Searching a Bargain: Power of Strategic Commitment*

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Abstract

This paper investigates the impacts of reputation (in contact with inflexibility) on imperfectly competitive search markets where the sellers announce their initial demands prior to the buyer's visit and the buyer directs his search for a better deal. The buyer facing multiple sellers can negotiate with only one at a time and can switch his bargaining partner with some cost. The introduction of commitment types that are inflexible in their demands, even with low probabilities, makes the equilibrium of the resulting multilateral bargaining game essentially unique. A modified war of attrition structure is derived in the equilibrium. The model unites and smooths out Bertrand and Diamond price competition models and eliminates their inexplicable predictions.

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1. Introduction

Consider markets in which buyer is either a venture capitalist who would like to buy a poorly managed but promising small business, an investor who plans to purchase a real estate, or a highly skilled worker searching for a job. The common characteristics of the trading mechanism that we often observe in these *imperfectly competitive search markets* are that there are a few sellers in competition over the buyers and each seller usually suggests or posts his price so that the buyer can pay the offered price, buy the good or the service, and finalize the trade.[1] However, the general practice in these markets is that the buyers negotiate with the sellers with the hope of getting a deal better than the solicited "buy-it-now" prices. The two prominent features of imperfectly competitive search markets, bargaining and uncertainty about the players' flexibility, constitute the core of the analysis in this paper.

Models that have close resemblance to Diamond (1971) and Bertrand (1883) assume that market participants are inflexible in their posted or announced terms of trade. On the other hand, bargaining models presume that agents are perfectly flexible in their demands. The former models ignore an evident aspect of imperfectly competitive markets, negotiation, and the latter models disregard the power of commitment. The potential benefits of commitment are clear: Since one's opponent is convinced, his best strategy is to yield if possible. Thus, a model that does not unite search, negotiation, competition and commitment in a sensible manner is prone to fail explaining price formation in imperfectly competitive search markets.

The main motivation of this paper is to investigate the market participants' equilibrium behavior in imperfectly competitive search markets given that the sellers and the buyers have the opportunity of building "*reputation*" on inflexibility through negotiation. In order to illustrate the effects of reputation, I construct a simple set-up with two sellers and a buyer[2], in which flavors of Bertrand and Diamond price competition models coexist: The sellers bid their demands simultaneously, but the buyer believes that he can get a better deal if he searches for it.

However, analysis suggests that equilibrium outcomes are in contrast to the predictions of Bertrand and Diamond: Price undercutting is not an optimal strategy for the sellers even if the search friction approaches zero (thus marginal cost pricing is not necessarily the unique equilibrium outcome) and monopoly pricing does not occur for positive search frictions. Hence, the model I use in this paper unites and smooths out these celebrated price competition models and eliminates their inexplicable predictions.

[1]The buyers can solicit the sellers' posted prices through on-line search or by requesting quotes without visiting the sellers' stores or engaging in some negotiation process

[2]Section 5 generalizes the model and extend the results for the case of multiple sellers.

Furthermore, analysis establishes that each player builds reputation on inflexibility and opting out of negotiation is credible for rational bargainers. I also show that reputation concern of the players overwhelms their behaviors so that equilibrium has a war of attrition structure (each player is indifferent between accepting his opponents' initial demand and waiting for acceptance). As a result, the equilibrium of the haggling process is unique and robust in the sense that it is "independent" of the exogenously assumed bargaining protocols (unlike more familiar but relatively less sophisticated models). Since the equilibrium is relatively simple and unique, the approach I suggest in this paper could facilitate further applications in various other fields such as labor economics, industrial organization and market microstructure.[3]

Following Kreps and Wilson (1982) and Milgrom and Roberts (1982), I assume that each of three players suspects that the opponents might have some kind of irrational commitment forcing them to insist on a specific allocation. Obstinate (or commitment) types take an extremely simple form. Parallel to Myerson (1991), and Abreu and Gul (2000) a commitment player always demands a particular share and accepts an offer if and only if it weakly exceeds that share. An obstinate seller, for example, never offers a price below his original posted price, and never accepts an offer below that price. Similarly, an obstinate buyer always offers a particular amount, and will never agree to pay more. Thus, a rational player must choose either to mimic an inflexible type, or reveal his rationality and continue negotiation with no uncertainty regarding his actual type. Therefore, reputation of a player is the posterior probability (attached to this player) of being the obstinate type.

Building reputation on inflexibility by haggling with a seller is an investment for the buyer, which increases his continuation payoff of visiting another seller. Having a higher outside option in return increases the bargaining power of the buyer relative to his current partner. Therefore, once the buyer attains sufficient level of reputation, he leaves his current bargaining partner unless his demand is accepted. Thus, in equilibrium, the buyer receives positive expected surplus as opposed to Diamond's prediction. On the other hand, sellers can influence the growth rate of the buyer's reputation. If the sellers's posted prices are lower, then the buyer's reputation grows slower. Therefore, the sellers can counterbalance the buyer's threat of opting out by posting sufficiently low prices (but not necessarily zero). By doing so, the sellers can bring time pressure to bear on

[3]The model and the outcomes could facilitate a fruitful ground to answer further questions such as how reputation on inflexibility would affect the structure of decentralized markets (in which no agent is assumed to be a price-taker and no auctioneer is assumed to be present). Specifically, if it is an option, do the sellers prefer to announce their prices before or after the buyer's visit? Does the buyer prefer to be engaged in a haggling process or to invite the sellers for an auction? These questions and many more deserve comprehensive considerations and are beyond the scope of this paper. However, they constitute the immediate items of my research agenda.

the buyer to ensure that he will not have enough time to build his reputation against his current partner. If the buyer needs more time to build his reputation before going to the other seller, then it is less likely that his opponent will concede to him earlier. Therefore, if the sellers post their prices sufficiently low, then they can "lock" the buyer in their stores and impel him to make an agreement promptly. As a result, each seller can attain positive expected surplus in equilibrium as opposed to Bertrand's prediction.

Section 2 explains the model and the two-stage competitive-bargaining game in continuous-time. Equilibrium strategies of the second-stage war of attrition game for single commitment types are characterized in Section 3. Section 4 examines the case with multiple commitment types and the equilibrium prices and demands that the players would announce in stage 1. Section 5 presents some limiting results. The convergence of discrete-time bargaining problem to the "protocol-free" continuous-time problem is explored in Section 6. Section 7 elaborates on the assumptions of the model and their impacts on the results. Finally, Section 8 makes some closing remarks.

Related Literature

Versions of the haggling game and the market environment that I analyze in this paper have been investigated in different contexts to explain various different economic phenomena. In this section, I would like to highlight the contribution of this paper by relating it to some of the brilliant works in the literature. Also, admitting that it is far from being a complete survey, this literature review underlines the directions that my work may complement our understanding on playing fields of negotiators.

To start with, this paper is directly related to reputation and bargaining literature initiated by Myerson (1991) on one-sided reputation building, as well as works by Abreu and Gul (2000) and Kambe (1999) with two-sided versions of it. Compte and Jehiel (2002) consider a discrete-time bilateral bargaining problem in Abreu-Gul setting and explore the role of (exogenous) outside options. They show that if both agents' outside options dominate yielding to the commitment type, then there is no point in building a reputation for inflexibility and the unique equilibrium is again the Rubinstein outcome. The results I present in Section 3 of this paper suggest that in equilibrium, the buyer makes the "take it or leave it" ultimatum to seller 1 (the first seller the buyer visits) when the buyer's initial reputation is sufficiently high (that is, the value of the buyer's outside option of going to the other store to bargain with the other seller is high). Thus, analogous to Compte and Jehiel (2002), rational seller 1 immediately reveals his type, but the rational buyer does not. This strategy is an equilibrium in the discrete-time setting that I analyze in Section 6 because the buyer's outside option depends on his reputation when he leaves the first store: If the buyer reveals his rationality in store 1, then he "loses" his outside option.

Another paper in the bargaining and reputation literature, that is closely related to my work, is Atakan and Ekmekci (2009). They consider a two-sided search market with a large number of buyers and sellers who wait to be matched (randomly) to an opponent to bargain over the unit surplus, so the bargaining parties' outside options are endogenous. Atakan and Ekmekci analyze the steady state of this market, and in agreement with my results, they show that the endogenous outside options of the rational agents are never large enough to deter the effect of commitment types. Atakan and Ekmekci (2009) examine "large" markets. That is, negotiations are anonymous, and thus market participants do not have incentive to invest on their reputations for future negotiations. However, I investigate "smaller" markets in this paper, where there are only a few competing sellers. Thus, negotiation process is used not only to reach an agreement with an opponent, but also serves as an investment for future negotiations.

The potential benefits of commitment in strategic and dynamic environments are clear. Shelling (1960) asserts that one way to model the possibility of commitment is to explicitly include it as an action players can take. However, an important question Shelling leaves unanswered is whether commitment can be rationalized in equilibrium. The answer is affirmative if revoking a commitment is costly, according to Crawford (1982) and Muthoo (1996), or if commitment is a costly action, according to Ellingson and Miettinen (2008). In contrast to these models, I assume in this paper that rational bargainers can mimic commitment types, that are not rational in the usual sense, if they find it optimal.[4] Parallel to the works of Abreu and Gul (2000), I show that "full" commitment does not need to be an equilibrium strategy. Propositions 3.1 and 6.1 suggest that each player will revoke his commitment with positive probability. Although I do not take commitment as a costly action for players, revoking commitment is costly because it means yielding to opponent's demand. However, this cost is endogenously determined and implied in equilibrium.

Non-cooperative analyses of bargaining, using extensive form games, are sometimes criticized because the conclusions are sensitive to the exact description of the games, such as the timing for offers, counter-offers and exit options. On the contrary, real life negotiations seem very shapeless processes. In this regard, my paper adds to the literature aiming to understand whether credible commitment to certain promises or threats would wash out technical specifications of the bargaining procedures, and if so, how extensive forms of non-cooperative bargaining games would be. Caruana, Eirav and Quint (2007) consider a multilateral bargaining game with a fixed deadline and allow players to revise

[4]Abreu ad Sethi (2003) supports the existence of commitment types from evolutionary perspective and show that if players incur a cost of rationality, even if it is very small, the absence of such "irrational" types is not compatible with evolutionary stability in bargaining environment.

their demands often. However, revisions are costly and this cost increases as the deadline gets closer. Thus, earlier offers serve as a commitment mechanism because changing demand later becomes increasingly expensive. The unique equilibrium is "independent" of the bargaining protocols, where the agreement is reached immediately and the revision costs are avoided. Caruana and Einav (2008) work with a similar model where the number of players and their offers are restricted to two. The extensive form game takes a relatively simpler form (war of attrition), leaving the outcome independent of the bargaining procedures. Likewise, Chatterjee and Samuelson (1987) consider bilateral bargaining environment where each player has two types (regarding the valuation of the good) and knows his own but not his opponents' type. Moreover, players' offers are binary; the buyer, for example, either offers the demand of his weakest type or the demand of the seller's strongest type. Thus, the bargaining game is modeled as a war of attrition game, and so equilibrium strategies are similar to Abreu and Gul (2000). Samuelson (1992) investigates a similar case with an infinite number of buyers and sellers to explain why opting out would occur even though there is positive expected gain from continued bargaining. He shows, in Lemma 1, that a seller will opt out if the percentage of high-type buyers in the population is sufficiently high. On the other hand, I show in Section 6 that reputation concern of the players in a competitive-bargaining environment overwhelms their behaviors so that the unique equilibrium has a war of attrition structure, and thus the equilibrium outcome is independent of the bargaining procedure.[5] Furthermore, unlike Samuelson (1992), I show that opting out (or disagreement) may occur in equilibrium for any values of the primitives.

This paper also adds to the literature initiated by Rubinstein and Wolinsky (1985). It is conventional wisdom that centralized and decentralized markets are Walrasian under frictionless conditions. Walrasian theory suggests that equilibrium will be achieved through a process of tâtonnement; given the supply and demand, the market will clear itself. This view implies that the institutional structure of a frictionless market that includes the particulars of the trading procedure has no or little impact on the market outcome. Since the Walrasian theory itself has nothing to say on this line, it remains an interesting and open question whether all frictionless markets are indeed Walrasian. Following the Rubinstein's seminal paper, varied contributions shed light on decentralized homogeneous goods markets where the price is determined as the perfect equilibrium of a bargaining game between sellers and buyers. My paper adds to this literature by noticing, in Section 5, that when players have reputation concerns, frictionless competitive markets need not be Walrasian.

Rubinstein and Wolinsky (1985) consider a market, in steady state, where at each

[5] Abreu and Gul (2000) and Atakan and Ekmekci (2009) reach similar conclusions in different settings.

period, finite (but large number of) buyers and sellers are matched with an exogenous matching mechanism to negotiate over the price, and new players enter as some leave the market after agreement. Their main result suggests that the unique outcome is not Walrasian even when search and bargaining frictions vanish. Gale (1986a,b) objects to this result by arguing that supply and demand in such market setups should be treated in terms of "flows" (not "stocks") of agents into the market at any period, and then shows that the bargaining approach indeed supports the Walrasian equilibrium. Binmore and Herrero (1989) support this point and show that frictionless markets will clear period by period. That is, the short side of the market will appropriate the whole surplus if and only if entry into the market is negligible relative to exit from it. Satterthwaite and Shneyerov (2007) reinforce this finding by achieving an analogous result when there is incomplete information (regarding the players' valuations) on both sides.

Rubinstein and Wolinsky (1990) show that the controversial result in their earlier paper does not occur if there is no new entry into the market. In this case, players' fear that they may not find a bargaining partner tomorrow if they reject their current offer today forces the long side of the market to compete fiercely, thus yielding a Walrasian outcome as frictions vanish. Bester (1988) employs a model similar to Rubinstein and Wolinsky (1985) with an infinite number of buyers and sellers and shows that if there is uncertainty regarding the sellers' product quality, then relative speed of convergence for bargaining friction and search friction determines whether the limit approaches Walrasian outcome. However, if the quality uncertainty is not in play along with the other frictions, then the market outcome is clearly Walrasian, as argued in Bester (1989).

In contrast, Shaked and Sutton (1984) examines a labor market with one firm and multiple workers (similar to the one I investigate in this paper), showing that the unique equilibrium outcome is non-Walrasian. This conclusion is correct under the assumption that the firm cannot switch its bargaining partner unless some time ($T > 1$ periods), which is exogenously set, passes. However, it is hard to motivate whether a firm would commit itself to such haggling protocols in a competitive environment. In this paper, however, I show that the buyer's reputation concern may lock him in with a seller. In equilibrium, when the buyer has a low initial reputation, he cannot leave his bargaining partner before his reputation reaches a certain point (optimal departure time). Moreover, Propositions 5.1 and 5.2 support this point even stronger. As the uncertainty regarding the players' rationality vanish, i.e. probabilities z_b and z_s converge to zero at the same rate, the buyer has to continue to be "*weak*", implying that the optimal departure time is always positive in equilibrium. However, as the number of sellers approaches infinity, we cannot sustain non-Walrasian outcomes in equilibrium unlike Shaked and Sutton (1984).

Therefore, building a model of reputation with considerable sophistication is not only important to match some stylized facts but also overturns a number of conclusions reached in their absence. In this respect, my paper also contributes to market microstructure literature. Although negotiating over prices with sellers is common in many markets, it is not clear how a haggling price policy can help a firm gain a strategic advantage or whether it is even sustainable in a competitive market. Riley and Zeckhauser (1983), Bester (1993), Wong (1995), Desai and Purohit (2004), and Camera and Delacroix (2004) compare negotiated prices with posted prices and show that each argument has specific merits.[6] It is common to all of these significant works that price posting requires irreversible commitment. That is, sellers either post price and act absolutely inflexible in their demands or do not post a price but behave completely flexible and bargain with each buyer. In this paper, however, I show that dedication to such extreme strategies (absolute flexibility or inflexibility) that postulate pure commitment is not optimal in a competitive market environment. Very roughly, rational players prefer to randomize (in a sense) these two strategies optimally. Clearly, the model I investigate in this paper facilitates a fruitful ground to answer further questions in this line of research.

2. The Competitive Bargaining Game in Continuous-Time

The Environment: Consider a market where there are two sellers having an indivisible homogeneous good and a single buyer who wants to consume only one unit.[7] All players are impatient (that is, they discount time) and the valuation of the good is one for the buyer and zero for the sellers. There is no informational asymmetry regarding the players' valuations and time preferences. The sellers make initial posted-price offers; the buyer can accept one of these costlessly (over the phone, say), or else visit one of the stores and try to bargain for a lower price. The buyer can negotiate only with the seller whom he is currently visiting. The buyer is free to walk out of one store and try the other, but at a cost (delay) of switching. The reader may wish to picture this market as an environment where the sellers' stores are located at opposite ends of a town while the buyer's position is midway in between the two. Thus, changing the bargaining partner

[6] Parallel approaches are extensively used in labor market literature to investigate the wage determination in competitive labor markets. See Rogerson, Shimer and Wright (2005) and the references therein.

[7] In Section 5, I consider the case where the number of sellers is some $N \geq 2$. I take the short side of the market as the demand side. The unique buyer assumption is consistent with markets where the buyer has some monopsony power, or each seller has a large number of goods to sell (so no competition between the buyers) and the buyers cannot convey information to one another (no interaction between the buyers).

is costly for the buyer because it takes time to move from one store to the other and the buyer discounts time. Finally, bargaining is complicated by the possibility of "reputations for inflexibility" – both the buyer and the sellers have some small, positive probability of being a commitment type who will never back down from his initial offer.

More formally, an obstinate (or commitment) type seller i is identified by a number $\alpha_i \in C \subset (0,1)$ and implementing a simple strategy: He always offers α_i, rejects any price offer strictly below it and accepts any price offer weakly above it. The initial probability that a seller is obstinate is denoted by z_s. The initial prior, z_s, and the time preference, r_s, are common for both sellers. Similarly, there is a small but strictly positive probability, z_b, that the buyer is a commitment type. Obstinate buyer with demand $\alpha_b \in C$ executes the following strategy: He always offers α_b to sellers, accepts any price offer less than or equal to α_b and rejects any price offer strictly above it. The rational buyer's time preference is r_b. Let $\pi(\alpha)$ denote the conditional probability that a player is type α given that he is obstinate. That is, π is a probability distribution on C.

Assuming that the sellers' are spatially separated, let δ denote the discount factor for the buyer that occurs due to the time, $\Delta > 0$, required to travel from one store to the other. That is, $\delta = e^{-r_b \Delta}$. Note that δ (the search friction) is the cost that the buyer incurs at each time he switches his bargaining partner.[8] Also note that, as the stores get very close to each other, the search friction vanishes and δ converges to one.

The Competitive Bargaining Game

Here I define the *competitive-bargaining game* in continuous-time. I then analyze two special cases in Section 3, in which each player has only one commitment type. The first part of Section 3 studies the case where the sellers' commitment demands are symmetric, and the second part investigates the case of asymmetric demands. These special cases both convey the flavor of the analysis and are furthermore the basic building blocks for the multiple type cases studied subsequently in Section 4.

The competitive-bargaining game between the sellers and the buyer is a two-stage, continuous-time game, and it proceeds as follows. Initially (in stage 1), each seller i simultaneously chooses a demand $\alpha_i \in C$. If he is rational, this is a strategic choice. If he is the obstinate type, then he merely declares the demand corresponding to his type. After observing both sellers' demands, α_1 and α_2, the buyer immediately accepts α (the minimum of α_1 and α_2) and finishes the game strategically if he is rational or because he is obstinate and of type α_b such that $\alpha_b \geq \alpha$. Or the buyer visits one of the sellers, seller

[8]One may assume a switching cost for the buyer that is independent of the "travel time" Δ, but this change would not affect our results. However, incorporating the search friction in this manner simplifies the notation substantially.

i say, and makes a counter offer $\alpha_b \in C$, which is observable by the other seller,[9] with $\alpha_b < \alpha_i$. Again, this may be because the buyer is rational and strategically demanding α_b or because the buyer is the obstinate type α_b. After the buyer declares his demand, the seller (who is currently visited by the buyer) can immediately accept the buyer's demand and finish the game or reject it, in which case the game proceeds to the second stage (the bargaining phase).

Upon the beginning of the second stage, the buyer and seller i immediately begin to play the following *concession game*: At any given time, a player either accepts his opponent's demand or waits for a concession. At the same time, the buyer decides whether to stay or leave store i. Concession of the buyer or seller i, while the buyer is in the store, marks the completion of the game. In case of simultaneous concession, surplus is split equally.[10] If the buyer leaves store i and goes to store j, the buyer and seller j start playing the concession game upon the buyer's arrival at that store. [11] I denote the two stage competitive-bargaining game in continuous-time by G.

The competitive bargaining game, G, is modeled as a modified war of attrition game. This model is justified in Section 6. There, I show that under some restrictions, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers converge to zero, and this limit is equivalent to the unique outcome of the game G.

Strategies of The Obstinate Types: Strategies of an obstinate player is simple; never back down from the initial offer. In addition to this, I assume that the obstinate buyer (regardless of his demand) is not enthusiastic enough to exert great efforts in moving back and forth between the sellers. That is, the obstinate buyer is a man who "plays it cool." To be more specific, I assume that the obstinate buyer understands the equilibrium and leaves his bargaining partner when he is convinced that his partner is also obstinate, and visits each seller with equal probabilities to announce his demand if it is not compatible with the lowest price announced by the sellers. Section 7 elaborates on these assumptions and their impacts on our results.

Strategies of the Rational Players: In the first stage of the competitive-bargaining game G, a strategy for rational seller i is a pure action $\alpha_i \in C$. Since the subsequent analysis is quite involved, I restrict sellers to play pure strategies in stage 1. However,

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[9] See footnote 30.

[10] This particular assumption is not crucial because simultaneous concession occurs with probability zero in equilibrium.

[11] After leaving store i and traveling part way to store j, the buyer could, if he wished, turn back and enter store i again. However, the buyer will never behave that way in equilibrium.

the buyer can employ mixed strategies. A strategy for the buyer in stage 1 consists of three parts $\sigma^b_{s_1} = (\sigma_{(\alpha_1,\alpha_2)}, \mu_{\alpha_1}, \mu_{\alpha_2})$. Given the posted prices of the sellers, α_1 and α_2, $\sigma_{(\alpha_1,\alpha_2)}$ is a probability measure over the sellers, i.e. $\sigma_{(\alpha_1,\alpha_2)}(i)$ denotes the probability that the buyer visits seller i first. Again, given that the posted prices of the sellers are α_1 and α_2, and that the rational buyer visits seller i first, the strategy μ_{α_i} is a probability distribution over $C_{\alpha_i} = \{x \in C | x \leq \alpha_i\}$. For the buyer, visiting seller i and choosing α_i corresponds to immediate concession. However, I require that for $i = 1, 2$, $\mu_{\alpha_i}(\alpha_i) = 0$. That is, both conceding at $t = 0$ (the beginning of the second stage) and choosing α_i in stage 1 correspond to immediate concession.

Given the first stage strategies of the players $\left(\alpha_1, \alpha_2; \sigma^b_{s_1}\right)$, the probability that seller i is the obstinate type conditional on posting price α is z_s if $\alpha = \alpha_i$ and 1 otherwise. Furthermore, let $\hat{z}_b(i, \alpha_b)$ denote the probability that the buyer is the commitment type conditional on him visiting seller i first and demanding $\alpha_b < \alpha_i$. Thus, given the first stage strategies, the Bayes' rule implies that

$$\hat{z}_b(i, \alpha_b) = \frac{\frac{1}{2} z_b \pi(\alpha_b)}{\frac{1}{2} z_b \pi(\alpha_b) + (1 - z_b)\mu_{\alpha_i}(\alpha_b)\sigma_{(\alpha_1,\alpha_2)}(i)} \tag{1}$$

A nonterminal history of length t, h_t, summarizes the initial demands chosen by the players in stage 1, the sequence of stores the buyer visits and the duration of each visit until time t (inclusive). For each $i = 1, 2$, Let $\hat{H}^i_t$ be the set of all nonterminal histories of length t such that the buyer is in store i at time t. Also, let H^i_t denote the set of all nonterminal histories of length t with which the buyer just enters store i at time t.[12] Finally, set $\hat{H}^i = \bigcup_{t \geq 0} \hat{H}^i_t$ and $H^i = \bigcup_{t \geq 0} H^i_t$.

The buyer's strategy in the second stage $\sigma^b_{s_2} = (\sigma_{s_2}, \mathscr{F}^1_b, \mathscr{F}^2_b)$ has three parts. The first part σ_{s_2} determines the buyer's location at any given history. For the other parts, let $\mathbb{I}$ be the set of all intervals of the form $[T, \infty]$ ($\equiv [T, \infty) \cup \{\infty\}$)for $T \in \mathbb{R}_+$, and $\mathbb{F}$ be the set of all right-continuous distribution functions defined over an interval in $\mathbb{I}$.

Therefore, for $i = 1, 2$, $\mathscr{F}^i_b : H^i \to \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F^{i,T}_b : [T, \infty] \to [0, 1]$. Similarly, seller i's strategy in the second stage $\mathscr{F}_i : H^i \to \mathbb{F}$ maps each history $h_T \in H^i$ to a right-continuous distribution function $F^T_i : [T, \infty] \to [0, 1]$.

Player i's reputation $\hat{z}_i : \hat{H}^i \to [0, 1]$, $i \in \{1, 2, b\}$, is a function specifying the probability with which other players believe that player i to be a commitment type. At the beginning of the game $\hat{z}_b = z_b$ and $\hat{z}_1 = \hat{z}_2 = z_s$. In equilibrium, the buyer's reputation in stage 1 will be updated as in Equation (1). Let $h_t \in \hat{H}^i$ and let $T \leq t$ be the biggest number with which $h_T \in H^i$. Therefore, in equilibrium, the buyer and seller i's

[12]That is, there exits $\epsilon > 0$ such that for all $t' \in [t - \epsilon, t)$, $h_{t'} \notin \hat{H}^i_t$ but $h_t \in \hat{H}^i_t$.

reputations in stage 2 evolve according to Bayes' rule:

$$\hat{z}_b(h_t) = \frac{\hat{z}_b(h_T)}{1 - F_b^{i,T}(t)}, \qquad \hat{z}_i(h_t) = \frac{\hat{z}_i(h_T)}{1 - F_i^{T}(t)}$$

The buyer's reputation, $\hat{z}_b(h_t)$, reaches 1 when $F_b^{i,T}(t)$ reaches $(1 - \hat{z}_b(h_T))$. Note that $F_b^{i,T}(t)$ is the sellers' belief about the buyer's play during the concession game with seller i. That is, it is the strategy of the buyer from the point of view of the sellers. For this reason, the distribution function $F_b^{i,T}(t)$ never reaches 1 since the buyer is the obstinate type with probability $\hat{z}_b(h_T)$ once he arrives at store i. Thus, in equilibrium, we must have $\lim_{t\to\infty} F_b^{i,T}(t) = 1 - \hat{z}_b(h_T)$. Same arguments apply to the sellers' strategies.

Given $F_b^{i,T}$, seller i's expected payoff of conceding to the buyer at time t (conditional on not reaching a deal before time t where $T \le t$,) is

$$\begin{aligned} U_i(t, F_b^{i,T}) := {} & \alpha \int_T^t e^{-r_s y} dF_b^{i,T}(y) \\ & + \frac{1}{2}(\alpha + \alpha_b)[F_b^{i,T}(t) - F_b^{i,T}(t^-)]e^{-r_s t} + \alpha_b[1 - F_b^{i,T}(t)]e^{-r_s t} \end{aligned} \tag{2}$$

with $F_b^{i,T}(t^-) = \lim_{y \uparrow t} F_b^{i,T}(y)$.

In a similar manner, given F_i^T, the expected payoff of the buyer who concedes to seller i at time t is

$$\begin{aligned} U_b^i(t, F_i^T) := {} & (1 - \alpha_b) \int_T^t e^{-r_b y} dF_i^T(y) \\ & + \frac{1}{2}(2 - \alpha - \alpha_b)[F_i^T(t) - F_i^T(t^-)]e^{-r_b t} + (1 - \alpha)[1 - F_i^T(t)]e^{-r_b t} \end{aligned} \tag{3}$$

where $F_i^T(t^-) = \lim_{y \uparrow t} F_i^T(y)$.[13]

3. Single Commitment Types

I now turn to the analysis of equilibrium in case each player has only one obstinate type.

A. Symmetric Obstinate types for the Sellers

Suppose now that the sellers' obstinate types are identical, who demands some $\alpha \in (0, 1)$, and it is incompatible with the buyer's commitment type $\alpha_b \in (0, 1)$, i.e. $\alpha > \alpha_b$. The equilibrium of the competitive-bargaining game in the second stage is unique. A short descriptive summary of the equilibrium strategies is as follows (see *Figure 1*). At time 0, the buyer enters, for example, store 1 with probability $\sigma_\alpha(1)$. The buyer's reputation

[13]Expected payoffs are evaluated at time T.

at the time he enters store 1 is $\hat{z}_b(1) = z_b/(z_b + 2(1 - z_b)\sigma_\alpha(1))$. If the buyer's initial reputation, z_b is high enough (relative to z_s), then the rational buyer makes his offer α_b, and leaves store 1 immediately if his demand is not accepted. However, for small values of z_b, the rational buyer starts playing the concession game with the seller until time $T_1^d > 0$. The value of the (deterministic) departure time from store 1, T_1^d, depends on the primitives.[14]

During the concession game, the rational buyer and seller 1 concede by choosing the timing of acceptance randomly according to distribution functions $F_b^1 : [0, T_1^d] \to [0, 1]$ and $F_1 : [0, T_1^d] \to [0, 1 - z_s]$, respectively. At time T_1^d, the buyer leaves store 1 for sure, if the game has not yet ended, and goes directly to store 2. At the the time the buyer leaves store 1, reputations of the buyer and seller 1 reach $\hat{z}_b(T_1^d) = \hat{z}_b/(1 - F_b^1(T_1^d))$ and one, respectively.

Denote by ω_i the time that the buyer starts negotiating with seller i. That is, $\omega_1 = 0$ and $\omega_2 = T_1^d + \Delta$ where Δ is the travel time between the stores. For notational simplicity, I manipulate the subsequent notation and denote ω_2 by 0. That is, I reset the clock once the buyer arrives in store 2 (but not the players' reputations).[15] Once the buyer arrives at store 2, the buyer and seller 2 play the concession game until T_2^e, when both players' reputations reach 1. That is, by time T_2^e the game ends with certainty. The buyer and the second seller concede with $F_b^2 : [0, T_2^e] \to [0, 1 - \hat{z}_b(T_1^d)]$ and $F_2 : [0, T_2^e] \to [0, 1 - z_s]$. Therefore, in equilibrium, the buyer visits each store at most once.

Proposition 3.1. *In the unique (sequential) equilibrium of the competitive-bargaining game G,*

1. *the rational buyer visits each store at most once, and*

2. *assuming that the buyer visits seller 1 first, the players' equilibrium strategies in the concession game are as follows:* $F_b^1(t) = 1 - c_b^1 e^{-\lambda_b t}$, $F_b^2(t) = 1 - e^{-\lambda_b t}$, $F_1(t) = 1 - z_s e^{\lambda(T_1^d - t)}$ *and* $F_2(t) = 1 - z_s e^{\lambda(T_2^e - t)}$ *where* $\lambda = \frac{(1-\alpha)r_b}{\alpha - \alpha_b}$ *and* $\lambda_b = \frac{\alpha_b r_s}{\alpha - \alpha_b}$.

I defer the proofs of all the results in this section to Appendix A.

In equilibrium, the buyer's continuation payoff is no more than $1 - \alpha$ if he reveals his rationality.[16] Since the obstinate buyer leaves a seller when he is convinced that his bargaining partner is the commitment type, leaving the first seller "earlier" (or "later")

[14] When there are two sellers, building reputation on inflexibility by negotiating with the first seller is an investment for the buyer, which increases his continuation payoff in the second store. In equilibrium, the rational buyer leaves the first store when his discounted expected payoff in the second store is at least as high as his continuation payoff in the first store. Therefore, in equilibrium, if z_b is low relative to z_s, the rational buyer needs to build up his reputation before leaving the first store.

[15] Thus, with some manipulation of the notation, I define each player's distribution function as if the concession game in each store starts at time 0.

[16] Arguments similar to the proof of Proposition A.2 in the Appendix C yields this result.

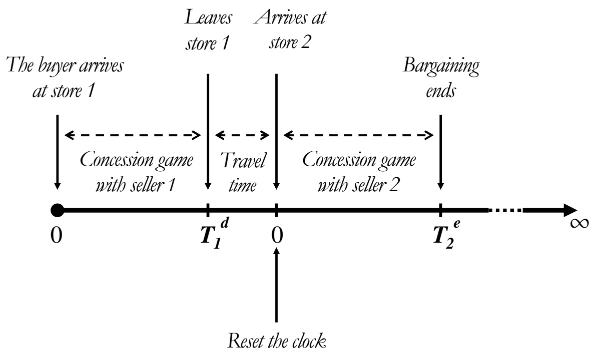

Figure 1: The time-line of the buyer's equilibrium strategy

than this time would reveal the buyer's rationality. Therefore, in equilibrium, the rational buyer never leaves a seller as long as there is positive probability that this seller is a rational type, and he immediately leaves otherwise. Clearly the buyer does not revisit a seller once he knows that this seller is the obstinate type.

In equilibrium both the buyer and the sellers concede by choosing the timing of acceptance randomly with a constant hazard rate (or instantaneous acceptance rate). Characterization of the distribution functions $(F_i, F_b^i)_i$ uses arguments in Hendricks, Weiss and Wilson (1988) and is analogous to the proof of Lemma 1 in Abreu and Gul (2000). The departure time, T_1^d, the time that the game G ends in store 2, T_2^e, and the *initial probabilistic concession* of the buyer in store 1 at time 0, $(1 - c_b^1)$ are all functions of the primitives and σ_α and are determined in equilibrium.

Since the buyer is indifferent between conceding and waiting at all times during the concession game with, for example, seller i, his expected payoff during the concession game with this seller is equal to what he can achieve at time 0 (the time that the buyer enters store i), i.e.[17]

$$v_b^i = F_i(0)(1 - \alpha_b) + (1 - F_i(0))(1 - \alpha) \tag{4}$$

Note that in equilibrium, at most one player makes an initial probabilistic concession, namely $F_i(0)F_b^i(0) = 0$ (see Lemma A.3 in Appendix A.)[18] The buyer is called ***strong*** if the first seller he visits makes an initial probabilistic concession and ***weak*** otherwise. Thus, if the buyer is weak, his expected payoff in the concession game with the first seller is $1 - \alpha$.

[17]Similarly, seller i's expected payoff in the concession game is $F_b^i(0)\alpha + (1 - F_b^i(0))\alpha_b$.

[18]Note that, $F_i(0)$ differs depending on whether the buyer is visiting seller i before or after visiting the other seller.

In equilibrium where the buyer first visits seller 1, the rational buyer leaves the first seller when he is convinced that this seller is obstinate. Walking out of store 1 is optimal for the buyer if his discounted continuation payoff in the second store, δv_b^2, is no less than $1-\alpha$, payoff to the buyer if he concedes to the obstinate seller 1. Let X_s denote the reputation of the buyer required in order to leave a store in equilibrium. Assuming that $\hat{z}_b(1) < X_s$ (i.e., the buyer needs to build up his reputation before walking out of store 1), the game ends in store 2 at time $T_2^e = -\log(X_s)/\lambda_b$.[19] Thus, in equilibrium, given the value of $F_2(0)$ and the buyer's continuation payoff in store 2 (Equation 4), X_s must solve

$$1-\alpha = \delta[1-\alpha_b - z_s(\alpha-\alpha_b)X_s^{-\lambda/\lambda_b}]$$

implying that $X_s = \left(\frac{z_s}{A}\right)^{\frac{\lambda_b}{\lambda}}$ and $A = \frac{\delta(1-\alpha_b)-(1-\alpha)}{\delta(\alpha-\alpha_b)}$.[20]

When $\hat{z}_b(1) \geq X_s$, the buyer's discounted continuation payoff in store 2 (evaluated at time 0) is higher than his continuation payoff in store 1. In this case, the buyer prefers going to store 2 and playing the concession game with this seller over conceding to seller 1 (and receiving payoff of $1-\alpha$). Thus, in equilibrium the buyer leaves store 1 immediately at time 0. Since the rational seller 1 knows that the buyer does not need to build reputation but rather plans to leave his store immediately, he accepts the buyer's demand at time 0.

Lemma 3.1. *In equilibrium that the rational buyer visits seller 1 first and $\hat{z}_b(1) > X_s$ holds, rational seller 1 immediately accepts the buyer's demand and finishes the game at time 0 with probability one. In case seller 1 does not concede to the buyer, the buyer infers that seller 1 is the obstinate type, so he immediately leaves store 1 and never comes back to this store again. The buyer goes directly to seller 2. The concession game with the second seller may continue until the time $T_2^e = -\log(\hat{z}_b(1))/\lambda_b$ and the players concede according to the following strategies: $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - z_s/[\hat{z}_b(1)]^{\lambda/\lambda_b}e^{\lambda t}$.*

In equilibrium, when $\hat{z}_b(1) \geq X_s$ holds, the buyer's expected payoff (evaluated at time 0) of visiting seller 1 first is given by

$$\begin{aligned} V_b(1) &= (1-z_s)(1-\alpha_b) + \delta z_s\left[v_b^2\right] \\ &= (1-\alpha_b)\left[1 - z_s(1-\delta) - \frac{\delta z_s^2}{[\hat{z}_b(1)]^{\lambda/\lambda_b}}\right] + (1-\alpha)\frac{\delta z_s^2}{[\hat{z}_b(1)]^{\lambda/\lambda_b}} \end{aligned} \tag{5}$$

[19] According to Proposition 3.1, $F_b^2(T_2^e) = 1 - X_s$, which implies the value of T_2^e.

[20] Note that X_s is well-defined when the cost of traveling is small, i.e., δ close to 1. Otherwise, the rational buyer will never leave the first seller.

Lemma 3.2. *In equilibrium that the rational buyer visits seller 1 first and $\hat{z}_b(1) \leq X_s z_s^{\lambda_b/\lambda}$ holds, rational seller* 1 *leaves store 1 at time $T_1^d = -\log(z_s)/\lambda$ for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller* 2 *may continue until the time $T_2^e = -\log(X_s)/\lambda_b$ and the players concede according to the following strategies: $F_b^1(t) = 1 - \hat{z}_b(1)/(X_s z_s^{\lambda_b/\lambda})e^{-\lambda_b t}$ and $F_1(t) = 1 - e^{-\lambda t}$ in store 1, and $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda t}$ in store 2.*

In equilibrium, $\hat{z}_b(1) \leq z_s^{\lambda_b/\lambda} X_s$ implies that the buyer's initial reputation is very low and thus he needs to spend significant amount of time to build up his reputation before leaving the first seller. In this case, $F_1(0) = 0$, i.e. the buyer does not receive an initial probabilistic gift from seller 1, implying that the buyer is weak and so his expected payoff during the concession game with seller 1, v_b^1, is $1 - \alpha$. Therefore, the buyer's expected payoff of visiting seller 1 first, $V_b(1)$, is also $1 - \alpha$.

Lemma 3.3. *In equilibrium that the rational buyer visits seller 1 first and $X_s z_s^{\lambda_b/\lambda} < \hat{z}_b(1) \leq X_s$, rational seller* 1 *leaves store 1 at time $T_1^d = -\log[\hat{z}_b(1)/X_s]/\lambda_b$ for sure, if the game has not yet ended, and goes directly to store 2. The concession game with seller* 2 *may continue until the time $T_2^e = -\log(X_s)/\lambda_b$ and players concede according to the following strategies: $F_b^1(t) = 1 - e^{-\lambda_b t}$ and $F_1(t) = 1 - z_s[X_s/\hat{z}_b(1)]^{\lambda/\lambda_b} e^{-\lambda t}$ in store 1, and $F_b^2(t) = 1 - e^{-\lambda_b t}$ and $F_2(t) = 1 - Ae^{-\lambda t}$ in store 2.*

In this particular case, the buyer's expected payoff of visiting seller 1 first, evaluated at time 0, is

$$V_b(1) = (1 - \alpha_b)\left[1 - \frac{z_s^2}{A[\hat{z}_b(1)]^{\lambda/\lambda_b}}\right] + (1 - \alpha)\frac{z_s^2}{A[\hat{z}_b(1)]^{\lambda/\lambda_b}} \tag{6}$$

When $\hat{z}_b(1)$ takes intermediate values, the buyer receives an initial probabilistic gift from the first seller (i.e., $F_1(0) > 0$). Since the buyer's reputation is not high enough (in comparison with the first case -Lemma 3.1), the buyer needs to spend some time to build up his reputation. As a result, the buyer's expected payoff in this case is smaller than that of the first case. However, relative to the second case (Lemma 3.2), the buyer needs to spend less time in store 1 and his expected payoff is higher. Namely, the buyer's expected payoff in the game G is inversely proportional to the time he needs to spend in the first store he visits.

Note that the unique equilibrium outcome is always inefficient. This inefficiency is due to delay in agreement and uncertainty about the types of the players and it does not disappear even when the search friction vanishes, i.e. $\delta \to 1$.

Last three Lemmas show that the buyer's expected payoff increases with $\hat{z}_b$ (See Equations (5) and (6). Therefore, if the buyer's initial reputation z_b is high, then in equilibrium the rational buyer visits each seller with equal probabilities. Otherwise, the buyer has incentive to deviate and visit (with probability one) the seller whom he picks with lower probability according to σ_α. However, when z_b is weak, the buyer can choose each store with different probabilities as long as these probabilities are not too distinct from 1/2.

Proposition 3.2. *In equilibrium, the buyer visits seller 1 first with probability* $\sigma_\alpha(1) = \frac{1}{2}$ *whenever the buyer is strong, i.e.* $z_b > X_s z_s^{\lambda_b/\lambda}$. *Otherwise,* $\sigma_\alpha(1) \in [D, 1-D]$ *where* $D = \frac{z_b(1-X_s z_s^{\lambda_b/\lambda})}{2(1-z_b)X_s z_s^{\lambda_b/\lambda}}$.

B. Asymmetric Obstinate Types for the Sellers

This section characterizes the unique equilibrium strategy of the competitive-bargaining game G when the sellers' commitment types are different. Without loss of generality, I assume that $\alpha_1 > \alpha_2 > \alpha_b$. Furthermore, I assume that the search friction and z_s are small enough so that $(1-\alpha_1) < \delta(1-\alpha_2)$ and $z_s\alpha_2 < \alpha_b$. In this case, the structure of the equilibrium strategy drastically changes (relating to the case where $\alpha_1 = \alpha_2$). In equilibrium, the bargaining phase never ends with the buyer's concession to the seller who has the higher demand (seller 1). If the buyer ever visits store 1, the rational seller 1 concedes to the buyer (upon the buyer's arrival at this seller) because the buyer has the tendency to opt out instantly from the concession game in store 1.

More formally, consider the case where the buyer is in store 1 and playing the concession game with this seller. This means that the buyer is indifferent between, on the one hand, accepting seller 1's demand, thus receiving the instantaneous payoff of $1-\alpha_1$, and on the other hand, waiting for the concession of the seller. However, if the buyer leaves (immediately) seller 1 and goes directly to the second store to accept the demand of seller 2, his discounted (instantaneous) payoff will be $\delta(1-\alpha_2)$. Thus, if the buyer ever visits store 1 in equilibrium, then he will never accept seller 1's demand because by assumption, we have $(1-\alpha_1) < \delta(1-\alpha_2)$. Therefore, in equilibrium, the buyer does not concede to nor spend time with seller 1 given that he ever visits store 1. As a result, it must be the case that rational seller 1 instantaneously accepts the buyer's demand with probability one upon his arrival, and the buyer immediately leaves store 1 if seller 1 does not concede to him.

Since the buyer and seller 1 play an equilibrium strategy that impels seller 1 to reveal his type immediately, the buyer's expected payoff of visiting this seller is $(1-z_s)(1-\alpha_b) + \delta z_s \hat{v}_b^2$. I denote by $\hat{v}_b^2$ the buyer's expected payoff in store 2 when he visits this store knowing that seller 1 is the obstinate type. Thus, if the buyer initially chooses to visit seller 2, then he concedes to this seller and receives the instantaneous payoff of $1-\alpha_2$, if and only if $1-\alpha_2 \geq \delta[(1-z_s)(1-\alpha_b)+\delta z_s \hat{v}_b^2]$.

This inequality holds when $z_s \geq \bar{z}$ holds, where $\bar{z}$ converges one as δ approaches one.[21] However, assuming that initial priors are small enough, we have $z_s < \bar{z}$, implying that if the buyer visits seller 2 first in equilibrium, then the buyer strictly prefers leaving this seller immediately upon his arrival (given that seller 2 does not accept the buyer's demand and finish the game). Hence, rational seller 2 must concede to the buyer at time 0 with probability one. The next result characterizes the second-stage equilibrium strategies of the competitive-bargaining game G.

Proposition 3.3. *Suppose that commitment types are such that $\alpha_b < \alpha_2 < \alpha_1$. Then the unique sequential equilibrium of the competitive-bargaining game G is the following:*

(*i*) *If the buyer visits seller 1 first, then rational seller 1 immediately accepts the buyer's demand and finishes the game at time 0 with probability one. In case seller 1 does not concede to the buyer, the buyer infers that seller 1 is the obstinate type, so he immediately leaves store 1 and never comes back to this store again. The buyer goes directly to seller 2 to play the concession game with this seller. The concession game with seller 2 may continue until the time $T_2^e = \min\{-\frac{\log \hat{z}_b(1)}{\lambda_b}, -\frac{\log z_s}{\lambda_2}\}$ where $\lambda_2 = \frac{(1-\alpha_2)r_b}{\alpha_2-\alpha_b}$, $\lambda_b = \frac{\alpha_b r_s}{\alpha_2-\alpha_b}$ and $\hat{z}_b(1)$ is the posterior probability that the buyer is the obstinate type conditional on seller 1 is visited first. Moreover, players concede according to the following strategies: $F_2(t) = 1 - z_s e^{\lambda_2(T_2^e-t)}$ and $F_b^2(t) = 1 - \hat{z}_b(1)e^{\lambda_b(T_2^e-t)}$ for all $t \geq 0$.*[22]

(*ii*) *If the buyer visits seller 2 first, then rational seller 2 immediately accepts the buyer's demand upon his arrival. Otherwise, the buyer leaves seller 2 immediately at time 0 (knowing that seller 2 is the obstinate type), and goes directly to seller 1. Rational seller 1 instantly accepts the buyer's demand with probability one upon the buyer's arrival. In case seller 1 does not concede, the buyer immediately leaves this seller, directly returns to seller 2, accepts the seller's demand α_2 and finalizes the game.*

Therefore, in equilibrium, when the buyer visits seller 1 first, he sends a *take it or leave it* ultimatum to this seller. If seller 1 does not accept the buyer's demand, then the buyer will go to the second seller. In this case, an agreement might be reached with seller 2, but possibly after some delay. On the other hand, when the buyer visits seller 2 first, he sends the same ultimatum to both sellers (first to seller 2 and then to 1). If no seller accepts the buyer's demand, then the buyer will come back to seller 2 and accept

[21] See the proof of Proposition 3.3 in Appendix A

[22] Note that with some manipulation of the notation, I reset the clock once the buyer enters store 2.

his demand α_2.[23] Hence, the buyer visits seller 1 first only when he is strong relative to seller 2 (i.e., $\hat{z}_b(1)$ is sufficiently higher than z_s) so that the initial probabilistic concession he will receive from seller 2 is high enough. This implies that in equilibrium, the rational buyer will visit seller 1 first with a very low probability. The following result summarizes the last argument formally.

Proposition 3.4. *In the unique equilibrium of the competitive-bargaining game G, the buyer visits seller 1 first with a very small probability. That is,* $\sigma_{(\alpha_1,\alpha_2)}(1) = \frac{z_b(1-\bar{A}_2)}{2\bar{A}_2(1-z_b)}$ *where* $\bar{A}_2 = [(\alpha_2 - \alpha_b)/1 - \alpha_b - \delta(1-\alpha_2)]^{\lambda_b/\lambda_2}$.

4. Multiple Commitment Types

In this section, I assume that the sellers and the buyer has multiple types. For simplicity, suppose that the set of types is common for all players and it is $C = \{\frac{1}{m}, \frac{2}{m}, ..., \frac{m-1}{m}\}$ where m is a large (but finite) positive integer. Also, suppose that the probability distribution, π, on C is uniform for all players. The following analyses focuses on the case where z_b and z_s are small and δ is close to one.[24]

For the rest of this section, I fix the search friction δ and the set of obstinate types C. Instead of characterizing the equilibrium demand selection of the players in stage 1 for fixed primitives, I will characterize the set of all equilibrium prices for all possible values of z_b and z_s sufficiently small. Next section will examine the properties of this set as these probabilities vanish. Hence, for any $z_b, z_s \in (0,1)$ small enough, let $\mathrm{G}(z_b, z_s)$ denote the competitive-bargaining game G where the initial reputations of the sellers and the buyer are z_b and z_s, respectively. Denote by $E(C) \subseteq C \times C$ the set of equilibrium prices (of the sellers). More formally, a pair of demands $(\alpha_1, \alpha_2) \in C^2$ is an element of $E(C)$ if there exit $z_b, z_s \in (0,1)$ small enough such that α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the competitive-bargaining game $\mathrm{G}(z_b, z_s)$.

Proposition 4.1. *There exists no sequential equilibrium in which the sellers declare different demands in stage 1.*

I defer the proofs of all the results in this section to Appendix B.

In equilibrium, the rational buyer will visit each store first with a positive probability and this is true independent of the sellers' posted prices. However, when the sellers

[23] Seller 2's immediate concession to the buyer (and receiving the payoff of α_b) is optimal because otherwise the seller can achieve at most $\alpha_2 z_s$ (since the buyer revisits seller 2 only if seller 1 is the obstinate type) and we have $\alpha_2 z_s < \alpha_b$ by assumption.

[24] More specifically, I assume that $z_b, z_s < \frac{1}{m}$ and $\delta > \frac{(m-1)^2}{m(m-1)-1}$.

demands are different, in equilibrium, rational sellers will accept the buyer's demand (regardless of what it is) immediately after his arrival at their stores (Proposition 3.3). Therefore, the buyer's optimal demand choice in this case will be $\frac{1}{m}$ (the minimal element of C.) On the other hand, Proposition 3.4. demonstrates that the rational buyer will pick the seller who posts the highest price with a very low probability, implying that this seller can benefit by deviating and posting $\frac{1}{m}$ in stage 1.

Therefore, in equilibrium, the symmetric sellers' demand selection is also symmetric. Let α be an equilibrium selection of the sellers in stage 1 for some small z_b and z_s. Recall that $C_\alpha = \{x \in C | x < \alpha\}$ is the set of all demands that are incompatible with α. Therefore, in equilibrium, the rational buyer must randomize over some (or all) demands in C_α.[25] However, if α_b is in the support of μ_α, then all demands in C that are smaller than α_b must also be in the support.

Proposition 4.2. $E(C) = \{(\alpha, \alpha) | \alpha \in C\} \setminus \{\frac{2}{m}\}$.

Since the sellers are *ex-ante* identical, it is natural to suspect that in equilibrium both sellers should choose the same demand. However, it is surprising that almost all obstinate demands in C can be supported in equilibrium (for some z_b and z_s), even though the sellers compete in the Bertrand fashion. Given that both sellers choose the same demand α that is higher than $\frac{2}{m}$, the buyer's strategy choosing each seller with equal probabilities and declaring a demand uniformly drawn from the set C_α is an equilibrium strategy when, for example, z_s and z_b are such that the buyer is weak at all demands in C_α. The sellers' expected payoff under this strategy is greater than $\frac{1}{2m_\alpha}\sum_{x \in C_\alpha} x$, (where m_α is equal to the cardinality of C_α), and it is higher than $\frac{1}{m}$.[26] However, if a seller deviates and posts $\frac{1}{m}$ instead of α, his expected payoff will be $\frac{1}{m}$.

On the other hand, if a seller deviates to α' such that $\frac{1}{m} < \alpha' < \alpha$, then the buyer infers that this seller is the obstinate type with certainty. In this case, the buyer uses the deviating seller's price as an "outside option" to increase his bargaining power against the other seller. Thus, the buyer prefers to visit (first) the seller whom he knows he can negotiate and possibly get a much better deal. Hence, deviating to α' is not optimal for the sellers either because it would yield payoff strictly less than $z_s\alpha'$ (the buyer will visit deviating seller's store if the other seller is an obstinate type), which is very small since by assumption we have $z_s < \frac{1}{m}$.

Obstinate demands that are "close" to the minimal element in C (in our case it is only $\frac{2}{m}$) cannot be supported in equilibrium where the sellers are restricted to choose

[25] Depending on the strength of the buyer and the denseness of the set C in $[0, 1]$.

[26] In such an equilibrium where the sellers are strong for all demand selections of the buyer within the set C_α while π and μ_α are uniform, a seller's expected payoff is greater than $(1 - z_s)$ times one half (since the buyer pick each seller with equal probabilities) of the expected value of $\alpha_b \in C_\alpha$.

pure strategies in stage 1. The reason for this is obvious: If both sellers post $\frac{2}{m}$ in stage 1, then they can achieve their highest expected payoff when they are strong, i.e. when z_b is very weak relative to z_s. However, even in this case, the sellers' payoffs will be strictly less than $\frac{1}{m}$, assuming that the buyer visits sellers with equal probabilities, because the buyer is obstinate with some small, but positive, probability z_b and thus in equilibrium the sellers may have to concede to the buyer's demand (which is $\frac{1}{m}$) with a positive probability. For this reason, the sellers do have incentive to deviate and post price $\frac{1}{m}$ instead of $\frac{2}{m}$. However, this problem does not survive when the sellers post a price that is sufficiently higher than $\frac{1}{m}$ because in such a case, with the right selection of initial beliefs z_b and z_s, the sellers can be ensured to have expected payoff higher than $\frac{1}{m}$.

5. The Limiting Case of Complete Rationality

Given the search friction δ and the set C, let $E^\infty(C)$ denote *the set of equilibrium prices of the competitive-bargaining game* G *as initial priors vanish.* More formally, a pair of demands $(\alpha_1, \alpha_2) \in C^2$ is an element of $E^\infty(C)$ if for any $z_s, z_b \in (0,1)$ small enough, where α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the competitive-bargaining game $\mathrm{G}(z_b, z_s)$, we have the following: Take any sequences $\{z_s^n\}$ and $\{z_b^n\}$ (where $z_s^0 = z_s$, $z_b^0 = z_b$ and for all $n \geq 0$, $z_s^n = K z_b^n$ for some finite $K > 0$) of the prior beliefs converging to zero.[27] Then α_1 and α_2 are equilibrium demand selections of the sellers in the first stage of the competitive-bargaining game $\mathrm{G}(z_b^n, z_s^n)$ for all $n \geq 0$. Finally, recall that the buyer and the sellers discount the time with the interest rates r_b and r_s, respectively. The following result characterizes the set of equilibrium prices as the initial priors vanish.

Proposition 5.1. $E^\infty(C) = \{(\alpha,\alpha) \in E(C) \mid \alpha \leq \frac{r_b - \frac{2}{m} r_s}{r_b + 2r_s}\} \setminus \{\frac{2}{m}\}$.

I defer the proofs of all the results in this section to Appendix B.

Fixing the primitives δ and C, equilibria exist supporting any symmetric opening prices from two sellers (except $\frac{2}{m}$); but in the limit as the likelihood of commitment types goes to 0, some of these equilibria vanish and others survive. Let μ_α^n denote the buyer's equilibrium demand selection in stage 1 for the game $G(z_b^n, z_s^n)$. For large values of n, we must have

$$\hat{z}_b(i, \alpha_b) \leq \left(\frac{(z_s^n)^2}{A} \right)^{\frac{\alpha_b r_s}{(1-\alpha) r_b}}$$

for each i and α_b in the support of μ_α^n whenever $\alpha > \frac{1}{m}$. If this inequality does not hold, the buyer's equilibrium strategy $\mu_\alpha^n(\frac{1}{m})$ must converge to 1; as the likelihood of

[27]That is, $\forall \epsilon > 0$, $\exists M > 0$ such that $|z_s^m - 0| < \epsilon$, $\forall m > M$.

commitment types goes to 0, the buyer benefits more if he would demand $\frac{1}{m}$. However, as $\mu^n_\alpha(\frac{1}{m})$ approaches 1, each seller's expected payoff converge to $\frac{1}{2m}$; since the buyer is strong, in equilibrium, he must visit each seller with equal probabilities. Therefore, for large values of n, each seller has incentive to deviate and announce the price as $\frac{1}{m}$ in stage 1 instead of α, which contradicts μ^n_α being an equilibrium strategy.

Therefore, since the buyer is weak for all demands in the support of μ^n_α, the equilibrium strategy μ^n_α must have full support on C_α. Also, taking the limit of the above inequality as z^n_b and z^n_s converge to zero at the same rate will give $2\alpha_b r_s + \alpha r_b \leq r_b$ for all $\alpha_b \in C_\alpha$, which gives the desired result.

Corollary 5.1. *Let $m \to \infty$ and $\delta \to 1$ at the same rate. Then the set $E^\infty(C)$ approaches the interval $[0, \frac{r_b}{r_b+2r_s}]$.*

As a special case, when the buyer's and the sellers' interest rates are common, i.e. $r_b = r_s$, then in the limit, $E^\infty(C)$ approaches $\{(\alpha,\alpha)|\ \alpha \in [0, \frac{1}{3}]\}$. Notice that higher impatience for the rational buyer (higher r_b) will increase the maximum price attainable in equilibrium. On the contrary, increasing impatience for the sellers (higher r_s) decreases the maximum price that can be supported in the limit.

The final result of this section examines a straightforward extension of the model to the case with $N \geq 2$ identical sellers. Namely, let G_N denote the competitive-bargaining game where the number of sellers is N. The game G_N is identical to G except the number of players. Symmetrically define $E^\infty(N, C)$ to be the set of equilibrium prices of the competitive-bargaining game G_N as initial priors vanish.[28] Therefore,

Proposition 5.2. *Let $m \to \infty$ and $\delta \to 1$ at the same rate. Then the set $E^\infty(N, C)$ approaches the interval $[0, \frac{r_b}{r_b+Nr_s}]$.*

6. The Discrete-Time Model and Convergence

In this section, I consider the second-stage competitive-bargaining game in discrete time and investigate the structure of its equilibria as players can make their offers increasingly frequent. I show that given the symmetric posted prices in stage 1, the second stage equilibrium outcomes of the competitive-bargaining game in discrete-time converge to a unique limit, independent of the exogenously given bargaining protocols, as time between offers approach to 0, and this limit is equivalent to the unique outcome of the continuous-time game investigated in Section 3.

[28] I assume that $m \gg N$.

To be more specific, I suppose that each player has a single commitment type and the sellers' type, $\alpha \in (0,1)$ is incompatible with the buyer's type $\alpha_b \in (0,1)$. That is, $\alpha_b < \alpha$. Upon the buyer's arrival at store i, beginning of stage 2 -the bargaining phase,- the buyer and seller i bargain in discrete time according to some protocol g^i that generalizes Rubinstein's alternating offers protocol. A bargaining protocol g^i between the buyer and seller $i \in \{1,2\}$ is defined as $g^i : [0,\infty) \rightarrow \{0,1,2,3\}$ such that for any time $t \geq 0$, an offer is made by the buyer if $g^i(t) = 1$ and by seller i if $g^i(t) = 2$.[29] Moreover, $g^i(t) = 3$ implies a simultaneous offer whereas $g^i(t) = 0$ means no offer is made at time t. An infinite horizon bargaining protocol is denoted by $g = (g^1, g^2)$. The bargaining protocol g is discrete. That is, for any seller $i \in \{1,2\}$ and for all $\bar{t} \geq 0$, the set $I^i := \{0 \leq t < \bar{t} | g^i(t) \in \{1,2,3\}\}$ is countable. Notice that this definition for a bargaining protocol is very general and accommodates non-stationary, non-alternating protocols.

An offer $x \in (0,1)$ denotes the share the seller is to receive. If the proposer's opponent accepts his offer, the game ends with agreement x where $u_i(x,t,i) = xe^{-tr_s}$ denotes the payoff to the seller i, $u_j(x,t,i) = 0$ is the payoff to the seller $j \in \{1,2\}$ with $j \neq i$ and finally $u_b(x,t,i) = (1-x)e^{-tr_b}$ is the payoff to the buyer. If the proposer's opponent rejects his offer, the game continues. Prior to the next offer, the buyer decides whether to stay or leave the store. If the buyer decides to stay, the next offer is made at time $t' := \min\{\hat{t} > t | \hat{t} \in I^i\}$, for example, by the buyer if $g^i(t') = 1$. The two-stage competitive-bargaining game in discrete-time is denoted by $\mathrm{G}\langle g, (z_i, r_i)_{i \in \{b,s\}} \rangle$ (or $\mathrm{G}(g)$ in short). The competitive-bargaining game $\mathrm{G}(g)$ ends if the offers are compatible. In the event of strict compatibility the surplus is split equally. Throughout the game, both sellers can perfectly observe the buyer's moves.[30] Thus, the players' actual types remain to be the only source of uncertainty in the game.

I am particularly interested in equilibrium outcome(s) of the competitive-bargaining game $\mathrm{G}(g)$ in the limit where the players can make sufficiently frequent offers. Therefore, for $\epsilon > 0$ small enough, let $\mathrm{G}(g_\epsilon)$ denote discrete-time competitive-bargaining game where the buyer and the sellers bargain, in stage two, according to the protocol $g_\epsilon = (g^1_\epsilon, g^2_\epsilon)$ such that for all $t \geq 0$ and $i \in \{1,2\}$, both seller i and the buyer have the chance to make

[29]Time 0 denotes the beginning of the bargaining phase.

[30]On the one hand, assuming that each seller is completely unaware of the buyer's move in negotiating with the other seller is in the extreme. When stakes are high, the negotiation becomes (to some degree) public and bargainers can scrutinize their opponents' moves throughout the negotiation process rather easily. YouTube's flirt with Google and Yahoo before Google has acquired YouTube for $1.65 billion and Yahoo's negotiation with Microsoft and AOL Time Warner are just two examples on this account. On the other hand, in this paper, I consider another extreme case where the buyer's actions are perfectly observable. Clearly, in some circumstances, the sellers may not be able to attain all the information nor will the buyer convey it perfectly. Extending the model to introduce some informational imperfections may naturally result in different equilibrium behaviors during the negotiation process. These issues deserve comprehensive considerations and transcend the focus of this particular paper.

an offer, at least once, within the interval $[t, t+\epsilon]$ in the bargaining protocol g^i_ϵ.[31] In this sense, the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$ converges to continuous time as $\epsilon \to 0$.[32]

Now, let σ_ϵ denote a sequential equilibrium of the discrete-time competitive-bargaining game $\mathrm{G}(g_\epsilon)$. The random outcome corresponding to σ_ϵ is a random object θ_ϵ which denotes any realization of an agreed division as well as a time and store at which agreement is reached.

The next result shows that in the limit as ϵ converges to zero $\theta_\epsilon \to \theta$ in distribution, where θ is the unique equilibrium distribution of the continuous-time game G (with obstinate types α and α_b). Therefore, the outcome of the discrete-time competitive-bargaining game, independent of the bargaining protocol g_ϵ, converge in distribution to the unique equilibrium outcome of the competitive-bargaining game analyzed in Section 3-A.

Proposition 6.1. *As ϵ converges to 0, θ_ϵ converges in distribution to θ.*

I defer the proof to Appendix C.

7. The Obstinate Buyer

An obstinate player is a man of unyielding perseverance. Sellers may manifest such a steadfast attitude because they might be confined to do so. A company may be inflexible in a wage negotiation due to some regulations within the company. For example, a car dealer, a sales clerk or a realtor may be restricted by the owner regarding how flexible he can be in his demands while negotiating with a buyer. A fresh college graduate who is competing with other candidates for a specific job opening may commit to a certain salary because he wants to pay his student loan without too much financial difficulty.

Steady persistence in adhering to a course of action as assumed for an obstinate (type) buyer would be reasonable when, for example, the "buyer" is looking to advance his position. A worker (negotiating with more than one firm) may accept the new job offer if it provides a significant jump in his salary or title relative to the position he is already holding. On the other hand, a successful investor (venture capitalist) who only has assets that have high profit margins in his portfolio may commit to buy a small business only if it is a real bargain because otherwise it may not be worth including in

[31] More formally, either $g^i(\hat{t}) = 3$ for some $\hat{t} \in [t, t+\epsilon]$, or $g^i(t') = 1$ and $g^i(t'') = 2$ for some $t', t'' \in [t, t+\epsilon]$.

[32] One may assume that the travel time is discrete and consistent with the timing of the bargaining protocols so the buyer never arrives a store at some non-integer time.

his portfolio. An entrepreneur who is running a successful small business may commit to his initial demands while negotiating with investors to sell his business or a franchise because of overly optimistic expectations about the future of his business.

I assume that the obstinate buyer (1) does not discount time, (2) incurs a positive (but very small) switching cost ($\epsilon_b > 0$) every time he switches his bargaining partner, (3) understands the equilibrium and leaves his bargaining partner when he is convinced that his partner is also obstinate, and finally (4) visits each seller with equal probabilities to announce his demand if it is not compatible with the lowest price announced by the sellers.

According to (1), the time of an agreement is not a concern for the obstinate buyer, and thus he does not feel the need to distinguish himself from the rational buyer who wishes to reach an agreement as quickly as possible. Since the obstinate buyer does not discount time, i.e. $r_b = 0$, we have $\delta = 1$. Therefore, ϵ_b is the only search friction that the obstinate buyer is subject to and it has no crucial impact on our analysis.[33]

The statement in (3) can be interpreted as an implication rather than an assumption. Since the obstinate buyer does not value time (statement (1)), he is indifferent between staying with his current partner or visiting the other seller at any time (ignoring the switching cost). However, if he leaves his current partner before being convinced that he is obstinate, he will revisit this seller later if he exhausts all his hope to reach an agreement with the other seller. Therefore, since the switching cost ϵ_b is positive, the obstinate buyer will switch his partner just once and thus leaves a store when he is convinced that his opponent is the obstinate type.

Moreover, since the sellers share a common initial prior of being the commitment type and assuming that π is uniformly distributed over C, the obstinate buyer is initially indifferent about which store to visit first regardless of the sellers' announced demands. The assumption made in (4), however, is a simplification assumption that can be generalized with no impact on the main message of our results.[34]

Finally, one may think that coexistence of some other commitment types for the buyer (the ones who value time and wish to reach an agreement quickly) could change our results, but this is not necessarily the case. For example, consider a house owner who is negotiating with more than one person to sell his house in order to pay his urgent dept. The buyer (the house owner in this case) may have to commit to a certain price. But in this case, he will clearly not fit the model of the obstinate buyer I described above simply

[33]The switching cost ϵ_b works as a tie-breaking device: When the buyer's continuation payoff of negotiating with each seller is the same, then the buyer will prefer to stay with his current bargaining partner.

[34]For example, one may assume that there are multiple types for the obstinate buyer (regarding the initial store selection) such that some always choose a fix seller and some visit the sellers according to their announcements while the rest are possibly a combination of these two.

because he needs to reach an agreement as quickly as possible. If the buyer has some commitment as in this case, then he may wish to convince the sellers immediately about his actual intention, so he may go back and forth between the sellers multiple times. However, when time is a crucial factor for a bargainer, he usually needs to compromise between two things: Time of the agreement and the share he will receive. Therefore, the commitment of the buyer who runs back and forth between the sellers intensely would be credible from the point of view of the sellers if he is committed to a demand relatively lower than the one who would "play it cool." Thus, coexistence of such commitment types with the ones I assume here will not alter the results[35] if we assume that the buyer's commitment to high demands is interpreted by the sellers such that the buyer must be the one who will "play it cool." For this reason, I restrict my attention only to those commitment types that I described above.

8. Concluding Remarks

This paper develops a reputation-based model to highlight the influence of posted prices and obstinacy on imperfectly competitive search markets. The introduction of obstinate types that are completely inflexible in their demands and offers, even with low probabilities, makes the equilibrium of the multilateral bargaining game essentially unique. The equilibrium allocation does not depend on the fine details of the bargaining protocols, nor do the sellers extract all the surplus of the buyer because of the positive search friction. Instead, it depends on the posted prices and initial reputations as well as the time preferences of the players. The equilibrium has a war of attrition structure that engenders inefficiency due to possible delay in reaching an agreement. Although the sellers compete in the spirit of Bertrand, the equilibrium outcomes are in contrast to Bertrand's prediction.

Appendix A

Proof of Proposition 3.1. For any $i \in \{1, 2\}$ and history $h_{T_i} \in H^i$, consider a pair of equilibrium distribution functions $(F_b^{i,T_i}, F_i^{T_i})$. I next study the properties of these distribution functions on their domain $[T_i, T_i']$ where $T_i' \leq \infty$ depends on the buyers' equilibrium strategy σ_{s_2}. Proofs of the following results directly follow from the arguments in Hendricks, Weiss and

[35] Particularly the limiting results in Sections 4 and 5.

Wilson (1988) and is analogous to the proof of Lemma 1 in Abreu and Gul (2000), so I skip the details.

Lemma A.1. *If a player's strategy is constant on some interval $[t_1, t_2] \subseteq [T_i, T_i')$, then his opponent's strategy is constant over the interval $[t_1, t_2 + \eta]$ for some $\eta > 0$.*

Lemma A.2. *F_b^{i,T_i} and $F_i^{T_i}$ do not have a mass point over $(T_i, T_i']$.*

Lemma A.3. $F_i^{T_i}(T_i)F_b^{i,T_i}(T_i) = 0$

Therefore, according to Lemma A.1 and A.2, both $F_i^{T_i}$ and F_b^{i,T_i} are strictly increasing and continuous over $[T_i, T_i']$. Let

$$U_i(t, F_b^{i,T_i}) = \int_{T_i}^{t} \alpha e^{-r_s x} dF_b^{i,T_i}(x) + \alpha_b e^{-r_s t}(1 - F_b^{i,T_i}(t))$$

denote the expected payoff of rational seller $i \in \{1, 2\}$ who concedes at time $t \geq T_i$ and

$$U_b(t, F_i^{T_i}) = \int_{T_i}^{t} (1 - \alpha_b)e^{-r_b x} dF_i^{T_i}(x) + (1 - \alpha)e^{-r_b t}(1 - F_i^{T_i}(t))$$

denote the expected payoff of the rational buyer who concedes to seller i at time $t \geq T_i$. Therefore, the utility functions are also continuous on $[T_i, T_i']$.

Then, it follows that $D^{i,T_i} := \{t | U_i(t, F_b^{i,T_i}) = \max_{s \in [T_i, T_i']} U_i(s, F_b^{i,T_i})\}$ is dense in $[T_i, T_i']$. Hence, $U_i(t, F_b^{i,T_i})$ is constant for all $t \in [T_i, T_i']$. Consequently, $D^{i,T_i} = [T_i, T_i']$. Therefore, $U_i(t, F_b^{i,T_i})$ is differentiable as a function of t. The same arguments also hold for $F_i^{T_i}$. The differentiability of $F_i^{T_i}$ and F_b^{i,T_i} follows from the differentiability of the utility functions on $[T_i, T_i']$. Differentiating the utility functions and applying the Leibnitz's rule, we get $F_i^{T_i}(t) = 1 - c_i e^{-\lambda t}$ and $F_b^{i,T_i}(t) = 1 - c_b^i e^{-\lambda_b t}$ where $c_i = 1 - F_i^{T_i}(T_i)$ and $c_b^i = 1 - F_b^{i,T_i}(T_i)$ such that $\lambda_b = \frac{\alpha_b r_s}{\alpha - \alpha_b}$ and $\lambda = \frac{(1-\alpha)r_b}{\alpha - \alpha_b}$.

Now, consider the equilibrium strategies after the history that the buyer visits seller 1 first. In equilibrium we have $F_2(0) > 0$. Otherwise, the buyer's discounted continuation payoff in store 2, $\delta v_b^2 = \delta[F_2(0)(1 - \alpha_b) + (1 - F_2(0))(1 - \alpha)]$, is equal to $\delta(1 - \alpha)$. In this case, the rational buyer prefers to concede to seller 1 instead of traveling store 2. By Lemma A.3, we know that $F_2(0)F_b^2(0) = 0$. Hence, we have $F_b^2(0) = 0$, implying that $c_b^2 = 1$. Moreover, in equilibrium the first seller's reputation reaches one at time T_1^d, i.e. $F_1(T_1^d) = 1 - z_s$, and the second seller's reputation reaches one at time T_2^e, i.e. $F_1(T_2^e) = 1 - z_s$. Thus we have $c_1 = z_s e^{\lambda T_1^d}$ and $c_2 = z_s e^{\lambda T_2^e}$ as required.

There is no sequential equilibrium of the game G such that the buyer visits a store multiple times. Suppose on the contrary that there is a strategy in which, without loss of generality, the buyer visits store 1 twice. Then, the buyer must be the strong player in his second visit to seller 1. That is, seller 1 should concede to the buyer with a positive probability at the time the buyer enters his store for the second time. Otherwise the buyer would prefer to concede to seller 2 and finish the game before making the second visit to store 1 (because $\delta < 1$). Thus, since seller 1 is the weak player, his expected payoff is α_b when the buyer visits his store for the second time. However, in equilibrium this continuation payoff contradicts the optimality of seller 1's strategy because, to eliminate a further delay seller 1 would prefer to accept the buyer's offer (for sure) when the buyer first attempts to leave his store.

Finally, I want to argue that in equilibrium the buyer and seller 1 cannot play a strategy that extends the concession game in store 1 beyond the time T_1^d with some positive probability. Suppose on the contrary that each player chooses such a strategy, and these strategies establish an equilibrium. Conditional on players delaying the end of the concession game in store 1 for some extra $\hat{t}$ unit of time after T_1^d, the buyer should be indifferent between conceding to seller 1 and waiting for concession at any time $t \in [T_1^d, T_1^d + \hat{t}]$. That is, the buyer must concede to seller 1 with a constant hazard rate during this extra time.

However, in equilibrium, buyer's expected payoff in store 2 is a continuous and increasing function of his own reputation and of time. Therefore, we have $1 - \alpha < \delta v_b^2(t)$ for all $t \in (T_1^d, T_1^d + \hat{t}]$. That is, at any time $t > T_1^d$, the buyer's discounted continuation payoff in store 2 will be strictly higher than his instantaneous payoff in store 1. However, this contradicts the optimality of the equilibrium strategy.

Therefore, conditional on each player executing a strategy that extends the concession game in store 1 after the time T_1^d, and these strategies constitute an equilibrium, the buyer should not concede to seller 1 with a positive (constant) hazard rate after T_1^d. However, this requirement implies that rational seller 1 must accept the buyer's offer by the time T_1^d with probability 1, which contradicts the initial assumption. Moreover, since the buyer's equilibrium strategy F_b^1 cannot have a discontinuity point over the interval $(0, T_1^d]$, the buyer cannot make a positive concession at time T_1^d. Thus, the event that the buyer leaves seller 1 at time T_1^d must occur with probability one in equilibrium.

Proof of Lemma 3.1-3.3. Consider an equilibrium strategy after a history where the buyer visits store 1 first at time 0. Let $\tau_i = \inf\{t \geq 0 | \ F_i(t) = 1 - z_s\}$ denote the time that seller i's reputation reaches 1. Similarly, let $\tau_b^1 = \inf\{t \geq 0 | \ F_b^1(t) = 1 - \hat{z}_b(1)\}$ denote the time that the buyer's reputation reaches 1 in store 1.

By Proposition 3.1 we have $F_b^2(t) = 1 - e^{-\lambda_b t}$. Therefore, $F_b^2(T_2^e) = 1 - \hat{z}_b(T_1^d)$ implies that $T_2^e = -\frac{\log \hat{z}_b(T_1^d)}{\lambda_b}$. Therefore, the buyer's continuation payoff in store 2 is $v_b^2 = 1 - \alpha_b - z_s[\hat{z}_b(T_1^d)]^{-\frac{\lambda}{\lambda_b}}(\alpha - \alpha_b)$. Hence, T_1^d must solve $1 - \alpha = \delta v_b^2$, implying that $\hat{z}_b(T_1^d) = \left(\frac{z_s}{A}\right)^{\frac{\lambda_b}{\lambda}} := X_s$ where $A = \frac{1-\alpha_b - \frac{1-\alpha}{\delta}}{\alpha - \alpha_b}$.

Note that for large values of δ, we have $z_s^{\lambda_b/\lambda} < X_s < 1$. Moreover, since $\hat{z}_b(T_1^d) = \frac{\hat{z}_b(1)}{1 - F_b^1(T_1^d)} = X_s$ and $F_b^1(T_1^d) = 1 - c_b^1 e^{-\lambda_b T_1^d}$, in equilibrium we have

$$c_b^1 e^{-\lambda_b T_1^d} = \frac{\hat{z}_b(1)}{X_s} \tag{7}$$

Next we will find the buyer's equilibrium departure time from the first seller (i.e. seller 1). But first note that there are two critical times that we need to consider. These are $\tau_1 = -\frac{\log z_s}{\lambda}$ and $\tau_b^1 = -\frac{\log \hat{z}_b(T_1^d)}{\lambda_b}$. The former denotes the time that seller 1's reputation reaches one and the latter denotes the time that the buyer's reputation reaches one in store 2 if he leaves store 1 at time T_1^d.

First, consider the case where $c_1 \neq 1$. Lemma A.3 implies that $c_b^1 = 1$ and Equation (7) yields

$$T_1^d = -\frac{\log(\hat{z}_b(1)/X_s)}{\lambda_b}$$

Remark that T_1^d is well defined, i.e. $T_1^d > 0$, whenever $\hat{z}_b(1) < X_s$. Note that $T_1^d < \tau_1$ whenever $-\frac{\log(\hat{z}_b(1)/X_s)}{\lambda_b} < -\frac{\log z_s}{\lambda}$ implying that $\hat{z}_b(1) > X_s z_s^{\lambda_b/\lambda}$. It is also true that $T_1^d < \tau_b^1$ because $X_s < 1$ for δ large enough.

However, if $c_1 = 1$, i.e. seller 1 does not make an initial probabilistic acceptance, it should be true that $T_1^d \geq \tau_1$. In equilibrium, the buyer does not bear the cost of delay and thus it must be true that $T_1^d = \tau_1$. Note that for $T_1^d = \tau_1$ to be true in equilibrium it must be that $\tau_1 \leq \tau_b^1$. Moreover, since $T_1^d = -\frac{\log z_s}{\lambda}$, Equation (7) implies that $c_b^1 = \frac{\hat{z}_b(1)}{X_s z_s^{\lambda_b/\lambda}}$. It is well defined whenever (i) $\hat{z}_b(1) \leq c_b^1$ implying $X_s z_s^{\lambda_b/\lambda} \leq 1$ which is true for δ large enough, and (ii) $c_b^1 \leq 1$ implying that $\hat{z}_b(1) \leq X_s z_s^{\lambda_b/\lambda}$. Note also that we need to have $X_s < 1$ and $A > 0$, which holds whenever $\delta > (1-\alpha)/(1-\alpha_b-(\alpha-\alpha_b)z_s)$.

Therefore, for all values of $\hat{z}_b(1)$ satisfying $\hat{z}_b(1) < X_s z_s^{\lambda_b/\lambda}$, seller 1 does not make any initial concession, and the buyer leaves store 1 at $T_1^d = \tau_1$. Notice that for these values of $\hat{z}_b(1)$, we have $\tau_1 \leq \tau_b^1$ as required. Thus, for all $\hat{z}_b(1) < X_s$, the buyer leaves store 1 at time $T_1^d = \min\{-\frac{\log z_s}{\lambda}, -\frac{\log(\hat{z}_b(1)/X_s)}{\lambda_b}\}$. However, if $\hat{z}_b(1) \geq X_s$ then at time $t = 0$ the buyer's discounted expected payoff in store 2, δv_b^2, is larger than $1-\alpha$. Therefore, for such values of $\hat{z}_b(1)$, the buyer immediately leaves store 1 at time 0. As a result, rational seller 1 concedes to the buyer with probability 1 at time 0.

Since $F_1(T_1^d) = 1 - c_1 e^{-\lambda T_1^d} = 1 - z_s$, we have $c_1 = z_s e^{\lambda T_1^d}$. Moreover, for $\hat{z}_b(1) \geq X_s$, we have $c_b^1 = 1$. Otherwise it must be that $c_b^1 = \frac{\hat{z}_b(1)}{X_s z_s^{\lambda_b/\lambda}}$. Therefore, it can be summarized that

$$c_b^1 = \begin{cases} \frac{\hat{z}_b(1)}{X_s} e^{\lambda_b T_1^d}, & \text{if } \hat{z}_b(1) < X_s \\ 1, & \text{otherwise} \end{cases}$$

On the other hand, we have $c_2 = z_s e^{\lambda T_2^e}$ and $c_b^2 = 1$ where $T_2^e = -\frac{\log \hat{z}_b(T_1^d)}{\lambda_b}$, and $\hat{z}_b(T_1^d)$ is either X_s or $\hat{z}_b(1)$.

Proof of Proposition 3.2. Let $z_b > X_s z_s^{\lambda_b/\lambda}$. If $\sigma_\alpha(i) > 1/2$, then $\hat{z}_b(j) > \hat{z}_b(i)$ where $i, j \in \{1, 2\}$ and $j \neq i$ and $\hat{z}_b(j) > z_b$. Since the buyer's expected payoff is increasing with his reputation (see Equations 5 and 6), the buyer strictly prefers to visit seller j first, contradicting with $\sigma_\alpha(i) > 1/2$ being an equilibrium strategy.

Now, let $z_b \leq X_s z_s^{\lambda_b/\lambda}$. If the buyer is strong relative to $\hat{z}_b(i)$, then in equilibrium he must be strong with respect to $\hat{z}_b(j)$ too. Moreover, in equilibrium, we must have $\hat{z}_b(1) = \hat{z}_b(2)$, but this is possible only if $\sigma_\alpha(1) = \frac{1}{2}$. However, in this case we will have $\hat{z}_1 = \hat{z}_2 = z_b$ and it is less than $X_s z_s^{\lambda_b/\lambda}$, implying that the buyer cannot be strong. Therefore, for each i, we have $\hat{z}_b(i) \leq X_s z_s^{\lambda_b/\lambda}$. By Baye's rule we have $\hat{z}_b(i) = \frac{z_b}{z_b + 2(1-z_b)\sigma_\alpha(i)}$ which implies the desired result.

Proof of Proposition 3.3. Since we have $1-\alpha_1 < \delta(1-\alpha_2)$, in equilibrium the buyer finds it optimal to go to store 2 instead of conceding to seller 1 at any given time $t \geq 0$. Moreover, the buyer waits in store 1 if he believes that seller 1's concession will come after a short delay. But postponing concession is not optimal for rational seller 1 because the buyer will never accept the seller's demand. Thus, in equilibrium, the buyer leaves seller 1 immediately if seller 1 does not accept the buyer's demand α_b, and rational seller 1 concedes to the buyer with probability 1 upon his arrival.

Upon arrival at store 2 (after visiting seller 1), the buyer and seller 2 play the concession game. By Lemma A.3, we know that $F_2(0)F_b^2(0) = 0$. First, suppose that $F_b^2(0) = 0$. It implies that $c_b^2 = 1$ and thus $F_b^2(t) = 1 - e^{-\lambda_b t}$. Therefore, $F_b^2(T_2^e) = 1 - \hat{z}_b(1)$, implying that $1-e^{-\lambda_b T_2^e} = 1-\hat{z}_b(1)$ if and only if $T_2^e = -\frac{\log \hat{z}_b(1)}{\lambda_b}$. On the other hand, if $F_2(0) = 0$, then we have $c_2 = 1$ and thus $F_2(t) = 1 - e^{-\lambda t}$. Therefore, $F_2(T_2^e) = 1 - z_s$, implying that $1 - e^{-\lambda T_2^e} = 1 - z_s$ if and only if $T_2^e = -\frac{\log z_s}{\lambda}$. Hence, the game ends at time $T_2^e = \min\{-\frac{\log \hat{z}_b(1)}{\lambda_b}, -\frac{\log z_s}{\lambda}\}$.

Moreover, players concede according to the strategies $F_2(t) = 1-c_2 e^{-\lambda_2 t}$, $F_b^2(t) = 1-c_b^2 e^{-\lambda_b t}$ where $c_2 = z_s e^{\lambda_2 T_2^e}$ and $c_b^2 = \hat{z}_b(1)e^{\lambda_b T_2^e}$. Note that the buyer's expected payoff (evaluated at time 0) in this subgame is $V_b(1) = (1 - z_s)(1 - \alpha_b) + \delta z_s u_b^2$ where $u_b^2 = (1 - F_2(0))(1 - \alpha_2) + F_2(0)(1 - \alpha_b)$. In particular, if the buyer is strong relative to seller 2, i.e., $\hat{z}_b(1) \geq z_s^{\lambda_b/\lambda_2}$ then $u_b^2 = z_s[\hat{z}_b(1)]^{-\lambda_2/\lambda_b}(1 - \alpha_2) + (1 - z_s[\hat{z}_b(1)]^{-\lambda_2/\lambda_b})(1 - \alpha_b)$. Otherwise, we have $u_b^2 = 1 - \alpha_2$.

Now consider an equilibrium strategy after a subgame that the buyer visits seller 2 first at time 0. If the buyer concedes to seller 2, his instantaneous payoff is $1 - \alpha_2$. However, if the buyer leaves store 2 at some time $t \geq 0$ and goes to store 1, we know from previous arguments that concession in store 1 will immediately finish upon arrival of the buyer. So, the buyer would directly come back to store 2 if seller 1 is the obstinate type. Thus, the buyer's continuation payoff if he leaves store 2 at time t is $(1 - z_s)(1 - \alpha_b) + \delta z_s v_b^2$ where $v_b^2 = (1 - F_2(0))(1 - \alpha_2) + F_2(0)(1 - \alpha_b)$ denotes the buyer's expected payoff in his second visit to store 2.[36]

$v_b^2 > 1 - \alpha_2$ requires that seller 2 offers positive probabilistic gift to the buyer on his second visit. In this case, seller 2's expected payoff must be α_b. However, optimality of the equilibrium strategy implies that rational seller 2 should accept the buyer's offer with probability 1 when the buyer attempts to leave his store for the first time. Hence, it must be that in equilibrium $v_b^2 = 1 - \alpha_2$. Therefore, the buyer's payoff if he leaves store 2 at time 0 is $\bar{V}_b = \delta\left[(1 - z_s)(1 - \alpha_b) + \delta z_s(1 - \alpha_2)\right]$.

If $\bar{V}_b$ is larger than $1 - \alpha_2$, then the buyer leaves store 2 immediately at time 0 instead of conceding to seller 2. However, $\bar{V}_b > 1 - \alpha_2$ implies that $z_s < \frac{1-\alpha_b-\frac{1-\alpha_2}{\delta}}{1-\alpha_b-\delta(1-\alpha_2)} := \bar{z}$. Note that $\bar{z} > 0$ since by assumption we have $\delta(1 - \alpha_b) > (1 - \alpha_2)$. Moreover, since we consider small values of z_s, we have $z_s < \bar{z}$, so the buyer finds it optimal to leave store 2 immediately at time 0.

On the other hand, if seller 2 is weak, i.e. $\hat{z}_b(2) \geq z_s^{\lambda_b/\lambda_2}$, then optimality of the equilibrium

[36] Notice that there is no equilibrium in which the buyer visits store 1 multiple times and store 2 more than twice.

strategy implies that rational seller 2 must accept the buyer's offer at time 0 with probability 1. However, if the buyer is weak relative to seller 2, i.e. $\hat{z}_b(2) < z_s^{\lambda_b/\lambda_2}$, then seller 2's payoff when the buyer arrives at store 2 for the second time is no more than $\delta^2 z_s \alpha_2$. However, by assumption we have $\alpha_b \geq z_s\alpha_2$, implying that rational seller 2 immediately concedes to the buyer with probability 1, and gets α_b, upon the buyer's arrival at time 0.

Proof of Proposition 3.4. Suppose that the buyer is strong relative to seller 2, i.e. $\hat{z}_b(1) > z_s^{\lambda_b/\lambda_2}$. If the buyer first visits seller 1, then his expected payoff is $V_b(1) = (1-z_s)(1-\alpha_b)+\delta z_s u_b^2$ where $u_b^2 = z_s[\hat{z}_b(1)]^{-\lambda_2/\lambda_b}(1-\alpha_2)+(1-z_s[\hat{z}_b(1)]^{-\lambda_2/\lambda_b})(1-\alpha_b)$. However, if he chooses to visit seller 2 first, his expected payoff is $V_b(2) = (1-z_s)(1-\alpha_b)+z_s\delta[(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)]$. Hence, the buyer is indifferent between which seller to visit first if, and only if $V_b(1) = V_b(2)$, i.e. $(1-[\hat{z}_b(1)]^{-\lambda_2/\lambda_b})(1-\alpha_b) = (1-\alpha_2)(\delta-[\hat{z}_b(1)]^{-\lambda_2/\lambda_b})$, implying that $\hat{z}_b(1) = \left(\frac{\alpha_2-\alpha_b}{1-\alpha_b-\delta(1-\alpha_2)}\right)^{\lambda_b/\lambda}$. Since the Bayes' rule implies that $\hat{z}_b(1) = \frac{1/2z_b}{1/2z_b+(1-z_b)\sigma_{(\alpha_1,\alpha_2)}(1)}$, we have the desired result.

Suppose now that the buyer is weak relative to seller 2, that is $\hat{z}_b(1) \leq z_s^{\lambda_b/\lambda_2}$. If the buyer first visits store 1, then $V_b(1) = (1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)$. However, if the buyer first visits store 2, then the buyer's expected payoff, $V_b(2)$, as given above. Thus, the buyer is indifferent between the sellers if, and only if $V_b(1) = V_b(2)$ which holds whenever $z_s = \frac{1-\alpha_b-(1-z_s)(1-\alpha_b)-\delta z_s(1-\alpha_2)}{1-\alpha_b-\delta[(1-z_s)(1-\alpha_b)+\delta z_s(1-\alpha_2)]}$. However, this equality cannot be true when z_s is small and δ is close enough to 1. Hence, in equilibrium, the rational buyer will always pick $\sigma_{(\alpha_1,\alpha_2)}(1)$ in such a way that he becomes strong relative to seller 2 when he visits store 1 first.

Appendix B

Proof of Proposition 4.1. Suppose for a contradiction that $(\alpha_1,\alpha_2) \in E(C)$, and without loss of generality that $\alpha_1 > \alpha_2 \geq \frac{1}{m}$. According to Propositions 3.3 and 3.4, the buyer will visit seller 1 first with the probability $\sigma_{(\alpha_1,\alpha_2)}(1) \in (0,1)$, demand $\frac{1}{m}$ and leave immediately if seller 1 does not concede to the buyer. This is the equilibrium strategy independent of the initial beliefs z_s and z_b. Therefore, at any $G(z_s,z_b)$ the payoff to seller 1 is no more than $\frac{1}{m}\sigma_{(\alpha_1,\alpha_2)}(1)+z_s\frac{1}{m}(1-\sigma_{(\alpha_1,\alpha_2)}(1))$. However, seller 1 could profitably deviate and demand $\frac{1}{m}$ in stage 1, yielding a payoff more than $\frac{1}{2m}$ that is higher than what seller 1 would get by posting α_1.

Proof of Proposition 4.2. First I will show that $\{(\alpha,\alpha)|\alpha \in C\} \setminus \{\frac{2}{m}\} \subseteq E(C)$. For this purpose, take any $\alpha \in C$ and suppose that both sellers choose α in stage 1, the buyer chooses each seller with probability 1/2, and in the subgames following the first stage, each player uses the equilibrium strategies given in Propositions 3.1 and Lemmas 3.1-3.3. Also assume that the buyer's strategy μ_α (randomization over the set C_α) is uniform. Then choose the initial priors z_b, z_s (sufficiently small) such that the buyer is weak at all $\alpha_b \in C_\alpha$. More formally, the posterior belief that the buyer is the obstinate type conditional on the buyer chooses seller i and demand $\alpha_b \in C_\alpha$, i.e. $\hat{z}_b(i,\alpha_b) = \frac{(1/(m-1))z_b}{(1/(m-1))z_b+(1-z_b)(1/m_\alpha)}$ (where m and m_α are the cardinalities of the sets C and C_α, respectively), is strictly less than $(\frac{z_s^2}{A_{\alpha_b}})^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for all $\alpha_b \in C_\alpha$ with $A_{\alpha_b} = \frac{1-\alpha_b-\frac{1-\alpha}{\delta}}{\alpha-\alpha_b}$. Thus, the buyer is weak for any α_b in the support of his strategy μ_α, implying that the buyer's expected payoff is $1-\alpha$ in the game G. Since the buyer will never deviate to a demand above α, he has no profitable deviation.

On the other hand, the sellers do not have a profitable deviation either: If, for example, seller 1 deviates to a demand different than α, then the buyer will assign probability one that seller 1 is the obstinate type. In this case, the buyer will visit seller 1 first if the seller deviates to the demand $1/m$. Otherwise, the buyer will visit seller 1 after being convinced that seller 2 is the obstinate type as well.[37] Therefore, deviating to some α' such that $1/m < \alpha' < \alpha$ is not a profitable deviation for seller 1 because we assume that z_s is very small. Likewise, deviating to α_{min} is not profitable either because the seller's ex-ante expected payoff of declaring the demand α is $u_\alpha = \frac{1}{2m_\alpha}\sum_{x\in C_\alpha}[x+(\alpha-x)F_b^1(0,x)]$ where $F_b^1(0,x) = 1 - z_b\left(\frac{A}{z_s^2}\right)^{xr_s/(1-\alpha)r_b}$ (since the sellers are strong for all $x \in C_\alpha$), and it is higher than $1/m$ for large values of m.

The obstinate demand $\frac{2}{m}$ cannot be supported in equilibrium (for any positive and small z_b and z_s) because if sellers post $\frac{2}{m}$ in stage 1, then they can achieve their highest expected payoff when z_b is very weak relative to z_s. However, even in this case, the sellers' payoffs will be strictly less than $\frac{1}{m}$, assuming that the buyer visits sellers with equal probabilities, because the buyer is obstinate with some small, but positive, probability z_b and thus in equilibrium the sellers may have to concede to the buyer's demand (which is $\frac{1}{m}$) with a positive probability. For this reason, the sellers have incentive to deviate and post price $\frac{1}{m}$ instead of $\frac{2}{m}$. Hence, by Proposition 4.1, we have $E(C) = \{(\alpha,\alpha)|\alpha \in C\} \setminus \{\frac{2}{m}\}$.

Proof of Proposition 5.1. First, I will show that there exist no sequences of $\{z_b^n\}, \{z_s^n\}$ (converging to zero at the same rate) and $\alpha \in C$ with $2/m < \alpha$ such that (α,α) is the equilibrium demand selection of the sellers in the game $G(z_b^n, z_s^n)$ and the buyer is strong for some $\alpha_b \in C_\alpha$ for each $n \geq 0$.

Suppose that for all $n \geq 0$, the buyer is strong with the posterior belief that he is the obstinate type conditional on him visiting seller i first and choosing the demand $1/m$, i.e. $\hat{z}_b^n(i, 1/m)$. Then, there exists some demand $\alpha_b \in C_\alpha$ with $1/m < \alpha_b$ such that μ_α^n assigns positive weight. Suppose not. In this case, the sellers' expected payoff is no more than $1/2m$. Thus, either seller can benefit by deviating and posting price of $1/m$ (instead of α) in stage 1. Therefore, if the buyer is strong with $\hat{z}_b^n(i, 1/m)$, then the buyer must be strong at all $\hat{z}_b^n(i,\alpha_b)$ where $\mu_\alpha^n(\alpha_b) > 0$. Also note that, if $\alpha_b \in C_\alpha$ is in the support of μ_α^n, then so does a demand $\hat{\alpha}_b \in C_\alpha$ with $\hat{\alpha}_b \leq \alpha_b$.

Let $1/m < \alpha_b$ be a demand in the support of μ_α^n for each $n \geq 0$. To simplify the subsequent notation I will drop the n terms and denote the sequences by $\{z_b\}, \{z_s\}$. The buyer's expected payoff of declaring his demand as α_b (when he is strong) is either

[37] If seller 1 deviates to α' such that $1/m < \alpha' < \alpha$, then the buyer will visit seller 2 first for sure to make a "take $1/m$ or I will leave you" offer. This choice is optimal for the buyer because rational seller 2 will immediately accept the buyer's demand $1/m$ (since we have $\delta(1-\alpha') > 1-\alpha$ by assumption).

$$\left(1 - \frac{z_s^2}{A[\hat{z}_b(i,\alpha_b)]^{\lambda/\lambda_b}}\right)(1-\alpha_b) + \frac{z_s^2}{A[\hat{z}_b(i,\alpha_b)]^{\lambda/\lambda_b}}(1-\alpha)$$

or

$$\left[1 - z_s(1-\delta) - \frac{\delta z_s^2}{[\hat{z}_b(i,\alpha_b)]^{\lambda/\lambda_b}}\right](1-\alpha_b) + \frac{\delta z_s^2}{[\hat{z}_b(i,\alpha_b)]^{\lambda/\lambda_b}}(1-\alpha)$$

Remark that as z_b (equivalently $\hat{z}_b(i,\alpha_b)$) and z_s converge to zero at the same rate, both of the these payoffs converge to $1-\alpha_b$.[38] However, this implies that the buyer would strictly prefer to declare smaller demands for sufficiently smaller values of z_s and z_b, contradicting that in equilibrium there exists $\alpha_b > 1/m$ in the support of μ_α^n (for all n). One possibility to avoid contradiction is that as n increases, μ_α^n may attach higher probability on $1/m$ and thus smaller probabilities on other demands. However, in this case, the seller's expected payoff will take values close to $1/2m$ for large values of n (since the buyer is strong, in equilibrium he visits each seller with equal probabilities.) This, again, yields a contradiction because the sellers clearly prefer to deviate and post price of $1/m$ instead of α.

Therefore, in equilibrium for $n \geq 0$ large enough, we must have $\hat{z}_b^n(i,\alpha_b) \leq \left(\frac{(z_s^n)^2}{A}\right)^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for each $i \in \{1,2\}$, α_b in the support of μ_α^n (otherwise, the unique equilibrium is such that both sellers choose $1/m$). Moreover, since the buyer is weak at all demands in the support of μ_α^n, then μ_α^n must have full support on C_α. Taking the log of both sides and rearranging the terms of the above inequality yields

$$\frac{2\alpha_b r_s}{(1-\alpha)r_b} - 1 \leq \frac{\ln[\frac{1}{2}Kz_s^n\pi(\alpha_b) + (1-Kz_s^n)\mu_\alpha^n(\alpha_b)\sigma_\alpha^n(i)]}{|\ln z_s^n|} - \frac{\ln(\frac{1}{2}K\pi(\alpha_b))}{|\ln z_s^n|} - \frac{\alpha_b r_s \ln(A)}{(1-\alpha)r_b|\ln z_s^n|}$$

for any $i \in \{1,2\}$. The limit of the right hand side as z_s^n converges to zero is 0, implying that we must have $2\alpha_b r_s \leq (1-\alpha)r_b$ for all $\alpha_b \in C_\alpha$ as required.

Proof of Proposition 5.2. The same arguments used in the proofs of Propositions 4.1 and 4.2 suffice to show that $E(N,C) = \{(\alpha,...,\alpha) \in C^N | \alpha \in C\} \setminus \{\frac{2}{m}, \frac{3}{m}, ..., \frac{N}{m}\}$ where $m \gg N$. That is, in equilibrium all the sellers will choose the same demand in stage 1. Also, the arguments in the proof of Proposition 5.1 will show that there exist no sequences of prior beliefs $\{z_b^n\}, \{z_s^n\}$ (converging to zero at the same rate) and $\alpha \in C$ with $N/m < \alpha$ such that $(\alpha,...,\alpha)$ is an equilibrium demand selection of the sellers in the game $G_N(z_b^n, z_s^n)$ and the buyer is strong for some $\alpha_b \in C_\alpha$ (in the support of μ_α^n) for each $n \geq 0$.

Recall that the claim of the Proposition 5.1 (i.e. $(\alpha,\alpha) \in E^\infty(C) \Leftrightarrow 2\alpha_b r_s \leq (1-\alpha)r_b$) relies solely on the fact that in equilibrium (when there are two sellers) we must have $\hat{z}_b(i,\alpha_b) \leq \left(\frac{z_s^2}{A}\right)^{\frac{\alpha_b r_s}{(1-\alpha)r_b}}$ for each α_b in the support of μ_α and for each admissible sequence of $\{z_b, z_s\}$ converging to zero at the same rate. Namely, the buyer must be weak (and continues to be weak)

[38]Remark that as the initial priors converge to zero at the same rate, the strength of the buyer may alter. For example, the buyer remains to be the strong player for sufficiently small priors if and only if $1 < \lambda/\lambda_b < 2$ holds. Therefore, the first equation gives the buyer's expected payoff for small values of z_s and z_b only when the last inequality holds, implying that the value of the first equation approaches to $1-\alpha_b$ for vanishing priors. Similar arguments apply for the second equation.

as initial priors converge to zero. Therefore, we need to find the condition such that the buyer is weak under the initial priors z_b and z_s when the declared demands in stage 1 are $\alpha_b, \alpha \in C$ with $\alpha_b < \alpha$.

For the ease of exposition, I will derive this condition for the 3-sellers case, which can be extended to N-sellers case by iterating the same process (the following arguments are straightforward extensions of the approach that I use in the proof of Proposition 3.1). For this reason, suppose now that there are tree sellers all of which choose the same demand α in stage 1 and the buyer declares his demand as $\alpha_b < \alpha$. Without loss of generality, I assume that the buyer visits seller 1 first and seller 3 last (if no agreement have been reached with the sellers 1 and 2). Therefore, let T_i^d denote the time that the buyer leaves seller $i \in \{1, 2\}$.

The buyer leaves seller 2 when his discounted continuation payoff in store 3, i.e. $v_b^3(T_2^d) = 1 - \alpha_b - z_s[\hat{z}_b(T_2^d)]^{-\lambda/\lambda_b}(\alpha - \alpha_b)$, equals to $1 - \alpha$. This equality implies that the buyer must leave the second store when his reputation reaches to X_s, namely $\hat{z}_b(T_2^d) = X_s = (\frac{z_s}{A})^{\lambda_b/\lambda}$. Therefore, the buyer's expected payoff in store 2 at the time he enters this store is $v_b^2(T_1^d) = 1 - \alpha_b - z_s \left[\frac{X_s}{\hat{z}_b(T_1^d)}\right]^{\lambda/\lambda_b} (\alpha - \alpha_b)$. Note that if the buyer is weak, then $\hat{z}_b(T_1^d) < X_s$ is always the case. If, on the contrary, $\hat{z}_b(T_1^d) \geq X_s$ holds, then at the time the buyer leaves store 1, his expected payoff in store 2 will be strictly higher than his continuation payoff in store 1 (i.e., $1 - \alpha$). This is possible in equilibrium only if $T_1^d = 0$, implying that the buyer is strong.

Thus, similar to the previous arguments, the buyer leaves seller 1 when his discounted continuation payoff in store 2, i.e. $v_b^2(T_1^d)$, equals to $1 - \alpha$. Then we have $\hat{z}_b(T_1^d) = \left(\frac{z_s^2}{A}\right)^{\lambda_b/\lambda}$. Since $\hat{z}_b(T_1^d) = \frac{\hat{z}_b(1,\alpha_b)}{1-F_b^1(T_1^d)}$, the previous equality implies that $\frac{\hat{z}_b(1,\alpha_b)}{c_b^1 e^{-\lambda_b T_1^d}} = \left(\frac{z_s^2}{A}\right)^{\lambda_b/\lambda}$. If the buyer does not make an initial acceptance, i.e. $c_b^1 = 1$, then $\frac{\hat{z}_b(1,\alpha_b)}{(z_s^2/A)^{\lambda_b/\lambda}} = e^{-\lambda_b T_1^d}$ which implies that $T_1^d = \frac{-\log(\hat{z}_b(1,\alpha_b)/(z_s^2/A)^{\lambda_b/\lambda})}{\lambda_b}$. Therefore, if the buyer is weak, then we must have $T_1^d \geq \frac{-\log z_s}{\lambda}$ implying that

$$\hat{z}_b(1, \alpha_b) \leq \left(\frac{z_s^3}{A}\right)^{\lambda_b/\lambda} \tag{8}$$

The last inequality implies that (as z_b and z_s converge to zero at the same rate) an equilibrium demand, α, chosen by the sellers must satisfy $3\alpha_b r_s + \alpha r_b \leq r_b$ for all α_b in C_α. Therefore, as $m \to \infty$ and $\delta \to 1$ at the same rate, we will have $E^\infty(N, C) \to [0, \frac{r_b}{r_b+3r_s}]$ which proves the claim for $N = 3$. Iterating the above arguments suffice to prove the claim for any finite N.

Appendix C

I first present a serious of results which I will later use in the proof of Proposition 6.1.

Proposition A.1. *As ϵ converges to zero, in any sequential equilibrium of the discrete-time competitive-bargaining game $G(g_\epsilon)$ in stage two after any history h_t such that the buyer is in store $i \in \{1, 2\}$ and unknown to be rational while seller i is known to be rational, the payoff to the buyer is no less than $1 - \alpha_b - \epsilon$ and the payoff to seller i is no more than $\alpha_b + \epsilon$ (payoffs are evaluated at time t).*

The proof of this result is the same with the proof of Theorem 8.4 in Myerson (1991) and Lemma 1 in Abreu and Gul (2000) and very similar to the proof of Proposition A.2: I show that the payoff to the buyer if he continues to stay in store i and mimics the obstinate type converges to $1-\alpha_b$ as ϵ converges to zero. Given this, we can conclude that in any sequential equilibrium, the buyer chooses not to reveal his type and he stays in store i unless his expected payoff of doing the opposite exceeds $1-\alpha_b$. I first show that the game ends (by seller i's acceptance of the buyer's offer α_b) with probability 1 in finite time, given history h_t, if the buyer continues to stay in store i and mimics the obstinate type. Finally, I show that as players make offers frequent enough ($\epsilon \to 0$), the game ends immediately with (almost) no delay. Therefore, I skip the proof.

With a similar spirit, Proposition A.2 claims that as ϵ converges to 0, at any sequential equilibrium of the game $G(g_\epsilon)$ after the history h_t such that the buyer is in store $i \in \{1,2\}$ and known to be rational while both sellers are not known to be rational, the buyer makes immediate agreement with seller i, and the payoff to seller i (which depends on the details of the bargaining protocol g_ϵ) cannot be lower than α in the limit.

Before presenting the proof of Proposition A.2, I prove two Lemmas that I use extensively later:

Lemma A.4. *Let $\epsilon \to 0$ and let h_t be a history such that the buyer is in store $i \in \{1,2\}$, known to be rational, seller i is unknown to be rational and seller $j \in \{1,2\}$, $j \neq i$ is known to be the obstinate type. Then, for any sequential equilibrium of the game $G(g_\epsilon)$ in stage two after the history h_t, the payoff to the buyer is no more than $1-\alpha+\epsilon$ and the payoff to the seller i is no less than $\alpha-\epsilon$ (payoffs are evaluated at time t).*

Proof. Given that seller j is the obstinate type, the buyer's continuation payoff in store j is at most $1-\alpha$. Therefore, the buyer has no incentive to leave store i to get a price better than α. Given this, seller i does not reveal his type unless he gets a payoff higher than α by doing the opposite. Hence, the payoff to the buyer is no more than $1-\alpha$ as ϵ converges to zero. □

Lemma A.5. *Let ϵ converge to 0 and let h_t be a history such that the buyer is in store $i \in \{1,2\}$ and known to be rational while both sellers are unknown to be rational. Then in any sequential equilibrium of the game $G(g_\epsilon)$ in stage two after the history h_t it cannot be the case that seller i finishes the game at time t at some price $x < \alpha - \epsilon$ with probability one.*

Proof. Suppose for a contradiction that rational seller i makes a deal with the buyer at some price $x < \alpha$ at time t with probability 1. Given that this is an equilibrium strategy, both seller j and the buyer assign probability 1 to the event that seller i is the obstinate type if the seller does not accept the buyer's offer. But then, according to Lemma A.4, the buyer accepts the price α and finishes the game immediately at time t^* where $t < t^* \leq t+\epsilon$.

However, for arbitrarily small ϵ, rational seller i would prefer to deviate from his equilibrium strategy and wait until time t^* by mimicking the obstinate type so that he can get the payoff of

α which is higher than x.[39] Hence, in equilibrium after the history h_t seller i delays the game with a positive probability. □

Proposition A.2. *As ϵ converges to zero, in any sequential equilibrium of the discrete-time competitive-bargaining game $G(g_\epsilon)$ in stage two after any history h_t such that the buyer is in store $i \in \{1, 2\}$ and known to be rational while both sellers are not known to be rational, the payoff to the buyer is no more than $1 - \alpha + \epsilon$ and the payoff to the seller i is no less than $\alpha - \epsilon$ (payoffs are evaluated at time t).*

Proof. Without loss of generality, suppose that the buyer is in store 1 at time t after the history h_t. I will show that as seller 1 continues to mimic the obstinate type, the payoff to the buyer converges to $1 - \alpha$ and the payoff to seller 1 converges to α, as ϵ converges to zero. For the remainder of this proof, assume that seller 1 continues to mimic the obstinate type, while the buyer and seller 2 execute their equilibrium strategies.

For $i \in \{1, 2\}$, let $\hat{z}_i(h_t)$ denote the probability that seller i is the obstinate type at time t after the history h_t. By Bayes' rule, $\hat{z}_i(h_t)$ is either zero or higher than z_s. By our assumption, however, for each i, we must have $\hat{z}_i(h_t) \geq z_s$.

If the buyer continues to stay in store 1 for long enough according to his equilibrium strategy while seller 1 continues to act irrationally, we know by proposition A.1 that the payoff to the buyer converges to $1 - \alpha$ as ϵ converges to zero, and this proves the claim of the proposition. It is, however, possible that in equilibrium the buyer does not stay in store 1 long enough if seller 1 continues to mimic the obstinate type. This implies that the buyer leaves store 1 at some time $t' \geq t$. Note that the second seller's reputation at time t' is still $\hat{z}_2(h_t)$.

The buyer's decision of leaving store 1 at time t' implies that $\hat{z}_2(h_t) \leq \frac{\delta+\alpha-1}{\delta\alpha} = \hat{\rho} < 1$ This is true because, if the buyer goes to store 2 and seeks an agreement with seller 2, the highest payoff he could achieve is $\delta[1 - \hat{z}_2(h_t) + (1 - \alpha)\hat{z}_2(h_t)]$. But leaving store 1 and going to store 2 at time t' is optimal for the buyer only if $1 - \alpha \leq \delta[1 - \hat{z}_2(h_t) + (1 - \alpha)\hat{z}_2(h_t)]$ which implies the desired result.

According to his strategy, if the buyer continues to stay in store 2 long enough, conditional on seller 2 mimicking the obstinate type, we know again by Proposition A.1 that the payoff to the buyer converges to $1 - \alpha$ as ϵ converges to zero. This implies that $1 - \alpha$ is the highest payoff the buyer can attain in store 2. If this is the case, however, the buyer does not leave store 1 at time t', which contradicts our supposition. Therefore, it must be the case that the buyer leaves store 2 as well, conditional on seller 2 continuing to mimic the obstinate type, at some time t'' where $t'' > t'$.

According to his equilibrium strategy, seller 2 may be playing a strategy that ends the game while the buyer is in store 2. However, according to Lemma A.5, we know that seller 2 will not play a strategy that will end the game with a price less than α (in the limit) with probability one. If rational seller 2 is playing a strategy which ends the game with a price higher than α,

[39] Receiving α at time t^* is equivalent to receiving $\alpha e^{-r_s(t^*-t)}$ at time t, which is arbitrarily close to α as ϵ converges to 0.

then he buyer does not leave store 1 at time t', which contradicts our supposition. Therefore, it must be the case that seller 2 is playing a strategy that extends the game, i.e. seller 2 will mimic the obstinate type with a positive probability, until time t''.

Conditional on the buyer arriving to store 1 once more, the same arguments show that the buyer shall leave store 1 once again as seller 1 continues to mimic the obstinate type (because otherwise, the payoff to the buyer will be at most $1-\alpha$ and this contradicts our supposition that the buyer leaves store 2 when seller 2 continues to mimic the obstinate type).

Therefore, conditional on both sellers extending the game and the buyer leaving store 1 twice, we have $\hat{z}_2(h_t) \leq \hat{\rho}^2$, so that extending the game by going back and forth between the sellers (twice) is more profitable for the buyer than seeking an immediate agreement with irrationally behaving seller 1. Similarly, for the game that lasts until the k^{th} departure of the buyer, it must be true that $\hat{z}_2(h_t) \leq \hat{\rho}^k$. Choosing k such that $\hat{\rho}^k < z_s$ establishes contradiction since, as argued earlier, $\hat{z}_2(h_t) \geq z_s$.

Therefore, as seller 1 continues to mimic the obstinate type, seller 2 will continue to play a strategy which extends the game with positive probability (immediate consequence of Lemma A.5). The buyer, however, will travel back and forth between the sellers only for some finite time in order to get a deal better than α. This implies that the buyer will end up at some store $i \in \{1,2\}$ at some finite time $\bar{t}$. That is, the buyer does not leave store i after time $\bar{t}$ while seller i continues to mimic the obstinate type.

This implies that the buyer's continuation payoff in store i is at most $1-\alpha$, evaluated at time $\bar{t}$. This leads to a contradiction because, given that the buyer's continuation payoff in his final destination is less than $1-\alpha$, the buyer should not have left store j when seller j continues to act irrationally. Hence, repeating this argument backward, we can conclude that the buyer does not delay the game, but instead seeks an immediate agreement with seller 1 at time t. □

Proof of Proposition 6.1. This proof is adapted from the proof of Proposition 4 in Abreu and Gul (2000). Let $\mathrm{G}(g_{\epsilon_n})$ be a sequence of discrete-time competitive-bargaining games and $\sigma_n = (\sigma_n^b, \sigma_n^1, \sigma_n^2)$ (drop the term ϵ to ease the notation) be the corresponding sequence of sequential equilibria. Then, for each $i \in \{1,2\}$ and $T \in H^i$ define $F_n^{i,T} : [T, T'] \to [0,1]$, where $T' \leq \infty$ depends on the buyer's equilibrium strategy σ_n^b, and $F_n^{i,T}(t)$ is the cumulative probability that seller i takes an action not consistent with the obstinate type in the interval $[T,t]$, conditional on the buyer and the other seller having acted like a obstinate type until time t. Similarly, define $F_n^{b,i,T} : [T, T'] \to [0,1]$ where $F_n^{b,i,T}(t)$ is the cumulative probability that the buyer takes an action not consistent with the obstinate type in the interval $[T,t]$, conditional on the buyer is in store i in this time interval according to σ_n^b and both sellers having acted as if they are obstinate until time t.

To prove the Proposition, arbitrarily choose some $\bar{n} \geq 0$, an equilibrium strategy $\sigma_{\bar{n}}$, $i \in \{1,2\}$ and a history $h_T \in H^i$. Then, I show that as $n \geq \bar{n}$,

Step (1) Every subsequence of $F_n^{i,T}$ and $F_n^{b,i,T}$ have a convergent subsequence: Similar to Steps 1 and 2 in the proof of Proposition 4 in Abreu and Gul (2000), define G_n^i such that

$$G_n^{i,T}(t) = \frac{F_n^{i,T}(t)}{F_n^{i,T}(T')} \quad \text{whenever} \quad F_n^{i,T}(T') \neq 0$$

for all $t \leq T'$ where $T' \leq \infty$ depends on the buyer's equilibrium strategy σ_n^b and $F_n^{i,T}(T') = 1 - \hat{z}_s(T)/\hat{z}_s(T')$. Note that $\{F_n^{i,T}(T')\}$ is a bounded real sequence, which is bounded below by 0 and above by $1 - z_s$ for all n. The same arguments hold for the buyer. Moreover, by Helly's selection Theorem (See Billingsley (1986)), the sequence $G_n^{i,T}$ has a subsequence $G_{n_k}^{i,T}$ which converges to a right continuous, non-decreasing function G_i^T at every continuity point of G_i^T. Let $F_{n_k}^{i,T} = F_{n_k}^{i,T}(T')G_{n_k}^{i,T}$. Since the real sequence $F_{n_k}^{i,T}(T')$ is bounded below 0 and bounded above $1 - z_s$ for any n, there must exist a subsequence $F_{n_{k_j}}^{i,T}(T')$ which converges to some real number $F_i^T(T')$. Therefore, $F_{n_{k_j}}^{i,T} = F_{n_{k_j}}^{i,T}(T')G_{n_{k_j}}^{i,T}$ implies that $F_i^T = F_i^T(T')G_i^T$. Apply the same arguments to the buyer and renumber the sequence n_{k_j} will yield the desired result.

Step (2) the limit points of $(F_n^{i,T}, F_n^{b,i,T})$ do not have common points of discontinuity in the domain $[T, T']$; The proofs of this claim utilizes the exact methods used in the proof of steps 3-6 of Proposition in Abreu and Gul (2000). Therefore, I do not represent the proof here to prevent duplication.

Step (3) if $(F_n^{i,T}, F_n^{b,i,T})$ converges to $(F_i^T, F_b^{i,T})$ and if the limit functions do not have common points of discontinuity then $(F_i^T, F_b^{i,T})$ is an equilibrium of the competitive-bargaining game in the interval $[T, T']$.

The following arguments prove Step 3 and complete the proof of the Proposition 6.1. Recall that σ_n is the equilibrium strategy of the game $G(g_{\epsilon_n})$. For any $t > 0$ and $\hat{\epsilon} > 0$ define a strategy $\tilde{\sigma}_n^i$ to be a strategy of seller i within the interval $[T, T']$ as follows: Seller i behaves according to σ_n^i until time t_n where t_n is the last time the buyer makes an offer prior to $t + \bar{\epsilon}$ (for some $\bar{\epsilon} > 0$) and at time t_n seller i accepts the buyer's offer α_b. Let U_n^i denote the utility function of seller i in the game $G(g_{\epsilon_n})$. Then there exist finite integers N_1, N_2, N_3 and $\bar{\epsilon} > 0$ sufficiently close to 0, such that $t + \bar{\epsilon}$ is a continuity point of $F_b^{i,T}$ and

$$U^i(t, F_b^{i,T}) - U^i(t + \bar{\epsilon}, F_b^{i,T}) < \hat{\epsilon}, \tag{9}$$

$$U^i(t + \bar{\epsilon}, F_b^{i,T}) - U^i(t_n, F_n^{b,i,T}) < \hat{\epsilon} \quad \forall n \geq N_1, \tag{10}$$

$$U^i(t_n, F_n^{b,i,T}) - U^i(\tilde{\sigma}_n^i, \sigma_n^b) < \hat{\epsilon} \quad \forall n \geq N_2, \tag{11}$$

$$U_n^i(\tilde{\sigma}_n^i, \sigma_n^b) - U_n^i(\sigma_n^i, \sigma_n^b) \leq 0 \qquad \forall n, \tag{12}$$

$$U_n^i(\sigma_n^i, \sigma_n^b) - U_n^i(F_n^{i,T}, F_n^{b,i,T}) < \hat{\epsilon} \quad \forall n \geq N_2 \tag{13}$$

$$U^i(F_n^{i,T}, F_n^{b,i,T}) - U^i(F_i^T, F_b^{i,T}) < \hat{\epsilon} \quad \forall n \geq N_3. \tag{14}$$

Equation (9) follows immediately from the definition of U^i. That is, $U^i(., F_b^{i,T})$ is continuous at continuity points of $F_b^{i,T}$. If t is not a continuity point of $F_b^{i,T}$, for $\bar{\epsilon}$ small enough the left-hand side of (9) is strictly negative (similar logic to the proof of step 3 of Abreu and Gul (2000):

if the buyer makes a mass acceptance at time t, seller i would prefer conceding at time $t + \bar{\epsilon}$ over conceding at time t). Since $t + \bar{\epsilon}$ is a continuity point of $F_b^{i,T}$, (10) follows from Step 6 of Abreu and Gul (2000). Equation (11) follows from the definition of $\tilde{\sigma}_n^i$ and Proposition A.2. Equation(12) is the consequence of the fact that (σ_n^i, σ_n^b) is equilibrium. Equation (13) is an application of Proposition A.1; seller i can never get more than α_b after revealing his rationality. Moreover, in equilibrium, since his opponent makes offers frequently, he can reveal himself to be rational in a manner that guarantees α_b. Equation (14) follows from Steps 3-6 of Abreu and Gul (2000).

Choosing $n \geq \max\{N_1, N_2, N_3\}$ and adding Equations (9)-(14) will yield

$$U^i(t, F_b^{i,T}) - U^i(F_i^T, F_b^{i,T}) < 5\hat{\epsilon}$$

Since this inequality is true for any $\hat{\epsilon} > 0$, it must be the case that

$$U^i(t, F_b^{i,T}) - U^i(F_i^T, F_b^{i,T}) \leq 0.$$

Hence, F_i^T is a best response to $F_b^{i,T}$. Symmetric arguments imply that $(F_i^T, F_b^{i,T})$ is a Nash equilibrium of the concession game within the interval $[T, T']$. Note that if seller i is the first to reveal his type, he can guarantee α_b by accepting the buyer's offer. This would yield the buyer a payoff of $1 - \alpha_b$. If seller i reveals his type in some other way, then by Proposition A.1 he is still, in the limit, guaranteed α_b. This happens only if agreement is reached immediately at these terms. Analogous arguments are valid for the buyer. Therefore, convergence in expected payoffs implies convergence in distribution within the interval $[T, T']$.

After an arbitrary history h_T and continuation strategy $\sigma_{\bar{n}}^b$, I proved the convergence in interval $[T, T']$. So, for given $\sigma_b^{\bar{n}}$, the strategy of the discrete-time competitive-bargaining game $\mathrm{G}(g_{\epsilon_n})$, i.e. $\mathscr{F}_n = (\mathscr{F}_n^b, \mathscr{F}_n^1, \mathscr{F}_n^2)$ (the function which maps histories to the set of right-continuous distribution functions), converge to the the strategy of the continuous-time competitive-bargaining game G, $\mathscr{F} = (\mathscr{F}^b, \mathscr{F}^1, \mathscr{F}^2)$, history by history (i.e., interval by interval) in the product topology.

Given that $\mathscr{F}_n$ converges to $\mathscr{F}$ history by history, similar arguments in the proof of Proposition 3.1 suffice to show that for sufficiently large n, the buyer visits each store at most once according to the equilibrium strategy of the game $\mathrm{G}(g_{\epsilon_n})$. As a result, convergence in distribution in all subgames implies that the buyer's timing and location decisions together with the distribution functions, $\mathscr{F}_n$, converge to the unique equilibrium of the competitive-bargaining game G.

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