Cost Enabled Choice of Pricing Rule when Buyers’ Information is Private*

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Abstract

The prices a consumer knows are her private information and determine her acceptance/rejection decision to a seller’s price. Posted prices (with an implicit take it or leave it offer) may not then be a seller’s best strategy. Inviting every buyer to reveal her private information is attractive, especially for a low cost seller, as it helps to tailor the price to the consumer and reduces the proportion of rejections. This paper seeks to explore endogenous choices of pricing rules between heterogeneous cost sellers in such a market. I restrict the investigation to the comparison of posted price (take it or leave it) with two alternative rules or interactions that have comparable transaction simplicity. The price matching interaction proves to be desirable, solving important informational issues and adverse selection present in the other two. But the highest cost sellers are unable to adopt this interaction in a market with free entry and through their rejection of matching, fall back to posted price (take it or leave it). In any endogenous equilibrium therefore there would be a mixture of sellers using posted price (take it or leave it) and price matching, with price matching possible.

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only below a threshold cost (relative to the market). The adoption of price matching by a seller increases the price it posts. However, the distribution of prices in the market is not necessarily higher because the seller with the lowest price has incentives to post a low price to minimize the number of rivals price matching.

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1 Introduction

The popularity of posted prices is indisputable, and therefore justifies the emphasis it receives in economic applied and theoretical research on pricing. At the same time the persistence of price variation in markets is increasingly being accepted as a stylistic fact, which suggests two important points. One, that not all sellers can be earning zero profits if multiple prices co-exist in a market. Two, that a prospective buyer to a particular seller may be aware of any subset of the multiple rival prices in a market. With the possibility then of some sellers earning positive profits and buyers having private information regarding the rival market, a posted price trading interaction with an implicit take-it-or-leave-it offer may neither be optimal for a seller nor need it be credible.

The optimality comes into question because different prospective buyers may have different subsets of rival-price information, and to serve them all with one price may be sub-optimal. The credibility is questionable because if the seller earns positive profit at the price it posts and has no capacity constraints, then a buyer’s credible refusal of that price gives the seller incentive to offer a second lower price rather than lose the buyer. It is therefore not subgame perfect. And the two are inter-related issues because a buyer’s credible refusal of a price is a function of her value and her information regarding rival prices.

Posted prices with take-it-or-leave-it ultimatums therefore cannot be considered as the only rules of price determination. Rather a seller choosing this must be making that decision after considering and rejecting alternative pricing rules or trading interactions. Moreover, it is common to find sellers settling for trade interactions that use posted prices but are not limited to them; i.e. without the take-it-or-leave-it ultimatum, and with some predefined deviations from the posted price. Examples of these are quantity discounts, loyalty discounts and redeemable points, rebate coupons; i.e.

\[1\] A credible refusal is one that is true if verified, as in Wolinsky (1983) rather than an incredible threat of refusal motivated solely with getting a better bargain.

\[2\] There may be possible strategic public benefits to all sellers from a commitment to a single posted price, as illustrated by Bester (1994).
forms of price discrimination. The extent of some of these measures may be limited in some markets, but one or more of these are used in many consumer markets ranging from groceries to air travel.

I investigate the endogenous choice of trading interactions or pricing rules for different types of sellers in such large consumer markets with free entry, closely resembling retail with the good being sold considered to be perfectly homogeneous between different sellers. Despite differences of determination of selling price, prospective buyers’ information of rival prices can be expected to be an important variable in predicting purchases. I explore how the presence of a variety of information types of buyers affects different sellers’ incentives and ability to price discriminate by endogenously choosing a trading interaction that enables it. Png & Hirshleifer (1987) were the first to illustrate how the policy of price matching is a desirable pricing interaction to adopt in the presence of such buyer information heterogeneity. Sellers in their model are however identical and therefore the desirability of price discrimination for sellers arises indiscriminately. This paper asks the important question of the extent and likelihood of adoption of a pricing policy that enables price discrimination if the market is not homogeneous, such that the ability of a seller to do so, is a function of its own cost and is a function of rival policies and prices (which are in turn best responses). Moreover I address the important concern that the question of adoption of a particular trading interaction must be considered together with the choice of a posted price (if the interaction depends on one), especially given heterogeneity. Discussing these two choices separately ignores the cross-effects of the incentives created by each.

I therefore analyze a market where different sellers can each choose from amongst three possible pricing interactions with buyers. Each seller can

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3 An institutional argument in the debate between one price for all buyers and price discrimination is put forth by North (1991). Although he refers to price discrimination of a particular kind where sellers and buyers explicitly haggle to decide the price, North (1991) claims that a preference for a single (posted) price may have emerged to avoid the transaction costs of reaching an agreement and the agency costs of monitoring sales agents by owners. His argument can be justified with the observation that sellers who are also owners tend to price discriminate more often (as seen in bazaars and souks) rather than sellers who are agents. However, acknowledging the surge of technology that has enabled cheap monitoring of sales collection, the agency costs should have been lowered, and we should expect to see an increase in the use of discriminatory pricing rules even by sellers who are not owners.

4 The ability to price discriminate in their model also critically depends on the difference in price elasticity of demand between the differently informed buyers. Differences in elasticities although essential for price discrimination in a monopoly, should not be necessary in multiple seller markets if buyers differ by information.
either post a single price and leave each prospective buyer with a take it or leave it ultimatum, or can invite a single bid per buyer and retain the option to accept or refuse to sell at it, or can use price matching i.e. post one price but allow buyers deviations from that posted price as a function of their information. The reason I restrict the investigation to these three pricing interactions is because they all have comparable costs of execution.

I first discuss what the market would look like if it were restricted to each of the first two interactions. Next, allowing sellers endogenous choice between these two interactions I establish their choice. I find that no seller would choose to invite a bid per buyer as doing so signals low cost and thus adversely selects unprofitable bids. This result is an extension of what Perry (1986) found in a monopoly buyer-seller setup with private values and costs. The replacement of multiple value types of buyer (as in that paper) with multiple information types of buyers, and the extension to a large market does not alter the result because it does not change each buyer’s incentive to an extremely low bid, given the information that such an invitation of bids conveys.

Thus eliminating one of the possible three interactions, I illustrate that the posted price with take it or leave it ultimate is a special case of price matching such that the seller chooses zero price matching. Moreover the deviations from posted price allowed in a price matching interaction are of the same nature as a buyer’s bid, the only difference being that the bid now needs to exist as a rival price and is therefore contingent on the buyer’s information and thus on the rest of the market. The question of endogenous choice of trading interaction then shrinks to posting a price with or without allowing matching of rival prices.

I therefore examine whether sellers have incentives to deviate from the market where they each only post prices (with take it or leave it ultimatums), by using price matching; and if so then which types of sellers are able to do this. I find that from any equilibrium in the market restricted to take it or leave it posted prices, at least some sellers with costs in the lower tail will have incentive to adopt the price matching interaction. I also find that the adoption of price matching by a seller raises its posted price because its matching policy substitutes, in part, for an actually lower price. On the other hand, I find that sellers with costs in the upper tail of the cost distribution cannot afford to credibly adopt price matching interactions. This is because their price-cost margins are low (as the distribution of costs is bounded above by the value of the buyers), and because most prices in the market are lower than their costs; matching rival prices would reduce profits rather than add to them.
With each seller thus making an individual choice of price and pricing rule, the resulting outcome is a market that sees a mix of sellers adopting take it or leave it posted prices and adopting price matching. Adoption of price matching moreover is observed only for sellers below a threshold cost, and thus the freedom to adopt such a trading interaction awards benefits in the direction of cost advantages in the market given existing information imperfections.

This finding is important because it suggests that arguments that presuppose that all sellers would desire to price match cannot be made regarding markets where sellers’ costs can vary. As there is no concerted decision making and each seller acts independently, the question that needs to be asked is which seller(s), if any would desire to adopt price matching given the rest of the market. Rather, that there would be sellers adopting price matching as well as others who use take it or leave it posted prices seems both sensible and reflective of consumer retail markets.

Moreover, the lowest priced seller is unable to effectively price discriminate through price matching. The only motive driving the lowest price in an equilibrium therefore is the desire to maximize the probability of purchase for every prospective buyer, regardless of information. It turns out that this probability is itself a function of the level of this lowest price because that endogenously determines the extent of the adoption of price matching by rivals in the market. The level of the lowest price in a market that allows a choice of pricing rules, is then not necessarily higher than in a market restricted to posted prices. This is because the lowest price is itself determined by an incentive to stay low to discourage rivals from price matching.

The model thus illustrates that price matching is a measure used by sellers with relatively low costs, to price discriminate between buyers with different sets of information about rival prices. It is then an adaptation by such sellers to use their cost advantage to exploit buyers’ informational imperfections. The ability to do so, squeezes the profits of high cost sellers, as lower cost sellers sell at a larger range of possible prices because they can discriminate.

The paper also illustrates how the mechanism of price matching solves the issue of price discrimination by inviting buyers to invoke a match that is similar to inviting buyers to bid, but avoiding the market failure due to adverse bids by tying them to rival prices (thus also revealing their information). It thus solves the private information issue while avoiding associated adverse selection.

This paper contributes both to the wider literature on the optimality,
credibility and efficiency of pricing rules in imperfectly informed markets\textsuperscript{5} (Bester, 1988 & 1994; Gale, 1988; Wolinsky, 1983; Perry, 1986; Camera & Delacroix, 2004), and to the literature on the price matching policy (Hviid & Shaffer, 1999; Janssen & Parakhonyak, 2013; Moorthy & Winter, 2006; Png & Hirshleifer, 1987) which so far has not addressed heterogeneity in large markets.

\section{The Model Market}

Consider a market with a large number of sellers, given by $I$, where $I \gg 2$; and a unit mass of buyers such that the number of sellers although large, is far smaller than the number of buyers. The sellers, being the short side of the market, have the authority to dictate trading interactions to buyers who each responds by choosing the best deal available to her. Assume that every buyer desires to buy at most one unit of the homogeneous good; and that no seller is capacity constrained.

Sellers differ in their constant marginal costs of production, and have no fixed costs. Let the marginal cost of each seller be drawn from the support $(\bar{s}, \bar{s})$, and assume that no two sellers have identical costs. The seller with the lowest cost realization in the market will be referred to as having cost $s_1$ and likewise other sellers will often be ranked by cost. Sellers are assumed to be aware of rival costs and rival prices. Buyers on the other hand, do not know sellers’ costs but will be assumed to know the distribution of costs in the market\textsuperscript{6} which is given by the cumulative distribution function $F(s)$.

To focus the model on buyer differences in information the buyers that constitute the market for which sellers vie, are assumed to have identical value, $v$. Free entry in the market would then allow entry till the marginal entrant’s marginal cost is equal to $v$. That is,

$$v = \bar{s}, \ldots, \.getElementsByClassName("\"\")}, \ldots, s_2 > s_1; \quad (1)$$

Buyers however differ in how many stores they have information about.

\textsuperscript{5}Chatterjee (2013) is a good summary of similar research in the bargaining literature in large markets with imperfect information.

\textsuperscript{6}Although this assumption may not accurately depict the nature of consumer information very well, it is used here only to be able to quantify buyers’ bids if and when they have the option to make them; the basic results of the paper do not depend on this assumption.
Each buyer accurately observes pricing rules and prices of a subset of stores in the market, and does not know the distribution of all prices in the market unless she knows the identity (location) of all prices. The identity of the stores known by a buyer is assumed to be purely random and independent of store prices and policies, such that a seller’s pricing rule and/or price determines only the probability of purchase, given buyers who know the seller. This assumption may be restrictive in market settings where buyers ‘know’ of stores (their prices and policies) because of various store decisions like size, reputation, advertising, historical prices, or even current pricing rules. The point of making this assumption is to abstract away from history, reputation, store advertisement, and all promotional and other strategies that attract buyers, and ask the question: what price and interaction is best for a seller to offer a prospective buyer, given incentives arising from market heterogeneity. The buyer chooses to purchase one unit of the good from the store (amongst those known) offering her the lowest effective price, as long as it doesn’t exceed her value for the good.

The size of the set of a buyer’s information (of stores known) is denoted as her type, which is exogenously given. Buyer types can be thought of as different degrees of shopping around, before purchase. Some buyers may enjoy shopping around, and may therefore be aware of a larger subset of market prices, whereas others may be reluctant or unable to shop extensively and would therefore be aware of a small set of market prices; i.e. buyers either value information differently, or have different costs of acquiring information, or belong to different network sizes of information spread, etc.

For any given seller, a buyer who includes the seller in her set of information will be referred to as a prospective buyer. If a seller were to know the type of a prospective buyer, denoted by the variable $k$, but not the identity of the stores known by the buyer, the seller could form an expectation of the buyer’s outside option (the minimum price known to her). I term this expectation the expected minimum price (EMP)\(^7\) of a buyer of a given type. The EMP of a type of buyer is statistically equivalent to the first order statistic of rival market prices, given the extent of sampling ($k$) or the buyer’s type. For notational ease, the EMP of a buyer of type $k$, considered by seller $i$ will be denoted by $\hat{p}(k, Q(p_{-i}))$; where $Q(p_{-i})$ is the true distribution of all rival prices with respect to a seller $i$.

\(^7\)This was used by Stigler (1961) to propose the strategic determination of optimal level of market search, by a rational buyer. By taking search to be non-strategic, the model here instead uses the EMP to typify each buyer and to feed back into the strategic interaction of price determination between buyer and seller.
The support of \( k \) is therefore \([0, \frac{I - 1}{I}]\) from the point of view of any seller, i.e. \( k \) can range from knowing zero rival stores, to the other extreme of knowing all rival stores (minus the seller itself). Note that the total number of sellers known by a buyer of type \( k \) is therefore \( Ik + 1 \) (including the seller itself). To interpret, a buyer \( j \) of information type \( k_j \), with respect to any seller, is aware of \( Ik_j \) number of rival prices in the market. For example, the least informed buyer \((k = 0)\) is unaware of all other stores in the market, while a perfectly informed buyer \((k = \frac{I - 1}{I})\) would know all rival prices \((Ik = I - 1)\). Moreover for example for a given seller, a prospective buyer of type \( k = \frac{2}{3} \) in a market of five sellers, knows three rival stores at random amongst the given four in the market, irrespective of the true prices and their distribution; i.e. each rival has a \(\frac{3}{4}\)th chance of being known by this buyer.

The cumulative distribution function of \( k \) is given by \( G(k) \), which is interpreted as the proportion of prospective buyers of type less than or equal to \( k \). The density of \( k \) is assumed to be continuous and Uniform. Therefore \( G(k) = \frac{kl}{I - 1} \in [0, 1] \).

Sellers thus face competition from rivals, but the level of competition faced varies with the (private) type of buyer in question. What kind of a trading interaction and price should a seller strategically offer, such that the types of buyers it targets, are persuaded to purchase; and the types that purchase, at the prices that they purchase, maximize its expected profit from such sales? Also, how is a seller’s cost relative to the market important in deciding its optimal trading interaction?

Both the posted price and/or the decision of a pricing rule are assumed to be taken together by a seller, and simultaneously by all sellers. Sellers are moreover assumed to know buyers’ value and therefore the size of the trade surplus, but a buyer is unaware of individual seller’s cost and therefore of the size of the trade surplus with any seller. The model thus closely resembles a consumer market, especially retail.

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8 This can be thought of as a finer information categorization of buyers as compared with Png & Hirshleifer (1987).
9 The seller in question (considering the buyer’s ‘other’ or rival information) is the remaining seller that the buyer knows.
10 The distribution is treated as continuous, such that for example, if \( k = \frac{2}{3} \) then it is interpreted as knowing one rival store for sure, and knowing the second rival store with probability \( \frac{1}{4} \).
11 An intuitive justification is that sellers expect each type of buyer to have equal probability, in the absence of more information.
3 Pricing Rules Considered

The three possible trading interactions or pricing rules that will be analyzed in this setup are - (1) the seller posts a price and gives the buyer the choice to accept or refuse (take it or leave it and purchase from elsewhere), (2) the seller invites a price from every buyer and retains the right to accept or refuse (each buyer separately) to sell at that price, (3) the seller posts a price and leaves the buyer with the choice of accepting, refusing, or invoking an existing lower rival price, at which the seller pre-commits to sell. Interaction (1) will be referred to as Take-it-or-leave-it (TILI); (2) will be referred to as Optimal Bid (OB); and (3) will be referred to as Price Matching (PM). In OB, a seller need not post any price. In PM on the other hand, the posted price may not necessarily be the price at which all sales are made. It is simply an upper bound on the price that will be charged; the lower bound being the minimum of all market prices. The claim is not that these three interactions exhaust all possibilities of trade, but that other types of interactions or rules (for eg. auctions, one-on-one haggling with back-and-forth bids and offers, etc.) would involve higher transaction or execution costs (time or effort), given the private information in the market.\footnote{Price beating is not considered here although the transaction cost in that would presumably be the same as in PM. It is left out because efficient price beating requires knowing exactly the level of a price cut that makes consumers behaviorally prefer (strictly) the beating store; thus introducing discrete price cuts or beating refunds that lack continuity and can thus make the mathematics cumbersome. A beating refund would also act as a tie-breaker if its optimal size is known (which may differ between buyers).}

The first two of these interactions are compared by Perry (1986) in a single buyer-single seller setting with multiple types (costs) of sellers and buyers (values). Extending it to large markets, this model replaces value types for buyers with information types, and modifies buyers' acceptance/rejection decisions accordingly.

For any store posting a price, prospective buyers compare the posted price with their information set (posted prices and policies of other stores that they are aware of). A well informed buyer who has information on a lower price elsewhere, would try to get that price in the current store if she is to purchase in this store at all. Otherwise she would simply buy from the lower priced store. TILI does not involve negotiations beyond the posted price. Therefore to attract such a buyer, the TILI posted price needs to be at least equal to (or less than), the buyer's outside option. But that may compromise on the high profit the seller could earn from another buyer whose outside option may be a much higher price.
This problem may have a solution in the price matching policy, such that the posted price (which is seen by both types of consumers) is set at a level to serve the less informed consumer, i.e. a high posted price, and secret discounts are given to more informed buyers. Just like in Png & Hirshleifer (1987) however, there must exist lower prices in the market that still give the seller positive profit on average, to use PM thus to effectively discriminate.

The model first needs to characterize prices at which buyers of different types will be able to make purchases in the market. Notice first that if all sellers adopt TILI, there will be no single-price equilibrium. That is, there will be some dispersion (variance) in prices. This is because with a common price for all sellers, a buyer regardless of type \( k \), would have the same EMP (equal to each seller’s posted price) and would purchase from any one of the stores known. That is, no store can be sure of a prospective buyer purchasing from it because there exist rivals known to the buyer who also offer the same price.

Any seller with a cost lower than this common price, would profit by an \( \epsilon \) price cut by beating the outside option of all its prospective buyers - thus ensuring that they each purchase from it for sure. On the other hand, if the common price is low enough for the lowest cost seller to have no incentive to undercut rivals (i.e. it is already selling at zero profit), then a higher cost seller can improve expected profit (or eliminate expected loss) by deviating to a higher price. Therefore we have

Lemma 1: If sellers with different costs each offer TILI and buyers are Uniformly distributed between different degrees of information as described, there cannot be a pure strategy equilibrium in which all sellers list the same price.

Proof: Consider all sellers listing the same price, \( p \). All buyers would be equally divided between all sellers; i.e. seller \( i \) earns \( \frac{(p-s_i)}{I} \).

If \( p > s_i \), for some seller \( i \), it can with an \( \epsilon \)-price cut, ensure that each prospective buyer purchases from it, increasing sales by a great deal while the price cut is of degree zero. The expected profit with such a price cut is as follows, such that the integral is the expected proportion of buyers who know a seller\(^{13}\)

\[
(p - \epsilon - s_i) \int_0^{I-1} \frac{Ik + 1}{I-1} \, dk = \left( \frac{I + 1}{2I} \right) (p - \epsilon - s_i)
\]

\(^{13}\)Buyer type \( k \) would know a seller with probability \( \frac{Ik + 1}{I} \), and that is multiplied with the density of \( k \).
For all $I > 2$, the above deviation profit exceeds the profit from a common price $(\frac{p-s}{I})$, for some infinitesimally small price cut $\epsilon > 0$.

If however, $p \leq s$ such that for no seller does a small profitable price cut exist, then $(\frac{p-s}{I}) \leq 0$. This implies that all sellers with cost higher than $s$, earn losses at the common price, and can therefore profitably deviate to a higher price. Therefore $p$ cannot be an equilibrium. QED.

An implication of the lemma is that in an exclusively TILI market, buyers having different information types and sellers varying in costs necessarily implies a dispersion of prices in equilibrium. The rest of this paper uses Lemma 1 as the basis of assuming a variation in posted prices in any equilibrium in the market that uses posted prices.

Keeping in mind the nature of simultaneous play by all sellers, it is assumed that for any seller making a choice of which types of buyers to target sales toward, $Q(p-i)$ is considered exogenous. Therefore $\hat{p}(k, Q(p-i))$ from a seller’s point of view, is a function only of the type, $k$ of a buyer. That is, $\hat{p}(k_j, Q(p-i))$ is what seller $i$ ex ante expects buyer $j$’s EMP to be, without information on the identity of stores the buyer actually knows.

A dispersion of prices in the markets from Lemma 1 gives a strict inverse relationship between the EMP and the type of a buyer:

$$\frac{\partial (\hat{p}(k, Q(p-i)))}{\partial (k)} < 0, \ \forall \ k \in [0, \frac{I-1}{I}).$$

That is, a higher type of buyer (with information on a larger number of rival stores in the market) has a strictly lower EMP. As a result, if a store lists a price higher than $\hat{p}(k_j, Q(p-i))$, the store will ex ante expect to fail to persuade a buyer of type $k_j$ (and all higher types) to purchase from it. On the other hand, with a price lower than or equal to $\hat{p}(k_j, Q(p-i))$, seller $i$ hopes to successfully sell to a buyer of type $k_j$, and all lower types, because it is equal to or lower than their EMP. This is stated as the following lemma, without proof.

**Lemma 2:** In a market with price dispersion, to target to sell to a buyer of type $k_j$ at posted price, a store must list price at or below $\hat{p}(k_j, Q(p-i))$. Lower types of buyers ($k \leq k_j$) are also expected to purchase.

Effectively then, in posting a price a seller actually chooses which types of buyers to target selling to. Equivalently, in posting a price a seller chooses
the probability of making a sale, given the distribution of buyer types. There is the usual trade-off between a higher price or a higher probability of making a sale. It is now possible to construct the various trade interactions a seller can offer. Equilibria will be characterized as every seller choosing a strategy that maximizes expected profit from a buyer, given rivals’ strategies and the known distribution of buyer types, keeping in mind that every seller can choose one of the three interactions or rules. To keep the model tractable, I first analyze the market if every seller were to use TILI, and if every seller were to use OB. Then comparing between the two for each seller, I establish each seller’s choice between TILI and OB. And finally, because TILI and PM differ only in allowing matching of rival prices\(^{14}\), I question each seller’s incentive to deviate from an all-TILI market by adopting PM, and thereby describe the market that would result in which both TILI and PM are used.

3.1 Optimal Prices in TILI

Consider the market if all sellers offer TILI. In a market with an existing price dispersion, a seller’s prospective buyer of a certain type can be characterized with an outside option that ex ante equals her EMP. Although the seller does not know the type of the buyer, knowing the distribution of types of buyers in the market it can through its choice of posted price, target to sell to certain buyer types. It would then try to maximize its expected profit from sales to the types of buyers it targets selling to. Therefore, by posting a price, a seller is in effect choosing the marginal buyer type who will be attracted to purchase (intuitively it can also be thought as if each seller is choosing its rank in the market price distribution). Let \( k_i \) be defined as the marginal target buyer type (and lower types, by Lemma 2) that a seller \( i \) chooses to target, with a posted price \( \hat{p}(k_i, Q(p_{-i})) \). The optimal choice of \( k_i \) must be seller \( i \)'s best response to its rival prices in the market.

The expected profit from every buyer\(^{15}\) in an all-TILI interaction would be:

\[
\pi^{TILI} = G(k_i)[\hat{p}(k_i, Q(p_{-i})) - s_i], \tag{3}
\]

with belief that buyer types \( k \leq k_i \) accept \( p = \hat{p}(k_i, Q(p_{-i})). \)

\(^{14}\)That is, TILI results if given a choice of PM a seller chooses zero price matching.

\(^{15}\)The proportion of buyers that know any seller is random and therefore a function only of the number of sellers, and therefore plays no role in the optimization function.
That is, choosing a target buyer type at the margin requires that the seller lists a price no larger than that type’s EMP. The first order derivative with respect to $k_i$ gives:

$$\frac{\partial \pi^{TILI}}{\partial k_i} = G'(k_i)[\hat{p}(k_i, Q(p-i)) - s_i] + G(k_i) \frac{\partial \hat{p}(k_i, Q(p-i))}{\partial k_i}$$

Setting this marginal profit equal to zero, gives us the following FOC\(^{16}\), the solution to which gives the optimal $k_i$ which will be denoted as $k_i^{TILI}$.

$$\hat{p}(k_i, Q(p-i)) - s_i = -\frac{G(k_i)}{G'(k_i)} \frac{\partial \hat{p}(k_i, Q(p-i))}{\partial k_i}$$

(4)

The LHS of the FOC is simply the profit from a successful trade or the seller’s price-cost margin, and it decreases in $k_i$, as selling to a more informed buyer implies that the price that can then be charged, must be lowered (from (2)). The RHS on the other hand, is the product of the change in price needed to target a higher buyer type, and the density weighted probability of making a successful trade\(^{17}\).

For all $k$ as defined, $G(k) \geq 0$; and by (2), the partial derivative on the RHS is negative at all $k < \frac{I-1}{I}$. Therefore for all sellers targeting $k < \frac{I-1}{I}$, the FOC requires that the price-cost margin is non-negative at the optimal $k_i$. Also because (2) is not defined\(^{18}\) at $k = \frac{I-1}{I}$, we need to talk about posting the lowest price in the market as being that seller’s best response separately from the above FOC.

**Lemma 3:** The optimal $k_i$, or $k_i^{TILI}$ is unique for each seller.

**Proof:** From the Uniform distribution of $k$, we have

$$\frac{G(k)}{G'(k)} = k \in [0, \frac{I-1}{I}]$$

\(^{16}\)The second order condition is satisfied for a maximum, as the second derivative is $2G'(k_i)\frac{\partial \hat{p}(k_i, Q(p-i))}{\partial k_i} + G(k_i)\frac{\partial^2 \hat{p}(k_i, Q(p-i))}{\partial k_i^2}$ which can be reduced to $\frac{I}{I-1} \frac{\partial \hat{p}(k_i, Q(p-i))}{\partial k_i} + \frac{k_i}{I-1} \frac{\partial^2 \hat{p}(k_i, Q(p-i))}{\partial k_i^2}$, a negative expression because $2I > Ik, \forall k$ as $k$ is defined strictly in the interval $[0, \frac{I-1}{I}]$ and the first and second derivative of the EMP are negative and positive respectively.

\(^{17}\)This is also known as the Mill’s ratio.

\(^{18}\)The right hand limit is undefined as the EMP is not continuous at full information.
That is, \( \frac{G(k)}{G'(k)} < 1 \), \( \forall k \).

From statistical properties of the EMP (first order statistic) of a random sample (without replacement) from a population, we have \( \frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} \) increases in \( k_i \) (i.e. becomes less negative\(^{19}\)). That is, \( \hat{p}(k_i, Q(p_{-i})) \) is a convex (to the origin) decreasing function of \( k_i \), which implies that increases in \( k_i \) have larger effects on reducing \( \hat{p}(k_i, Q(p_{-i})) \) when the base \( k_i \) is small.

\[
\frac{\partial^2 \hat{p}(k_i, Q(p_{-i}))}{\partial k_i^2} > 0, \quad \forall k_i < \frac{I - 1}{I}
\]

(6)

Moreover, this second derivative is always no greater than the absolute value of the slope of the EMP. Otherwise, the slope of the EMP would be positive at some \( k < 1 \), which is impossible. Therefore,

\[
\left| \frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} \right| \geq \frac{\partial^2 \hat{p}(k_i, Q(p_{-i}))}{\partial k_i^2}, \quad \forall k_i < \frac{I - 1}{I}
\]

(7)

From (5), (6) and (7) we have

\[
\left| \frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} \right| > \frac{G(k_i)}{G'(k_i)} \frac{\partial^2 \hat{p}(k_i, Q(p_{-i}))}{\partial k_i^2} \geq 0
\]

(8)

Notice that the derivative of the RHS of (4) with respect to \( k_i \) is

\[
\frac{\partial {\text{RHS}}}{\partial k_i} = -\frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} - \frac{G(k_i)}{G'(k_i)} \frac{\partial^2 \hat{p}(k_i, Q(p_{-i}))}{\partial k_i^2}
\]

Using (2), it follows from (8) that the RHS of (4) strictly increases in \( k_i \). The LHS of (4) strictly decreases in \( k_i \), from (2). Therefore, the optimal \( k_i \), or the intersection of the RHS and LHS, is unique. QED.

Notice also, that this optimal decision is a function of the seller’s cost, \( s_i \), and is a best response to its rival prices in the market, \( Q(p_{-i}) \). That is, \( k_i^{\text{TILI}}(s_i, Q(p_{i-1}^{\text{TILI}})) \) is seller \( i \)’s best response function, such that its posted

\(^{19}\)Stigler (1971) mentions (quotes Solow) such an expectation for sampling with replacement; the properties of the derivatives of the expectation carry through to sampling without replacement.
price then is \( \hat{p}(k_{Ti}^{TILL}, Q(p_{Ti}^{TILL})) \). Because optimal \( k_i \) for a seller is not simply a function of its cost, the following lemmas are needed to address some important questions in characterizing the market equilibrium.

**Lemma 4:** For any sellers \( a \) and \( b \), in equilibrium, \( p_a^{TILL} = p_b^{TILL} \) iff \( k_a^{TILL} = k_b^{TILL} \).

**Proof:** Step 1: First, note that \( p_a^{TILL} = p_b^{TILL} \) is equivalent to \( Q(p_{a}^{TILL}) = Q(p_{b}^{TILL}) \), i.e. if two sellers list the same price in equilibrium, then the set of market prices excluding each of them (but including the price of the other) is the same for both. Similarly, \( p_a^{TILL} > p_b^{TILL} \) is equivalent to \( Q(p_{a}^{TILL}) \) first order stochastically dominating (FOSD) \( Q(p_{b}^{TILL}) \), because exactly one price in \( Q(p_{a}^{TILL}) \) exceeds that in \( Q(p_{b}^{TILL}) \) while all other prices are identical in both sets.

Step 2: There are exactly three possibilities in comparing the prices of the two sellers, \( p_a^{TILL} = p_b^{TILL} \), \( p_a^{TILL} < p_b^{TILL} \), and \( p_a^{TILL} > p_b^{TILL} \). Let \( k_a^{TILL} = k_b^{TILL} = k^{TILL} \), i.e. the target buyers for both sellers have the same degree of information about the rest of the market. This implies that if these target buyers face the same ‘rival’ market, they should have the same EMP. And the one facing the market with exactly one lower (higher) price should have a strictly lower (higher) EMP. That is,

\[
\hat{p}(k^{TILL}, Q(p_{a}^{TILL})) = \hat{p}(k^{TILL}, Q(p_{b}^{TILL})), \text{ iff } Q(p_{a}^{TILL}) = Q(p_{b}^{TILL});
\]

and

\[
\hat{p}(k^{TILL}, Q(p_{b}^{TILL})) > \hat{p}(k^{TILL}, Q(p_{a}^{TILL})), \text{ iff } Q(p_{b}^{TILL}) \text{ FOSD } Q(p_{a}^{TILL});
\]

But \( Q(p_{a}^{TILL}) = Q(p_{b}^{TILL}) \) is equivalent to \( p_a^{TILL} = p_b^{TILL} \), from Step 1, above. And \( Q(p_{b}^{TILL}) \) FOSD \( Q(p_{a}^{TILL}) \) is equivalent to \( p_a^{TILL} > p_b^{TILL} \). Therefore,

\[
\hat{p}(k^{TILL}, Q(p_{a}^{TILL})) = \hat{p}(k^{TILL}, Q(p_{b}^{TILL})), \text{ iff } p_a^{TILL} = p_b^{TILL}; (9)
\]

\( p_a^{TILL} > p_b^{TILL} \)
and

\[ \hat{p}(k^{TILI}, Q(p_{-a}^{TILI})) > \hat{p}(k^{TILI}, Q(p_{-b}^{TILI})), \text{ iff } p_{a}^{TILI} > p_{b}^{TILI}; \]  

(10)

But (10) is a contradiction because by definition, \( p_i \) is equal to \( \hat{p}(k^{TILI}, Q(p_{TILI} - b)) \), and therefore \( \hat{p}(k^{TILI}, Q(p_{-a}^{TILI})) > \hat{p}(k^{TILI}, Q(p_{-a}^{TILI})) \) is by definition equivalent to \( p_{a}^{TILI} > p_{b}^{TILI} \). Therefore, \( k_{TILI}^a = k_{TILI}^b = k^{TILI} \) implies that in equilibrium, neither price can be greater than the other, i.e. they must be equal.

Step 3: For the converse, let \( p_{a}^{TILI} = p_{b}^{TILI} \). By definition, this means

\[ \hat{p}(k^{TILI}, Q(p_{-a}^{TILI})) = \hat{p}(k_{b}^{TILI}, Q(p_{-b}^{TILI})). \]  

(11)

And, from Step 1 above, this is equivalent to:

\[ Q(p_{-a}^{TILI}) = Q(p_{-b}^{TILI}). \]  

(12)

Equations (11) and (12) are both reconcilable only if \( k_{TILI}^a = k_{TILI}^b \) because different information types would lead to different EMPs from the same set of prices. QED.

**Lemma 5:** For any sellers \( a \) and \( b \), in equilibrium, \( p_{a}^{TILI} > p_{b}^{TILI} \) iff \( k_{TILI}^a < k_{TILI}^b \).

**Proof:** Consider, \( p_{a}^{TILI} > p_{b}^{TILI} \). The inequality is equivalent to \( \hat{p}(k_{a}^{TILI}, Q(p_{-a}^{TILI})) > \hat{p}(k_{b}^{TILI}, Q(p_{-b}^{TILI})) \) and \( Q(p_{-b}^{TILI}) \ FOSD \ Q(p_{-a}^{TILI}) \). But these are both reconcilable together only if \( k_{TILI}^a > k_{TILI}^b \), because the same information types or a \( k_{TILI}^a \) higher than \( k_{TILI}^b \) would lead to \( \hat{p}(k_{a}^{TILI}, Q(p_{-a}^{TILI})) < \hat{p}(k_{b}^{TILI}, Q(p_{-b}^{TILI})) \), given \( Q(p_{-b}^{TILI}) \ FOSD \ Q(p_{-a}^{TILI}) \).

For the converse, let \( k_{TILI}^a < k_{TILI}^b \) therefore. From the three possibilities: \( p_{a}^{TILI} = p_{b}^{TILI} \), \( p_{a}^{TILI} < p_{b}^{TILI} \), and \( p_{a}^{TILI} > p_{b}^{TILI} \), the first two are ruled out because each gives rival sets of prices from which a lower information type cannot lead to the same or a lower EMP, leading to a contradiction.

Therefore, \( k_{TILI}^a < k_{TILI}^b \implies p_{a}^{TILI} > p_{b}^{TILI} \). QED.

Notice that Lemmas 4 & 5 do not really use the FOC in (4), but rather, are results of the properties of the EMP function and the assumption that
in equilibrium each seller chooses a target buyer type at the margin, and lists the EMP of that target buyer given the prices that other sellers list. The following theorem therefore uses the FOC and interprets the lemmas with respect to the sellers’ cost heterogeneity.

**Theorem 1:** In an all-TILI market equilibrium, for any sellers \(a\) and \(b\), if \(s_a < s_b\), then it must be that \(k_a^{TILI} > k_b^{TILI}\) and therefore \(p_a^{TILI} < p_b^{TILI}\).

**Proof:** Take any two sellers \(a\) and \(b\), such that \(s_a < s_b\).

First, suppose that in equilibrium, \(k_a^{TILI} = k_b^{TILI} = k^{TILI}\). Then from Lemma 3, \(p_a^{TILI} = p_b^{TILI}\). From each seller’s FOC as given by (4), and using \(p_i^{TILI} = \hat{p}(k_i^{TILI}, Q(p_i^{TILI} - a))\), \(\forall i\), we therefore have \(p_a^{TILI} - s_a = p_b^{TILI} - s_b\).

But \(p_a^{TILI} = p_b^{TILI}\), while \(s_a < s_b\), which is a contradiction. Therefore, \(k_a^{TILI} \neq k_b^{TILI}\).

Consider next, in equilibrium, \(k_a^{TILI} < k_b^{TILI}\), which therefore also gives \(p_a^{TILI} > p_b^{TILI}\) from Lemma 4. But because \(p_a^{TILI} > p_b^{TILI}\), seller \(a\) faces a first order stochastically dominated rival market price distribution compared with that faced by seller \(b\); i.e. for a common \(k\), we have \(\hat{p}(k, Q(p_a^{TILI} - b)) < \hat{p}(k, Q(p_b^{TILI} - b)), \forall k < \frac{I-1}{I}\), with approximate equality when \(k = \frac{I-1}{I}\). An implication of this is that \(\hat{p}(k, Q(p_a^{TILI} - b))\) falls more steeply in \(k\), than does \(\hat{p}(k, Q(p_b^{TILI} - b))\). That is,

\[
|\frac{\partial \hat{p}(k, Q(p_a^{TILI} - b))}{\partial k}| < |\frac{\partial \hat{p}(k, Q(p_b^{TILI} - b))}{\partial k}|, \forall k < \frac{I-1}{I}. \tag{13}
\]

On the other hand, from \(s_a < s_b\) and \(p_a^{TILI} > p_b^{TILI}\), we have \(p_a^{TILI} - s_a > p_b^{TILI} - s_b\). From the FOC (4), and using \(\frac{G(k)}{G'(k)} = k\), the RHS of the FOCs of sellers \(a\) and \(b\) can be related in equilibrium as follows:

\[
k_a^{TILI} |\frac{\partial \hat{p}(k_a^{TILI}, Q(p_a^{TILI} - b))}{\partial k_a^{TILI}}| > k_b^{TILI} |\frac{\partial \hat{p}(k_b^{TILI}, Q(p_b^{TILI} - a))}{\partial k_b^{TILI}}|. \tag{14}
\]

Also from (2) and (8), we have that the RHS of (4) increases in \(k\); which gives for prices faced by seller \(b:\)

\[
k_b^{TILI} |\frac{\partial \hat{p}(k_b^{TILI}, Q(p_b^{TILI} - a))}{\partial k_b^{TILI}}| > k_a^{TILI} |\frac{\partial \hat{p}(k_a^{TILI}, Q(p_a^{TILI} - b))}{\partial k_a^{TILI}}|. \tag{15}
\]
Together, (14) and (15) imply \( k_a^{\text{TILI}} \left| \frac{\partial \hat{p}(k_a^{\text{TILI}}, Q(p_{TILI}^{TILI}))}{\partial k_a^{\text{TILI}}} \right| > k_b^{\text{TILI}} \left| \frac{\partial \hat{p}(k_b^{\text{TILI}}, Q(p_{TILI}^{TILI}))}{\partial k_b^{\text{TILI}}} \right| \), which contradicts (13), given \( k_a^{\text{TILI}} < k_b^{\text{TILI}} \). Therefore if \( s_a < s_b \), equilibrium cannot have \( k_a^{\text{TILI}} \) less than \( k_b^{\text{TILI}} \). It must therefore be that \( k_a^{\text{TILI}} > k_b^{\text{TILI}} \), which is equivalent to \( p_a^{\text{TILI}} < p_b^{\text{TILI}} \) (from Lemma 5). QED.

Theorem 1 tells us that in an exclusive TILI equilibrium sellers will maintain their cost ranks with their posted prices. This is similar to what Carlson & McAfee (1983) found although theirs was a model of strategic sequential search where consumers search knowing ex ante the exact distribution and values of the prices in the market. Another important result that follows from the theorem, with an extension of the same argument, is that in equilibrium a lower cost seller must have a higher price-cost margin.

**Corollary 1:** If for sellers \( a \) and \( b \), \( s_a < s_b \), then it must be that in an all-TILI equilibrium seller \( a \) has a larger price-cost margin; i.e. \( p_a^{\text{TILI}} - s_a > p_b^{\text{TILI}} - s_b \).

**Proof:** Let \( s_a < s_b \). From Theorem 1, we therefore have \( k_a^{TILI} > k_b^{TILI} \), or equivalently \( p_a^{TILI} < p_b^{TILI} \). This implies that for a common \( k \), we have \( \hat{p}(k, Q(p_{TILI}^{a})) > \hat{p}(k, Q(p_{TILI}^{b})) \), \( \forall k < \frac{I-1}{I} \), and the two are approximately equal when \( k = \frac{I-1}{I} \). Also \( \hat{p}(k, Q(p_{TILI}^{a})) \) falls more steeply in \( k \), than does \( \hat{p}(k, Q(p_{TILI}^{b})) \). That is,

\[
\left| \frac{\partial \hat{p}(k, Q(p_{TILI}^{a}))}{\partial k} \right| > \left| \frac{\partial \hat{p}(k, Q(p_{TILI}^{b}))}{\partial k} \right|, \forall k < \frac{I-1}{I}. \tag{16}
\]

There are two possibilities regarding the relative price cost margins: \( p_a^{\text{TILI}} - s_a < p_b^{\text{TILI}} - s_b \) and \( p_a^{\text{TILI}} - s_a > p_b^{\text{TILI}} - s_b \). Consider the first. From the FOC (4), we therefore have

\[
k_a^{\text{TILI}} \left| \frac{\partial \hat{p}(k_a^{\text{TILI}}, Q(p_{TILI}^{a}))}{\partial k_a^{\text{TILI}}} \right| < k_b^{\text{TILI}} \left| \frac{\partial \hat{p}(k_b^{\text{TILI}}, Q(p_{TILI}^{b}))}{\partial k_b^{\text{TILI}}} \right|. \tag{17}
\]

We also know from (2) and (8) that the RHS of the FOC increases in \( k \); i.e. \( k_b^{\text{TILI}} \left| \frac{\partial \hat{p}(k_b^{\text{TILI}}, Q(p_{TILI}^{b}))}{\partial k_b^{\text{TILI}}} \right| < k_a^{\text{TILI}} \left| \frac{\partial \hat{p}(k_a^{\text{TILI}}, Q(p_{TILI}^{a}))}{\partial k_a^{\text{TILI}}} \right| \). But this together with (17), is a contradiction to (16). Therefore, it must be that \( p_a^{\text{TILI}} - s_a > p_b^{\text{TILI}} - s_b \). QED.
Because equilibrium behavior can be defined by the FOC above only for optimal choice of \( k \) less than \( \frac{I-1}{I} \), the model still needs to define the behavior of the seller targeting to sell to all prospective buyers; such that its marginal target buyer is \( k = \frac{I-1}{I} \). This seller must list the (strictly) lowest price in the market and only by doing so, chooses to sell to buyers of all information types. Given Theorem 1 an equilibrium would exist only if the lowest cost seller in the market has such a best response\(^{21}\) while all others choose lower buyer types to target, according to their FOCs, and such that their posted prices maintain their cost ranks.

Therefore in any equilibrium the all-TILI market has multiple prices. On the other hand, the seller targeting \( k = 0 \), lists the highest price in the market, as it chooses to sell only to consumers unaware of all rival prices. This buyer will buy at any price, as long as it does not exceed her value for the good. Therefore the seller targeting only this buyer must sell at \( v \). However, \( G(k = 0) = 0 \)\(^{22}\) i.e. for a seller targeting a completely ignorant buyer the RHS of the FOC \( (4) \) is zero. This implies that this decision is optimal only when the LHS is zero as well. In other words, the seller is indifferent between selling and not selling, and makes zero profit in its optimal decision. This defines the limit to entry in the market; i.e. \( s = v \). In turn, the number of sellers in the market depends on \( s - s_1 \) or equivalently on \( v - s_1 \). The following summarizes the price equilibrium in an all-TILI market:

**Result 1:** If sellers can only offer TILI, the marginal entrant (seller) in the market has cost \( s = v \) and posts price equal to \( v \), making zero profit. All other sellers post prices strictly greater than their cost but not exceeding \( v \).

This makes sense because there are a variety of partly informed buyers and posting price equal to cost is for all sellers, dominated by a higher price serving the less informed buyers; while posting a price equal to the value of the buyers will always lose out to some rival price. Only the marginal entrant (the highest cost seller) with cost equal to buyers’ value cannot afford to sell at positive profit.

\(^{21}\)A strictly lower price than all rivals turns out to be its best response only if at such a price it includes another group of buyers (with possibly lower value) who are not served by rivals.

\(^{22}\)The assumption of a continuous distribution for \( k \) leaves no mass probability at any point.
Notice that it hasn’t been claimed that an equilibrium exists in the all-TILI market. The question of interest is whether the equilibrium - if it exists - is robust to a wider definition of sellers’ strategies; i.e. when each could choose a pricing rule too and not just the optimal price to post.

3.2 Inviting Optimal Bids

I extend Perry’s (1986) argument to a market with multiple buyers and sellers if sellers were to invite bids from buyers and then choose to accept or decline to purchase at the bid price. Consider a seller offering OB to every prospective buyer, while all other sellers also offer the same. A buyer would then expect a seller to accept the bid as long as the bid is no lower than the seller’s cost, given the assumption of no capacity constraints for the sellers.

Believing this, every buyer would make a bid that maximizes her expected utility from purchase. Assuming all buyers know the distribution of sellers’ costs (equivalently, they may have the same beliefs about the cost distribution), the optimal bid of each buyer would be the same to every seller and would solve the following:

\[ b^* = \arg\max_b F(b)(v_h - b); \quad \text{with belief that seller type } s \text{ accepts } b \geq s \]

which gives the FOC

\[ (v - b^*)F'(b^*) = F(b^*) \]  \hspace{1cm} (18)

The LHS is the marginal benefit of quoting a higher bid because it increases the chance of successful purchase by \( F''(b^*) \), and the buyer gets a payoff equal to \( v - b^* \) from a successful purchase; and the RHS is its marginal cost; i.e. the probability that the seller is overpaid because its cost is lower than the bid.

Because the buyer believes such an optimal bid \( b^* \) is unacceptable to sellers with higher costs in the market (it involves selling at a loss), she expects to be refused by all sellers with cost, \( s_i > b^* \). This also implies that buyers expect (correctly) that these sellers can be making no sales in an all-OB market. That is, in an all-OB market only sellers with cost \( s_i \) lower than \( b^* \) would sell, and would then make profit \( b^* - s_i \) per buyer.
3.3 Choosing Between TILI and OB

From section 3.1 above, sellers (other than the highest cost seller) make positive profits in an all-TILI market equilibrium. And from section 3.2 above, only sellers with cost lower than $b^*$ can make positive profit in an all-OB market. Therefore, choosing between TILI and OB could lead only low cost sellers to prefer OB such that selling at $b^*$ involves a profit higher than that earned from TILI.

Buyers should expect this; i.e. if buyers know that sellers have a choice between offering OB and offering TILI, they expect only low cost sellers to prefer offering OB and selling at $b^*$ compared with selling in a TILI interaction. The observation of OB by a buyer therefore signals to a buyer that the seller’s cost is no higher than some threshold $\hat{s}$ where $\hat{s} < b^*$; thus truncating the distribution of believed seller costs (if OB is offered) accordingly.

A buyer’s optimal bid then, is appropriately low, no longer attractive even to sellers who the buyer earlier believed would prefer OB from actually doing so. This truncates the distribution further, and the argument unravels the optimal bid all the way down to $s$ or $s_1$; still failing to sustain equilibrium as at this bid no seller prefers to offer OB. A kind of market failure occurs because of the adverse selection of bids by offering OB. Therefore no seller prefers OB to TILI, and this theorem is stated without proof.\footnote{The proof in Perry (1986) also renders a repetition unnecessary.}

**Theorem 2:** No seller will choose to offer OB when the option of offering TILI is available to all.

Thus eliminating the possibility of voluntary adoption of OB by all sellers, the question of choice of pricing rules is reduced to posting a price with or without accommodations to match rival prices.

3.4 Price Matching as a Deviation from TILI

The adoption of a price matching policy by a seller is not a decision separable from its choice of posted price. A seller would choose a PM posted price to optimize its expected profits from a PM interaction, and therefore a PM posted price could differ from a TILI posted price. Moreover while choosing an optimal PM posted price, a seller has the inherent option to choose zero price matching, in which case the seller effectively ends up choosing TILI.
and not PM. This is more important in a market where sellers differ in cost because for example the highest cost seller does not want to unconditionally commit to matching all rivals’ prices. Therefore it is unreasonable to analyze the market restricted to PM for each seller. It thus makes better sense to investigate the choice between the two, starting from an all-TILI market instead, in the form of a possible deviation.

Let the rest of the market still offer TILI such that the rival market price distribution remains unchanged from that discussed in the last section; therefore the function \( \hat{p}(k_i, Q(p_{-i})) \) also remains unchanged for each \( k_i \), for a given seller\(^{24}\). Giving a seller the choice of PM thus adds additional terms to its seller’s profit function representing sales due to matching. The PM (alone) profit function is therefore the TILI function with added terms for expected price matching,

\[
\pi_i^{PMalone} = G(k_i)[\hat{p}(k_i, Q(p_{-i}))-s_i] + \delta_1 \int_{k_i}^{k_{i+1}} G'(k)[\hat{p}(k, Q(p_{-i}))-s_i]dk.
\]  

(19)

By offering PM, a seller’s profit is made up of two components: the profit from selling to the target and lower types of buyers at the posted price, and that from selling to higher types of buyers at their invoked matches. That is, PM opens the store to all higher (than target) types of buyers; promising to sell to them at their best known price which can be approximated by the seller ex ante, as the buyers’ EMPs. Only lower prices existing in the market are invoked, by buyers who are aware of these.

The discount factor \( \delta_1 \) adjusts for the uncertainty that a match will be invoked where a case for matching exists. In the extreme state of matching being costless, the match would be expected to be invoked with a maximum probability\(^{25}\) of 1/2, because the strategy of playing PM alone, implies no other seller in the buyer’s information set offers a match. However, if some or all buyers have any hassle costs of matching (Hviid & Shaffer 1999), the probability of a match being invoked will be strictly less than half. Therefore, \( \delta_1 \leq \frac{1}{2} \). Notice also that if \( \delta_1 \) is really small, offering PM becomes irrelevant as matches will hardly ever be invoked. The variable \( \delta_1 \) is inversely related to any hassle costs for consumers in invoking and claiming price matches. That is, a seller’s benefit from offering PM alone is significantly

\(^{24}\)As before, this analysis is true only for a seller other than the lowest cost seller, whose price defines the floor of the market price distribution.

\(^{25}\)In case the buyer is considered to be indifferent between invoking matching and purchasing at the lower priced store.
larger, if the matching process is made painless for consumers.\footnote{This paper does not however model $\delta_1$ as a choice variable because it is difficult to separate a consumer’s perception of matching in the general market, from a particular seller’s reputation for easy matching.}

Notice that PM is a linear combination of TILI and OB with a buyer type specific floor\footnote{The realized best outside option in fact becomes such a floor for any buyer, but because this is private information, ex ante judgments by a seller use the EMP for every buyer type as a proxy for this.} automatically imposed on OB by requiring the price to exist as a rival price, and the probability of whether a buyer takes the posted price or bids a lower rival price also a function of the buyer’s type. That is, offering PM to a buyer type $k$ (where type is still private) is for the seller, equivalent to making a sale with expected price equal to $\lambda(\text{posted price}|\text{posted price} \leq \text{EMP}) + (1-\lambda)(\text{EMP}|\text{posted price} > \text{EMP})$; where $\lambda$ is a decreasing function of $k$. This is equivalent to a linear combination of TILI and a censored OB, because in TILI the buyer must take the posted price or leave, and offering OB leads to the lowest bid from the buyer that she can make. That is in theory, the policy of price matching is equivalent to announcing to buyers the following - “you can buy at our posted price, or if you don’t like that we invite your bid as long as it exists in the market as a rival price”.

Moreover, even if such an offer alone still signals a low cost similar to what a similar choice of OB does, the buyer is restrained from exploiting that information. That is, such an offer still leads the buyer to bid the lowest price possible and thus retains some adverse selection. But this is much reduced because the lowest price known (depending on the buyer’s information of rival prices) may still be much greater than the cost the buyer attributes to the seller (given a low cost signal).\footnote{Such a signal also differs greatly from a signal of lowest price, as claimed in Moorthy & Winter (2006) as it gives the buyer no additional information regarding the association of price and cost. This is especially the case as in large markets, multiple sellers are seen to offer such policies, and the existence of invoked price matching is evidence that buyers do not associate them with the lowest price.}

The FOC for the optimization problem would give

$$G'(k_i)[\hat{p}(k_i, Q(p_{-i})) - s_i] + G(k_i) \frac{\partial \hat{p}(k, Q(p_{-i}))}{\partial k_i} - \delta_1 G'(k_i)[\hat{p}(k_i, Q(p_{-i})) - s_i] = 0,$$
which can be reduced to

\[(1 - \delta_1)[\hat{p}(k_i, Q(p_{-i})) - s_i] = -\frac{G(k_i)}{G'(k_i)} \frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} . \quad (20)\]

Comparing with (4), the FOC above is different only in the factor \((1 - \delta_1)\) multiplying the LHS. Intuitively the tradeoff in targeting a marginally higher buyer type is similar to that in TILI except that now types higher than the target are not lost but are also expected to purchase invoking a match each, but may not do so with probability \((1 - \delta_1)\).

From (4), it is known that

\[\hat{p}(k_i, Q(p_{-i})) - s_i] = -G(k_i) \frac{\partial \hat{p}(k_i, Q(p_{-i}))}{\partial k_i} \text{ at } k_i^{TILI}. \]

And given that \(\frac{1}{2} \leq 1 - \delta_1 < 1\), we have that offering PM alone enables a seller to lower its target buyer type and therefore raise its optimal posted price, i.e.

\[k_i^{PMalone} < k_i^{TILI} \quad (21)\]

The choice to deviate from an all-TILI market to offer PM is characterized in the following theorem.

**Theorem 3:** In a market where all other sellers offer TILI, a seller would prefer to deviate and offer PM alone only if it expects to earn an average positive profit from the transactions in which it matches other prices. That is, \(k_i^{PMalone} < k_i^{TILI}\) exists only if

\[
\delta_1 \int_{k_i^{PMalone}}^{k_i^{TILI}} G'(k)|\hat{p}(k, Q(p_{-i})) - s_i|dk > 0.
\]

**Proof:** Let \(\delta_1 \int_{k_i^{PMalone}}^{k_i^{TILI}} G'(k)|\hat{p}(k, Q(p_{-i})) - s_i|dk \leq 0\), given that the chosen target buyer type is \(k_i^{PMalone}\). The seller would then rather withdraw the offer to match and simply offer TILI without changing its posted price. But that implies that the seller’s profit is the function, \(G(k_i)|\hat{p}(k_i, Q(p_{-i})) - s_i|\), which cannot be maximized at \(k_i^{PMalone}\) because its argument maximizer is \(k_i^{TILI}\) from (4). That is, if \(\delta_1 \int_{k_i^{PMalone}}^{k_i^{TILI}} G'(k)|\hat{p}(k, Q(p_{-i})) - s_i|dk \leq 0\), then the optimization problem for the seller leads it to choose \(k_i^{TILI}\) and not \(k_i^{PMalone}\). Therefore, a deviation to PM in a market where all others offer TILI is profitable for a seller only if \(\delta_1 \int_{k_i^{PMalone}}^{k_i^{TILI}} G'(k)|\hat{p}(k, Q(p_{-i})) - s_i|dk > 0\). QED.

But if this is true for a seller, then it must be that the seller’s cost is smaller than some threshold level, because only then can the sales with
invoked matching add positive profit on average (as prices are bounded above by the highest cost).

**Corollary 2:** From an all TILI market, there exists a threshold cost $s^t$ such that $v > s^t > s_1$ and no seller with cost greater than this threshold has incentive to deviate and offer PM.

**Proof:** Theorem 1 gives a correlation between cost ranks and price ranks in a purely TILI market. And Theorem 3 gives that a seller prefers to deviate from a TILI market to offer PM, only if its expected profit from delivering lower prices known to consumers, on average, is positive.\(^{29}\) Moreover from Result 1 and Corollary 1, all (but the highest cost) sellers in an all-TILI market make positive profit and the price-cost margin is higher for a lower cost seller. Therefore at least some low cost sellers will always find posted prices by their lower cost rivals in an all-TILI market to be above their own cost, and thus deviating to PM alone is profit improving for them if the expected profit from matching EMPs above own cost is greater than the expected loss of matching prices lower than own cost.

But it is clear that for the seller with cost $s = v$, offering PM is not profit improving as all rival prices are below its cost. For other higher cost sellers, from Corollary 1 the price-cost margins are lower. Plus, a higher cost seller targets fewer buyer types with its posted price. Together therefore a higher cost seller has a smaller margin of positive profit to offset the losses that may result in some matching interactions. And because a higher cost is further from the lowest price in the market, a higher cost seller expects losses more often in matching interactions. This holds despite the hike in price that accompanies adopting PM alone, because $\delta_1$ is the same constant for all sellers.

Therefore there exists a threshold $s^t$ such that $v > s^t > s_1$ and no seller with cost higher than this threshold could prefer to deviate and offer PM alone. QED.

Offering PM alone then sends a similar signal to buyers as inviting OB alone sent; that the seller has low cost. But with PM, sending such a signal simply indicates that the seller is willing to negotiate below its posted price. This does not harm the seller’s interest because the seller clearly specifies that it would agree to sell only at other existing prices in the market. Thus, it does not adversely select extremely low bids, as these (bids which are

\(^{29}\)Notice that this is a necessary but not a sufficient condition for a seller to prefer to deviate and offer PM.
also invoked matches) are tied to rival prices and thereby to the buyer’s information. It thus serves to sell at the EMP or the outside option of each buyer aware of a rival’s (lower) price; rather than lose the buyer. The ability to thus price discriminate however exists as shown in Theorem 3 only as long as doing so with every buyer leaves the seller with a positive expected matching profit.

Not only does PM enable a seller to price discriminate between different buyers where OB failed to do so, it is also a credible strategy unlike TILI wherein refusal by a buyer gave the seller incentives to sell below the posted price. In PM if a buyer refuses (both the posted price and her best known rival price) the seller has no incentive to lower the price anymore because the buyer herself has no bargaining power below her best known rival price. The seller can then confidently expect the buyer’s refusal to be an incredible threat; PM is thus a subgame perfect interaction with every buyer. But it can be used as a deviation from an all-TILI market only by sellers with cost below a threshold. Therefore with sellers choosing the pricing interactions, the market would have a mix of sellers offering TILI and PM.

3.5 Mixed Market: TILI and PM

When all firms in the market have the option to offer choose a pricing rule they would either use TILI or PM. The distribution of prices is then different from that in an all-TILI market. Let this new price distribution be $\tilde{Q}(p)$. The EMP function that follows henceforth, will be a function of $\tilde{Q}(p)$, i.e. $\tilde{p}(k, \tilde{Q}(p_{-i}))$ and is not comparable with the TILI market EMP.

Let all firms have a choice between offering PM or offering TILI. Each seller $i$ takes the rest of the market as given, i.e. the price distribution $\tilde{Q}(p_{-i})$ as given, and the number of other firms offering PM or TILI as given. For any given seller then, the profit from choosing to offer PM is as follows, while those offering TILI in this market would have only the first

\footnote{As in previous sections, this holds for all but the lowest cost seller whose price defines the range of prices in the market.}
component as their profit function.

\[ \pi_i^{PM} = G(k_i)\{\hat{p}(k_i, \tilde{Q}(p_{-i})) - s_i\} \int_0^{k_i} \frac{h(p_L, k)G'(k)dk}{G(k_i)} \]

\[ + \int_{k_i}^{I+1} \delta(p_L, k)G'(k)\{\hat{p}(k, \tilde{Q}(p_{-i}))(k) - s_i\}dk \]  

(22)

When more than one seller offers PM, \( \delta(p_L, k) \) is the probability of a buyer invoking a match at the given seller’s store, will be an inverse function of the number of sellers offering PM and therefore indirectly a function of the lowest price in the market, call that \( p_L \). Similarly \( h(p_L, k) \) is the probability of a buyer type \( k \) purchasing from the lowest price known, even though rival sellers offer matches. This is less than one, because a price that is the best known by a buyer may not ensure purchase if the buyer uses it to invoke a match with another seller offering PM. Also both these probabilities are inversely related to \( p_L \), and are inversely related to the buyer type \( k \) in question. That is, as the lowest price in the market increases, the number of sellers offering to match lower rival prices would be expected to increase and therefore the probability of a buyer purchasing without matching decreases and so does the probability of any one seller receiving a match request; and a more informed buyer has a higher probability of knowing other rivals offering matches, and therefore the probability that any one seller sells to this buyer (with or without matching) is lower.

\[ \frac{\partial \delta(p_L, k)}{\partial p_L}, \frac{\partial h(p_L, k)}{\partial p_L} < 0; \quad \frac{\partial \delta(p_L, k)}{\partial k}, \frac{\partial h(p_L, k)}{\partial k} < 0 \]

Both these probabilities are also functions of buyers’ perception of the ease of matching (or buyers’ willingness to match), which is considered here to be exogenous. And just like we reasoned in price matching alone, even in this mixed market because of some possible hassle cost we have \( h(p_L, k) > \delta(p_L, k) \), although the difference between the two diminishes as the buyer (information) type \( k \) gets large. That is,

\[ h(p_L, k) > \delta(p_L, k); \quad \forall k, \quad h(p_L, k) - \delta(p_L, k) \downarrow \text{as } k \uparrow \]  

(23)

\[ ^{31} \text{This is because the highest price in the market is } v, \text{ and the number of sellers offering PM depends on the range of prices in the market.} \]
The FOC for a seller choosing $TILI$ in this mixed market is as follows

$$
\hat{p}(k_i, \tilde{Q}(p-i)) - s_i = -\frac{\partial \hat{p}(k_i, \tilde{Q}(p-i))}{\partial k_i} \frac{\int_0^{k_i} h(p_L, k) dk}{h(p_L, k_i)} ,
$$

(24)

while the FOC for choosing $PM$ in the mixed market is:

$$
\hat{p}(k_i, \tilde{Q}(p-i)) - s_i = -\frac{\partial \hat{p}(k_i, \tilde{Q}(p-i))}{\partial k_i} \frac{\int_0^{k_i} h(p_L, k) dk}{h(p_L, k_i) - \delta(p_L, k_i)} ,
$$

(25)

The FOC (25) differs from (24) only in the factor $h(p_L, k_i) - \delta(p_L, k_i)$ dividing the RHS. Given our assumption (23), an analogy to (21) exists even in the mixed market, and the target buyer type a seller would choose in this market, if offering $PM$ would be smaller than that chosen if offering $TILI$; i.e. $k_i^{PM} < k_i^{TILI}$.

**Result 2:** The act of offering $PM$ raises any given seller’s posted price (by lowering its target buyer type at the margin), both when he may be the only one offering $PM$ and if there exist others who do so as well.

Also notice that the factor $\frac{\int_0^{k_i} h(p_L, k) dk}{h(p_L, k_i)}$ on the RHS of (24) increases in $k_i$. This FOC therefore has identical properties as the FOC in the all-$TILI$ market. Therefore a similar result to Theorem 1 is obtained here as well; i.e. all sellers choosing not to match in this mixed market post prices that can be ranked by their costs. The same holds for the FOC (25) as $\frac{\int_0^{k_i} h(p_L, k) dk}{h(p_L, k_i) - \delta(p_L, k_i)}$ increases in $k_i$. We therefore have the following corollary directly from Theorem 1.

**Corollary 3:** Within the group of sellers who choose to offer $TILI$, posted prices can be ranked by their costs. This is also true within the group of sellers choosing to offer $PM$.

Notice also that the FOCs again give the result that the market would accommodate sellers with cost as high as $v$ such that the highest cost seller makes zero profit and all others post prices strictly above cost, expecting to make positive profit, whether price matching or not. And again, a seller would offer $PM$ only if at its optimal $k_i^{PM}$, it expects to earn positive profits on average from all transactions involving price matching. Therefore again,
sellers with costs above a threshold cost\textsuperscript{32} are unable to offer PM in a mixed market.

**Theorem 4:** In a mixed market a seller would offer $\tilde{P}M$ only if it expects to earn an average positive profit from all transactions in which it matches other prices; equivalently, only if its cost is below $\tilde{s}'$.

**Proof:** The argument is analogous to the proofs of Theorem 3 and Corollary 2. If a seller does not expect a positive expected profit from all matching transactions, then it would do better to withdraw its matching policy. But in that case it is maximizing the same function as it would when offering TILI and therefore it would post a price that optimizes that. This proves the first statement.

Moreover, because the RHS of the FOCs in (24) and (25) increase in $k_i$ and using Corollary 3, we have from Corollary 1 that price-cost margins shrink in costs within the two groups. That is, a higher cost seller makes smaller profits, and a higher cost is further away from the lowest price in the market. Therefore, there exists $\tilde{s}'$ such that $v > \tilde{s}' > s_1$ and no seller with cost greater than $\tilde{s}'$ would offer to price match. QED.

An equilibrium exists only if the lowest cost seller’s best response is to post the lowest price in the market (i.e. it targets to sell to all types of buyers) and it lacks the incentive to price higher and use rival prices to have matches invoked, while others price according to their FOCs. In an equilibrium therefore, the lowest cost seller optimizes $\max_{p_L} h(p_L, k = \frac{I-1}{I})(p_L - s_1)$; where $p_L$ is its price (lowest in the market)\textsuperscript{33}.

Notice that because rival sellers are offering to match, not all prospective buyers need purchase from the lowest priced seller, but the probability $h(p_L, k = \frac{I-1}{I})$ that they do so, is itself a function of its (lowest) price because it determines the number of rivals that choose to offer matching. Internalizing this effect then, the lowest cost seller chooses an optimal $p_L$ setting the FOC $h'(p_L, k = \frac{I-1}{I})(p_L - s_1) + h(p_L, k = \frac{I-1}{I})$ equal to zero; given that the ceiling on $p_L$ is marked by the lowest rival price. Thus while sellers who adopt price matching raise their prices to extract more surplus from uninformed buyers and more effectively price discriminate, the seller posting the lowest price in the market has incentives to post a low price and thus discourage adoption of price matching by the rest of the market.

\textsuperscript{32}This threshold cost for the mixed market is different from that in the all TILI market.

\textsuperscript{33}An equilibrium is more likely if the range of costs is large and $h(p_L) - \delta(p_L)$ is large for a perfectly informed buyer.
because its optimization trades off a higher price with a higher probability of selling to all types of buyers.

4 Conclusion

The paper illustrates that price matching has evolved as an endogenous choice within markets in answer to a need and ability to price discriminate on the basis of buyers’ differences in information. The policy rule of matching lower rival prices solves in part the incredibility and sub-optimality of selling via a single posted price; while also reducing considerably, the adverse selection involved in giving buyers’ the controls to tailor the price according to their bids.

I find that the highest cost seller never finds it profit improving to price match. Even other sellers with high costs relative to the market are found to be unable to use the policy. For the sellers who do use it, their conditioning of the policy on consumers invoking it (rather than making it concrete or absolute by posting price equal to the lowest price in the market) makes the decision to adopt price matching itself endogenous to the market price distribution. The strategic nature of threat or commitment of price matching is thus lost. It is important to note in this context that the model assumes that hassle cost exists for some or all consumers, thus weakening the strategic threat argument. This assumption however avoids imposing limits, levels or even homogeneity of such cost and is therefore a reasonable representation of transaction or execution costs involved in verifying buyers’ information.

The price discrimination is enabled by eliciting a buyer’s private information regarding the rest of the market (captured by the buyer’s lowest known price or her ‘outside option’), while committing \(^{34}\) to sell at that price (subject to verification that it exists as a rival price). At the same time, it leaves the buyer with a surplus (if any) just large enough to convince her to purchase, given the competition that her best known rival price poses to the seller. Moreover, by requiring this price to exist in the market, the rule detaches it from the buyer’s perception of the seller’s cost; thus avoiding extreme adverse selection.

\(^{34}\)Such commitment resembles what Camera & Delacroix (2004) point out as being necessary for buyers to reveal information through a quote/bid. In this case its not just necessary but complying with the commitment is also the seller’s best response given that the decision to price match is taken only if matching interactions give positive expected profit.
Although adoption of price matching by sellers in such large markets still gives them incentives to hike their prices, because it substitutes for an actually lower price, it is not clear that all prices would be higher. This is because the extent of adoption of price matching in a market is found to be a function itself of the range of existing prices, and the seller posting the lowest price has an incentive to discourage such adoption by its rivals by posting a low price.

Lastly an important caveat is that the model treats price matching only where an offer must be market wide. A policy to match sub-samples of rival prices would be more complicated and the claims in this paper do not extend to that.
References:


