In this work, we introduce a novel flow representation for finite games in strategic form. This representation allows us to develop a canonical direct sum decomposition of an arbitrary game into three components, which we refer to as the potential, harmonic and nonstrategic components. We analyze natural classes of games that are induced by this decomposition, and in particular, focus on games with no harmonic component and games with no potential component. We show that the first class corresponds to the well-known potential games. We refer to the second class of games as harmonic games, and study the structural and equilibrium properties of this new class of games.

Intuitively, the potential component of a game captures interactions that can equivalently be represented as a common interest game, while the harmonic part represents the conflicts between the interests of the players. We make this intuition precise, by studying the properties of these two classes, and show that indeed they have quite distinct and remarkable characteristics. For instance, while finite potential games always have pure Nash equilibria, harmonic games generically never do. Moreover, we show that the nonstrategic component does not affect the equilibria of a game, but plays a fundamental role in their efficiency properties, thus decoupling the location of equilibria and their payoff-related properties. Exploiting the properties of the decomposition framework, we obtain explicit expressions for the projections of games onto the subspaces of potential and harmonic games. This enables an extension of the properties of potential
and harmonic games to nearby games. We exemplify this point by showing that the set of approximate equilibria of an arbitrary game can be characterized through the equilibria of its projection onto the set of potential games.