Nonlinear Pricing with Asymmetric Competition in Absence of Private Information

Yong Chao, Guofu Tan, and Adam Chi Leung Wong

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A Stylized Model

- Two firms: A dominant firm (D or Firm 1) and a rival firm (R or Firm 2).
- Both firms can produce a homogeneous product at constant marginal cost $c \geq 0$, but R has capacity $k \in (0, \infty)$. 

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- The timing of the game:
  1. D offers a tariff $\tau(\cdot)$, which specifies payment $\tau(Q)$ that B has to make if she chooses to buy $Q \geq 0$ units from D.
  2. R offers a unit price $p$.
  3. B chooses quantities she buys from the firm(s).
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- Objective: Determine subgame perfect equilibrium (SPE) and study the implications of the equilibrium.
In a number of recent antitrust cases in the U.S., E.U., Canada, and China,

- a plaintiff (an antitrust agency or a rival firm) alleged that a dominant firm used pricing schemes such as conditional rebates/discounts to its downstream buyers to fully or partially exclude its rival firm(s) and that such an exclusion had harmed competition and consumer welfare.
Motivation

In a number of recent antitrust cases in the U.S., E.U., Canada, and China,

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Those antitrust cases share some common features:

1. A firm that is “dominant” in market share, capacity, product lines, profits, and so on.
2. A much smaller firm (or recent entrants) that has limited capacity, narrower product line, or limited distribution channels.
3. The dominant firm typically offers more complex pricing schemes (e.g., rebates/discounts conditional on volumes) than its rival(s).
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- Price discrimination based on heterogeneous preferences of downstream customers (e.g., private information)?

- Strategic exclusion (foreclosure)? Focus of this paper (even without private information).
Messages from Our Analysis

1. Nonlinear pricing (NLP) (or a menu of offers based on quantities) can arise in the presence of competition but in the absence of asymmetric information.
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2. NLP may be used strategically by a dominant firm to restrict its competitor’s choices, reduce its competitor’s profits, and decrease consumer welfare and possibly efficiency.
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We solve subgame perfect equilibria using mechanism design techniques.
Assumption 1: $u : \mathbb{R}_+ \rightarrow \mathbb{R}$ is twice continuously differentiable, satisfying $u(0) = 0$, $u''(\cdot) < 0$, $u'(0) > c$.

- Let

$$D(p) \equiv \arg\max_{\tilde{q} \geq 0} \{u(\tilde{q}) - p\tilde{q}\},$$

$$\pi(p) \equiv (p - c)D(p).$$

- A unique, efficient quantity $q^e = D(c) > 0$. Assume $q^e$ finite.
Conditional Payoff of the Buyer

Given the two firms’ offers $\tau \in \mathcal{T}$ and $p \in \mathcal{P}$, the buyer’s maximization problem can be decomposed into two sub-problems:
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- Given \( Q \) units purchased from D, B chooses \( q \) units from R to solve
  \[
  V(Q, p) \equiv \max_{q \in [0, k]} \left\{ u(Q + q) - pq \right\},
  \]
  and

- B chooses \( Q \) units to solve
  \[
  \max_{Q \geq 0} \left\{ V(Q, p) - \tau(Q) \right\}.
  \]
Properties of the Buyer’s Surplus Function

\[ V(Q, p) \equiv \max_{q \in [0,k]} \{ u(Q + q) - pq \} \]

- Conditional on buying \( Q \) from D, B’s demand for R’s product:
  
  \[ q^*(Q, p) = \text{Proj}_{[0,k]}(D(p) - Q). \]
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- Both \( V \) and \( V_p \) increase with \( Q \), and \( V \) decreases with \( p \).

- Spence-Mirrlees single-crossing property holds: Let
  \[ U(T, Q; p) = V(Q, p) - T. \]
  Then
  \[ -\frac{\partial U}{\partial Q} \frac{\partial Q}{\partial U} = V_Q \]

  increases with \( p \).
Offering Two Points

Nonlinear Pricing with Asymmetric Competition
Single-crossing of the Surplus Curves

\[ BS = V(Q', p) - T' \]

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\[ Q' < Q \]
R's profit, conditional on B's purchase from D being $Q$, is

$$\pi(Q, p) \equiv (p - c) \text{Proj}[0,k](D(p) - Q)$$

$$= \text{Proj}[0,(p-c)k](\pi(p) - (p - c)Q).$$
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$$= \text{Proj}_{0, (p - c)k}(\pi(p) - (p - c)Q).$$
Every tariff $\tau \in \mathcal{T}$ firm D might offer induces a continuation subgame in which R and B sequentially choose their actions. When choosing $\tau$, firm D understands that R and B would play a SPE in the continuation subgame:

Given $\tau$, the buyer would optimally choose some $Q(p)$ from D, contingent on any possible price $p$. The payment is $\tau(Q(p)) - T(p)$. On the other hand, given that B’s optimal purchase from D is $Q(p)$, and hence from R is $\text{Proj}[0,k](D(p)Q(p))$, R would optimally choose some price $\bar{p}$.
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Dominant Firm’s Problem

In the spirit of the Revelation Principle (imagining D asks B to report R’s price), solving SPE for the whole game is equivalent to solving the following constrained optimization problem:

(OP1): Maximize $T(\bar{p}) - cQ(\bar{p})$

over quantity function $Q : \mathcal{P} \rightarrow \mathbb{R}_+$, payment function $T : \mathcal{P} \rightarrow \mathbb{R}$, and recommendation of R’s price $\bar{p} \in \mathcal{P}$, subject to

$V(Q(p), p) T(p) V(Q(\tilde{p}), p) \leq p$, $\tilde{p} \in \mathcal{P}$.

$V(Q(\bar{p}), \bar{p}) T(p) V(Q(\bar{p}), p) \leq p$, $\bar{p} \in \mathcal{P}$. 

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Nonlinear Pricing with Asymmetric Competition
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\[(BIC): V(Q(p), p) - T(p) \geq V(Q(\bar{p}), p) - T(\bar{p}) \quad \forall p, \bar{p} \in \mathcal{P}\]

\[(BIR): V(Q(p), p) - T(p) \geq V(0, p) \quad \forall p \in \mathcal{P}\]
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\[(\text{BIR}): V(Q(p), p) - T(p) \geq V(0, p) \quad \forall p \in \mathcal{P} \]

\[(\text{RIC}): \pi(Q(\bar{p}), \bar{p}) \geq \pi(Q(p), p) \quad \forall p \in \mathcal{P}. \]
Theorem 1: A SPE is equivalent to a solution to (OP1).

We have a virtual Principal-Agents model in which

- the Principal (dominant firm) offers a direct revelation mechanism, \( Q : \mathcal{P} \to \mathbb{R}_+ \) and \( T : \mathcal{P} \to \mathbb{R} \), to one Agent (the Buyer),
- the second Agent (the Rival firm) takes an action (price) \( p \in \mathcal{P} \), and
- the Buyer reports the Rival firm’s price \( p \in \mathcal{P} \) to the Principal.
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- the second Agent (the Rival firm) takes an action (price) $p \in \mathcal{P}$, and
- the Buyer reports the Rival firm’s price $p \in \mathcal{P}$ to the Principal.

Theorem 2: Characterization of the solution to (OP1).
Remark: In equilibrium,

(i) firm D offers a nonlinear tariff schedule \( \tau(\cdot) \), or \( Q : \mathcal{P} \to \mathbb{R}_+ \) and \( T : \mathcal{P} \to \mathbb{R} \), which is shown to be strictly increasing and strictly convex on \([Q_0, \bar{Q}]\), where \( 0 \leq Q_0 < \bar{Q} \).
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(ii) firm R chooses $p = \bar{p}$, and

(iii) the buyer purchases $\bar{Q} = Q(\bar{p})$ units from firm D and $D(\bar{p}) - \bar{Q}$ units from firm R, respectively.
Equilibrium Tariff

\[ T \]

\[ \bar{T} \]

\[ \tau(\bullet) \]

\[ Q_0 \]

\[ Q \]

Yong Chao, Guofu Tan, and Adam Chi Leung Wong

Nonlinear Pricing with Asymmetric Competition
Proposition: For $k \in (0, q^e)$, as compared with Linear Pricing (LP), the adoption of NLP by the dominant firm 

(a) decreases the rival’s supply (strictly below its capacity level), increases its own profits, and reduces the joint surplus of its rival and the buyer; 

(b) reduces the price, market share and profits of the rival firm, and decreases the buyer surplus, when $k$ is relatively small.
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Quantity-Forcing (All-or-Nothing):\((Q, T)\) or \((0, 0)\)
The Choices of Buyer and Rival under Quantity-Forcing

Figure: B’s Surplus and R’s Profits as Functions of R’s Price

\[ BS = V(Q, p) - T \]
\[ BS = V(0, p) \]
\[ \pi(Q, p) \]
The optimal Quantity-Forcing (QF) program is to choose $(Q, T; p, x)$ to

Maximize $T - cQ$

subject to

$V(Q, x) - T = V(0, x)$

$\pi(Q, p) \geq \pi(Q, p') \quad \forall p' \geq x$

$\pi(Q, p) \geq \pi(0, x)$.

Here $x$ is the maximum (threat) price that firm R could potentially choose, below which the buyer purchases only from firm D.
Offering One More Point

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Nonlinear Pricing with Asymmetric Competition
One More Option Improves Firm D’s Profits

Nonlinear Pricing with Asymmetric Competition
Intuition: By offering another bundle, firm D can constrain its rival’s offers and hence extract more surplus from the buyer without initially affecting the rival’s profits.

Moreover, firm D can modify the two quantities and payments to further improve its own profits and reduce the rival’s profits.
Optimal Two Options

\[ BS = V(Q, p) - T \]
\[ BS = V(0, p) \]
\[ BS = V(Q', p) - T' \]

\[ \pi(Q, p) \]
A Continuum of Points

\[ T \]

\[ \bar{T} \]

\[ \tau(\bullet) \]

\[ 0 \quad Q_0 \quad \bar{Q} \]
Dominant Firm’s Constrained Optimization Problem

(OP1): Maximize \( T(\bar{p}) - cQ(\bar{p}) \)
over \( Q : \mathcal{P} \to \mathbb{R}_+, \quad T : \mathcal{P} \to \mathbb{R}, \) and \( \bar{p} \in \mathcal{P} \), subject to

(BIC): \( V(Q(p), p) - T(p) \geq V(Q(\tilde{p}), p) - T(\tilde{p}) \quad \forall p, \tilde{p} \in \mathcal{P} \)

(BIR): \( V(Q(p), p) - T(p) \geq V(0, p) \quad \forall p \in \mathcal{P} \)

(RIC): \( \pi(Q(\bar{p}), \bar{p}) \geq \pi(Q(p), p) \quad \forall p \in \mathcal{P} \).
Lemma: $Q : \mathcal{P} \to \mathbb{R}_+$ and $T : \mathcal{P} \to \mathbb{R}$ satisfy (BIC) if and only if

- Monotonicity of $Q$ (BIC-1): For any $p_1, p_2 \in \mathcal{P}$ with $p_1 \leq p_2$, either $Q(p_1) \leq Q(p_2)$ or $D(p_1) \leq Q(p_2)$ or $Q(p_1) + k \leq D(p_2)$; and

- Transfer (BIC-2): For any $p \in \mathcal{P}$,
  \[ T(p) = T(c) - V(Q(c), c) + V(Q(p), p) - \int_c^p V_p(Q(\tilde{p}), \tilde{p})d\tilde{p}. \]
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Lemma: (BIC) implies:

1. $Q(p)$ is nondecreasing in $p$ on
   $$\{p \in \mathcal{P} : D(p) - k \leq Q(p) \leq D(p)\};$$

2. The demand for R’s product, $\text{Proj}_{[0,k]}(D(p) - Q(p))$, is nonincreasing in $p$ on $\mathcal{P}$. 

Yong Chao, Guofu Tan, and Adam Chi Leung Wong
(BIR) constraint:

\[ V(Q(p), p) - T(p) \geq V(0, p) \quad \forall p \in \mathcal{P}. \]
Characterizing (BIR) Constraints

(BIR) constraint:

\[ V(Q(p), p) - T(p) \geq V(0, p) \quad \forall p \in \mathcal{P}. \]

**Lemma:** Given (BIC), (BIR) holds if and only if 
\[ V(Q(c), c) - T(c) \geq V(0, c). \] If (BIR) is satisfied and is binding at some \( p \in \mathcal{P} \), then it must be binding at \( p = c \).
(BIR) constraint:

\[ V(Q(p), p) - T(p) \geq V(0, p) \quad \forall p \in \mathcal{P}. \]

**Lemma:** Given (BIC), (BIR) holds if and only if 
\[ V(Q(c), c) - T(c) \geq V(0, c). \] If (BIR) is satisfied and is binding at some \( p \in \mathcal{P}, \) then it must be binding at \( p = c. \)

Thus, (BIC) and (BIR) imply that, for all \( p \in \mathcal{P}, \)

\[
T(p) = V(Q(p), p) - V(0, c) - \int_{c}^{p} V_p(Q(\bar{p}), \bar{p}) d\bar{p}
= \int_{0}^{Q(p)} V_Q(Q, c) dQ + \int_{c}^{p} [V_p(Q(p), \bar{p}) - V_p(Q(\bar{p}), \bar{p})] d\bar{p}.
\]
Using (BIC) and (BIR), Firm D’s profit can be written as

\[ \Pi_1 = \int_0^{Q(\bar{p})} [V_Q(Q, c) - c]dQ + \int_{\bar{Q}}^{\bar{p}} [V_p(Q(\bar{p}), p) - V_p(Q(p), p)]dp \]

\[ = \int_0^{Q(\bar{p})} [\text{Proj}_{[\hat{u}'(Q + k), \hat{u}'(Q)]}(c) - c]dQ \]

\[ + \int_{\hat{Q}}^{\bar{p}} [\text{Proj}_{[0,k]}(D(p) - Q(p)) - \text{Proj}_{[0,k]}(D(p) - Q(\bar{p}))]dp. \]
Equivalent Optimization

Using (BIC) and (BIR), we can write (OP1) as

\[(\text{OP2}): \text{Maximize } \Pi_1\]

over \( \mathcal{Q} : \mathcal{P} \rightarrow \mathbb{R}_+, \bar{p} \in \mathcal{P} \), and firm 2’s profit \( \Pi_2 \geq 0 \), subject to the monotonicity of \( \mathcal{Q} \) and

\[(\text{RIC}): \Pi_2 \geq (p - c) \operatorname{Proj}_{[0,k]}(D(p) - Q(p)) \quad \forall p \in \mathcal{P},\]

\[\Pi_2 = (\bar{p} - c) \operatorname{Proj}_{[0,k]}(D(\bar{p}) - Q(\bar{p})).\]
Theorem 2: There exists at least one equilibrium. In any equilibrium, \((\Pi_2, \bar{p}, \bar{Q}, x_0, Q_0)\) solves

\[
\bar{p} - c = e \cdot (x_0 - c) > 0, \tag{1}
\]

\[
\Pi_2 = \pi(\bar{p}) - (\bar{p} - c)\bar{Q} = \pi(x_0) - (x_0 - c)Q_0, \tag{2}
\]

\[
\pi'(\bar{p}) = \bar{Q}, \tag{3}
\]

\[
Q_0 = \max\{D(x_0) - k, 0\}. \tag{4}
\]
An equilibrium mechanism \([Q(\cdot), T(\cdot)]\) is given by

\[
Q(p) = \begin{cases} 
D(p) - \frac{\Pi_2}{p-c} & \text{if } p \in [x_0, \bar{p}] \\
Q_0 & \text{if } c \leq p < x_0 \\
\bar{Q} & \text{if } \bar{p} < p \leq u'(0)
\end{cases} \tag{5}
\]

and

\[
T(p) = u(Q_0 + k) - u(k) + \int_{x_0}^{p} \tilde{p} dQ(\tilde{p}) \quad \forall p \in [x_0, \bar{p}]. \tag{6}
\]

An equilibrium tariff \(\tau(\cdot)\) is given by

\[
\tau(Q) = \begin{cases} 
 u(Q_0 + k) - u(k) + \int_{Q_0}^{Q} x(\tilde{Q}) d\tilde{Q} & \text{if } Q \in [Q_0, \bar{Q}] \\
0 & \text{if } Q = 0 \\
\infty & \text{otherwise}
\end{cases} \tag{7}
\]

where \(x(\cdot)\) on \([Q_0, \bar{Q}]\) is the inverse of \(Q(\cdot)\) on \([x_0, \bar{p}]\).
Equilibrium Non-linear Pricing

Nonlinear Pricing with Asymmetric Competition
Happy Birthday John
Proposition 1: Under some regularity assumptions, the equilibrium is essentially unique.
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Regularity assumptions: For instance,

\[ u'(q) - c \] is strictly log-concave in \( q \) on \([0, q^e]\). \tag{8} \]

or

\[ -(p - c)D'(p) \] is strictly increasing in \( p \) on \( \mathcal{P} \). \tag{9}
Proposition 2: In any equilibrium,

\[
\Pi_2 + BS = \int_{x_0}^{\infty} \min\{D(p), k\} dp \\
= u(\min\{D(x_0), k\}) - x_0 \cdot \min\{D(x_0), k\} \\
= u(D(x_0) - Q_0) - x_0 \cdot (D(x_0) - Q_0).
\] (10)

That is, what firm R and the buyer jointly earn in equilibrium is equal to their joint outside option under the counterfactual situation that firm R’s unit cost was raised to \(x_0\).
Proposition 3: In any equilibrium,

1. Firm D’s tariff $\tau$ is strictly increasing and strictly convex on $[Q_0, \bar{Q}]$, where $0 \leq Q_0 < \bar{Q}$.

2. Marginal prices at $\bar{Q}$ are equal: $\tau'(\bar{Q}) = \bar{p}$, but the average price for firm D might not be lower, depending on the size of $k$.

3. Firm R supplies strictly below its capacity level: $D(\bar{p}) - \bar{Q} < k$. 
Proposition 4: There is a unique $\hat{k}$ such that $Q_0 = 0$ in equilibrium iff $k \geq \hat{k}$. This $\hat{k}$ satisfies $D(p^m) < \hat{k} < q^e$.
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$$D(p^m) < \hat{k} < q^e.$$ 

The set of equilibria is independent of $k$ on $[\hat{k}, \infty]$, and:

1. $\Pi_2, \bar{p}, x_0, \bar{p} - x_0$, and $D(\bar{p}) - \bar{Q}$ are increasing in $k$ on $(0, \hat{k}]$.
2. $\Pi_1, \bar{Q}, Q_0, TS$ are decreasing in $k$ on $(0, \hat{k}]$.
3. $\Pi_2 + BS$ and $BS$ are increasing in $k$ when $k$ is small, and are decreasing in $k$ when $k$ is close to but below $\hat{k}$. 

Yong Chao, Guofu Tan, and Adam Chi Leung Wong
Proposition 5: For $k \in (0, q^e)$, as compared with LP, the adoption of NLP by the dominant firm

(a) decreases the rival's supply (strictly below its capacity level), increases its own profits, and reduces the joint surplus of its rival and the buyer;

(b) reduces the price, market share and profits of the rival firm, and decreases the buyer surplus, when $k$ is relatively small.
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(b) reduces the price, market share and profits of the rival firm, and decreases the buyer surplus, when $k$ is relatively small.
Suppose $u(q) = q - q^2/2$ and $c = 0$. 

The equilibrium outcome is as follows:

- $\bar{p} = \min\{k, \hat{k}\}$
- $x_0 = \bar{p}$
- $\bar{Q} = 1 - 2\bar{p}$
- $Q_0 = 1 + \frac{1}{2}(1 + e^2)\bar{p}$
- $\bar{y}_2 = \bar{p}^2$
- $TS = \frac{1}{2}\bar{p}^2$
- $\bar{y}_1 = \begin{cases} \frac{1}{2}(1 + e^2) & \text{if } k < \hat{k} \\ h(1 - k) & \text{if } k \geq \hat{k} \end{cases}$

- $BS = \begin{cases} e^2(e^2 + 2) & \text{if } k < \hat{k} \\ e^4 + e^2k & \text{if } k \geq \hat{k} \end{cases}$
Suppose $u(q) = q - q^2/2$ and $c = 0$. Then $D(p) = 1 - p$, $\pi(p) = p(1 - p)$, $\pi'(p) = 1 - 2p$ for $p \in [0, 1]$. 
An Example: Linear Demand and Zero Marginal Cost

Suppose \( u(q) = q - q^2/2 \) and \( c = 0 \). Then \( D(p) = 1 - p \), \( \pi(p) = p(1 - p) \), \( \pi'(p) = 1 - 2p \) for \( p \in [0, 1] \).

Let

\[ \hat{k} = \frac{e^2}{1 + e^2}. \]

The equilibrium outcome is as follows:

\[
\bar{p} = \frac{1}{e} \min\{k, \hat{k}\}, \quad x_0 = \frac{\bar{p}}{e}, \quad \bar{Q} = 1 - 2\bar{p},
\]

\[
Q_0 = 1 - \frac{1 + e^2}{e} \bar{p}, \quad \Pi_2 = \bar{p}^2, \quad TS = \frac{1 - \bar{p}^2}{2},
\]

\[
\Pi_1 = \begin{cases} 
\frac{1}{2(1 + e^2)} & \text{if } k \geq \hat{k} \\
\frac{1}{2} \left[ (1 - k)^2 + \left( \frac{k}{e} \right)^2 \right] & \text{if } k \leq \hat{k}
\end{cases},
\]

\[
BS = \begin{cases} 
\frac{e^2(e^2 - 2)}{2(1 + e^2)^2} & \text{if } k \geq \hat{k} \\
k - \frac{4 + e^2}{2e^2} k^2 & \text{if } k \leq \hat{k}
\end{cases}.
\]
Tariff Schedules under LP, QF, and NLP: Linear Demand and $c = 0$
Tariff Schedules under LP, QF, and NLP: Linear Demand and $c = 0$
### Table 1: Linear Demand $q = 1 - p$ with $c = 0$

<table>
<thead>
<tr>
<th></th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>BS</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LP</strong></td>
<td>0.4</td>
<td>0.2</td>
<td>0.16</td>
<td>0.08</td>
<td>0.18</td>
<td>0.42</td>
</tr>
<tr>
<td><strong>QF</strong></td>
<td>0.8472</td>
<td>0.0764</td>
<td>0.3218</td>
<td>0.0058</td>
<td>0.1694</td>
<td>0.4971</td>
</tr>
<tr>
<td><strong>NLP</strong></td>
<td>0.8528</td>
<td>0.0736</td>
<td>0.3227</td>
<td>0.0054</td>
<td>0.1692</td>
<td>0.4973</td>
</tr>
</tbody>
</table>

$k = 0.2$

<table>
<thead>
<tr>
<th></th>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$\pi_1$</th>
<th>$\pi_2$</th>
<th>BS</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LP</strong></td>
<td>0.05</td>
<td>0.9</td>
<td>0.0025</td>
<td>0.045</td>
<td>0.4513</td>
<td>0.4987</td>
</tr>
<tr>
<td><strong>QF</strong></td>
<td>0.2929</td>
<td>0.3536</td>
<td>0.0429</td>
<td>0.125</td>
<td>0.2696</td>
<td>0.4375</td>
</tr>
<tr>
<td><strong>NLP</strong></td>
<td>0.3519</td>
<td>0.324</td>
<td>0.0596</td>
<td>0.105</td>
<td>0.2829</td>
<td>0.4475</td>
</tr>
</tbody>
</table>

$k = 0.9$
Theorem 3: For $k \in (0, q^e)$ and for the NLP vs NLP game, (a) SPE exists, and (b) any $(\Pi_1, \Pi_2, BS)$ is a SPE payoff vector iff $\Pi_2, BS \geq 0$, and

$$\Pi_1 = [u(q^e) - c \cdot q^e] - [u(k) - c \cdot k], \quad (11)$$

$$\Pi_2 + BS = u(k) - c \cdot k. \quad (12)$$
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Remark (Multiple SPE): Firm D’s profit is uniquely determined. The joint surplus between R and B is also uniquely determined but not the division of the surplus.
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Remark (Multiple SPE): Firm D’s profit is uniquely determined. The joint surplus between R and B is also uniquely determined but not the division of the surplus.

Question: Equilibrium selection?

Proposition 6: Switching from any SPE of the NLP vs LP game to any SPE of the NLP vs NLP game, both $TS$ and $\Pi_2 + BS$ increase but $\Pi_1$ decreases.
Assume that the buyer has a one-dimensional private information (or type) $\theta \in [\underline{\theta}, \bar{\theta}] \equiv \Theta$. Let $u(q, \theta)$ denote the buyer's gross surplus if she consumes $q$ units and her type is $\theta$. 
Assume that the buyer has a one-dimensional private information (or type) $\theta \in [\underline{\theta}, \overline{\theta}] \equiv \Theta$. Let $u(q, \theta)$ denote the buyer’s gross surplus if she consumes $q$ units and her type is $\theta$. Define

$$D(p, \theta) \equiv \operatorname{argmax}_{\tilde{q} \geq 0} \{u(\tilde{q}, \theta) - p\tilde{q}\},$$

$$V(Q, p, \theta) \equiv \max_{q \in [0, k]} \{u(Q + q, \theta) - pq\},$$

$$\pi(Q, p, \theta) \equiv (p - c) \operatorname{Proj}_{[0,k]}(D(p, \theta) - Q).$$
(OP-PI): Maximize $\Pi_1 = \int_{\theta}^{\tilde{\theta}} [T(\bar{p}, \theta) - cQ(\bar{p}, \theta)]dF(\theta)$

over $Q: \mathcal{P} \times \Theta \rightarrow \mathbb{R}_+$, $T: \mathcal{P} \times \Theta \rightarrow \mathbb{R}_+$, and $\bar{p} \in \mathcal{P}$, subject to
(OP-PI): Maximize $\Pi_1 = \int_{\Theta}^\bar{\theta} [T(\bar{p}, \theta) - cQ(\bar{p}, \theta)]dF(\theta)$

over $Q : \mathcal{P} \times \Theta \rightarrow \mathbb{R}_+$, $T : \mathcal{P} \times \Theta \rightarrow \mathbb{R}_+$, and $\bar{p} \in \mathcal{P}$, subject to

(BIC): $V(Q(p, \theta), p, \theta) - T(p, \theta) \geq V(Q(\bar{p}, \tilde{\theta}), p, \theta) - T(\bar{p}, \tilde{\theta}) \ \forall p, \bar{p} \in \mathcal{P}, \tilde{\theta} \in \Theta$. 

Yong Chao, Guofu Tan, and Adam Chi Leung Wong

Nonlinear Pricing with Asymmetric Competition
(OP-PI): Maximize \( \Pi_1 = \int_{\theta}^{\bar{\theta}} [T(\bar{p}, \theta) - cQ(\bar{p}, \theta)]dF(\theta) \)

over \( Q: \mathcal{P} \times \Theta \to \mathbb{R}_+ \), \( T: \mathcal{P} \times \Theta \to \mathbb{R}_+ \), and \( \bar{p} \in \mathcal{P} \), subject to

(BIC): \( V(Q(p, \theta), p, \theta) - T(p, \theta) \geq V(Q(\bar{p}, \bar{\theta}), p, \theta) - T(\bar{p}, \bar{\theta}) \) \( \forall p, \bar{p} \in \mathcal{P}, \bar{\theta} \in \Theta \)

(BIR): \( V(Q(p, \theta), p, \theta) - T(p, \theta) \geq V(0, p, \theta) \) \( \forall p \in \mathcal{P}, \theta \in \Theta \)
(OP-PI): Maximize $\Pi_1 = \int_\theta^\bar{\theta} [T(\bar{p}, \theta) - cQ(\bar{p}, \theta)]dF(\theta)$

over $Q: \mathcal{P} \times \Theta \to \mathbb{R}_+$, $T: \mathcal{P} \times \Theta \to \mathbb{R}_+$, and $\bar{p} \in \mathcal{P}$, subject to

(BIC): $V(Q(p, \theta), p, \theta) - T(p, \theta) \geq V(Q(\bar{p}, \bar{\theta}), p, \theta) - T(\bar{p}, \bar{\theta}) \ \forall p, \bar{p} \in \mathcal{P}$,

(BIR): $V(Q(p, \theta), p, \theta) - T(p, \theta) \geq V(0, p, \theta) \ \forall p \in \mathcal{P}, \ \theta \in \Theta$,

(RIC): $\int_\theta^\bar{\theta} \pi(Q(\bar{p}, \theta), \bar{p}, \theta)dF(\theta) \geq \int_\theta^\bar{\theta} \pi(Q(p, \theta), p, \theta)dF(\theta) \ \forall p \in \mathcal{P}$. 
Monopoly price discrimination based on asymmetric information, e.g., Maskin and Riley (RAND 1984), Rochet and Stole (RES 2002)

Competing principals with common agency, e.g., Berheim and Whinston (JPE 1998), Stole (JEMS 1995)

Long-term contracts between the incumbent and the buyer, e.g., Aghion and Bolton (AER 1987) and the related literature

Simple pricing mechanisms such as All-units Discounts, e.g., Chao and Tan (2014)