely leading to democratization, depends on whether the cost of losing office, \( R \), is smaller than the cost of electoral fraud, \( Z \). In most cases, the former is larger than the latter, which prevents democratization even when election is introduced. However, when the cost of losing office is lower than the cost of committing electoral fraud, authoritarian rulers may introduce fair elections and be willing to accept defeat.

Now consider an further extension of the above model. Politicians decide whether to introduce elections at date-0, and then play the date-1 game repeatedly. We make the assumption that the politicians are myopic, i.e., they base their decisions just on the period payoff defined in (6). Then, if the cost of committing electoral fraud increases overtime, authoritarian elections featured by electoral fraud in the beginning, may lead to democratization in the future. This could be illustrated by Mexico’s democratization process. The hegemonic Institutional Revolutionary Party (PRI) in Mexico kept itself in office by systematic vote buying and ballot stuffing since its founding in 1929. However, as the PRI suffered a great fiscal loss from the 1994 currency crisis, it could not afford the
cost of committing electoral fraud. The opposition then won in a series of gubernatorial and congressional elections, and finally ended PRI’s 70-year rule by winning the 2000 presidential election (Magaloni, 2006).

4.3 Authoritarian Election and Economic Growth

Researchers have found that authoritarian regimes that hold elections outperform pure authoritarian regimes in economic growth and domestic investment. Gandhi (2008) finds that authoritarian regimes that have legislatures experience more rapid economic growth. Wright (2008a) shows that the presence of elected legislatures is correlated with faster investment and economic growth in military and single-party regimes, but not in personalist regimes. Gehlbach and Keefer (2012) find that the competitiveness of legislative elections in autocracies is robustly correlated with higher investment. In terms of health-care performance, Blaydes and Kayser (2011) find that hybrid regimes, which combine elements of autocracy and democracy, perform as well as democracies and better than pure autocracies in daily calorie provision. However, among autocracies with multi-party elections, Michalik (2015) finds that competitiveness of legislative election is negatively correlated with public health care spending.

In Proposition 4, we find that mild electoral control always leads to higher effort and hence higher public good provision. Autocrats use the cost of election fraud as a commitment device. Both politicians in the model work harder to avoid the costly fraud, and complementarity in their efforts reinforce the positive effect. It is well-established that provision of public goods, such as infrastructure, education, and protection of property rights, contributes to higher economic growth (Acemoglu and Robinson, 2005b; Barro, 1991; Roller and Waverman, 2001). The autocrats, in turn, benefit from holding election. In the next paragraph, we use Singapore as an illustrative example for our analysis.

Soon after its establishment, Singapore’s People’s Action Party (PAP) secured every seat in the parliament in the 1968 general election. Since then, the PAP has been the dominant power in Singapore politics. But, unlike some authoritarian parties that suppressed elections and outlawed opposition parties, the PAP continued to hold parliamentary elec-
tions. The opposition parties never won more than a few seats to have a substantial direct impact on policy. However, they formed a continuous pressure for the PAP to provide effective governance (Mauzy and Milne, 2002). Aiming to gain legitimacy by economic performance, the PAP has built one of the most open and least corrupt market economies in the world. During the last 50 years, Singapore has experienced a phenomenal growth in economy, along with remarkable development in national healthcare and education (Ghesquière 2007). Although the PAP used to set obstacles for opposition parties by threats of defamation suits and restrictions on fund-raising, it obtained its overwhelming victory in elections mainly through efficient governance rather than electoral fraud (Mauzy and Milne, 2002). Election, as an incentive scheme, promoted economic growth and authoritarian rule in Singapore.

4.4 Some Other Facts

Researchers also find that autocracies with elections do not necessarily democratize. Brownlee (2009) shows that although authoritarian regimes which hold elections are more likely to transit to democracy after incumbents are overthrown, they are not more likely to collapse in the first place. Donno (2013) shows that competitive authoritarian elections are more prone to democratization only when domestic and international actors choose to actively pressure the regime. Scholars also find elections are unrelated to democratization in various regions, such as Postcommunist Eurasia (Kaya and Bernhard, 2013), Latin America (McCoy and Hartlyn, 2009), and the Middle East (Lust-Okar, 2009). Our model offers two perspectives in understanding this phenomenon. On the one hand, dictators are less likely to move to democracy if authoritarian elections are already in place and effective in providing incentives for public good provision. On the other hand, the general public are less likely to demand democracy if authoritarian elections effectively promote economic growth and public welfare.

Empirical studies also find autocracies rich in oil and other natural resources are less likely to hold elections. Gandhi and Przeworski (2007) find that authoritarian regimes with ratio of mineral exports (including oil) exceeding 50% are less likely to introduce legislature
and multi-party elections. Wright (2008a) shows that oil wealth is associated with the existence of legislatures in military and single-party regimes, but not in personalist regimes and monarchies. Wahman and Basedau (2015) find that incumbent parties/presidents in countries with higher oil income relative to GDP win more seats in multi-party elections. In a subnational analysis of Nigeria, they also show oil production corresponds with higher incumbent-party support in gubernatorial, senatorial, and lower-house elections. Our model suggests the introduction of election is related to moral-hazard-in-teams problem within the government. Given a high fiscal income from natural resources, however, politicians’ jobs become easier. They need less collective endeavour to produce the same amount of public good. Moreover, politicians face less pressure to engage in the cumbersome collective task of tax collecting. Thus, they are less likely to introduce elections to motivate themselves.

5 Conclusion

In this paper, we model authoritarian rulers’ decision to introduce election. We show that authoritarian rulers will not introduce election when the cost of electoral fraud is too high. When the cost of electoral fraud is low, however, we find conditions under which authoritarian rulers will introduce election as an incentive scheme. As such, we offer a new perspective to understand authoritarian elections.

The voter in our model focuses on motivating higher effort from politicians. An extension to our model might introduce unobserved types of incumbents. The voter then faces a potential conflict in motivating effort for different teams of politicians. When the cost of electoral fraud is small, politicians with lower abilities will be less likely to introduce election than in the original model. But more examination is needed to speak for politicians with higher abilities. Another extension might allow other forms of interactions between the voter and the politicians. For example, if the voter could contest an unfair election, his optimal satisfaction level will be different from the characterization of the level in our model. The politicians’ decision to introduce election will alter correspondingly.

---

\footnote{Politicians with lower abilities need to commit electoral fraud more often due to an increase in voter’s satisfaction level. Moreover, the motivating effect of election for them is weakened.}
Researchers have shown that the moral-hazard-in-teams problem can be alleviated in various cases. For example, Kandel and Lazear (1992) show that peer pressure originating from guilt, shame and norms can motivate effort. Rotemberg (1994) shows that altruism among agents also mitigates the problem. Gervais and Goldstein (2007) suggest that an agent’s overconfidence can benefit all team members. It would be interesting to study how these factors which alleviate the moral hazard problem among politicians interact with the incentive provided by election.

References


Appendix

Proof of Lemma 1. Consider the function

\[ W^D(a_1, a_2) := y(a_1, a_2) - \frac{c_1(a_1)}{a_1} - \frac{c_2(a_2)}{a_2}, \]

which is strictly concave by our assumptions on the functions \( y, c_1, \) and \( c_2. \) Denote the unique maximizer of the function \( W^D \) by \((a_1^d, a_2^d)\). We know that \((a_1^d, a_2^d)\) is interior and uniquely satisfies the first order conditions:

\[ \alpha_i \frac{\partial y}{\partial a_i} (a_i^d, a_j^d) - c'_i(a_i^d) = 0, \quad i, j \in \{1, 2\}, \quad j \neq i \]

Therefore, \((a_1^d, a_2^d)\) must also be the unique equilibrium of the diarchy game.

Proof of Proposition 1. Suppose by way of contradiction that there exist a strictly increasing and unbounded sequence of the cost of electoral fraud \( \{Z_n\}_{n \in \mathbb{N}} \) and a corresponding sequence of equilibria of the election game \( \{(a_1^n, a_2^n, y_n^*)\}_{n \in \mathbb{N}} \) such that none of \( a_1^n (y_n^*), a_2^n (y_n^*) \to \infty \) as \( n \to \infty \). Take a subsequence if necessary, we conclude that the sequence \( \{(a_1^n, y_n^*), a_2^n (y_n^*)\}_{n \in \mathbb{N}} \) is bounded. We would like to show that for an arbitrary \( \bar{y}^c \in \mathbb{R} \), at least one of \( a_1^n (\bar{y}^c), a_2^n (\bar{y}^c) \to \infty \) as \( n \to \infty \). This will contradict the optimality of \( \{(y_n^*)\}_{n \in \mathbb{N}} \).

Suppose not, take a subsequence again if necessary, \( \{\bar{y}^c - y(a_1^n (\bar{y}^c), a_2^n (\bar{y}^c))\}_{n \in \mathbb{N}} \) is bounded. As \( f \) is strictly positive and continuous, it has a strictly positive minimum on a compact set. Thus, there exists a \( \delta \in \mathbb{R}_{++} \) such that for all \( n \in \mathbb{N}, \) \( f [\bar{y}^c - y(a_1^n (\bar{y}^c), a_2^n (\bar{y}^c))] \geq \delta > 0 \). The first order necessary condition (3) implies that for each \( i \in \{1, 2\}, \) either \( \frac{\partial y(a_1^n (\bar{y}^c), a_2^n (\bar{y}^c))}{\partial a_i} \to 0 \) or \( c''(a_i^n (\bar{y}^c)) \to \infty \). Either case implies that for both \( i \in \{1, 2\}, a_i^n (\bar{y}^c) \to \infty, \) which is what we claim.

Thus, we must have at least one of \( a_1^n (y_n^*), a_2^n (y_n^*) \to \infty \) as \( n \to \infty \). Then, by assumption, at least one of the politicians must receives negative payoff and prefer the diarchy game as \( u_i^D (a_i^d, a_j^d) > 0 \). Under the unanimity rule, the election game will not be chosen.

□
**Proof of Proposition 2.** Consider the function

$$W^E(a_1, a_2) := y(a_1, a_2) - \frac{c_1(a_1)}{(\alpha_1 + r_1 f(\theta^*)) Z} - \frac{c_2(a_2)}{(\alpha_2 + r_2 f(\theta^*)) Z},$$

(8)

which is strictly concave by our assumptions on the functions $y$, $c_1$, and $c_2$. Denote the unique maximizer of the function $W^E$ by $(a_1^*, a_2^*)$. For each $i \in \{1, 2\}$, $a_i^*$ is interior and uniquely satisfies the first order condition

$$(\alpha_i + r_i f(\theta^*) Z) \frac{\partial y(a_i^*, a_j^*)}{\partial a_i} - c_i'(a_i^*) = 0$$

(9)

Let $\tilde{y}^* = y(a_1^*, a_2^*) + \theta^*$. By construction, $a_i^*$ satisfies the first order necessary condition (3) for politician $i$’s problem, which is also sufficient for $a_i^*$ to be optimal when $Z$ is small. Next, by the implicit function theorem, politicians’ equilibrium effort functions $(a_1^e, a_2^e)$ are unique and continuously differentiable in $\tilde{y}$ when $Z$ is small enough. Suppose there exists $\tilde{y} \in \mathbb{R}$ such that either $a_1^e(\tilde{y}) > a_1^*$ or $a_2^e(\tilde{y}) > a_2^*$. Suppose $a_1^e(\tilde{y}) > a_1^*$, then $c_1'(a_1^e(\tilde{y})) > c_1'(a_1^*)$.

As $f(\tilde{y} - y(a_1^e(\tilde{y}), a_2^e(\tilde{y})) < f(\theta^*)$, we must have

$$\frac{\partial y(a_1^e(\tilde{y}), a_2^e(\tilde{y}))}{\partial a_1} > \frac{\partial y(a_1^*, a_2^*)}{\partial a_1}.$$

Therefore, by concavity and supermodularity of $y$, $a_2^e(\tilde{y}) > a_2^*$. Repeating the previous argument, we have $\frac{\partial y(a_1^e(\tilde{y}), a_2^e(\tilde{y}))}{\partial a_2} > \frac{\partial y(a_1^*, a_2^*)}{\partial a_2}$, which is impossible by strict concavity and supermodularily of $y$. To see why, for an arbitrary pair $(\bar{a}_1, \bar{a}_2) \in (\bar{a}, \infty)^2$, consider the problem

$$\max_{(a_1, a_2) \in [a_1^*, \bar{a}] \times [a_2^*, \bar{a}]} \frac{\partial y(a_1, a_2)}{\partial a_2}$$

(10)
So, a system cannot be satisfied unless

\( \lambda : \frac{\partial y(a_1, a_2)}{\partial a_1} \geq \frac{\partial y(a_1^*, a_2^*)}{\partial a_1} \),

\( \mu_1 : a_1 \geq a_1^* \),

\( \mu_2 : a_2 \geq a_2^* \),

\( \eta_1 : a_1 \leq \bar{a}_1 \),

\( \eta_2 : a_2 \leq \bar{a}_2 \),

where \( \lambda, \mu_1, \mu_2, \eta_1, \eta_2 \geq 0 \) are the Lagrange multipliers for the respective constraints. Denote the optimum of (10) \((a_1^0, a_2^0)\). We would like to show that \((a_1^0, a_2^0) = (a_1^*, a_2^*)\). The first order conditions can be represented as:

\[
\begin{pmatrix}
\frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_1^2} & \frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \\
\frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} & \frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_2^2}
\end{pmatrix}
\begin{pmatrix}
\lambda \\
1
\end{pmatrix}
= 
\begin{pmatrix}
\eta_1 - \mu_1 \\
\eta_2 - \mu_2
\end{pmatrix}
\tag{11}
\]

Suppose \( \frac{\partial y(a_1^0, a_2^0)}{\partial a_1} > \frac{\partial y(a_1^*, a_2^0)}{\partial a_1} \), then \( \lambda = 0 \). Since \( \frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_2^2} < 0 \), we must have \( \mu_2 > 0 \), so \( a_2^0 = a_2^* \). \( \frac{\partial y(a_1^0, a_2^0)}{\partial a_1} > \frac{\partial y(a_1^*, a_2^0)}{\partial a_1} \) implies that \( a_1^0 < a_1^* \), which is impossible. Suppose \( \frac{\partial y(a_1^0, a_2^0)}{\partial a_1} = \frac{\partial y(a_1^0, a_2^0)}{\partial a_1} \) and \( \mu_1 = \mu_2 = 0 \). By strict concavity, the Hessian matrix has full rank and is invertible. Then,

\[
\begin{pmatrix}
\lambda \\
1
\end{pmatrix}
= 
\begin{pmatrix}
\frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_1^2} \eta_1 - \frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \eta_2 \\
\frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \eta_2 - \frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \eta_1
\end{pmatrix}
\]

Since by strict concavity and supermodularity, \( \left( \frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_1^2} \right) \left( \frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_2^2} \right) - \left( \frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \right)^2 \) is positive definite,

\[\frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_1^2} \eta_1 - \frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \eta_2 < 0\]

and \( \frac{\partial^2 y(a_1^0, a_2^0)}{\partial a_2^2} \eta_2 - \frac{\partial y(a_1^0, a_2^0)}{\partial a_1 \partial a_2} \eta_1 < 0\), the system cannot be satisfied. Therefore, the system (11) cannot be satisfied unless \( \frac{\partial y(a_1^0, a_2^0)}{\partial a_1} = \frac{\partial y(a_1^*, a_2^0)}{\partial a_1} \) and there exists \( i \in \{1, 2\} \) such that \( \mu_i > 0 \). But then \((a_1^0, a_2^0) = (a_1^*, a_2^*)\). Therefore, \( \bar{y}^* \) must be the optimal choice for the
voter. Using the same argument, we can show that for all \( i \in \{1, 2\} \), \( a_i^* (\bar{y}^*) = a_i^d > a_i^d \).

**Proof of Proposition 3.** The proof is only a repetition of the arguments in Proposition 2 and is thus omitted.

**Proof of Proposition 4.** For \( Z \) small enough, denote the politician \( i \)'s first order condition at voter’s optimal choice \( \bar{y}^* \) by

\[
G_i (a_i, a_j, Z) = (\alpha_i + r_i f(\theta^*) Z) \frac{\partial y (a_i, a_j)}{\partial a_i} - c_i'(a_i) = 0. \tag{12}
\]

For a given \( Z \), denote \( a_i^c(\bar{y}^*) \) by \( \hat{a}_i^* (Z) \). We have \( \hat{a}_i^* (0) = a_i^d \). Differentiate (12), we have

\[
\frac{\partial G_i (a_i, a_j, Z)}{\partial a_i} = (\alpha_i + r_i f(\theta^*) Z) \frac{\partial y^2 (a_i, a_j)}{\partial a_i^2} - c_i''(a_i), \tag{13}
\]

\[
\frac{\partial G_i (a_i, a_j, Z)}{\partial a_j} = (\alpha_i + r_i f(\theta^*) Z) \frac{\partial^2 y (a_i, a_j)}{\partial a_i \partial a_j}, \tag{14}
\]

\[
\frac{\partial G_i (a_i, a_j, Z)}{\partial Z} = r_i f(\theta^*) \frac{\partial y (a_i, a_j)}{\partial a_i}. \tag{15}
\]

By the implicit function theorem, we have

\[
\begin{bmatrix}
\frac{\partial \hat{a}_1^*}{\partial Z} \\
\frac{\partial \hat{a}_2^*}{\partial Z}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial G_1 (\hat{a}_1^*, \hat{a}_2^*, Z)}{\partial a_1} & \frac{\partial G_1 (\hat{a}_1^*, \hat{a}_2^*, Z)}{\partial a_2} \\
\frac{\partial G_2 (\hat{a}_1^*, \hat{a}_2^*, Z)}{\partial a_1} & \frac{\partial G_2 (\hat{a}_1^*, \hat{a}_2^*, Z)}{\partial a_2}
\end{bmatrix}^{-1} \begin{bmatrix}
- \frac{\partial G_1 (\hat{a}_1^*, \hat{a}_2^*, Z)}{\partial Z} \\
- \frac{\partial G_2 (\hat{a}_1^*, \hat{a}_2^*, Z)}{\partial Z}
\end{bmatrix} \tag{16}
\]

since as \( Z \rightarrow 0 \),

\[
\begin{bmatrix}
\frac{\partial G_1 (a_1, a_2, Z)}{\partial a_1} & \frac{\partial G_1 (a_1, a_2, Z)}{\partial a_2} \\
\frac{\partial G_2 (a_1, a_2, Z)}{\partial a_1} & \frac{\partial G_2 (a_1, a_2, Z)}{\partial a_2}
\end{bmatrix} \rightarrow \begin{bmatrix}
\alpha_1 \frac{\partial^2 y (a_1^d, a_2^d)}{\partial a_1^2} - c_1'' (a_1^d) & \alpha_1 \frac{\partial^2 y (a_1^d, a_2^d)}{\partial a_1 \partial a_2} \\
\alpha_2 \frac{\partial^2 y (a_1^d, a_2^d)}{\partial a_2 \partial a_1} & \alpha_2 \frac{\partial^2 y (a_1^d, a_2^d)}{\partial a_2^2} - c_2'' (a_2^d)
\end{bmatrix}, \tag{17}
\]

which is invertible by strict concavity of the (8). The expected payoff of politician \( i \) can be
approximated by the first order expansion to (2), thus, for $Z$ small enough,

$$
E \left[ u^E_i \left( \hat{a}^*_i(Z), \hat{a}^*_j(Z) \right) \right] - E \left[ u^E_i \left( a^d_i, a^d_j \right) \right] \\
\approx \frac{\partial E \left[ u^E_i \left( a^d_i, a^d_j \right) \right]}{\partial a_i} \frac{d\hat{a}^*_i(0)}{dZ} Z + \frac{\partial E \left[ u^E_i \left( a^d_i, a^d_j \right) \right]}{\partial a_j} \frac{d\hat{a}^*_j(0)}{dZ} Z + \frac{\partial E \left[ u^E_i \left( a^d_i, a^d_j \right) \right]}{\partial Z} \frac{d\hat{a}^*_i(0)}{dZ} Z
$$

Thus, by rearranging terms, for each $i \in \{1, 2\}$, we can find $k^*_i \in \mathbb{R}_{++}$, independent of the distribution $F$, such that

$$
E \left[ u^E_i \left( \hat{a}^*_i(0), \hat{a}^*_j(0) \right) \right] > E \left[ u^E_i \left( a^d_i, a^d_j \right) \right]
$$

for $Z > 0$ small enough if $f(\theta^*) > k^*_i$. Define $K := \max \{k_1, k_2\}$. We are done.

**Proof of Proposition 5.** Using (13)-(17), we find that the first term in (18) is proportional to $f(\theta^*)$ when $Z = 0$. Thus, by rearranging terms, for each $i \in \{1, 2\}$, we can find $k^*_i \in \mathbb{R}_{++}$, independent of the distribution $F$, such that

$$
E \left[ u^E_i \left( \hat{a}^*_i(Z), \hat{a}^*_j(Z) \right) \right] > E \left[ u^E_i \left( a^d_i, a^d_j \right) \right]
$$

for $Z > 0$ small enough if $f(\theta^*) > k^*_i$. Define $K := \max \{k_1, k_2\}$. We are done.

Using (13)-(17), we find that the first term in (18) is proportional to $f(\theta^*)$ when $Z = 0$. Rearranging (18), we have

$$
E \left[ u^E_i \left( \hat{a}^*_i(Z), \hat{a}^*_j(Z) \right) \right] - E \left[ u^E_i \left( a^d_i, a^d_j \right) \right] \\
\approx \left( L \frac{f(\theta^*)}{F(\theta^*)} - \frac{r_i}{r_j} \right) Z
$$

where $L$ is independent of $f(\theta^*)$ and $\frac{r_i}{r_j}$. Therefore, $E \left[ u^E_i \left( \hat{a}^*_i(Z), \hat{a}^*_j(Z) \right) \right] > E \left[ u^E_i \left( a^d_i, a^d_j \right) \right]$ for $Z > 0$ small enough if $f(\theta^*) > k^*_i = \frac{r_i}{L \frac{f(\theta^*)}{F(\theta^*)} - \frac{r_i}{r_j}}$. As the two politicians have the same payoff function in the diarchy game, we know $L_1 = L_2$ by (13)-(17). Therefore, $K := \max \{k_1, k_2\}$ is linearly increasing in $\max \left\{ \frac{r_1}{r_2}, \frac{r_2}{r_1} \right\}$. Given that, it is obvious that $K$ is minimized when $r_1 = r_2 = 1/2$. ■
Proof of Proposition 6. Suppose \( \tilde{\theta} \sim N(\mu, \sigma^2) \), then

\[
\lim_{\theta \to -\infty} R(\theta) = \lim_{\theta \to -\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}} = \lim_{\theta \to -\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{(\theta - \mu)^2}{2\sigma^2}} = -\lim_{\theta \to -\infty} \frac{(\theta - \mu)}{\sigma^2} = +\infty.
\]

Therefore, we can pick \( \tilde{y}^{**} \) sufficiently small so that as \( Z \to 0 \), \( R(\tilde{y}^{**} - y(\tilde{a}^*_i, \tilde{a}^*_j)) > K \). We will use the first order approximation to the politicians’ payoffs to show that this is sufficient to ensure that both politicians would agree to introduce election when \( Z \) is small.

For \( Z \) small enough, denote the politician \( i \)'s first order condition at \( \tilde{y}^{**} \) by

\[
H_i(a_i, a_j, Z) = (a_i + r_if(\tilde{y}^{**} - y(a_i, a_j))) Z \frac{\partial y(a_i, a_j)}{\partial a_i} - c_i'(a_i) = 0. \tag{19}
\]

For a given \( Z \), denote \( a_i^c(\tilde{y}^{**}) \) by \( \tilde{a}_i^*(Z) \). We have \( \tilde{a}_i^*(0) = a_i^d \). Differentiate (12), we have

\[
\frac{\partial H_i(a_i, a_j, Z)}{\partial a_i} = (a_i + r_if(\tilde{y}^{**} - y(a_i, a_j))) Z \frac{\partial y^2(a_i, a_j)}{\partial a_i^2} \frac{\partial y(a_i, a_j)}{\partial a_i} + r_if'(\tilde{y}^{**} - y(a_i, a_j)) Z \left( \frac{\partial y(a_i, a_j)}{\partial a_i} \right)^2 - c_i''(a_i), \tag{20}
\]

\[
\frac{\partial H_i(a_i, a_j, Z)}{\partial a_j} = (a_i + r_if(\tilde{y}^{**} - y(a_i, a_j))) Z \frac{\partial^2 y(a_i, a_j)}{\partial a_i \partial a_j} - r_if'(\tilde{y}^{**} - y(a_i, a_j)) Z \left( \frac{\partial y(a_i, a_j)}{\partial a_i} \right) \left( \frac{\partial y(a_i, a_j)}{\partial a_j} \right), \tag{21}
\]

\[
\frac{\partial H_i(a_i, a_j, Z)}{\partial Z} = r_if(\tilde{y}^{**} - y(a_i, a_j)) \frac{\partial y(a_i, a_j)}{\partial a_i}. \tag{22}
\]

By the implicit function theorem, we have

\[
\begin{pmatrix}
\frac{\partial \tilde{a}_i^*}{\partial Z} \\
\frac{\partial \tilde{a}_j^*}{\partial Z}
\end{pmatrix} = \begin{pmatrix}
\frac{\partial H_1(\tilde{a}_i^*, \tilde{a}_j^*, Z)}{\partial a_1} & \frac{\partial H_1(\tilde{a}_i^*, \tilde{a}_j^*, Z)}{\partial a_2} \\
\frac{\partial H_2(\tilde{a}_i^*, \tilde{a}_j^*, Z)}{\partial a_1} & \frac{\partial H_2(\tilde{a}_i^*, \tilde{a}_j^*, Z)}{\partial a_2}
\end{pmatrix}^{-1} \begin{pmatrix}
\frac{\partial H_1(\tilde{a}_i^*, \tilde{a}_j^*, Z)}{\partial Z} \\
\frac{\partial H_2(\tilde{a}_i^*, \tilde{a}_j^*, Z)}{\partial Z}
\end{pmatrix} \tag{23}
\]

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since as $Z \to 0$,

\[
\begin{pmatrix}
\frac{\partial H_1(a_1,a_2,Z)}{\partial a_1} & \frac{\partial H_2(a_1,a_2,Z)}{\partial a_2} \\
\frac{\partial H_2(a_1,a_2,Z)}{\partial a_1} & \frac{\partial H_3(a_1,a_2,Z)}{\partial a_2}
\end{pmatrix}
\to
\begin{pmatrix}
\alpha_1 \frac{\partial^2 y(a_1^d,a_2^d)}{\partial a_1^2} - c_1''(a_1^d) & \alpha_1 \frac{\partial^2 y(a_1^d,a_2^d)}{\partial a_1 \partial a_2} \\
\alpha_2 \frac{\partial^2 y(a_1^d,a_2^d)}{\partial a_2^2} - c_2''(a_2^d) & \alpha_2 \frac{\partial^2 y(a_1^d,a_2^d)}{\partial a_2 \partial a_1}
\end{pmatrix},
\]

(24)

which is invertible by strict concavity of the (8). The expected payoff of politician $i$ can be approximated by the first order expansion to (2), thus,

\[
E\left[u_i^E\left(\tilde{a}_1^*(Z), \tilde{a}_2^*(Z)\right)\right] - E\left[u_i^E\left(a_1^d, a_2^d\right)\right]
\approx \frac{\partial E\left[u_i^E\left(a_1^d, a_2^d\right)\right]}{\partial a_i} \frac{\partial \hat{a}_i^*(0)}{\partial Z} + \frac{\partial E\left[u_i^E\left(a_1^d, a_2^d\right)\right]}{\partial a_j} \frac{\partial \hat{a}_j^*(0)}{\partial Z} Z
\]

\[
= \left(\alpha_i \frac{\partial y(a_1^d,a_2^d)}{\partial a_i} \frac{\partial \hat{a}_i^*(0)}{\partial Z} - \alpha_j \frac{\partial y(a_1^d,a_2^d)}{\partial a_j} \frac{\partial \hat{a}_j^*(0)}{\partial Z} \right) Z.
\]

(25)

Using (20)-(23), we find that the first term in (25) is proportional to $f\left(\tilde{y}^{**} - y\left(a_i^e(\tilde{y}^{**}), a_j^e(\tilde{y}^{**})\right)\right)$ when $Z = 0$. Thus, by rearranging terms, we conclude that $E\left[u_i^E\left(\tilde{a}_1^*(Z), \tilde{a}_2^*(Z)\right)\right] > E\left[u_i^E\left(a_1^d, a_2^d\right)\right]$ for $Z > 0$ small enough iff $R\left(\tilde{y}^{**} - y\left(a_i^e(\tilde{y}^{**}), a_j^e(\tilde{y}^{**})\right)\right) > k_i^*$, where $k_i^*$ is defined in the proof of Proposition 4. Thus, both politicians would agree to introduce election when $Z$ is small. Finally, since

\[
y\left(\tilde{a}_1^*(Z), \tilde{a}_2^*(Z)\right) - y\left(a_1^d, a_2^d\right) \approx \frac{\partial y(a_1^d,a_2^d)}{\partial a_1^e} \frac{\partial \hat{a}_1^*(0)}{\partial Z} + \frac{\partial y(a_1^d,a_2^d)}{\partial a_2^e} \frac{\partial \hat{a}_2^*(0)}{\partial Z} > 0,
\]

the voter is also better off. \blacksquare