Dynamic School Choice Problem

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Abstract

Both families and public school systems desire siblings to be assigned to the same school. Although students with siblings at the school have a higher priority than students who do not, public school systems do not guarantee sibling assignments. Hence, families with more than one child may need to misstate their preferences if they want their children to attend to the same school. In this paper, we study the school choice problem in a dynamic environment where some families have two children and their preferences and priority orders for the younger child depend on the assignment of the elder one. In this dynamic environment, we introduce a new mechanism which assign siblings to the best possible school together if parents desire them to attend the same school. We also introduce a new dynamic fairness notion which respects priorities in a dynamic sense. Finally, we show that it is possible to attain welfare gains when school choice problem is considered in a dynamic environment.

1 Introduction

In most of the public school systems, students with siblings at the school have higher priority than students who do not. By this priority structure, school systems allow siblings to attend the same school. This is one of the goals of the school choice plans (Pathak (2011)). It is also mentioned in the school choice literature that the factors which give an applicant higher priority at a school also lead that school to be more desirable for the agent (Erdil and Ergin (2008) and Pathak and Sonmez (2008)). That is, a school becomes more preferable to a family when applying for the younger sibling if the older sibling attends to that school.[1] Although siblings have higher priority they are not guaranteed to be assigned in the same school. Consider the following statement from Boston Public School website[2]:

> "We try to assign children in the same family to the same school if the parent requests it. However, sometimes a school doesn't have room for all the siblings who apply for it; so we can't guarantee sibling assignments."

[1] The 1998 kindergarten survey in Cambridge (Massachusetts) showed that 42% of the parents considered 'having sibling currently at school' as the most important reason in choosing a school.

[2] You can find similar statements from other disctricts' website in the Appendix.

In this paper, motivated by the statement above we will answer the following questions: Is it a dominant strategy for all applicants to act truthfully under the Deferred Acceptance Mechanism which is currently used in Boston? If not, can we introduce a new mechanism which is easy to adopt and has better features than the current one in terms of incentive compatibility? Is it possible to guarantee parents that their children will be assigned to the same school? Does the current mechanism violate priorities of some agents? Does the current mechanism cause easily recoverable welfare losses which are not due to the arbitrary tie breaking?

We answer all these questions by considering the school choice problem in a dynamic setting where the preferences and priorities of parents with multiple children evolve by the assignment of the elder one. After the seminal paper by Abdulkadiroglu and Sonmez (2003), school choice problem is studied in static models. To our knowledge, this will be the first paper studying the school choice problem in a dynamic environment. There are four related papers which study other matching problems in dynamic settings. Abdulkadiroglu and Loertscher (2007) consider a dynamic house allocation problem in which agents live two periods. They introduce a dynamic mechanism which gives an agent higher priority in the second period if he/she opts out in the first period. They show that dynamic mechanism provides welfare gains compared to the static mechanism in which priority order of second period does not depend on the first periods assignment. However in school choice problems agents can compete with different sets of agents in two separate periods. Moreover, there exists parents with only one child who applies once. Hence, their dynamic mechanism is not suitable for school choice problem. Kurino (2009) studies the house allocation problem with an overlapping generations structure. He shows that for both serial dictatorship and top trading cycles mechanisms a tie breaking rule favoring existing tenants is better than the one favoring new comers in terms of Pareto efficiency. However, in school choice problem priority orders are forced by the central authority and agents are first ranked based on their priorities than based on the tie breaking rule. Moreover, the OLG framework is not consistent with the school choice problem since most of the families do not have children in two consecutive periods. Unver (2010) studies the kidney exchange problem in a dynamic setting. In that paper an exchange model in which agents leave the market with the objects is considered. However, in school choice problem agents are temporarily assigned to school for a limited time. Kennes, Monte and Tumennasan (2011) study the Danish assignment problem in a dynamic environment. They consider agents who apply twice in an overlapping generations model. The problem differs from the dynamic school choice problem in four major points: (1) In day-care problem they assume that agents' preferences for second period do not depend on the assignment in the first period. However, in school choice problem having a sibling in a particular school makes that school more desirable when parents apply for the second child. For instance, in 1998 Cambridge kindergarten survey showed that 42% of parents mentioned that having an elder sibling is the most important reason for choosing a school. (2) The dynamic problem is considered as OLG. In school choice problem not all of the agents have children born in two consecutive years hence they do not apply in two consecutive years. Moreover, if all agents had children born in consecutive years then public school systems could guarantee that younger sibling can be assigned in the elder sibling's school when parents desire. (3) Day-care centers assumed to have strict priority orders over applicants. However, coarse priority ordering is one of the important feature of the school choice problem. (4) Preferences of agents are assumed to be time separable and it is also assumed that agents rank all school pairs strictly. However, for this assumption to be true in school choice problem parents must be able to rank 100 school pairs when there are 10 schools. Moreover applicants must have well defined cardinal preferences over all schools and there must not be any complementarity between schools. As mentioned in Pathak (2011) it is difficult for agents to have cardinal preferences over schools.

Dynamic school choice problem consists of parents and schools. For simplicity, we assume that parents can have one or two children. Parents can apply for the assignment of their children in any two periods. That is, they are not restricted to have children only in two consecutive years. When the first child is assigned to a school, parents obtain higher priority for the younger one in that school. The ties between parents who have the same priority are broken by random draws. In particular, every applicant is assigned a random number in each period and the agent with the lowest number is favored most in tie breaking. Parents with multiple children submit their preference list for the first child first and then based on the assignment of the first children and they submit their preference for second children when they apply again. Then based on this submission order we can consider the preference list as lexicographic order and it is ranked based on the first child's assignment. Except for some school pairs consist of the same school in both periods, we will consider the preference of agents as lexicographic order. School districts assign the students to the schools considering the submitted preferences and enforced school priorities . In 2005, Boston Public School system changed its old mechanism which was highly vulnerable to manipulation and has started using Deferred Acceptance mechanism with single tie-breaking (DA-STB). Works of Abdulkadiroglu, Sonmez, Roth and Pathak played a key role in this decision. In their papers they all argue that the Boston mechanism is not strategy-proof and naive players are hurt by truth-telling. DA-STB has really nice properties when a static problem is considered. It is fair, strategy-proof and there is no other strategy-proof mechanism whose outcome Pareto dominates the outcome of DA-STB (Abdulkadiroglu, Pathak, Roth (2009)).

In this paper, we will show that some of the good features are not valid in a dynamic environment. We have two additional assumptions to the assumptions that are commonly used in static environment. We assume that some parents may prefer their children attending the same school rather than attending separate schools.[3] Most of the results shown in this paper are also valid if we weaken this assumption. However, since we aim to guarantee assigning siblings to the same school, we should keep this assumption in the model. Finally, we also assume that school districts know the available seats in each period and the number of children applying is equal to the number of available seats. Under these two assumptions, we modify the current mechanism and get better results in terms of dynamic fairness, strategy-proofness and Pareto efficiency.

In section 2, we will go over the model which mimics the school choice problem in the real life. In section 3 we will illustrate the deficiencies of current mechanism in dynamic school choice problem. In section 4, we give 2 impossibility theorems in dynamic school choice problem then introduce a new mechanism which has better features in terms of fairness, strategy proofness and efficiency. In section 5, we provide our simulations results of comparing our mechanism and the current one in use.

2 Model

School choice problem is composed of a disjoint sets of schools and families. Let $S = \{s_1, s_2, ..., s_n, \emptyset\}$ be the set of schools and school s has q_s^t available seats in period t. Being unassinged is denoted by $\emptyset$ and $q_\emptyset^t = \infty$. We denote the set of families applying to central mechanism in period t as F^t. For simplicity, we assume that families either have 1 child or 2. The model can be extended to case where families have more than 2 children. Hence $F^t = F_s^t \cup F_m^t$ where F_s^t is the set of families having single child and F_m^t is the set of families having 2 children. Moreover let $F^{t,t'}$ be the set of families applying in period t and t' where $t < t'$. In period t, $i \in F_m^t$ can apply for the elder child if $i \in F^{t,t'}$ and i can apply for the younger one if $i \in F^{\widetilde{t,t}}$. We consider infinite and discrete time horizon. To be consistent, let $F^{t,\emptyset}$ be the set of families with one child applying in period t. Let $F = \{F^t\}_{t=0}^\infty$ be the set of all families in the school choice problem. Moreover, the set of all families with one child is $F_s = \{F_s^t\}_{t=0}^\infty$ and the set of all families with two children is $F_m = \{F_m^t\}_{t=0}^\infty$.

The total number of available seats in period t is $Q^t = \sum_{i=1}^{n} q_{s_i}^t$. The number of agents applying in period t is $|F^t| = |F_s^t| + |F_m^t|$ and we assume that $Q^t = |F^t|$.

Each family with one child $i \in F^{t,\emptyset}$ has a strict preference relation P_i on S and each family with two children $j \in F^{t,t'}$ has a strict preference relation P_j on S^2. The preferences of all agents are complete and transitive. In most of the public school assignment mechanisms used in US parents can submit only their preference list for the child that will start school in that period. Hence, the current mechanism considers the complete preferences of parents with multiple children as lexicographic. For instance, if family $i \in F^{t,t'}$ submit

[3] For instance, in Portland and Seattle families are allowed to submit a preference list in order to keep siblings in the same school. In addition to this, parents also submit different preference lists for each sibling. If school district cannot assign siblings to the one of the schools in the list then it consider the separate preference lists for each sibling.

preferences as $s_1 - s_2$ in period t and $s_2 - s_1$ in period t' then his complete preference is $(s_1, s_1)P_i(s_1, s_2)P_i(s_2, s_1)P_i(s_2, s_2)$. For the purpose of this paper we will stick to the assumption of lexicographic order except for some assignment pairs in which both children are assigned to the same school. That is, in the light of the evidences mentioned in introduction, we assume that there are some schools which parents prefer their both children to be assigned to these school together to be assigned in separate schools. Let A_i be the set of schools that $i \in F^{t,t'}$ prefers to be assigned in both periods to be assigned in separate schools.[4] Let P_i^t and $P_i^{t'}$ be the submitted preference by agent $i \in F^{t,t'}$ then

$$\begin{aligned} s &\in A_i \Longrightarrow (s,s)P_i(s',s'') \ \forall s', s'' \in S \text{ and } s' \neq s'' \\ sP_i^t s' &\Leftrightarrow (s,.)P_j(s',.) \\ s'P_i^{t'} s'' &\Leftrightarrow (s,s')P_i(s,s'') \ . \end{aligned}$$

Note that, if $i \in F^{t,t'}$ include school s' in A_i then he must also include all $sP_i^t s'$. We will use P^t for the preference profile submitted in period t and $P = \{P^t\}_{t=0}^{\infty}$ where $P_i = \{P_i^t, P_i^{t'}\}$ for $i \in F^{t,t'}$ and $P_i = P_i^t$ for $i \in F^{t,\emptyset}$.

In Boston, priority order for a school is determined by two criteria: distance from a school and sibling attending that school. Students living in the walk zone of school s and having sibling currently attending in school s are given the first priority for school s. Students having sibling in school s but not living in the walk zone of school s are given the second priority for school s. Students living in the walking zone of school s but do not have a sibling in school s are given the third priority for school s. Fourth priority is given to the remaining students. Let $r_s^t(i) \in \{1, 2, 3, 4\}$ be the priority class of agent i for school s in period t. More than 6,000 agents apply in each period hence it is inevitable to observe ties in each priority class. In the current mechanism ties are broken by random draw. In particular, in each period a number is assigned to each applicant and the agents in the same priority class are ordered based on the numbers assigned to them. Let $b^t(i)$ be the number assigned to agent $i \in F^t$ in period t. Let $\succ_s^t$ be the strict priority order of school s in period t based on priority classes and the random draw then

$$\begin{aligned} r_s^t(i) < r_s^t(i) &\Rightarrow i \succ_s^t j \\ r_s^t(i) = r_s^t(i) \& b^t(i) < b^t(j) &\Rightarrow i \succ_s^t j. \end{aligned}$$

In the rest of the paper, when we use priority order we mean the strict priority order.

A period t matching is defined as the assignment of school seats to agents in set F^t such that each agent is assigned to one school and the number of agents assigned to a school cannot be more than q_s^t. Formally, a matching in period t is a function $\mu_t : F^t \to S$ such

[4] In Seattle and Portland, parents are allowed to submit a list similar to A_i. School district first try to assign students one of the schools in that list and if it cannot then it assigns students in different schools.

that $\mu_t(i) = s$ and $\mu_t(i) = s'$ if only if $s = s'$ and $|\mu_t^{-1}(s)| \leq q_s^t$. Let $\mathcal{M}^t$ be the set of all matchings in period t. Let matching μ be the collection of all period t matchings from period 0 on. Formally, $\mu = \{\mu_t\}_{t=0}^{\infty}$ and if agent $i \in F^{t,t'}$ then we refer to $\mu(i) = (\mu_t(i), \mu_{t'}(i))$ as the life time matching of family i and $\mu(i) = \mu_t(i)$ if $i \in F^{t,\emptyset}$. Let $\mathcal{M}$ be the set of all matching.

As mentioned above, a students with sibling is given higher priority in the school s in which his elder brother/sister had been assigned. Hence, the priority order evolves depending on the previous periods assignments. Then $\succ_s^t (\mu_{0,t-1})$ denotes the priorities of school s in period t as a function of all previous matchings up to period t. We will use $\succ^t (\mu_{0,t-1})$ for the priority orders in period t.

Agent $i \in F$ *strictly prefers* matching μ to $\mu' \in \mathcal{M}$ if he strictly prefers $\mu(i)$ to $\mu'(i)$, $\mu(i) P_i \mu'(i)$. A matching μ *Pareto dominates* matching μ' if no agent in F strictly prefers matching μ' to μ and there is at least an agent i who strictly prefers μ to μ'. A matching μ is *Pareto efficient* if there is no matching $\mu' \in \mathcal{M}$ in which Pareto dominates matching μ.

The natural counterpart of *stability* in school choice problem is *fairness* (Balinski and Sonmez, 1999)[5]. We give two different definitions for fairness in the dynamic environment: static fairness and dynamic fairness.

Definition 1 *A matching μ is static fair if for all t there does not exist (s, i) pair such that one of the statement below is true*

(1) For $i \in F^{t,\emptyset} \cup F^{t,t'}$, $sP_i^t \mu_t(i)$ and for some $j \in \mu_t^{-1}(s)$ $i \succ_s (\mu_{0,t-1}) j$

(2) For $i \in F^{\tilde{t},t}$, $(\mu_{\tilde{t}}(i), s) P_i \mu(i)$ and for some $j \in \mu_t^{-1}(s)$ $i \succ_s (\mu_{0,t-1}) j$

This definition is exactly the same definition used in the literature . However, this definition ignores the complementarity between siblings assignments. For instance, $i \in F^{t,t'}$ may prefer (s, s) to (s', s) when $s' P_i^t s$ and for some $j \in \mu_t^{-1}(s')$ $i \succ_s (\mu_{0,t-1}) j$. One can think that priority of agent i for school s' was violated in period t. However, as a consequence of complementarity agent i will not object his assignment. Then we need a new definition for fairness.

For all $i \in F_m$ let $Z_i((s, s'))$ be the set of matchings in which except i all other agents priorities are respected when i is assigned to (s, s'). Formally, $\mu \in Z_i((s, s'))$ where $\mu(i) = (s, s')$ if for all $t \geq 0$ there does not exist (j, s) pair such that:

- for $j \in F^{t,\emptyset} \cup F^{t,t'}$, $s \ P_j^t \ \mu(j)$ and for some $k \in \mu_t^{-1}(s)$ $j \succ_s^t (\mu_{0,t-1}) k$
- for $j \in F^{\tilde{t},t}$, $(\mu_{\tilde{t}}(j), s) P_j \mu(j)$ and for some $k \in \mu_t^{-1}(s)$ $j \succ_s^t (\mu_{0,t-1}) k$.

Now by using set Z we can give the definition of dynamic fairness.

[5] In the rest of the paper fairness and stability are used interchangeably.

Definition 2 *A matching μ is dynamic fair if for all t there does not exist (s,i) pair and $\widetilde{\mu} \in Z(\widetilde{\mu}(i))$ such that one of the statement below is true*

(1) For $i \in F^{t,\emptyset}$, $sP_i^t\mu_t(i)$ and for some $j \in \mu_t^{-1}(s)$ $\; i \succ_s (\mu_{0,t-1})j$

(2) For $i \in F^{\widetilde{t},t}$, $(\mu_{\widetilde{t}}(i), s)P_i\mu(i)$ and for some $j \in \mu_t^{-1}(s)$ $\; i \succ_s (\mu_{0,t-1})j$

(3) For $i \in F^{t,t'}$, $\widetilde{\mu}(i)P_i\mu(i)$.

In this dynamic matching problem we assume that every agent $i \in F^t$ submits their period t preference, P_i^t, at the beginning of period t. Agent $i \in F^{t,t'}$ submits A_i at the beginning of period t. Note that A_i can be empty set for some agents. In the current mechanism, each agent is assigned a random number when they apply for each child. However, to have more information about the future priorities school districts can assign two random number for each child of family $i \in F^{t,t'}$ when family i first apply in period t. Hence, based on the assignment of agent $i \in F^{t,t'}$ in period t and the random number $b_{t'}(i)$ assigned in period t, the priorities of agent i in period t' can be determined before period when matching in period t is done.

A *mechanism* is a procedure which selects period t matching by using all available information about preferences and priorities by period t. Let $P^{t,\infty}$ be the all available information about preferences and $\succ^{t,\infty}$ be the available information about priorities at the beginning of period t. That is, a mechanism φ takes the preference profile of the agents and priority order of agents for schools, then selects a period t matching $\varphi(P^{t,\infty}, \succ^{t,\infty})$ for every $P^{t,\infty}$ and $\succ^{t,\infty}$. Let $\varphi_i(P^{t,\infty}, \succ^{t,\infty})$ denotes the period t assignment of agent $i \in F^t$ by mechanism φ for preference profile $P^{t,\infty}$ and priority ordering $\succ^{t,\infty}$ in period t. Since the priorities are forced by the central authority we will drop out it and use $\varphi(P^{t,\infty})$ instead of $\varphi(P^{t,\infty}, \succ^{t,\infty})$.

A mechanism φ is static fair (dynamic fair) if it selects static fair (dynamic fair) matching by using the available information in each period. A mechanism φ is Pareto efficient if it selects Pareto efficient matching by using the available information in each period.

A mechanism φ is *strategy-proof* if it is dominant strategy for all agents to submit their true preferences when all other agents act truthfully. Formally, let P be the preference profile when all agents submit true preference. A mechanism φ is *strategy-proof* if for every preference profile $P_i' \neq P_i$, $\varphi_i(P)R_i\varphi_i(P_i', P_{-i})$ for all agents $i \in F$ where at least as good as relation is denoted by R. Here, P_{-i} represents the true preference profile of agents except i and $\varphi_i(P)$ is the life time assignment of agent i.

3 Examples

In this section, we will go over a simple examples and illustrate the deficiencies of DA-STB in dynamic environment.

Example 1. There are 3 schools $S = \{s_1, s_2, s_3\}$ with 1 available seat in each period. By assumption, there will be 3 agents applying in each period. Let $f_1, f_2 \in F^{1,2}$, $f_3 \in F^{1,\emptyset}$

and $f_4 \in F^{0,2}$. That is, $F^1 = \{f_1, f_2, f_3\},\ F^2 = \{f_1, f_2, f_4\}$ and all other agents have only 1 child. The preferences of agents are given as:

$$s_1 P^1_{f_1} s_2 P^1_{f_1} s_3 \text{ and } A_{f_1} = \{s_1, s_2\}$$
$$s_2 P^1_{f_2} s_1 P^1_{f_2} s_3 \text{ and } A_{f_2} = \{s_2\}$$
$$s_3 P^1_{f_3} s_2 P^1_{f_3} s_1$$
$$s_1 P^0_{f_4} s_2 P^0_{f_4} s_3 \text{ and } A_{f_4} = \{s_1\}$$

In period 0, f_4 was assigned to s_1 so he has sibling priority for it. The priority orders known at the beginning of period 1 are:

$t = 1$	s_1	s_2	s_3	$t = 2$	s_1	s_2	s_3
Pri 1				Pri 1	f_4		
Pri 2				Pri 2			
Pri 3	f_3	f_1	f_3	Pri 3			
Pri 4	f_1, f_2	f_2, f_3	f_1, f_2	Pri 4		f_4	f_4

.

Current mechanism only considers the P^1 in period 1. If we take $b_1(f_1) < b_1(f_2)$ then the assignments in period 1 will be : $\mu_1(f_1) = s_1, \mu_1(f_2) = s_2$ and $\mu_1(f_3) = s_3$. If we only consider the problem in period 1 then the matching μ is static fair and truth-telling is the dominant strategy for all players. If we update the priority orders in period 2 then at the beginning of period 2 we have:

$t = 2$	s_1	s_2	s_3
Pri 1	f_4		
Pri 2	f_1	f_2	
Pri 3		f_1	
Pri 4	f_2	f_4	f_1, f_2, f_4

.

By only using the A sets for each applicant DA-STB can do the assignments. That is, since f_4 and f_2 have the highest priority in the schools that they want their children to attend together we do not need rest of the preference lists. Also the only available school left to f_1 will be s_3. Then the outcome of DA-STB in period 2 will be: $\mu_2(f_1) = s_3, \mu_2(f_2) = s_2$ and $\mu_2(f_4) = s_1$. If we only consider the problem in period 2 then the matching μ is static fair and truth-telling is the dominant strategy for all players.

Now suppose f_1 submit his preference as: $s_2 P'^1_{f_1} s_1 P'^1_{f_1} s_3$ and $A_{f_1} = \{s_2, s_1\}$. Then for any tie breaking rule in period 1 DA-STB will assign as: $\widetilde{\mu}_1(f_1) = s_2, \widetilde{\mu}_1(f_2) = s_1$ and

$\widetilde{\mu}_1(f_3) = s_3$. Based on $\widetilde{\mu}_1$ the updated priority structure in period 2 will be:

$t = 2$	s_1	s_2	s_3
Pri 1	f_4	f_1	
Pri 2	f_2		
Pri 3			
Pri 4	f_1	f_2, f_4	f_1, f_2, f_4

.

By only using the A sets for each applicant DA-STB can do the assignments: $\widetilde{\mu}_2(f_1) = s_2, \widetilde{\mu}_2(f_2) = s_3$ and $\widetilde{\mu}_2(f_4) = s_1$.

Agent f_1 benefits from misstating his preferences: $\widetilde{\mu}(f_1) P_{f_1} \mu(f_1)$. Hence, DA-STB is not strategy proof. ■

Although the current mechanism is not strategy-proof in the dynamic environment one can ask whether violation of strategy-proofness imply easy manipulability. For instance, the main reason for replacing the Boston mechanism with DA-STB was that Boston mechanism can easily be manipulated. Under the Boston mechanism, an applicant who initially has high priority for a school might lose the priority advantage when he/she does not rank that school at the top of his/her preference list and might end up in a lower ranked school. Hence, many agents may prefer to misstate their true preferences. For instance if an agent who belongs to the lowest priority group for a popular school may believe that he/she does not have a chance to be assigned to that popular school and he will not rank that school at the top of his preference list. Instead he will rank a school that he has walking zone priority. If we look at the dynamic problem, we can easily see agents may misstate their preferences in order to guarantee assignment of their both children in the same school. In the example, since f_1 belongs to the 4 priority class when his first child is assigned to s_1 he will be in the second priority class and it is possible that there will be another agent who had obtained highest priority in s_1 in previous periods. Moreover, if f_1 is assigned to s_1 he will lose the chance of obtaining highest priority for the other schools. If he was assigned to a less popular school that he lives in the walking zone then he will guarantee his second child to be assigned in that school too. To summarize, people will tend to misstate their preferences to guarantee the assignment of their children to the same school and stating true preferences may hurt the families. Moreover, if the public school system could guarantee to assign the siblings to the same school that parents desire whenever it is possible then f_1 would not misstate his preferences.

Example 2. We will change Example 1 to illustrate that there exists dynamic fair matching which Pareto dominates the outcome of DA-STB. Let $f_1 \in F^{1,2}$,$f_2, f_3 \in F^{1,\emptyset}$ and $f_4, f_5 \in F^{0,2}$. That is, $F^1 = \{f_1, f_2, f_3\}$ and $F^2 = \{f_1, f_4, f_5\}$. The preferences of agents are given as:

$$s_1 P^1_{f_1} s_2 P^1_{f_1} s_3 \text{ and } A_{f_1} = \{s_1, s_2\}$$
$$s_1 P^1_{f_2} s_2 P^1_{f_2} s_3$$
$$s_3 P^1_{f_3} s_2 P^1_{f_3} s_1$$
$$s_1 P^0_{f_4} s_2 P^0_{f_4} s_3 \text{ and } A_{f_4} = \{s_1\}$$
$$s_3 P^0_{f_5} s_2 P^0_{f_5} s_1 \text{ and } A_{f_4} = \{s_3\}$$

In period 0, f_4 was assigned to s_1 and f_5 was assigned to s_3 so they have sibling priority. The priority orders known at the beginning of period 1 are:

$t=1$	s_1	s_2	s_3
Pri 1			
Pri 2			
Pri 3	f_3	f_1	f_3
Pri 4	f_1, f_2	f_2, f_3	f_1, f_2

$t=2$	s_1	s_2	s_3
Pri 1	f_4		f_5
Pri 2			
Pri 3		f_5	
Pri 4	f_5	f_4	f_4

If we take $b_1(f_1) < b_1(f_2)$ then the assignments in period 1 will be : $\mu_1(f_1) = s_1, \mu_1(f_2) = s_2$ and $\mu_1(f_3) = s_3$. The updated priority order in period 2 will be:

$t=2$	s_1	s_2	s_3
Pri 1	f_4		f_5
Pri 2	f_1		
Pri 3		f_1,f_5	
Pri 4	f_5	f_4	f_1,f_4

Since f_4 and f_5 desire their second children to be assigned to the school that they have the highest priority we do not need rest of their preferences. Moreover, the only school left for f_1 is s_2 then DA-STB will assign f_1 to s_2. The outcome of DA-STB in period 2 is $\mu_2(f_1) = s_2, \mu_2(f_4) = s_1$ and $\mu_2(f_5) = s_3$. A trade between f_1 and f_2 in period 2 will improve the welfare of both agents and do not affect the assignment of the other agents. Let the new matching be $\widetilde{\mu}$: $\widetilde{\mu}(f_1) = (s_2, s_2)$, $\widetilde{\mu}(f_2) = s_1$, $\widetilde{\mu}(f_3) = s_3$, $\widetilde{\mu}(f_4) = (s_1, s_1)$, $\widetilde{\mu}(f_5) = (s_3, s_3)$. Matching $\widetilde{\mu}$ is dynamic fair and it Pareto dominates the outcome of DA-STB. ■

4 Guaranteed Deferred Acceptance Mechanism

Before going through the new mechanism we will first present two negative results. In Example 1, we show that DA-STB is not strategy-proof. In fact, there is no static fair mechanism which is also strategy-proof.

Theorem 1 *In dynamic school choice problem, there is no static fair mechanism which is also strategy-proof.*

Proof. Consider the following example. There are 3 schools $S = \{s_1, s_2, s_3\}$ with one available seat. Let $f_1, f_2 \in F^{t,t+1}$, $f_3 \in F^{t-1,t}$, $f_4 \in F^{t-1,t+1}$ and $f_5 \in F^{t-1,\emptyset}$. All other agents have only one child hence apply only once. The preferences of agents are given as:

$$
\begin{aligned}
& s_1 P^t_{f_1} s_2 P^t_{f_1} s_3,\ s_1 P^{t+1}_{f_1} s_2 P^{t+1}_{f_1} s_3 \text{ and } A_{f_1} = \{s_1\} \\
& s_2 P^t_{f_2} s_3 P^t_{f_2} s_1,\ s_2 P^{t+1}_{f_2} s_3 P^{t+1}_{f_2} s_1 \text{and } A_{f_2} = \{s_2\} \\
& s_3 P^{t-1}_{f_3} s_2 P^{t-1}_{f_3} s_1,\ s_3 P^t_{f_3} s_2 P^t_{f_3} s_1 \text{ and } A_{f_3} = \{s_3\} \\
& s_1 P^{t-1}_{f_4} s_2 P^{t-1}_{f_4} s_3 \text{ and } A_{f_4} = \{s_1\} \\
& s_3 P^{t-1}_{f_5} s_2 P^{t-1}_{f_5} s_1
\end{aligned}
$$

Let $r^{t-1}_{s_3}(f_5) = 3$, $r^{t-1}_{s_3}(f_3) = 4$, $r^{t-1}_{s_2}(f_3) = 4$, $r^{t-1}_{s_1}(f_4) = 3$, $r^t_{s_1}(f_1) = 4$, $r^t_{s_2}(f_1) = 3$, $r^t_{s_2}(f_2) = 4$ and $r^t_{s_3}(f_2) = 3$. Then in any static fair matching we have the following assignments: $\mu(f_1) = (s_1, s_3)$, $\mu(f_2) = (s_2, s_2)$, $\mu(f_3) = (s_2, s_3)$, $\mu(f_4) = (s_1, s_1)$ and $\mu(f_5) = s_3$. This is the only static fair matching.

If f_1 submits his preferences as: $s_2 P'^t_{f_1} s_1 P'^t_{f_1} s_3$, $s_2 P'^{t+1}_{f_1} s_1 P'^{t+1}_{f_1} s_3$ and $A_{f_1} = \{s_2\}$ then the only static fair matching is $\widetilde{\mu}(f_1) = (s_1, s_2)$, $\widetilde{\mu}(f_2) = (s_3, s_3)$, $\widetilde{\mu}(f_3) = (s_2, s_2)$, $\widetilde{\mu}(f_4) = (s_1, s_1)$ and $\widetilde{\mu}(f_5) = s_3$. Since $\widetilde{\mu}(f_1) P_{f_1} \mu(f_1)$ f_1 will misstate his preference and manipulate any static fair mechanisms. ■

Note that, in school choice problem applicants must be assigned in the periods that they apply. Moreover, public school system cannot know all the future agents and their preferences. It is only possible to collect information about future applicants if they had already applied for the elder sibling before. This feature of school choice problem make it impossible to have a dynamic fair mechanism.

Theorem 2 *In dynamic school choice problem, there is no dynamic fair mechanism.*

Proof. Consider the following example. There are 2 schools $S = \{s_1, s_2\}$ with one available seat. Let $f_1 \in F^{t,t+2}$, $f_2 \in F^{t,\emptyset}$ and $F^{\widetilde{t},t+1} = F^{\widetilde{t},t+2} = \emptyset$ for all $\widetilde{t} < t$. The preferences of agents are given as:

$$
\begin{aligned}
& s_1 P^t_{f_1} s_2 \text{ and } A_{f_1} = \{s_1, s_2\} \\
& s_1 P^t_{f_2} s_2
\end{aligned}
$$

Take $r^t_{s_1}(f_1) = r^t_{s_1}(f_2) = 4$, $r^t_{s_2}(f_1) = 3$. Let the tie breaking in period t favors f_1. First consider the case where $\mu_t(f_1) = s_1$. Suppose in period $t+1$ agents $f_3 \in F^{t+1,t+2}$, $f_4 \in F^{t+1,\emptyset}$ apply and their priorities and preferences are $r^{t+1}_{s_1}(f_3) = 3 < r^{t+1}_{s_1}(f_4)$ and $s_1 P^{t+1}_{f_3} s_2$. The possible assignments for these 4 agents are:

(1) $\mu(f_1) = (s_1, s_1)$, $\mu(f_2) = s_2$, $\mu(f_3) = (s_1, s_2)$, $\mu(f_4) = s_2$

(2) $\mu(f_1) = (s_1, s_1)$, $\mu(f_2) = s_2$, $\mu(f_3) = (s_2, s_2)$, $\mu(f_4) = s_1$

(3) $\mu(f_1) = (s_1, s_2)$, $\mu(f_2) = s_2$, $\mu(f_3) = (s_1, s_1)$, $\mu(f_4) = s_2$

(4) $\mu(f_1) = (s_1, s_2)$, $\mu(f_2) = s_2$, $\mu(f_3) = (s_2, s_1)$, $\mu(f_4) = s_1$

All 4 matchings above fail to be dynamic fair.

Now consider the case where $\mu'_t(f_1) = s_2$.Suppose in period $t+1$ agents agents $f_3 \in F^{t+1,\emptyset}$, $f_4 \in F^{t+1,\emptyset}$ apply and their priorities and preferences are $r^{t+1}_{s_1}(f_3) = 3 < r^{t+1}_{s_1}(f_4)$ and $s_1 P^{t+1}_{f_3} s_2$. The possible assignments for these 4 agents are:

(1) $\mu'(f_1) = (s_2, s_1)$, $\mu'(f_2) = s_1$, $\mu'(f_3) = s_1$, $\mu'(f_4) = s_2$

(2) $\mu'(f_1) = (s_2, s_2)$, $\mu'(f_2) = s_1$, $\mu'(f_3) = s_1$, $\mu'(f_4) = s_2$

(3) $\mu'(f_1) = (s_2, s_1)$, $\mu'(f_2) = s_1$, $\mu'(f_3) = s_2$, $\mu'(f_4) = s_1$

(4) $\mu'(f_1) = (s_2, s_2)$, $\mu'(f_2) = s_1$, $\mu'(f_3) = s_2$, $\mu'(f_4) = s_1$

All 4 matchings above fail to be dynamic fair. Hence, there is no dynamic fair mechanism in the given environment. ■

The two impossibility theorems show that we cannot have a dynamic fair mechanism moreover there is no static fair mechanism which is not vulnerable to manipulations. We should not expect a mechanism to act perfectly when there is uncertainty about the future applicants. We should criticize a mechanism if it does not use all information it has. In example 1, at the beginning of period 2 we have enough information that f_1 cannot be assigned to s_1 in any static (dynamic) fair matching. Moreover, we can be also sure that if f_1 is assigned to s_2 in period 1 then public school system can guarantee him to be assigned to the same school in period 2. Given that f_1 submits $A_{f_1} = \{s_1, s_2\}$ public school system can be sure that f_1 would prefer to be assigned s_2 in period 1 and this will not violate any other players' priority. We should criticize a mechanism if it fails to assign f_1 to s_1 in period 1.

We will weaken the definition of dynamic fairness in order to condition it only on the available information.

Definition 3 *Given the available information a matching μ is weakly dynamic fair if for all t there does not exist (s, i) pair and $\widetilde{\mu} \in Z_i(\widetilde{\mu}(i))$ such that one of the statement below is true*

(1) For $i \in F^{t,\emptyset}$, $sP^t_i \mu_t(i)$ and for some $j \in \mu^{-1}_t(s)$ $\; i \succ_s (\mu_{0,t-1})j$

(2) For $i \in F^{\widetilde{t},t}$, $(\mu_{\widetilde{t}}(i), s)P_i\mu(i)$ and for some $j \in \mu^{-1}_t(s)$ $\; i \succ_s (\mu_{0,t-1})j$

(3.1) For $i \in F^{t,t'}$, $\widetilde{\mu}(i)P_i\mu(i)$ and with the available information at the beginning of period t i will get $\widetilde{\mu}(i)$ and cannot get better for sure or

(3.2) $sP^t_i\mu_t(i)$ and for some $j \in \mu^{-1}_t(s)$ $\; i \succ_s (\mu_{0,t-1})j$

Base on this definition if we can introduce a weakly dynamic fair mechanism then in example 1 agent f_1 cannot be better off by misreporting. In example 1 a weakly dynamic fair mechanism will select the following matching: $\mu(f_1) = (s_2, s_2)$, $\mu(f_2) = (s_1, s_3)$, $\mu(f_3) = s_3$ and $\mu(f_4) = (s_1, s_1)$ hence f_1 will not be better off by misreporting.

In theorem 1 we show that there is no static fair mechanism which is strategy-proof. This impossibility theorem is also true for the weakly dynamic fair mechanisms.

Theorem 3 *There is no weakly dynamic fair mechanism which is also strategy-proof.*

Proof. Same as theorem 1. ■

Now we can introduce a new mechanism called Guaranteed Deferred Acceptance Mechanism with Single Tie Breaking Rule (gDA-STB) which is weakly dynamic fair and use all available information to assign siblings to the same school if parents want. But before we will give some other definitions that will be used.

The *lower set* of agent $i \in F_t$ for school $s \in S$ in period t is the set of agents who have lower priority order than agent i for school s. Let $L(i,s,t)$ be the lower set then

$$L(i,s,t) = \{j | j \in F_t \text{ and } i \succ_s (\mu_{-\infty,t-1}) j\}.$$

We are constructing the lower sets $L(i,s,t)$ at the beginning of period t after learning all applicants in period t and their priority order. Note that $L(i,s,t)$ does not take the preferences submitted in period t, P^t. It only considers the priorities so agents who will apply first time in period t cannot affect $L(i,s,t)$ by their preference list. One can easily see that if $|L(i,s,t)| \geq n - q_s^t$ then a seat in school s in period t is guaranteed to agent $i \in F_t$ by any mechanism which respects priorities.

We can know relative priority of $i \in F^{t,t'}$ for period t' compared to agents $j \in F^{\widetilde{t},t'}$ in period t for all $\widetilde{t} < t$. Then the upper set of agent $i \in F^{t,t'}$ for school $s \in S$ in period t' is the set of agents $j \in F^{\widetilde{t},t'}$ who have higher priority than agent i when agent i is assigned to school s in period t. Let $U(i,s,t')$ be the upper set of agent $i \in F^{t,t'}$ for school s then

$$U(i,s,t') = \{j | j \in F^{\widetilde{t},t'} \text{ and } j \succ_s (\mu_{-\infty,t-1} \cup (\mu_t(i) = s)) i\}.$$

As lower set, we are constructing upper set of agent $i \in F^{t,t'}$ at the begging of period t. Let $Top(s,t)$ be the set of agents who rank school s at the top of their period t preferences,

$$Top(s,t) = \{j | j \in F_t \text{ and } s P_j^t s' \; \forall s' \neq s \in S\}.$$

Families submit their period t preferences at the beginning of period t. However, if parents can submit the set of schools,A, that they want their both children to be assigned together rather than to be assigned to separate schools and elder sibling was assigned to one of the schools in A then we will be sure that family will prefer their first child's assignment at the top when they apply for the second child. Hence we can get the $Top(s,t)$ set partially for some schools and agents before period t.

When $i \in F^{t,t'}$ applies for his first child in period t we construct $Top(s,t')$ and $U(i,s,t')$ before we do the period t assignment. And if we know $|Top(s,t') \cap U(i,s,t')| \geq q_s^{t'}$ at the beginning of period t then agent $i \in F^{t,t'}$ cannot be assigned to school s in period t' by any mechanism which respects priorities. Not to cause any confession we will use subscript t in sets $Top(s,t')$ and $U(i,s,t')$ to emphasize that these sets are constructed at the beginning of period $t: Top_t(s,t')$, $U_t(i,s,t')$.

Now we can introduce Guaranteed Deferred Acceptance Mechanism with Single Tie Breaking Rule (gDA-STB). The most important feature of gDA-STB is utilizing all the available information in order to place more siblings to the same school. By doing this it also aim to reduce the level of easy manipulation. In the mechanism, each parent submits period t preferences at the beginning of period t. Parents with two children applying for the elder children submit their preferred set of schools that they would like their children to be assigned together rather than to be assigned to the different schools. That is agent $i \in F^{t,t'}$ submits set A_i at the beginning of period t. Note that A_i can be empty. Public school system assigns a random tie-breaking number to each $i \in F^{t,\emptyset}$ and two random tie-braking numbers to each $j \in F^{t,t'}$ at the begging of period t. Note that one of the random number assigned to $j \in F^{t,t'}$ will be used in period t and the other will be used in period t'. A random number for tie breaking in period t has been already assigned to $j \in F^{\widetilde{t},t}$ at the beginning of period $\widetilde{t}$. The outcome of gDA-STB is selected by the following algorithm:

Algorithm 1 *gDA-STB in Period t*

Step -2 Set $\widetilde{P}^t = P^t$

Step -1.0 Create set $B_{i,s} = \{1, 2, ..., Q^{t'}\}$ *for all* $i \in F^{t,t'}$ *and* $s \in A_i$

Step -1.1. If $r_s^{t'}(j) < r_s^t(i) - 2$ *for* $j \in F^{\widetilde{t},t'}$ *then remove* $b_{t'}(j)$ *from* $B_{i,s}$

Step 0. Construct $Top_t(s,t')$, $U_t(i,s,t')$ *for all* $i \in F^{t,t'}$, $s \in A_i$, $t' > t$

Step 0.1. If $|Top_t(s,t') \cap U_t(i,s,t')| \geq q_s^{t'}$ *then add* s *to set* $D(i)$

Step 0.2. If $|L(i,s,t)| \geq n - q_s^t$, $r_s^t(i) = 3$ *and* $b_{t'}(i)$ *is the among the lowest* $q_s^{t'}$ *in* $B_{i,s}$ *then add* s *to set* $E(i)$

Step 0.3. If $s \in D(i)$ *and if the next top ranked* $s' \in A_i$ *is in the set* $E(i)$ *then remove* s *from* $\widetilde{P}_i^t$

Do step 0 for all agents

Step 1. If there is an agent $i \in F^{t,t'}$ *and the top ranked school* s *in* $\widetilde{P}_i^t$ *is in* $E(i)$, *then assign* i *to* s, *update the priority order in period* t', *remove* i *from all priority list of school* $s' \neq s$ *in period* t *and go back to Step -1.*

Step 2. Run DA-STB by using $\widetilde{P}^t$

Intuitively, this algorithm finds the schools that siblings cannot be signed together and remove that schools if siblings are guaranteed to be assigned together to the next best school in A_i. This algorithm works perfectly if $|A_i| = 2$. If $|A_i| > 2$ for some $i \in F^{t,t'}$ then it can be the case that $s \in D(i)$ and $s'' \in E(i)$ and s is not removed from P_i^t since s' is neither in $D(i)$ nor in $E(i)$ given $(s,s)P_i(s',s')P_i(s'',s'')$. In that case, i may want to play for (s',s') but since (s',s') cannot be guaranteed at the beginning of period t it may be the case that i may be assigned another school in period t' hence removing s in that case may lead to problems. However, if i consent to any other school if there is at least one school in $E(i)$ there will not be any problem. A risk averse agent will prefer to consent.

We can easily see that gDA-STB is weakly dynamic fair.

Theorem 4 *gDA-STB is weakly dynamic fair.*

Proof. By definition. ■

In each period t in step 0 we do not consider the submitted preferences for other agents. In step1, we only consider submitted preference lists of agents who are guaranteed to be placed to the best choice possible. Hence, $i \in F^t$ cannot affect the steps -3 to 1 in period t and be better off by misstating his preferences in period t. In fact, the mechanism itself strategizes on the behalf of agents and drops out the schools that both siblings cannot be assigned and they can be assigned to the next best school together for sure. As mentioned in Abdulkadiroglu, Pathak and Roth (2006) the agents who do not strategize or strategize inadequately are harmed in a mechanism which is not strategy-proof. Hence the mechanism above try to strategize on behalf of each agents by using the all available information. Moreover, none of the agents can utilize all the available information that public school system has.

As mentioned in theorem 1 and 3, there is no static or weakly dynamic fair mechanism which is also strategy-proof. Hence both DA-STB and gDA-STB cannot be strategy-proof. Pathak and Sonmez (2011) introduce a method to compare two mechanisms which are not strategy-proof based on their vulnerability to manipulation. However, there is no dominance between two mechanisms. That is there are some problems in which gDA-STB can be manipulated but DA-STB cannot be and there are also some problem in which DA-STB can be manipulated and gDA-STB cannot. However, agents require much more information to manipulate gDA-STB. In fact in the problems in which gDA-STB can be manipulated and DA-STB cannot be, agents must know the random numbers assigned to the other agents. Moreover, they should know the preference profiles of agents who will apply in the future. As a future work, we will compare the two mechanisms focusing on the level of information needed to manipulate. But for now we can have the following proposition.

Proposition 1 *If agents do not have information about the random numbers assigned to other families then DA-STB is weakly more manipulable than gDA-STB.*

We will assume that every agent states the true preferences in the rest of the paper. Then we will check if there is a mechanism whose outcome Pareto dominates the outcome of DA-STB. In example 2 we have already showed that the outcome of DA-STB is Pareto dominated by a matching which is weakly dynamic fair. For simplicity we will have one more assumption for the preferences. We will assume that if the first child's assignment is not in set A_i then $P_i^{t'} = P_i^t$ and if it is in set A_i then $\mu_t(i)$ will be ranked at the top of $P_i^{t'}$ and the relative order of other schools will be the same as P_i^t. To have a welfare gain compared to DA-STB it must be the case that there must be at least an agent $i \in F^{t,t'}$ such that assignment of the second child must be same under DA-STB and the new mechanism and must be in set A_i. Moreover, we should find a trade between agents in period t in which they will agree to get the assignment of agent i and give him $\mu_{t'}(i)$. That is we will check if there is an agent who is guaranteed to be placed to $s \in A_i$ in period t' and cannot be placed to a better place and who was placed to s' in period t and $s' P_i^t s$. Then if there is such an agent $i \in P^{t,t'}$ we will check if there is an improvement cycle in which agent i will get s in period t. In order not to have computational burden, we will only focus on the possible welfare gains in two consecutive periods. Random number assignment and submission of preference list are the same as gDA-STB. Let $B = \{1, 2, 3..., Q^{t+1}\}$.

In the following algorithm we will use the stable improvement cycle introduced by Erdil and Ergin (2008). They define the stable improvement cycle as follows:

Definition 4 *A stable improvement cycle in period t consist of district agents $f_1, f_2, ...f_n = f_0 \in F^t$ such that*

(1) $\mu_t(f_l) \in S$ (each student in the cycle is assigned to a school)

(2) f_l desires $\mu_t(f_{l+1})$ to his match $\mu_t(f_l)$

(3) $f_l \in D_t(\mu_t(f_{l+1}))$ where $D_t(s)$ the set of students who has the highest priority among the students preferring s to their match in period t

If there is a stable improvement cycle in any period t agents trade the schools within that stable improvement cycle.

Algorithm 2 *mgDA-STB (modified gDA-STB) in Period t*

Step -1: Run the DA algorithm in step t and set $\widetilde{P}_i^{t+1} = P_i^{t+1}$ for all $i \in F^{\widetilde{t},t+1}$ where $\widetilde{t} < t+1$

Step 0: $D_t(s) = i$ if $s P_i^t \mu_t(i)$ and i has the highest priority among the agents who prefers s to their match in period t.

Step 1: Construct $Top(s, t+1)$, $U(i, s, t+1)$ for all $i \in F^{\widetilde{t},t+1}$, $s \in S$ by considering all matchings in period $\widetilde{t} < t+1$

Step 2: If $|Top(s, t+1) \cap U(i, s, t+1)| \geq q_s^{t+1}$ then remove s from $\widetilde{P}_i^{t+1}$ and update $Top(s', t+1)$ for all $s' \in S$

Do step 2 for all $i \in F^{\widetilde{t},t+1}$

Step 3: Assign agent i to school s if

(1) $r_s^{t+1}(i) \geq 2$

(2) $U(i, s, t+1) < q_s^{t+1}$

(3) s is top ranked school in $\widetilde{P}_i^{t+1}$

If all (1), (2), (3) hold then $\mu_{t+1}(i) = s$, remove i from $U(i, s', t+1)$ for $s' \neq s \in S$ and remove $b_{t+1}(i)$ from B

Do step 3 for all $i \in F^{\widetilde{t},t+1}$

Step 4: If $\mu_{t+1}(i) = \emptyset$ *then let* s_{top} *be the top ranked in* $\widetilde{P}_i^{t+1}$. *Then check the following conditions*

(1) $r_{s_{top}}^{t+1}(i) = 3$

(2) $|U(i, s_{top}, t+1)| < q_{s_{top}}^{t+1}$

(3) $b_{t+1}(i)$ *is the among the lowest* $q_{s_{top}}^{t+1} - |U(i, s_{top}, t+1)|$ *in* B

If all (1), (2), (3) hold then $\mu_{t+1}(i) = s_{top}$, *remove* i *from* $U(i, s', t+1)$ *for* $s' \neq s_{top} \in S$ *and remove* $b_{t+1}(i)$ *from* B

Do step 4 for all $i \in F^{\widetilde{t},t+1}$

Step 5: If $\mu_t(i) P_i^t \mu_{t+1}(i)$, $\mu_{t+1}(i) \in A_i$ *and* $i \succ_{\mu_{t+1}(i)}^t D_t(\mu_{t+1}(i))$ *then* $D_t(\mu_{t+1}(i)) = i$ *and check if there is a stable improvement cycle including* $\mu_{t+1}(i)$ *in period* t. *If yes do the trades.*

In first 4 steps we are finding selecting exactly the same matching that will be selecting by DA-STB. We are trying to assign agents who will apply for the second child in period to the best place without violating priorities and using all the available information we have. In step 5, we are looking that if there will be improvement cycle which again respect priorities. Then if we can find a stable improvement cycle in Step 5 then we can say that the outcome selected by mDA-STB Pareto dominates the matching selected by DA-STB. It is not static fair as DA-STB but if it violates period t priorities of an agent $i \in F^{t,t'}$ then we will be sure that agent i will strictly prefers his match selected by mDA-STB to match selected by DA-STB. We can present a more general algorithm which would improves the welfare of agents $i \in F^{t,t'}$ for $\forall t' > t+1$ but this will need more computational steps. Moreover, here we just want to illustrate that there is possible welfare gains and that can be attained easily. We can relax the assumptions on the preferences but then we need to ask individuals to submit their future preferences.

5 Simulation

In this section, we aim to replicate the Boston school choice problem by simulations. We measure the percentage of agents assigned to a better school under mgDA-STB. Since the outcome selected by mgDA-STB Pareto dominates the outcome of DA-STB all the other agents are assigned to the same school under both mechanism. We use similar procedure in creating the preference profiles of students and priorities for schools as Erdil and Ergin (2008).

We consider a model of 5 periods. There are 10 school with 20 seats in each period. We take the number of families with 2 children as 350 and 300 agents have 1 child. We then randomly assign each agents to the periods. We generate locational parameters of agents and schools by using i.i.d uniform distribution on [0,1]X[0,1] map. Based on location of agents and schools we determine the initial priority structure. If an agent with two children is assigned to a school then he gains a sibling priority. We update the priority structure by considering the assignments in each period.

To determine the preference profile of agents we first generate $Z_{i,s}$ for all $i \in I$ and $s \in S$. Here $Z_{i,s}$ represents agent i's taste for school s. We also generate $Z_{0,s}$ for all $s \in S$ to represents agents common taste for school s. We then select the number of schools A_i for each agent i with 2 children. Then we calculate agent i's utility from matching with school j from the following equation

$$U_{i,j} = -\beta d_{i,j} + (1-\beta)(\alpha Z_{0,j} + (1-\alpha) Z_{i,j})$$

Here $d_{i,j}$ is the distance between agent i and school j. Each agents ranks schools based on the utilities. If $U_{i,j} = U_{i,j'}$ for school j and j' we use random draw to determine the preference lists. If the assignment of elder children of agent i is one of the schools in A_i then we update the preference order by taking that school to top of the list and keep the relative order for the other schools.

We randomly draw a tie breaking rule in each period and run both DA-STB and mgDA-STB 1000 times for each $\alpha, \beta \in \{0, 0.1, 0.2, ..., 1\}$.

In simulations, the higher welfare gains attained when alpha is large and beta is small. When alpha is large and beta small agents have similar preferences over schools and higher welfare gains must be expect. Because in that case there will be overdemand to some schools and it will be more difficult for both siblings to be assigned in the same school. Moreover, stable improvement cycle will be longer as the preferences of all agents become more similar. For instance, when $\beta = 0$ and $\alpha = 1$ all agents have the same preferences over the schools. In that case DA-STB fails to assign siblings to the same school due to the overdemand.

In Figure 1, one can see average of the percentage of agents who prefers the outcome of mgDA-STB over DA-STB. Note that this is the percentage of all population.

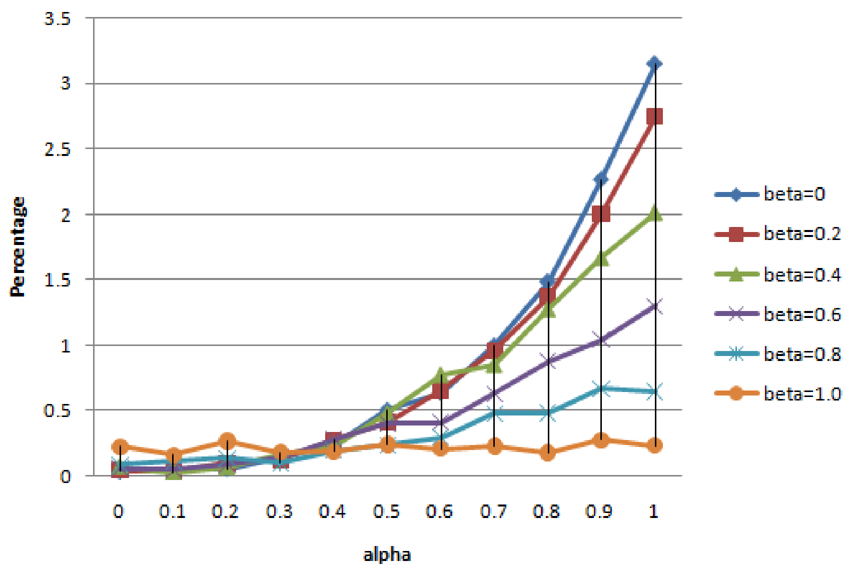

Figure 1. Simulations Results

6 Conclusion and Future Research

In this paper, we consider the school choice problem in a dynamic setting and show that some of the important results obtained in static problem are not valid in more realistic case. We then introduce a new fairness notion in the dynamic school choice problem. Then we introduce a new mechanism which gives better results compared to the DA-STB in terms of strategy-proofness and dynamic fairness. Moreover, we also show by simulations that up to 3% of the agents can be placed to better schools if the current mechanism is modified.

As a future work, we will try to relax the assumptions on the preference profiles. Moreover, we will focus on the necessary information requirements to manipulate the proposed mechanism and the current one. Then we will compare both vulnerability of the both mechanisms based on the required information.

7 Appendix

From Cambridge Public Schools Website:

> Q:"I have both sibling and proximity for my first choice school; am I guaranteed a seat in the school?"
>
> A:Although this is the highest preference an application could be given, there is no guarantee for a space, as there may not be enough open seats for all who apply."
>
> "The CPS will continue to use siblings as an assignment preference because

many parents/guardians believe that assigning siblings to the same school allows them to be more involved in their children's education."

From St. Paul Public School Website:

> Q: "Does sibling preference mean all of my students will be accepted to a magnet/citywide option school at the same time?"
>
> A: "No. The family must first have a student already enrolled in the program. Other guidelines such as available space and attendance area preferences are taken into consideration before sibling preference."

From San Francisco Public School Website

> "If placement in the older sibling's school is desired, it is strongly recommended that the older sibling's school be listed on the application form. It is recommended that other schools also be listed in the event there is a lack of space in the older sibling's school."