On Decentralizability of Multi-Agency Contracting with Bayesian Implementation*

Yu Chen†

Abstract

This note examines when the centralized mechanism design can be equivalently implemented by the decentralized menu design in generalized multi-agency games with Bayesian implementation. Our delegation principle identifies that Bayesian menu design is strategically equivalent to bilateral Bayesian mechanism design, which simplifies collective Bayesian mechanism design by ignoring relative information evaluation. Since our generalized multi-agency environment permits comprehensive interrelation among the agents and the principal, this delegation principle cannot be viewed as a straightforward aggregation of the delegation principle in single agency. Based on it, we take advantage of interim-payoff-equivalence to further provide conditions on the primitives for the overall equivalence between collective mechanism design, bilateral mechanism design, and menu design.

Keywords: multi-agency, Bayesian Nash equilibrium, mechanism design, menu design, interim-payoff-equivalence

JEL Classification: C72 D82 D86

---

*This paper is a revision of Chapter II of my Ph.D. dissertation. I would like to thank Frank Page and Robert Becker for advice, encouragement and comments. For additional helpful remarks, suggestion and comments, thanks are also due to Alessandro Pavan, Seungjin Han, Johannes Horner, Haomiao Yu, Yongchao Zhang, Rongzhu Ke, Xiang Sun, Geoffrey Woglom, Jun Zhang, Nian Yang, Jie Zheng, David Martimort, and the participants/audiences at University of Queensland, APET annual meeting 2014 and SAET annual conference 2014. However I am solely responsible for any errors.

†School of Economics, Nanjing University, 16 Jinyin Street, Nanjing, Jiangsu, China 210009. Email: yourchenyu@gmail.com.
1 Introduction

This note investigates when the centralized mechanism design can be equivalently implemented by the decentralized menu design in generalized multi-agency contracting games with Bayesian implementation. In such a game, a single principal (female) contracts with multiple agents (male). She needs to offer to the agents a contracting mechanism or menu, which defines a non-cooperative subgame for all agents to play simultaneously with Bayesian Nash equilibrium as its solution concept. Decentralizability of contracting suggests the feasibility of delegating to the agents the decision rights to specify contracts and that of simplifying information communication for specifying the contracts. Thus, decentralized contracting is a more practical procedure with a simple form. Decentralization of contracting in single-agency situations has attracted attention from a number of authors\(^1\). However, there are few comprehensive studies in this context under generalized multi-agency situations.

The multi-agency environment in our consideration has great generality. Our setting permits "full-blown" interdependence over the agents, including correlated types, cross constraints on the feasible contracts over the agents, contract externalities, and interdependent valuations. The message sets of different agents can be different or heterogeneous. The principal’s (expected) payoff is allowed to depend jointly on all the agents’ types and specified contracts. Hence, the impacts of the agents’ respective asymmetric information will be comprehensively interrelated. Moreover, our model is described in a general mathematical setting that is not necessarily restricted to finite or vector space structures.

Our first finding is the delegation principle for Bayesian implementation, which identifies that Bayesian menu design is only strategically equivalent to bilateral Bayesian incentive compatible (BIC)\(^2\) mechanism design, which simplifies the classic centralized collective BIC mechanism design by ignoring relative information evaluation. Bilateral (respectively, collective) mechanisms associate each contract for an individual agent with his individual report (respectively, with the joint reports of all agents). Since our multi-agency environment permits comprehensive interrelation among the agents and the principal, this delegation principle cannot be viewed as a straightforward aggregation of the delegation principle in single agency. It also provides a bridge to for the overall equivalence that collective BIC mechanism design can make the principal as well off as bilateral BIC mechanism design and Bayesian menu design. Based on it, our second finding is the sufficient conditions on the primitives for the overall equivalence in the quasi-separable environment via interim payoff equivalence.\(^3\) We additionally discuss the situation of cross constraints on feasible contracts over agents and that of participation constraints.

Our results generalize Han’s menu theorem (2006) in a "bilateral" environment with private

---

\(^1\)See Peters (2001), Martimort and Stole (2002), and Page and Monteiro (2003) among many others.

\(^2\)Due to the well-known revelation principle, we can restrict attention to BIC mechanisms out of general mechanisms.

\(^3\)Interim payoff equivalence is an important concept and analytical tool in the Bayesian implementation literature. Recently, Gershkov et al. (2013) take advantage of it to examine the equivalence of Bayesian and dominant strategy implementation.
valuations and without cross constraints on feasible contracts over agents. Han also defines all "bilateral" mechanisms on a single uniform message set across the agents. Moreover, Dequiedt and Martimort (2014) study bilateral mechanism design in a vertical contracting environment. They basically follow Han’s setting and claim the rationale of a priori using "bilateral" mechanisms in some real-life situations. Nonetheless, their analysis is not aimed at the discussion of the decentralizability of contracting. While Chen (2012) studies decentralization versus centralization under ex post implementation, this note focuses on Bayesian implementation, since finer informational structure concerning Bayesian updated beliefs is still sensible in the era of big data and provides more leeway for decentralizability via interim payoff equivalence.

2 Primitives

We consider a pure strategy multi-agency contracting game with one principal (short for PL) and \( n \) agents indexed by \( i \in \mathcal{N} = \{1, \cdots, n\} \). Throughout this paper, the symbols \( \mathcal{B}(X) \) is reserved for Borel \( \sigma \)-algebra of a certain space \( X \).

Agent \( i \) (short for \( A_i \)) has some private payoff type \( \theta_i \in \Theta_i \), where \( \Theta_i \) is a Borel space. We write \( \theta = (\theta_i)_{i \in \mathcal{N}} \in \Theta = \prod_{i=1}^{n} \Theta_i \) and \( \theta_{-j} = (\theta_i)_{i \in \mathcal{N}\backslash\{j\}} \in \Theta_{-j} = \prod_{i \neq j}^{n} \Theta_i \). Let \( \mu_i \) be a probability measure defined on \( \mathcal{B}(\Theta_i) \) and \( \mu \) be a probability measure on the associated product Borel \( \sigma \)-algebra \( \mathcal{B}(\Theta) \). \( \mu \) denotes the common prior over the agents’ types. Let \( \mu_{-i}(\cdot|\theta_i) \) be a conditional probability measure on \( \theta_i \) over \( \mathcal{B}(\Theta_{-i}) \). It denotes \( A_i \)’s interim belief about the other agents’ types after learning her own type \( \theta_i \). For each \( i \) and each closed subset \( A \) of \( \Theta_{-i} \), \( \mu_{-i}(A|\cdot) \) is continuous on \( \Theta_i \).

The contract\(^7\) available to \( A_i \) is \( k_i \in \mathcal{K}_i \). \( \mathcal{K}_i \) contains an element \( k_0 \) which denotes "no contracting." The feasible joint contract is \( k = (k_i)_{i \in \mathcal{N}} \in \mathcal{K} \subseteq \prod_{i=1}^{n} \mathcal{K}_i \). Write \( k_{-i} = (k_j)_{j \in \mathcal{N}\backslash\{i\}} \).

The set \( \mathcal{K} \) is assumed to be a compact metric space, since it may contain some realistic constraints on the contract profiles PL can offer to the agents. Chen (2012) provides considerable examples fitting these assumptions on (joint) contract sets in multi-agency situations, including finite contract sets, product-price pairs with budget constraint, contract sets for a single object (auction), and outcome (state)-contingent contract sets. Two typical examples are shown below.

**Example 1** Finite contract sets: There are only finitely many contracts in each \( \mathcal{K}_i \). \( \mathcal{K} \) is a compact metric space as any nonempty subset of \( \prod_{i=1}^{n} \mathcal{K}_i \).

**Example 2** Contract sets for single-object auction: Each bidder \( i \) is offered a pair \((x_i, p_i)\). \( x_i \) is \( i \)’s payment to the seller. \( p_i \) is the probability that \( i \) gets the object. The feasible joint contract

---

\(^4\)Major primitives and definitions of this note basically follow Chen’s work (2012).

\(^5\)Borel space is a Borel subset of a complete separable metric space.

\(^6\)One example for this assumption is that \( \mu_{-i} \) has conditional density \( f(\theta_{-i}|\cdot) \) which is continuous over \( \theta_i \).

\(^7\)Some authors may also call it outcome, alternative, or allocation.
set is
\[ \mathcal{K} = \{(x_1, \cdots, x_n, p_1, \cdots, p_n) \in \mathbb{R}^n \times \mathbb{R}^n : \sum_i p_i \leq 1, p_i \geq 0, 0 \leq x_i \leq I_i, \text{ for each } i\}, \]
where \( I_i > 0 \) is the wealth of bidder \( i \). Obviously, \( \mathcal{K} \) is a compact metric space.

Let \( v_i : \mathcal{K} \times \Theta \to \mathbb{R} \) denote \( A_i \)'s ex post payoff function defined over contract profiles and type profiles. Let \( u : \mathcal{K} \times \Theta \to \mathbb{R} \) denote PL's ex post payoff function over contract and type profiles. \( v_i \) is continuous on \( \mathcal{K} \times \Theta \), and \( u \) is continuous on \( \mathcal{K} \) and Borel-measurable on \( \Theta \). Moreover, \( u \) is \( \mu \)-integrably bounded, i.e. there exists a \( \mu \)-integrable function \( U : \Theta \to \mathbb{R} \) such that for almost every \( \theta \) with respect to \( \mu \), \( |u(k, \theta)| \leq U \) for all \( k \in \mathcal{K} \).

3 Mechanism Design and Menu Design

3.1 Contracting Games over Bayesian Mechanisms

The classic contracting procedure is the centralized mechanism design. Accordingly, the principal-agent contracting game over mechanisms unfolds as follows. At stage 1, PL proposes to the agents a mechanism, which is commonly observable. At stage 2, the agents unilaterally learn their own true types and simultaneously send reports to PL. At stage 3, through the pre-offered mechanism, PL assigns contracts to the agents according to their reports. At stage 4, after the agents' participation, the contracts are simultaneously executed.

The revelation principle helps restrict our attention to Bayesian incentive compatible direct mechanisms. Due to legal customs, technological restriction, or other feasible constraints, two classes of mechanisms may be available a priori to PL.

**Definition 1** A collective (respectively, bilateral) mechanism \( k = (k_i : \Theta \to \mathcal{K}_i)_{i \in \mathcal{N}} \) satisfying \((k_1(\theta), \cdots, k_n(\theta)) \in \mathcal{K} \) for each \( \theta \in \Theta \) (respectively, \( \overline{k} = (\overline{k}_i : \Theta \to \overline{\mathcal{K}}_i)_{i \in \mathcal{N}} \) satisfying \((\overline{k}_1(\theta_1), \cdots, \overline{k}_n(\theta_n)) \in \overline{\mathcal{K}} \) for each \( \theta \in \Theta \)), where \( k_i \) (respectively, \( \overline{k}_i \)) specifies a contract to \( A_i \) for each type report profile of all agents (respectively, of single \( A_i \)).

Let \( \mathcal{F}(\Theta, \mathcal{K}) \) and \( \overline{\mathcal{F}}(\Theta, \overline{\mathcal{K}}) \) respectively denote the collection of collective mechanisms and that of bilateral mechanisms. Collective mechanisms embody relative information (type reports) evaluation in nature, whereas bilateral mechanisms ignore it and merely reflect absolute information evaluation.

Each mechanism offered by PL induces a simultaneous-moving subgame for the agents in which Bayesian Nash equilibrium (BNE) is taken as the solution concept.

**Definition 2** A collective (respectively, bilateral) mechanism \( k \) is **Bayesian incentive compatible (BIC)** if it induces truthful reporting as the BNE for all the agents, i.e., for all \( i \in \mathcal{N} \),

---

*We allow not all the agents eventually participate.*
and all $\theta_i \in \Theta_i$,

$$\int_{\Theta_{-i}} v_i(k(\theta), \theta) \mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} v_i(k'(\theta_{-i}), \theta) \mu_{-i}(d\theta_{-i}|\theta_i),$$

for all $\theta'_i \in \Theta_i$ (respectively,

$$\int_{\Theta_{-i}} v_i(k(\theta), \theta) \mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} v_i(k_i(\theta_i), \theta_i) \mu_{-i}(d\theta_{-i}|\theta_i),$$

for all $\theta'_i \in \Theta_i$).

Thus, two corresponding PL’s optimization problems address contracting games over Bayesian mechanisms as follows.

(P1) collective BIC mechanism design problem:

$$\max_{k \in \mathcal{F}(\Theta, \mathcal{K})} \int_{\Theta} u(k(\theta), \theta) \mu(d\theta)$$

s.t. $k$ is BIC.

(P1’) bilateral BIC mechanism design problem:

$$\max_{k \in \mathcal{F}(\Theta, \mathcal{K})} \int_{\Theta} u(k(\theta), \theta) \mu(d\theta)$$

s.t. $k$ is BIC.

3.2 Contracting Games over Bayesian Menus

The decentralized contracting procedure is menu design, in which PL can design a joint menu, i.e., a subset of the feasible joint contract set, for the agents and allow them to pick the contracts from the joint menu on their own accord. Their choices must be made within the given menu and automatically satisfy the feasible constraint over the agents’ contract profiles.

Accordingly, the principal-agent contracting game over mechanisms unfolds as follows. At stage 1, PL proposes to the agents a joint contract menu, which is commonly observable. At stage 2, the agents unilaterally learn their own true types and simultaneously select the contracts from the pre-offered joint menu. At stage 3, after the agents’ participation, the contracts are simultaneously executed.

The possible contract menu for $A_i$ is a subset $C_i$ of $\mathcal{K}_i$. The joint feasible menu is

$$C \in P_f(\mathcal{K}) \subseteq \{(C_1, \cdots, C_n) | (C_1, \cdots, C_n) \subseteq \mathcal{K}\},$$

where $P_f(\mathcal{K})$ is a collection of nonempty, closed subsets of $\mathcal{K}$. Each $A_i$’s strategy is a function
\( \bar{k}_i : \Theta_i \rightarrow \mathcal{K}_i. \bar{k}_i \in \mathcal{F}_i \) denotes \( A_i \)'s contract selection according to his type. Let

\[
\bar{k} = (\bar{k}_i)_{i \in N}, \bar{k}(\theta) = (\bar{k}_i(\theta_i))_{i \in N}, \bar{k}_{-i}(\theta_{-i}) = (\bar{k}_j(\theta_j))_{j \in N \setminus \{i\}}.
\]

Under a joint menu \( C, \bar{k}(\theta) \in C \) for each \( \theta \in \Theta \). Let

\[
\bar{k} \in \mathcal{F}_c = \{ \bar{k} | \bar{k}(\theta) \in C \text{ for each } \theta \in \Theta \}.
\]

Each menu offered by PL induces a simultaneous-moved "generalized" subgame for the agents with BNE as the solution concept.

**Definition 3** A contract selection profile \( \bar{k} \) is a **BNE** under a joint menu \( C \) if for each \( i \in N, \theta_i \in \Theta_i \),

\[
\int_{\Theta_{-i}} v_i(\bar{k}(\theta), \theta) \mu_{-i}(d\theta_{-i}|\theta_i) \geq \int_{\Theta_{-i}} v_i(\bar{k}'(\theta_i), \bar{k}_{-i}(\theta_{-i}), \theta) \mu_{-i}(d\theta_{-i}|\theta_i),
\]

for all \( \bar{k}' \in \mathcal{F}_i \) satisfying \( (\bar{k}'(\theta_i), \bar{k}_{-i}(\theta_{-i})) \in C \) for some \( \theta_{-i} \in \Theta_{-i} \). Such a joint menu \( C \) is called a **Bayesian (joint) menu**.

PL can deduce that the agents will have the BNE contract selection profile in the subgame defined by a Bayesian menu and hence has an optimization problem to address this contracting game.

**(P2)** Bayesian menu design problem:

\[
\max_{C \in \mathcal{P}_f(\mathcal{K})} \int_{\Theta_{-i}} \max_{\bar{k} \in \mathcal{F}_c} u(\bar{k}(\theta), \theta) \mu(d\theta) \quad \text{s.t. } \bar{k} \text{ is the BNE under } C.
\]

In view of tie-breaking, PL may designate or recommend \( \bar{k} \) in her best interest for the agents with type profile \( \theta \) to follow.

### 3.3 Delegation Principle for Bayesian Implementation

When we examine the decentralizability of Bayesian Implementation in the generalized multi-agency environment, the first important observation is a complete characterization of all bilateral BIC mechanisms via Bayesian menus.

First consider the correspondence \( \Psi : \Theta \times \mathcal{P}_f(\mathcal{K}) \rightarrow \mathcal{K} \) defined by

\[
\Psi(\theta, C) = \{ \bar{k}(\theta) \in C : \bar{k} \text{ is the BNE under } C \}.
\]

It is deducible by PL and represents the \( \theta \)-type-profile agents’ joint BNE response to any menu offer \( C \).

**Definition 4** Given \( C \in \mathcal{P}_f(\mathcal{K}) \), \( \Psi(\cdot, C) \) is **well-defined** if \( \Psi(\theta, C) \) is nonempty for each \( \theta \).
Lemma 1 For any $C \in P_I(K)$ satisfying $\Psi(\cdot, C)$ is well-defined, $\Psi(\cdot, C)$ is a compact-valued Borel-measurable correspondence from $\Theta$ to $C$.

**Proof.** First show that the graph of $\Psi(\cdot, C)$ is closed. Fix $\theta$ and pick any arbitrary sequence $\{(\theta^l, \bar{k}(\theta^l))\}_l$ in the graph satisfying

$$\bar{k}(\theta^l) \in \Psi(\theta^l, C)$$

and $$(\theta^l, \bar{k}(\theta^l)) \to (\theta, \bar{k}(\theta))$$ as $l \to \infty$.

Thus it suffices to show that $\bar{k}(\theta) \in \Psi(\theta, C)$, that is, for each $i \in N$,

$$\int_{\Theta \setminus i} v_i(\bar{k}(\theta), \theta) \mu_{\cdot \setminus i}(d\theta_{\cdot \setminus i}|\theta_i) \geq \int_{\Theta \setminus i} v_i(\bar{k}_i(\theta_i), \bar{k}_{-i}(\theta_{-i}), \theta) \mu_{\cdot \setminus i}(d\theta_{\cdot \setminus i}|\theta_i),$$

for all $\bar{k}_i \in F_i$ satisfying $$(\bar{k}_i(\theta_i), \bar{k}_{-i}(\theta_{-i})) \in C$$ for some $\theta_{-i} \in \Theta_{-i}$. Thus,

$$\int_{\Theta \setminus i} v_i(\bar{k}(\theta^l), \theta^l) \mu_{\cdot \setminus i}(d\theta_{\cdot \setminus i}|\theta_i^l) \geq \int_{\Theta \setminus i} v_i(\bar{k}_i(\theta_i^l), \bar{k}_{-i}(\theta_{-i}), \theta^l) \mu_{\cdot \setminus i}(d\theta_{\cdot \setminus i}|\theta_i^l).$$

Since $v_i$ is joint continuous and is bounded in $\Theta$, Delbaen’s Lemma (1974)

$$\int_{\Theta \setminus i} v_i(\bar{k}(\theta), \theta) \mu_{\cdot \setminus i}(d\theta_{\cdot \setminus i}|\theta_i) \geq \int_{\Theta \setminus i} v_i(\bar{k}_i(\theta_i), \bar{k}_{-i}(\theta_{-i}), \theta) \mu_{\cdot \setminus i}(d\theta_{\cdot \setminus i}|\theta_i).$$

Therefore, the graph is closed in $\Theta \times C \times A$, i.e., $\Psi(\cdot, C)$ is closed-valued. Moreover, $\Psi(\cdot, C)$ is compact-valued as $K$ and $A$ are both compact metric spaces. Since $\Theta$ and $C$ are both Borel spaces, $\Psi(\cdot, C)$ is Borel-measurable by Theorem 3 in Himmelberg, Parthasarathy and Van Vleck (1976). □

Proposition 1 Given a contracting mechanism $\bar{k} \in \bar{F}(\Theta, K)$, the following statements are equivalent:

(i) $\bar{k}$ is BIC.

(ii) There exists a joint menu $C \in P_I(K)$ such that $\Psi(\cdot, C)$ is well-defined, and $\bar{k}$ is a Borel-measurable selection from $\Psi(\cdot, C)$, that is, $\bar{k}(\theta) \in \Psi(\theta, C)$ for all $\theta \in \Theta$.

**Proof.** (i)⇒(ii). Assume that $\bar{k} \in \bar{F}(\Theta, K)$ is BIC. Define

$$C = \prod_{i=1}^n \text{cl}\{\bar{k}_i(\theta_i) : \theta_i \in \Theta_i\} \cap K$$

where $\text{cl}$ denotes the closure.

First claim that $\bar{k}(\theta) \in \Psi(\theta, C)$ for all $\theta \in \Theta$. Suppose not. Then for some agent $j$, some
Moreover, we define the equivalent mechanism set induced by a joint menu for all $i \in \Theta$ such that
\[
\int_{\Theta - i} v_i(\overline{k}(\theta), \theta \mu_{-i}(d\theta | \theta_i)) \geq \int_{\Theta - i} v_i(\overline{k}_i(\theta_i), \overline{k}_i(\theta - i), \theta \mu_{-i}(d\theta | \theta_i)).
\]
Since there is some $\overline{k}_i$ satisfying $\overline{k}_i(\theta_i) = \overline{k}_i(\theta_i')$ for any $\theta_i' \in \Theta$, we have
\[
\int_{\Theta - i} v_i(\overline{k}(\theta), \theta \mu_{-i}(d\theta | \theta_i)) \geq \int_{\Theta - i} v_i(\overline{k}_i(\theta_i'), \overline{k}_i(\theta - i), \theta \mu_{-i}(d\theta | \theta_i)),
\]
for all $\theta_i'$. Thus, $\overline{k}$ is BIC. □

**Remark 1** The feasible bilateral BIC mechanism set is defined as
\[
\mathcal{IC} = \{ \overline{k} \in \mathcal{F}(\Theta, \mathcal{K}) : \overline{k} \text{ is BIC} \}
\]
(P1′) and (P2) can respectively be stated compactly as
\[
\max_{\overline{k} \in \mathcal{IC}} \int_{\Theta} u(\overline{k}(\theta), \theta \mu(d\theta)),
\]
and
\[
\max_{\overline{k} \in \mathcal{IC}} \int_{\Theta} u(\overline{k}(\theta), \theta \mu(d\theta)).
\]
Moreover, we define the equivalent mechanism set induced by a joint menu $C$
\[
\Sigma(C) = \{ \overline{k} \in \mathcal{F}(\Theta, \mathcal{K}) : \overline{k}(\theta) \in \Psi(\theta, C) \text{ for all } \theta \in \Theta \},
\]
and the overall equivalent mechanism set induced by the joint menu set

$$\Sigma_\Psi = \bigcup_{C \in P_f(K)} \Sigma_\Psi(C).$$

Thus, Proposition 1 is equivalent to say $\mathcal{IC}^I = \Sigma_\Psi$.

**Lemma 2** For each $C \in P_f(K)$ satisfying $\Psi(\cdot, C)$ is well-defined, there exists some $\bar{k} \in \Sigma_\Psi(C)$ such that

$$u(\bar{k}(\theta), \theta) = \max_{k(\theta) \in \Psi(\theta, C)} u(k(\theta), \theta),$$

for all $\theta \in \Theta$. Moreover, the function $\theta \mapsto \max_{k \in \Psi(\theta, C)} u(k, \theta)$ is Borel measurable.

**Proof.** Note that $\Theta$ and $K$ are Borel space. By Lemma 1, for each $C \in P_f(K)$ satisfying $\Psi(\cdot, C)$ is well-defined, $\Psi(\cdot, C)$ is Borel-measurable and compact-valued. We know $u$ is Borel-measurable and $u(\cdot, \theta)$ is continuous. Then by Theorem 2 in Himmelberg, Parthasarathy and Van Vleck (1976), there exists some Borel measurable selector $\bar{k}$ in $\Sigma_\Psi(C)$ for the set-valued function $\Psi(\theta, C)$ such that $u(\bar{k}(\theta), \theta) = \max_{k(\theta) \in \Psi(\theta, C)} u(k(\theta), \theta)$ for all $\theta \in \Theta$. Moreover, the function $\theta \mapsto \max_{k(\theta) \in \Psi(\theta, C)} u(k(\theta), \theta)$ is also Borel measurable. $\Box$

We now establish the delegation principle for Bayesian implementation. It shows that Bayesian menu design is strategically equivalent to bilateral Bayesian mechanism design even in a quite general multi-agency situation with comprehensive interrelations.

**Proposition 2** (Delegation Principle for Bayesian Implementation).

(i) If $\bar{K}^*$ solves the contracting problem over bilateral BIC mechanisms given by $(P1')$, then $C^* = \prod_{i=1}^{n} \text{cl}\{(\bar{K}^*_i(\theta_i) : \theta_i \in \Theta_i) \cap K \} \cap K$ solves the contracting problem over Bayesian menus given by $(P2)$.

(ii) If $C^*$ solves $(P2)$, then $\bar{K}^*$ satisfying $\bar{K}^* (\theta) \in \arg\max_{\bar{k}(\theta) \in \Psi(\theta, C^*)} u(\bar{k}(\theta), \theta)$ for each $\theta \in \Theta$ solves $(P1')$.

Moreover, the optimal objective values of the two problems are equal.

**Proof.** (i). By the proof of Proposition 1, $\bar{K}^*(\theta) \in \Psi(\theta, C^*)$, and

$$\int_{\Theta} \max_{\bar{k}(\theta) \in \Psi(\theta, C^*)} u(\bar{k}(\theta), \theta) \mu(d\theta) = \int_{\Theta} u(\bar{K}^*(\theta), \theta) \mu(d\theta).$$

Thus, for all $\bar{k} \in \mathcal{IC}^I$,

$$\int_{\Theta} u(\bar{k}(\theta), \theta) \mu(d\theta) \geq \int_{\Theta} u(\bar{K}^*(\theta), \theta) \mu(d\theta).$$
Then, by Proposition 1, \( IC' = \Sigma_{\Psi} \). Hence, for all \( \mathbf{k} \in \bigcup_{C \in P_f(K)} \Sigma_{\Psi}(C) \),

\[ \int_{\Theta} u(\mathbf{k}^*(\theta), \theta) \mu(d\theta) \geq \int_{\Theta} u(\mathbf{k}(\theta), \theta) \mu(d\theta). \]  

(2)

Moreover, by Lemma 2, for each \( C \in P_f(K) \), there exists some \( \mathbf{k}' \in \Sigma_{\Psi}(C) \) such that \( u(\mathbf{k}'(\theta), \theta) = \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \) for all \( \theta \). Thus, by (2), for each \( C \),

\[ \int_{\Theta} u(\mathbf{k}'(\theta), \theta) \mu(d\theta) \geq \int_{\Theta} \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \mu(d\theta). \]

Therefore,

\[ \int_{\Theta} \max_{\tilde{k}(\theta) \in \Psi(\theta, C^*)} u(k(\theta), \theta) \mu(d\theta) \geq \int_{\Theta} \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \mu(d\theta) \]

for all \( C \). Hence, \( C^* \) solves the given contracting game over menus. Clearly,

\[ \max_{C \in P_f(K)} \int_{\Theta} \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(k(\theta), \theta) \mu(d\theta) = \int_{\Theta} u(\mathbf{k}^*(\theta), \theta) \mu(d\theta) \]

\[ = \max_{\mathbf{k} \in IC'} \int_{\Theta} u(\mathbf{k}(\theta), \theta) \mu(d\theta). \]

(ii). By Lemma 2, for each \( C \), there exists some \( \mathbf{k} \) such that

\[ u(\mathbf{k}(\theta), \theta) = \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \]

for any \( \theta \). Now consider \( \mathbf{k}^* \in \Sigma_{\Psi}(C^*) \) satisfying \( \mathbf{k}^*(\theta) \in \arg\max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \) for all \( \theta \). For each \( C \), by hypotheses,

\[ \int_{\Theta} u(\mathbf{k}^*(\theta), \theta) \mu(d\theta) = \int_{\Theta} \max_{\tilde{k}(\theta) \in \Psi(\theta, C^*)} u(\tilde{k}(\theta), \theta) \mu(d\theta) \]

\[ \geq \int_{\Theta} \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \mu(d\theta) = \int_{\Theta} u(\mathbf{k}(\theta), \theta) \mu(d\theta), \]

for all \( \mathbf{k} \in \Sigma_{\Psi}(C) \) satisfying \( u(\mathbf{k}(\theta), \theta) = \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \) for any \( \theta \). Hence,

\[ \int_{\Theta} u(\mathbf{k}^*(\theta), \theta) \mu(d\theta) = \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} \int_{\Theta} u(\tilde{k}(\theta), \theta) \mu(d\theta). \]  

(3)

We know

\[ IC' = \Sigma_{\Psi} = \bigcup_{C \in P_f(K)} \Sigma_{\Psi}(C). \]  

(4)
Hence, by (3) and (4),

\[
\max_{C \in P_f(K)} \int \max_{\tilde{k}(\theta) \in \Psi(\theta, C)} u(\tilde{k}(\theta), \theta) \mu(d\theta) = \int \max_{\tilde{k}(\theta) \in \Psi(\theta, C^*)} u(\tilde{k}(\theta), \theta) \mu(d\theta) = \int \max_{\tilde{k}(\theta) \in \Psi(\theta, C^*)} u(\tilde{k}(\theta), \theta) \mu(d\theta) = \max_{\tilde{k}(\theta) \in \Psi(\theta, C^*)} \int u(\tilde{k}(\theta), \theta) \mu(d\theta).
\]

Therefore, \( \tilde{k}^* \) solves the given contracting game over mechanisms. \( \square \)

4 Availability of Decentralization in Bayesian Implementation

An immediate implication of our delegation principle is that contracting is decentralizable in the realistic environments when the centralized mechanisms are ad hoc restricted to be bilateral. But when will a PL have incentive to rely on decentralization (or bilateral centralization) if the centralized mechanisms are allowed to be collective? Another implication of our delegation principle is that we can transfer this question to when the optimal bilateral BIC mechanism (and equivalently optimal Bayesian menu) brings to PL the same objective value (expected payoff) as the optimal collective BIC mechanism does.

It is more desirable to find certain economically intuitive conditions on the primitives for this overall equivalence. A key idea is that such simplification of the contracting procedure tends to be offset by finer information structure (common knowledge) of the primitives of the contracting environment. We can adopt this idea to identify a class of economically intuitive conditions in which the overall equivalence holds by introducing interim payoff equivalence in Bayesian implementation.

4.1 Interim-Payoff-Equivalent Mechanisms

First we invoke the quasi-separable environment as an extension of the separable environment introduced in Chung and Ely (2006), which is useful in many economic applications and previous studies. In the quasi-separable environment, interdependent valuations and correlated types are permitted. Each agent can separate his direct utility from his own contract and type and valuation from all agents’ types in a linear form of his payoff.

**Definition 5** A contracting game is played in a quasi-separable environment if (1) \( K_i = \prod_{j=1}^{m_i} K_{ij} \) for each \( i \in \mathcal{N} \) and some \( m_i \in \mathbb{Z}^+ \). Its element is \( k_i = \prod_{j=1}^{m_i} k_{ij} \). Each \( K_{ij} \) is compact and connected, and (2) for each \( i \in \mathcal{N}, j \in \{1, \cdots, m_i\}, \) \( v_i(k, \theta) \equiv v_i(k_i, \theta) = \sum_{j=1}^{m_i} h_{ij}(k_{ij}, \theta_i)w_{ij}(\theta) + \)
\( q_i(\theta) \) for some continuous functions \( h_{ij} : \mathcal{K}_{ij} \times \Theta_i \rightarrow \mathbb{R}, \ w_{ij} : \Theta \rightarrow \mathbb{R} \) satisfying either \( w_{ij} \) is non-negative or \( w_{ij} \) is non-positive, and \( q_i : \Theta \rightarrow \mathbb{R} \) satisfying \( q_i(\theta_i, \cdot) \) are integrable with respect to \( \mu_{-i} \) for each \( \theta_i \). We call \( w_{ij} \) the multiplicative valuation of \( k_{ij} \) and \( q_i \) the additive valuation.

One can actually identify a bilateral mechanism interim-payoff-equivalent (with respect to the agents) to any collective mechanism through the Bayesian (interim) belief structure in the quasi-separable environment with no cross constraint on the feasible contract profiles. We will discuss the situation allowing cross constraint on the feasible contract profiles in section 5.1.

**Definition 6** Given any collective mechanism \( k \in \mathcal{F}(\Theta, \mathcal{K}) \), a bilateral mechanism \( \tilde{k} \in \mathcal{F}(\Theta, \mathcal{K}) \) is called interim-payoff-equivalent (IPE) to \( k \) (with respect to \( A_i \)) if for any \( \theta_i \in \Theta_i \),

\[
\int_{\Theta_{-i}} v_i(\tilde{k}_i(\theta_i), \theta) \mu_{-i}(d\theta_{-i}|\theta_i) = \int_{\Theta_{-i}} v_i(k_i(\theta), \theta) \mu_{-i}(d\theta_{-i}|\theta_i).
\]

**Proposition 3** In a quasi-separable environment, if \( \mathcal{K} = \prod_{i=1}^n \mathcal{K}_i \), then for any collective (respectively, collective BIC) mechanism \( k \) with its \( i \)-th coordinate \( k_i = \prod_{j=1}^m k_{ij} \), there exists a bilateral (respectively, bilateral BIC) mechanism \( \tilde{k} \) IPE to \( k \) (with respect to all agents).

**Proof.** For each \( i \), given \( k_i, \theta_i \), \( j \), since \( h_{ij}(\cdot, \theta_i) \) is continuous, compactness and connectedness of \( \mathcal{K}_{ij} \) implies that the range of \( h_{ij}(\cdot, \theta_i) \) is also compact and connected in \( \mathbb{R} \) and therefore should be a closed interval \([a_{ij}(\theta_i), b_{ij}(\theta_i)]\), and \( h_{ij}(\cdot, \theta_i) \) is onto from \( \mathcal{K}_{ij} \) to its range. Then define the inverse correspondence of \( h_{ij}(\cdot, \theta_i) \) as \( \rho^{-1}_{ij, \theta_i} : [a_{ij}(\theta_i), b_{ij}(\theta_i)] \rightarrow \mathcal{K}_{ij} \) by

\[
\rho^{-1}_{ij, \theta_i}(x) = \{k_{ij} \in \mathcal{K}_{ij} : h_{ij}(k_{ij}, \theta_i) = x\}.
\]

By the closed map lemma, \( h_{ij}(\cdot, \theta_i) \) is a continuous closed map. Thus, \( \rho^{-1}_{ij, \theta_i} \) is a measurable correspondence. Since \( h_{ij}(\cdot, \theta_i) \) are continuous, \( \rho^{-1}_{ij, \theta_i} \) must be nonempty closed-valued. Kuratowski-Ryll-Nardzewski Selection Theorem implies that \( \rho^{-1}_{ij, \theta_i} \) must admit a Borel-measurable selector, say \( \varphi_{ij, \theta_i} : [a_{ij}(\theta_i), b_{ij}(\theta_i)] \rightarrow \mathcal{K}_{ij} \). Now define \( \phi_{i, k_{ij}} : \Theta_i \rightarrow \mathbb{R} \) by

\[
\phi_{i, k_{ij}}(\theta_i) = \frac{\int_{\Theta_{-i}} h_{ij}(k_{ij}(\theta), \theta_i) w_{ij}(\theta) \mu_{-i}(d\theta_{-i}|\theta_i)}{\int_{\Theta_{-i}} w_{ij}(\theta) \mu_{-i}(d\theta_{-i}|\theta_i)}.
\]

Obviously, \( \phi_{i, k_{ij}} \) is a Borel-measurable function of \( \theta_i \). Moreover, for all \( \theta_i \),

\[
a_{ij}(\theta_i) \leq h_{ij}(k_{ij}(\theta), \theta_i) \leq b_{ij}(\theta_i).
\]
Thus, we can define a function $K_{ij} : \Theta_i \rightarrow C_{ij}$ by

$$K_{ij}(\theta_i) = \varphi_{ij,\theta_i}(\phi_{ij,k_{ij}}(\theta_i)),$$

for each $\theta_i$.

Hence $K_{ij}$ is clearly a Borel-measurable function, and $K$ is a well-defined bilateral mechanism.

Next by the definitions above, for each $\theta_i$,

$$h_{ij}(K_{ij}(\theta), \theta_i) = \frac{\int_{\Theta_{\theta_i}} h_{ij}(\theta, \theta_i) w_{ij}(\theta) \mu_{-i} (d\theta_{-i} | \theta_i)}{\int_{\Theta_{\theta_i}} w_{ij}(\theta) \mu_{-i} (d\theta_{-i} | \theta_i)}.$$

Thus,

$$\int_{\Theta_{\theta_i}} h_{ij}(k_{ij}(\theta), \theta_i) w_{ij}(\theta) \mu_{-i} (d\theta_{-i} | \theta_i) = h_{ij}(K_{ij}(\theta), \theta_i) \int_{\Theta_{\theta_i}} w_{ij}(\theta) \mu_{-i} (d\theta_{-i} | \theta_i)$$

$$= \int_{\Theta_{\theta_i}} h_{ij}(K_{ij}(\theta), \theta_i) w_{ij}(\theta) \mu_{-i} (d\theta_{-i} | \theta_i).$$

In sum, $K_i$ is IPE to $k_i$. Moreover, since $K$ is IPE to $k$, and $k$ is BIC, $K$ is clearly BIC too.

**Remark 2** Quasi-separable environment is of significance for the interim payoff equivalence. First, if contract externalities are permitted, it is difficult for each $A_i$ to form a $K_j$ ($j \neq i$) coupled with $K_i$ such that $K$ IPE to $k$. Non-contract externalities raise the degree of freedom to find IPE bilateral mechanisms.

Moreover, if quasi-separable forms of the agents’ payoff functions or compactness and connectedness of the contract sets are violated, interim payoff equivalence may also fail. Consider a simple finite case as follows. $N = \{1,2\}$. $K_1 = \{0,1\}$. $\Theta_1$ is a singleton, so it is negligible. $\Theta_2 = \{L,H\}$. $\theta_2$ are equally distributed. Let $v_1(0,L) = v_1(1,H) = 1$, and $v_1(1,L) = v_1(0,H) = 0$. Then consider $k_1$ such that $k_1(L) = 0$ and $k_1(H) = 1$. $\int_{\Theta_2} v_1(k_1(\theta), \theta) \mu_2 (d\theta_2) = \frac{1}{2}(v_1(k_1(L),L) + v_1(k_1(H),H)) = 1$. But it is unlikely to find a (constant) bilateral $K_1(\theta_1) \in K_1$ such that $\int_{\Theta_2} v_1(k_1(\theta_1), \theta) \mu_2 (d\theta_2) = 1$.

### 4.2 Overall Equivalence Results

Proposition 3 can usher in a few results on the overall equivalence between collective and bilateral BIC mechanism designs and therefore Bayesian menu design thanks to our delegation principle.

The key for the overall equivalence is to test whether the optimal collective mechanism and its IPE bilateral BIC mechanism can bring the same expected payoff to PL.\(^\dagger\)

For the rest analysis, we assume that for each $i \in N$ the interim belief $\mu_{-i}(\cdot | \cdot)$ is derived from the prior $\mu$, that is, for any $\mu$-integrable functions $\phi : \Theta \rightarrow \mathbb{R}$, $\int_{\Theta} \phi(\theta) \mu (d\theta) = \int_{\Theta} \int_{\Theta_{\theta_i}} \phi(\theta) \mu_{-i}(d\theta_{-i} | \theta_i) \mu_i (d\theta_i)$.

\(^\dagger\)Yet our subsequent analysis centers on the conditions on the primitives for the overall equivalence.
4.2.1 Linearly Additive Payoff Relations

Based on Proposition 3, if additionally PL’s payoff exhibits a certain linearly additive separability with each component as a linear transformation of each agent’s payoff (given his own type) in the quasi-separable environment, the overall equivalence can be ensured.

**Corollary 1** In a quasi-separable environment, if

(i) \( K = \prod_{i=1}^{n} K_i \), and

(ii) \( u(k, \theta) = \sum_{i=1}^{n} \left[ \sum_{j=1}^{m_i} (a_{ij}(\theta_i)h_{ij}(k_{ij}, \theta_i)w_{ij}(\theta)) \right] + L(\theta) \) for some continuous functions \( L : \Theta \rightarrow \mathbb{R} \) and \( a_{ij} : \Theta_i \rightarrow \mathbb{R} \) for each \( i \in \mathcal{N} \), \( j \in \{1, \ldots, m_i\} \),

then for any optimal collective mechanism \( k^* \), there exists its IPE bilateral mechanism \( \bar{k}^* \) bringing to PL the same expected payoff.

**Proof.** Given the optimal collective mechanism \( k^* \) solving P1, Proposition 3 implies there must be a BIC bilateral mechanism \( \bar{k}^* \) IPE to \( k^* \). Then

\[
\int_{\Theta} u(k^*(\theta), \theta) \mu(d\theta)
= \int_{\Theta} \left( \sum_{i=1}^{n} \sum_{j=1}^{m_i} (a_{ij}(\theta_i)h_{ij}(k_{ij}^*(\theta_i), \theta_i)w_{ij}(\theta)) \right) \mu(d\theta)
= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta} a_{ij}(\theta_i) \int_{\Theta_i} h_{ij}(k_{ij}^*(\theta_i), \theta_i)w_{ij}(\theta) \mu_i(d\theta_i) \right) + \int_{\Theta} L(\theta) \mu(d\theta)
= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta} a_{ij}(\theta_i) \int_{\Theta_i} h_{ij}(\bar{k}_{ij}^*(\theta), \theta_i)w_{ij}(\theta) \mu_i(d\theta_i) \right) + \int_{\Theta} L(\theta) \mu(d\theta)
= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta} a_{ij}(\theta_i)h_{ij}(\bar{k}_{ij}(\theta), \theta_i)w_{ij}(\theta) \mu(d\theta) \right) + \int_{\Theta} L(\theta) \mu(d\theta)
= \int_{\Theta} u(\bar{k}^*(\theta), \theta) \mu(d\theta).
\]

Thus, \( \bar{k}^* \) brings to PL the same expected (ex ante) payoff as \( k^* \) does. \( \square \)

**Remark 3** It will be difficult to find conditions on the primitives for the equivalence between collective mechanisms and its IPE bilateral mechanisms if we allow non-separable relation between the agents’ payoffs and the principal’s payoff. Because \( k_i(\theta) \) and \( k_{-i}(\theta) \) may simultaneously be integrated out with respect to \( \theta_{-i} \) under \( \mu_{-i} \).

There are a few examples to which Corollary 1 is applicable.
Example 3 (Vertical Contracting 1) A manufacturer (PL) wants to sell his products to n retailers in segmented markets. She just controls the wholesale prices \( p = (p_1, \ldots, p_n) \) as the contract. \( p_i \in [0, \overline{p}_i] \). Retailer \( i \) receives a local baseline demand signal \( \theta_i \). The prediction about actual market demand for \( i \) is \( w_i(\theta) \). Let \( P_i(w_i(\theta)) \) be the continuous (estimated) inverse demand based on market demand \( w_i(\theta) \) for \( i \). Then \( i \)'s profit is \( P_i(w_i(\theta))w_i(\theta) - p_iw_i(\theta) \). It implies interdependent valuations.\(^{11}\) The manufacturer's production cost \( c \in \mathbb{R} \) does not rely on the wholesale prices. Her payoff function can hence be her revenue from wholesale: \( \sum_{i=1}^{n} p_iw_i(\theta) \).

Example 4 (Vertical Contracting 2) Now the contract for retailer \( i \) is \((q_i, t_i)\), where \( q_i \in [0, \overline{q}_i] \) is the quantity that the manufacturer sells to retailer \( i \) and \( t_i \in [0, \overline{t}_i] \) is the payment from retailer \( i \) to the manufacturer. Each retailer is a price taker. Let \( P_i(\theta_i)q_i \) denote \( i \)'s revenue, where the market price \( P_i \) is a positive continuous function of the private signal about local market demand \( \theta_i \). Then \( i \)'s profit is \( P_i(\theta_i)q_i - t_i \). The manufacturer has the production cost as \( b(\sum_{i=1}^{n} q_i) \) for some \( b \in \mathbb{R} \). So her payoff is \( \sum_{i=1}^{n} t_i - b(\sum_{i=1}^{n} q_i) \).

Example 5 (Efficient Allocation) Consider a classic resources allocation context. Contracts consist of the assignments of some divisible resources \( x = \prod_{i=1}^{n} x_i \in \prod_{i=1}^{n} [0, \overline{x}_i] \) and the payments (from the agents to the social planner) \( t = \prod_{i=1}^{n} t_i \in \prod_{i=1}^{n} [0, \overline{t}_i] \). \( i \) has a private evaluation \( \theta_i \) about his assignment. His quasi-linear payoff function

\[
v_i(x, t, \theta) = w_i(\theta)x_i - t_i,
\]

where \( w_i \) denotes the "interdependent" valuation of \( A_i \) about \( x_i \). Then the planner's payoff function is \( \sum_{i=1}^{n} [w_i(\theta)x_i] \).

4.2.2 Non-linearly Additive Payoff Relations

In some applications under the quasi-separable environment, if PL's payoff accordingly exhibits a certain non-linearly additive separability with each component as a concave transformation of each agent's direct utility (given his own type) plus a valuation term, the overall equivalence can be ensured.

Corollary 2 In a quasi-separable environment, if

(i) \( \mathcal{K} = \prod_{i=1}^{n} \mathcal{K}_i \),

(ii) \( w_{ij}(\theta) \equiv w_{ij}(\theta_i) \), for each \( i \in \mathcal{N}, j \in \{1, \ldots, m_i\} \), and

\(^{11}\)This may be a remarkable phenomenon in modern practice. It has recently attracted more attention in the context of vertical contracting or procurement. For instance, Han (2013) in his recent paper addresses such interdependent valuation in the analysis asymmetric first-price menu auctions in the procurement environment.
(iii) $u(k, \theta) \equiv \sum_{i=1}^{n} \sum_{j=1}^{m_i} G_{ij}(h_{ij}(k_{ij}, \theta_i), \theta_i) + L(\theta)$ for some continuous functions $L : \Theta \to \mathbb{R}$ and $G_{ij} : \mathbb{R} \times \Theta \to \mathbb{R}$ for each $i$, $j$ satisfying $G_{ij}(-, \theta_i)$ is a concave transformation for each $\theta_i$, then for any optimal collective mechanism $k^*$, there exists its IPE bilateral mechanism $\overline{k}$ bringing to $PL$ the same expected payoff.

**Proof.** By Proposition 3, there is a bilateral BIC mechanism $\overline{k}$ IPE to $k^*$. Due to hypothesis (ii), we have

$$h_{ij}(\overline{k}_{ij}^*(\theta_i), \theta_i) = \int_{\Theta} h_{ij}(k^*_{ij}(\theta), \theta_i) \mu_{-i}(d\theta|\theta_i)$$

(5)

Hence,

$$\int_{\Theta} u(k^*(\theta), \theta) \mu(d\theta)$$

$$= \int_{\Theta} \left( \sum_{i=1}^{n} \sum_{j=1}^{m_i} (G_{ij}(h_{ij}(k^*_{ij}(\theta), \theta_i), \theta_i) + L(\theta)) \right) \mu(d\theta)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta} G_{ij}(h_{ij}(k^*_{ij}(\theta), \theta_i), \theta_i) \mu_{-i}(d\theta|\theta_i) \right) \mu(d\theta)$$

$$+ \int_{\Theta} L(\theta) \mu(d\theta)$$

$$\leq \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta_i} \left( \int_{\Theta} h_{ij}(k^*_{ij}(\theta), \theta_i) \mu_{-i}(d\theta|\theta_i) \right) \mu_i(d\theta_i) \right) + \int_{\Theta} L(\theta) \mu(d\theta)$$

(By Jensen’s inequality.)

$$= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta_i} G_{ij}(h_{ij}(\overline{k}_{ij}^*(\theta_i), \theta_i), \theta_i) \mu_i(d\theta_i) \right) + \int_{\Theta} L(\theta) \mu(d\theta)$$

(By (5))

$$= \sum_{i=1}^{n} \sum_{j=1}^{m_i} \left( \int_{\Theta_i} \left( \int_{\Theta} h_{ij}(\overline{k}_{ij}^*(\theta_i), \theta_i) \mu_{-i}(d\theta|\theta_i) \right) \mu_i(d\theta_i) \right) + \int_{\Theta} L(\theta) \mu(d\theta)$$

$$= \int_{\Theta} u(\overline{k}^*(\theta), \theta) \mu(d\theta).$$

Since $k^*$ is the optimal solution to (P1), $\int_{\Theta} u(k^*(\theta), \theta) \mu(d\theta) = \int_{\Theta} u(\overline{k}^*(\theta), \theta) \mu(d\theta)$. $\square$

There is an example of procurement to which Corollary 2 is applicable.

**Example 6** *(Procurement)* A buyer *(PL)* needs procurement of two imperfectly substitutive goods respectively from two producers indexed by $i = 1, 2$. $i$ receives a production cost signal $\theta_i \in [0, 1]$. The quantity of the good purchased from $i$ is $x_i \in [0, x_i]$. The monetary payment to $i$ is $t_i \in [0, t_i]$. Each producer $i$’s payoff is $t_i - c_i(x_i, \theta_i)$, where $c_i(x_i, \theta_i) = \theta_i x_i^2$ is the production cost of $x_i$. The buyer’s payoff is $\sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} t_i$. $\ln x_i$ is the payoff the buyer can draw from consumption of $x_i$. Here $\ln x_i = \frac{\ln x_i^2}{2}$. It is an increasing concave transformation of $x_i^2$. 
5 Discussion

5.1 Cross Constraints on Feasible Contract Profiles

The aforementioned results do not address the cross constraints on feasible contract profiles, which imply that \( K \) is not directly equal to the product of the agents’ contract sets. But if the IPE bilateral BIC mechanism \( \tilde{k} \) still satisfies the feasible constraint, i.e., \( \tilde{k}(\theta) \in K \) for each \( \theta \), Proposition 3 will still hold without the hypothesis that \( K = \prod_{i=1}^{n} K_i \). Even as long as the BIC bilateral mechanism IPE to optimal collective mechanism \( k^* \) still satisfies the feasible constraint, Corollaries 1-2 still hold without the hypothesis (i). Here is a simple example to this end.

Example 7 One producer procures two input goods separately from two input suppliers denoted by \( i = 1, 2 \). Contracts for \( i \) consist of two parts: purchase \( x_i \in [0, \bar{x}_i] \) and payment \( t_i \in [0, \bar{t}_i] \). \( i \)'s payoffs are independently distributed. \( i \)'s payoff function \( v_i(t_i, x_i, \theta_i) = t_i - \theta_i x_i \). The producer’s payoff \( u(k, \theta) = x_1^\alpha x_2^{2-\alpha} \) denotes the Cobb-Douglas (monetary) production function, where \( \alpha \in (0, 1) \). There is a feasible constraint over \( x_i \): \( x_1 x_2^{1-\alpha} \leq \bar{q} \), where \( \bar{q} \) denotes the capacity limit. The producer should not purchase the bundle of \( (x_1, x_2) \) beyond the production capacity constraint. For each \( i \), collective (respectively bilateral) BIC assignment rule for \( i \) is \( x_i : \Theta \rightarrow \mathbb{R} \) (respectively, \( x_i : \Theta \rightarrow \mathbb{R} \)). Suppose optimal collective BIC assignment rule is \( x^* \). Then its IPE bilateral assignment rule \( \bar{x}^* \) can be defined by

\[
\bar{x}^*_i(\theta_i) = \int_{\Theta-i} x^*_i(\theta)\mu_{-i}(d\theta_{-i}), i = 1, 2.
\]

Clearly, if for each \( \theta \), \( x_1^\alpha(\theta)x_2^{1-\alpha}(\theta) \leq \bar{q} \), then \( x_1^\alpha(\theta_1)x_2^{1-\alpha}(\theta_2) \leq \bar{q} \).

Yet in some cases, IPE bilateral mechanisms may not necessarily preserve some cross constraints, especially for those linear combination inequality constraints, such as the feasible probabilistic assignments in auction design. It is generally hard to a priori provide conditions on the primitives for such preservation. Such preservation will need some requirement on the properties of the optimal collective mechanism per se. For instance, the symmetric mechanism design with ex ante identical agents may help to this end, especially in auction contexts.

5.2 Individual Rationality Constraints

The preceding analysis does not include the participation constraints modeled as the individual rationality conditions. Suppose that \( A_i \) has the commonly-observable reservation utility \( r_i(\theta_i) \in \mathbb{R} \) based on his type \( \theta_i \). We can consider the conventional Interim (Bayesian) Individual Rationality (IR) conditions for the collective BIC mechanism, bilateral BIC mechanisms, and BNE contract selection profile under a Bayesian menu. From the mathematical perspective, IR conditions serve similar to corresponding BIC conditions or BNE conditions in the interim constraints of the multi-agency contracting problems, as the interim payoffs under
those mechanisms or contract selections in the IR conditions remain the same, and \( r_i(\theta_i) \) in the IR conditions are just some constants given \( \theta_i \). Thus, it is not technically difficult to incorporate the individual rationality conditions in all the aforementioned results.

References


