Incentives and the Shadow of the Future: Dynamic Moral Hazard with Learning

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Abstract

Following Holmstrom's career concerns model, we study dynamic moral hazard, with symmetric ex ante uncertainty and learning. Unlike Holmstrom, uncertainty pertains to the difficulty of the job rather than the general talent of the agent, so that contracts are required to provide incentives. With one period commitment, the contracting game is a dynamic game with private monitoring, since effort is privately chosen. Our main findings are, in a sense, the opposite of Holmstrom's. Long term interaction allows the agent to increase his future continuation value by deviating and exploiting the consequent misalignment of beliefs, thereby increasing the cost of inducing high effort. We characterize optimal contracts without commitment and also with renegotiation and full commitment. As the period of interaction increases (or if the agent becomes patient), incentive provision becomes increasingly costly.

1 Introduction

We study a multi-period principal agent model, with moral hazard, and ex ante symmetric uncertainty. Our underlying setting is reminiscent of Holmstrom's career concerns model (1999).[1] The main difference is that uncertainty pertains to the difficulty of the job (or the job-specific ability of

[1] See also Dewatripont et al. (1999a, 1999b) and Meyer and Vickers (1997).

the agent), rather than the general ability of the agent, as in Holmstrom. This implies that explicit incentives must be provided, in order to induce the agent to put in effort. Consequently, we allow complete contracting so that the principal can commit to output-contingent wages, at least within each period.[2]

Consider the standard principal agent model with moral hazard and one period, where there is uncertainty regarding job difficulty, where the probability distribution over output signals depends both on effort and upon job difficulty. Specifically, suppose that the agent believes that the job is good (i.e. easy) with probability λ and bad with complementary probability. That is, the agent's *first order belief* equals λ. In the standard model with common priors and no higher order uncertainty, the principal's belief regarding the agent's belief are degenerate. That is, her *second order belief* assigns probability one to the event that the agent's first order belief is λ. Notice that the principal designs her optimal contract based on her second order belief. In particular, assuming that the principal is risk neutral, the optimal contract minimizes expected wage payments, given the individual rationality and incentive constraints, where λ defines the probability distributions underlying these constraints. Indeed, it is standard that both these constraints bind at the optimum.

This observation regarding beliefs is the key to our analysis in the dynamic context. Suppose that effort must be chosen from the set $\{0, 1\}$, and that the principal designs a contract to induce high effort in both periods (independent of the realization of the first period signal). Now suppose that the agent chooses high effort in period one, and suppose that some signal y^k is realized. The agent's first order belief is now given by Bayesian updating given the signal realization and high effort. Let us denote the agent's belief by μ_1^k (the superscript indexing the signal realization, and the subscript effort choice). The principal does not observe effort; however, since the agent chooses high effort with probability one in equilibrium, the principal's second order belief is degenerate and assigns probability one to μ_1^k. Assuming that the principal can only commit for one period, the second period contract minimizes expected wage payments subject the incentive and individual rationality constraints defined by μ_1^k.

Now suppose that the agent deviates in the first period to $e = 0$. Given

[2] Gibbons and Murphy (1992) consider the implications of explicit incentives via (restrictive) linear contracts in the career concerns setting.

signal realization y^k, the agent updates to a belief μ_0^k. However, the principal's second order beliefs will be incorrect since they assign probability one to the agent having belief μ_1^k. That is, the principal continues to believe that the agent has chosen high effort, and therefore his second order belief is both certain and wrong. In consequence, the contract that she chooses for the second period will be subject to the (incorrect) incentive and individual rationality constraints defined by μ_1^k.

Our focus is on the second period continuation value of the agent when he deviates to low effort in the first period. We shall show that under general conditions, the agent's continuation value *strictly increases* if he deviates to low effort in the first period. The intuition comes from the fact that the individual rationality constraint always binds given belief μ_1^k. This implies that if the agent is more pessimistic about the job (i.e. $\mu_0^k < \mu_1^k$), then the constraint is violated, while if the agent is more optimistic (i.e. $\mu_0^k > \mu_1^k$), then the individual rationality constraint holds strictly, and the agent makes a surplus above his reservation utility. Now, when the IR constraint is violated, the agent will simply refuse the contract and earn his reservation utility, and therefore suffers no loss. Since the agent accepts the payoff gains but can refuse the payoff losses, he will benefit as long as there is some signal y^k such that $\mu_0^k > \mu_1^k$, i.e. where he is more optimistic regarding the job than the principal thinks that he is.

We show that there always exists some signal y^k such that $\mu_0^k > \mu_1^k$. This follows from the martingale property of beliefs. The expectation of the agent's posterior, over all signal realizations, must equal his prior, λ, regardless of whether the agent performs the experiment $e = 1$ or the experiment $e = 0$. Since good signal have higher probability under $e = 1$ than under $e = 0$, this equality of expectations can only be satisfied if there are some signals such that $\mu_0^k > \mu_1^k$.

Since the agent's second period continuation value is higher when he deviates to low effort in period one, as compared to the case where he does not deviate, this implies that the incentive constraint in the first period must be modified. That is, the principal must provide greater incentives for high effort than he would need to do in a static context, where there was no second period.

The paper makes this essential argument in the context of a model with binary effort choice and an arbitrary finite set of signals. In the case where we have only two periods we have a complete characterization for a variety of contracting scenarios. The "no-intertemporal commitment" case is our benchmark model – the agent is assumed to be unable to transfer resources across periods, and we assume that both the principal and the agent can make only one period commitments. We characterize the optimal dynamic contract in this environment with symmetric (but possibly asymmetric) learning. We then consider the case where the principal (and the agent) can make commitments for the second period at the end of the first period, i.e. after the signal realization, but before the agent consumes his first period wage. We allow renegotiation of first period consumption by the agent, in conjunction with the negotiation of second period wages. Optimal consumptions must satisfy the Lambert-Rogerson type martingale condition on the inverses of the marginal utilities of the agent. We show that the agent can increase his continuation value by deviating in the first period, just as in the as in the no commitment case. Finally, we consider the case where the principal can commit also for the second period, and provide conditions under which commitment helps other than by enhancing inter-temporal risk sharing possibilities. We also examine the possible role of random effort, so that there is true asymmetric information in the second period. The final part of the paper offers some results on a general many period model. Our main finding is that increasing the period of interaction always increases the cost of effort provision, in sharp contrast to the repeated game literature (e.g. Radner, 1985).

Our work is related to the work of Lambert (1983), Rogerson (1985) and Malcomson and Spinnewyn (1988) on dynamic moral hazard without uncertainty regarding the agent's ability. While the focus in these papers is on risk sharing in dynamic context, our focus is on the nature of learning by the principal and the agent. In this respect, it is also related to some of the recent literature on games with private monitoring, albeit in a context where history permits learning about a payoff relevant state of the world. The issues we analyze here also arise in Bergemann and Hege (1998,2005), who consider venture capital financing.

2 The model and preliminaries

Our model combines moral hazard with uncertainty regarding job difficulty. Specifically, the job is either good (easy) or bad (hard), i.e. the job type is

$\alpha \in \{G, B\}$.The agent chooses effort $e \in \{0, 1\}$. Let $y \in Y = \{y_1, y_2, .., y_n\}$ denote the signal that is realized following effort choice. This depends, stochastically, on both the type and the effort chosen. Let $p^k_{e\alpha}$ be the probability of signal y^k given effort e and type $\alpha \in \{G, B\}$. Thus for each signal y^k, we have a 4-tuple $(p^k_{0B}, p^k_{1B}, p^k_{0G}, p^k_{1G})$. Given that μ is the probability that the agent is type G, define $p^k_{1\mu}$ (resp. $p^k_{0\mu}$) to be the probabiliity of signal k when effort level 1 (resp. 0) is chosen.

We shall distinguish two types of likelihood ratio, the likelihood ratio on efforts for a given type (or belief over types) and the likelihood ratio over types for a given effort choice. The former is relevant for providing effort incentives, while the latter determines Bayesian learning. Let $\ell^k_\alpha = \frac{p^k_{1\alpha}}{p^k_{0\alpha}}$ be the likelihood ratio for signal k for type α. Generalizing this, $\ell^k_\mu = \frac{\mu p^k_{1G}+(1-\mu)p^k_{1B}}{\mu p^k_{0G}+(1-\mu)p^k_{0B}}$ denote the likelihood ratio for signal k when μ is the probability that the agent is type G. Let $\ell^k_e = \frac{p^k_{eG}}{p^k_{eB}}$ be the likelihood ratio for signal k for effort level e.

Our main assumption, that is maintained throughout this paper, is as follows:

A1 All probabilities belong to $(0, 1)$. For some y^k, $p^k_{1G} \neq p^k_{0B}$ i.e. there exists some informative signal. For any informative signal y^k, p^k_{1B} and p^k_{0G} lie in the interior of the interval spanned by p^k_{1G} and p^k_{0B}, i.e $p^k_{1B}, p^k_{0G} \in (\min\{p^k_{1G}, p^k_{0B}\}, \max\{p^k_{1G}, p^k_{0B}\})$.

To provide some intuition for this assumption, let Y^H be the set of high signals, where $p^k_{1G} > p^k_{0B}$. Then this assumption implies that if $y^k \in Y^H, \ell^k_\alpha > 1$ for $\alpha \in \{G, B\}$ and $\ell^k_e > 1$ for $e \in \{0, 1\}$.That is, if a signal is more likely when a given type of agent chooses high effort, it is also more likely for a given effort level when the job is the good type. This implies that signals that are indicative of high effort are also indicative of the agent being the good type. Similarly, let Y^L be the set of low signals, where $p^k_{1G} < p^k_{0B}$. The assumption implies that if $y^k \in Y^L, \ell^k_\alpha < 1$ for $\alpha \in \{G, B\}$ and $\ell^k_e < 1$ for $e \in \{0, 1\}$, so that a low signal indicates low ability as well as low effort. Finally, we may have some uninformative signals when $p^k_{1G} = p^k_{0B}$, where all likelihood ratios are one, but since there is at least one informative signals, both Y^H and Y^L are non-empty. Let Y^U denote the set of uninformative signals, and let $\Pr(Y^U)$ denote the probability that an uninformative signal is realized – this does not depend upon effort choice or ability.

Let $R(e, \mu)$ denote the revenue of the principal, as a function of the effort level, e :

$$R(1,\mu) = \sum_k \left[\mu p_{1G}^k + (1-\mu)p_{1B}^k\right] y^k.$$

$$R(0,\mu) = \sum_k \left[\mu p_{0G}^k + (1-\mu)p_{0B}^k\right] y^k.$$

The derivative of $R(1,\mu)$ with respect to μ equals $\sum_k (p_{1G}^k - p_{1B}^k y^k$. A sufficient condition for this to be positive is that if $y \in Y^H$ and $y' \in Y^L$, then $y > y'$. This condition is implied by, but weaker than, the monotone likelihood ratio condition. Similarly, under this condition, the $R(0,\mu)$ is also increasing in μ.

The difference in revenue from inducing high effort and inducing low effort, is given by

$$R(1,\mu) - R(0,\mu) = \mu \sum_k (p_{1G}^k - p_{0G}^k)y^k + (1-\mu)\sum_k (p_{1B}^k - p_{0B}^k)y^k.$$

Examining this expression, we see that it is strictly positive, since $p_{1G}^k - p_{0G}^k > 0$ if $y^k \in Y^H$ and $R(1,\mu) - R(0,\mu)$ is linear in μ, and can be either increasing or decreasing.

We shall assume henceforth that the principal always want to employ the agent in any period regardless of μ. That is, $\max\{R(1,\mu=0), R(0,\mu=0)\}$ is sufficiently large. We shall also assume that $R(1,\mu)$ is large relative to $R(0,\mu)$ for every value of μ, so that the principal always wants to induce high effort.

For most of the paper we are interested in a two period model, where the agent has prior belief λ that he is good, and where this belief is common knowledge between the agent and the principal. We can allow for the possibility that the principal's belief regarding the agent's type differs from λ, but we do not do this now. First we shall consider a model without commitment, where the principal can only commit for one period at a time. Then we shall consider the implications of being able to commit to second period contracts as a function of signal realizations, at the initial date itself.

2.1 The static model

Suppose that the principal wants to induce $e = 1$.The principal's optimal contract depends upon second-order beliefs, i.e. his beliefs regarding the agent's beliefs regarding his own type. Let us suppose that the principal assigns probability one to the agent assigning probability μ to being the good type. Let w_k denote the wage paid in the event that signal y^k is realized. The incentive constraint corresponding to this belief is given by

$$\mu \sum_k (p_{1G}^k - p_{0G}^k) u(w_k) + (1-\mu) \sum_k (p_{1B}^k - p_{0B}^k) u(w_k) \geq c(1) - c(0). \quad (1)$$

The individual rationality constraint given this belief is given by

$$\mu \sum_k p_{1G}^k u(w_k) + (1-\mu) \sum_k p_{1B}^k u(w_k) - c(1) \geq \bar{u}. \quad (2)$$

The optimal contract that induces $e = 1$ minimizes expected wage payments subject to these constraints, and is standard. Let $\mathbb{W}(\mu) = (w_k(\mu))_{k=1}^n$ denote the profile of wages corresponding to this optimal contract. The important thing that matters for our purpose is that wages are increasing in $\tilde{\ell}_\mu^k$, the likelihood ratio corresponding to belief μ.In particular, if we compare two signals $y^l \in Y^L$ and $y^h \in Y^H$, then $w_l(\mu) < w_h(\mu)$ for any belief μ.

Our first results concern the utility and optimal behavior of an agent who is offered contract $\mathbb{W}(\mu)$, but who in fact has belief μ'.

Lemma 1 *If $\mu' > \mu$, the agent gets utility that is strictly greater than $\bar{u}$; he may or may not choose high effort. If $\mu' < \mu$, the agent quits since he gets a utility that is strictly lower than $\bar{u}$, regardless of his effort choice.*

Proof. Using the fact that the IC binds at belief μ, the payoff difference between choosing $e = 1$ as and $e = 0$ at belief μ' can be written as

$$\Delta(\mu'|\mu) = (\mu' - \mu) \sum_k \left[(p_{1G}^k - p_{0G}^k) - (p_{1B}^k - p_{0B}^k) \right] u(w_k(\mu)).$$

Since $\Delta(\mu'|\mu)$ is linear in $(\mu - \mu')$, there are three possibilities. If the term under the summation sign is zero, then $\Delta(\mu'|\mu) = 0$ for all μ' and the IC holds. Otherwise, either the IC binds strictly for all $\mu' > \mu$ and is violated for all $\mu' < \mu$ or vice versa. If the IC holds at μ', then using the fact that IR also binds at belief μ, the difference between the agent's payoffs at μ' and μ can be written as

$$U(e=1,\mu)-U(e=1,\mu')=(\mu'-\mu)\sum_k(p^k_{1G}-p^k_{1B})u\left(w_k(\mu)\right). \qquad (3)$$

Now, $p^k_{1G}-p^k_{1B}>0$ if $y^k\in Y^H$, and $p^k_{1G}-p^k_{1B}<0$ if $y^k\in Y^L$. Since wages are uniformly higher for signals in Y^H than for signals in Y^L, (3) has the same sign as $(\mu'-\mu)$. If the IC is violated at μ', the agent will choose $e=0$ if he stays on the job. However, since the IC binds at belief μ, $U(e=1,\mu)=U(e=0,\mu)$. Using the fact that IR also binds at belief μ, the difference in payoffs at μ' and μ can be written as

$$U(e=0,\mu')-U(e=1,\mu)=(\mu'-\mu)\sum_k(p^k_{0G}-p^k_{0B})u(w_k(\mu)). \qquad (4)$$

Now, $p^k_{0G}-p^k_{0B}>0$ if $y^k\in Y^H$, and $p^k_{0G}-p^k_{0B}<0$ if $y^k\in Y^L$. Since wages are uniformly higher for signals in Y^H than for signals in Y^L, (4) has the same sign as $(\mu'-\mu)$. ■

3 The dynamic model

We now consider different versions of the dynamic model – first, where all commitments are only for one period, then the case where renegotiation is possible after the realization of the first period signal, and finally, the full commitment case. We shall focus on the case of deterministic effort, where the principal seeks to induce $e=1$ with probability one – however, at the end we consider the possible role of randomization, i.e. why the principal may seek to induce high effort with probability less than one.

3.1 One period commitment

We have two time periods, $t=1,2$. The agent lives for two periods, and we shall assume that neither the principal nor the agent can commit in period one regarding the contract in period two. Furthermore, we shall also assume that no renegotiation is possible between principal and agent after the signal realization in period one. One interpretation of the model is that there are two short term principals, one arriving in period one and the second arriving in period two, after consumption has taken place in period one. The principal in period two observes the public signal (output) in period one. This implies that wages paid have to satisfy incentive compatibility and individual rationality period by period.

3.1.1 The simple dynamic contract

Let $\lambda \in [0, 1]$ be the common prior probability that the agent is the good type. One simple conjecture on optimal contracts without commitment is as follows: in period one, the optimal contract is the solution to the static problem with beliefs λ. In period 2, the optimal contract is the solution to the static problem, but with updated beliefs corresponding to the signal realizations and $e = 1$. Let us call this contract the *simple dynamic contract.*

Suppose that effort in period one is observable ex post by the principal, before he offers the contract in period two, but that it is not verifiable. In this case, the simple contract is the optimal contract, since the agent cannot gain by deviating to low effort. However, we shall assume that effort is not observable. We now show that the simple dynamic contract cannot be an optimal contract, since the agent has a profitable deviation in the game that this contract induces. Suppose that the agent deviates in period one and chooses $e = 0$. Since his IR binds in the simple contract, the utility he gets in period one remains unaffected, and is indeed equal to his reservation utility, $\bar{u}$. However, his period two beliefs are now different from the principal's beliefs about the agent's beliefs. In particular, there is at least one signal realization such that he becomes more optimistic about his ability. Since the agent suffers no penalty when he becomes more pessimistic – he quits and gets his outside utility, which is the same as under the simple contract, the agent has a profitable deviation.

The fact that the agent always becomes more optimistic at some signal realization after deviating is a consequence of the martingale property of beliefs. For any effort level e that the agent chooses, the expectation of his posterior must equal his prior, λ. Thus his expected beliefs under $e = 0$ must equal his expected beliefs under $e = 1$. Since $e = 1$ makes signals in Y^H more likely than when $e = 0$ is chosen the equality of expectations can only be satisfied if there is some signal realization y such that $\mu_0^k > \mu_1^k$, where

μ_e^k is the posterior probability that the agent is the good type given signal realization y^k and effort choice e.

We now show this more formally. The agent's posterior beliefs at signal y^k when he has chosen $e = 1$ in the first period are given by

$$\mu_1^k = \frac{\lambda p_{1G}^k}{\lambda p_{1G}^k + (1-\lambda) p_{1B}^k} = \frac{\lambda \ell_1^k}{\lambda \ell_1^k + (1-\lambda)}.$$

His posterior belief at y^k after deviating to $e = 0$ are given by

$$\mu_0^k = \frac{\lambda \ell_0^k}{\lambda \ell_0^k + (1-\lambda)}.$$

Thus the agent is more optimistic about his ability after deviating on observing signal y^k if $\mu_1^k < \mu_0^k$.

Lemma 2 *There exists some k such that $\mu_0^k > \mu_1^k$.*

Proof. From the martingale property of beliefs, $\mathbf{E}(\mu_1^k) = \mathbf{E}(\mu_0^k) = \lambda$, i.e.

$$\sum_{k=1}^{n} p_{0\mu}^k \mu_0^k = \sum_{k=1}^{n} p_{1\mu}^k \mu_1^k,$$

which can be written as

$$\sum_{k=1}^{n} p_{0\mu}^k (\mu_0^k - \mu_1^k) = \sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k) \mu_1^k.$$

Since $\sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k) = 0$ (being the difference between two probability distributions), $\sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k)\lambda = 0$, so that

$$\sum_{k=1}^{n} p_{0\mu}^k (\mu_0^k - \mu_1^k) = \sum_{k=1}^{n} (p_{1\mu}^k - p_{0\mu}^k)(\mu_1^k - \lambda).$$

Under assumption A1, for any k, $(p_{1\mu}^k - p_{0\mu}^k)$ has the same sign as $(\mu_1^k - \lambda)$ – i.e. a signal that has higher probability under high effort is also informative of the job being easier. Since there is some informative signal, we conclude that $\sum_{k=1}^{n} p_{0\mu}^k (\mu_0^k - \mu_1^k) > 0$, i.e. the expectation of the difference in beliefs under the experiment $e = 0$ is strictly positive. Thus there must be some signal y^k such that $\mu_0^k > \mu_1^k$. ■

We have therefore shown that the expectation of the "false belief" held by the principal, μ_1^k, when the agent performs the experiment $e = 0$, is strictly smaller than the expectation of the true belief μ_0^k. Thus there must be some signal realization for which $\mu_0^k > \mu_1^k$. This immediately implies the following proposition:

Proposition 3 *Suppose that effort is not observable. The simple dynamic contract is never incentive compatible.*

Proof. If the simple dynamic contract is chosen, can deviate to $e = 0$. In this case, he gets the same first period utility. In period 2, there is at least one signal realization such that he has more optimistic beliefs and therefore gets a surplus – after a signal realization where he has a more pessimistic beliefs, then he quits and gets his reservation utility, which equals his utility under the optimal contract. ■

If principal wants to induce $e = 0$ in the second period, for some realization of his beliefs, then proposition 3 will not hold. For example, if effort and ability are complements, it might be optimal to induce low effort if the principal becomes more pessimistic about the agent's type. The optimal contract to induce low effort is a flat wage contract. In consequence, the agent's payoff does not depend upon his subjective beliefs regarding his ability. So if the principal induces $e = 0$ after some signal realization y^k, then the agent's continuation utility $V^+(y^k, 0) = 0$. Thus proposition 3 may not apply if there are some signals such that at the associated beliefs, the principal does not want to induce high effort. However, if there are multiple effort levels, so that the lowest level of effort is never optimal, and incentives always have to be provided, then it seems likely that something similar to this proposition will apply.

3.1.2 Characterizing optimal contracts

Suppose that the principal wants to induce $e = 1$ in both periods. Period 2 contracts are straightforward. Given that $e = 1$ is chosen, the principal's beliefs about the agent's beliefs are degenerate, and are given by μ_1^k after signal y^k. Thus the period two contract after signal y^k is given by $\mathbb{W}(\mu_1^k))$. Let w_{kj}^2 denote the wage paid under the optimal second period contract after second period signal realization y_j given first period signal realization y_k, and the belief μ_1^k.

Turning to period 1 contract, this must satisfy IR with the prior beliefs λ and also a modified IC given these beliefs. We turn to deriving this modified IC.

The agent's his continuation utility after signal y^k and deviation $e = 0$ in the event that he stays on the job is denoted by $V(y^k, 0)$ – we write it as function of μ_1^k to emphasize its dependence upon second period wages, which depend upon μ_1^k.

$$V(y^k, 0) = \begin{cases} \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{1G}^j - p_{1B}^j) u\left(w_{kj}^2\right) \text{ if } \Delta(\mu_0^k|\mu_1^k) \geq 0 \\ \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{0G}^j - p_{0B}^j) u(w_{kj}) \text{ if } \Delta(\mu_0^k|\mu_1^k) < 0. \end{cases} \tag{5}$$

Since the agent does better by quitting when $V < 0$,his actual continuation utility is given by

$$V^+(y^k, 0) = \max\{V(y^k, 0), 0\}.$$

Therefore the agent's expected continuation utility from choosing $e = 0$ in the first period is given by

$$\mathbf{E}(V^+(0)) = \sum_k V^+(y^k, 0)(\lambda p_{0G}^k + (1 - \lambda)p_{0B}^k).$$

The modified IC for the first period is therefore given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) \geq c(1) - c(0) + \mathbf{E}(V^+(0)). \tag{6}$$

The IR constraint is unaffected and is given by

$$\lambda \sum_k p_{1G}^k u\left(w_k^1\right) + (1 - \lambda) \sum_k p_{1B}^k u\left(w_k^1\right) - c(1) \geq \bar{u}. \tag{7}$$

Proposition 4 *The optimal dynamic contract that induces $e = 1$ in both periods is follows: i) in period 1, the contract wages minimize expected wage payments given the modified IC (6) and the IR (7), which hold with equality. ii) in period 2, the contract after signal realization y^k is given by the static contract $\mathbb{W}(\mu_1^k)$,corresponding to common beliefs μ_1^k.*

For future reference, let $(\tilde{w}_j^1)_{j=1}^n$ denote the profile of first period wages under the optimal contract set out in the proposition.

3.2 Renegotiation after the first period signal

The dynamic contract we have analyzed allows no renegotiation between principal and agent after the realization of the signal (output) in period one. One interpretation is that the principal has limited dynamic commitment possibilities, i.e. he cannot commit in period one to period two contracts even at the stage where the signal is realized – we could alternatively assume that the agent cannot commit at this point. Now let us consider the case where the principal can make such a commitment at the end of period one, before the agent consumes his wage. Alternatively, in the story with two distinct principals, this corresponds to the assumption that the principal in period 2 arrives at the end of period one, i.e. before the agent has consumed his wage. Suppose the signal realization is y_j, and the agent has been paid w_j^1 by principal one. In this case, prior to consumption, the principal may propose a renegotiation consisting of consumptions $(\hat{w}_j^1, (\hat{w}_{jk}^2)_{k=1}^n)$. We assume that the principal makes a take it or leave it offer to the agent, and that if the agent refuses, he takes the outside option.

The renegotiation offered must satisfy the following constraints. The incentive constraint is given by

$$\mu_1^j \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_{jk}^2\right) + (1-\mu_1^j) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_{jk}^2\right) \geq c(1) - c(0).$$

The agent will accept the offered on the equilibrium path (i.e. contingent on having chosen $e = 1$ in the first period) only if the following individual rationality constraint is satisfied

$$u(\hat{w}_j^1) + \mu_1^j \sum_k p_{1G}^k u\left(\hat{w}_{jk}^2\right) + (1-\mu_1^j) \sum_k p_{1B}^k u\left(\hat{w}_{jk}^2\right) - c(1) \geq u(w_j^1) + \bar{u}.$$

The optimal second period contract minimizes $\hat{w}_j^1 + \mu_1^j \sum_k p_{1G}^k w_{jk}^2 + (1 - \mu_1^j) \sum_k p_{1B}^k w_{jk}^2$ subject to these constraints. Now, by the same argument as in Rogerson (1985), it follows first and second period consumptions must satisfy a martingale condition on the inverses of marginal utilities. That is, we must have

$$\frac{1}{u'(\hat{w}_j^1)} = \sum_k \left[\mu_1^j p_{1G}^k + (1-\mu_1^j) p_{1B}^k\right] \frac{1}{u'\left(\hat{w}_{jk}^2\right)}. \tag{8}$$

Let us denote the consumptions that follow renegotiation from wage w_j^1, $\mathbb{W}(w_j^1, \mu_1^j)$. Thus the optimal second period contract after any signal realization must satisfy IR and IC with equality, and must also satisfy the martingale condition on the inverses of the marginal utilities.

We now examine the implications for the first period. Consider first the individual rationality constraint. If the agent chooses high effort in period one, then his continuation payoff when signal y_j is realized in period one is exactly equal to $u(w_j^1) + \bar{u}$, since the IR constraint binds in the second period. Thus the individual rationality constraint in period one is given by

$$\lambda \sum_k p_{1G}^k u\left(w_k^1\right) + (1-\lambda) \sum_k p_{1B}^k u\left(w_k^1\right) - c(1) + \bar{u} \geq 2\bar{u}.$$

That is, the IR constraint in the first period is exactly as in the previous analysis, where no renegotiation was possible.

Now let us consider the incentive constraint. If the agent deviates to low effort in period one, his continuation payoff after signal y_j conditional on staying on the job is equal to

$$\hat{V}(y^k, 0) = \begin{cases} \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{1G}^j - p_{1B}^j) u\left(\hat{w}_{kj}^2\right) & \text{if } \Delta(\mu_0^k | \mu_1^k) \geq 0 \\ \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{0G}^j - p_{0B}^j) u(\hat{w}_{kj}) & \text{if } \Delta(\mu_0^k | \mu_1^k) < 0. \end{cases}$$

This has the same qualitative form as in the case without renegotiation, except that the relevant wages are different. In particular, we see that the agent makes a positive rent when he is more optimistic, i.e. when $\mu_0^k > \mu_1^k$ and a negative rent when he is more pessimistic. Since the agent can always quit in latter instance, his actual rent is given by $\hat{V}^+(y^j, 0) = \max\{\hat{V}(y^k, 0), 0\}$. Define

$$\mathbf{E}(\hat{V}^{+}(0)) = \sum_k \hat{V}^{+}(y^k, 0)(\lambda p_{0G}^k + (1-\lambda)p_{0B}^k).$$

We now show that the rent in the case of renegotiation has the same sign as the rent in the first model, without renegotiation. Note that the incentive constraint in the case with renegotiation has exactly the same form as in the first model, since neither the first period wage nor consumption enter. Since the second period incentive constraint holds with equality, we may re-write this as

$$\mu_1^j \sum_k (p_{1G}^k - p_{1B}^k) u\left(w_{jk}^2\right) - \mu_1^j \sum_k (p_{0G}^k - p_{0B}^k) u\left(w_{jk}^2\right) = c(1) - c(0) - \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_{jk}^2\right).$$

The modified IC for the first period is therefore given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) \geq c(1) - c(0) + \mathbf{E}(\hat{V}^{+}(0)).$$

Thus, from the point of view of the principal in period one, the problem is formally very similar to the case where there is no renegotiation. Thus exactly the same analysis applies, as far as period one is concerned. We therefore have the following proposition.

Proposition 5 *Suppose the principal cannot commit at date 1 but can renegotiate after the realization of the signal at date 1. The optimal dynamic contract that induces $e = 1$ in both periods is follows: i) in period 1, the contract wages solve the modified IC and the IR with equality. ii) in period 2, the consumptions after signal realization y^k are given by $\mathbb{W}(w_j^1, \mu_1^j)$,corresponding to the first period contingent wage w_j^1 and common beliefs μ_1^k.*

3.3 Full Commitment

Finally, let us consider the case where the principal can commit at date one to a contract for both periods. We also assume that the agent can also commit to stay on the job for two periods. The incentive constraint in period two following signal realization y^j in period one is, as before, given by

$$\mu_1^j \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_{jk}^2\right) + (1-\mu_1^j) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_{jk}^2\right) \geq c(1) - c(0). \quad (9)$$

The individual rationality constraint at the beginning of period one is given by

$$\sum_j p_{1\lambda}^j u(w_j^1) + \sum_j \sum_k p_{1\lambda}^j p_{1\lambda}^k u\left(w_{jk}^2\right) - 2c(1) \geq 2\bar{u}.$$

$$\sum_j \left(\lambda p_{1G}^j + (1-\lambda) p_{1B}^j\right) \left[u(w_j^1) + \mu_1^j \sum_k p_{1G}^k u\left(w_{jk}^2\right) + (1-\mu_1^j) \sum_k p_{1B}^k u\left(w_{jk}^2\right)\right] - 2c(1) \geq 2\bar{u}.$$

This simplifies to

$$\sum_j \left(\lambda p_{1G}^j + (1-\lambda) p_{1B}^j\right) u(w_j^1) + \sum_j \sum_k \left(\lambda p_{1G}^j p_{1G}^k + (1-\lambda) p_{1B}^j p_{1B}^k\right) u\left(w_{jk}^2\right) - 2c(1) \geq 2\bar{u}. \quad (10)$$

We now turn to the incentive constraint in period one. If the agent deviates to $e = 0$ in period one and signal y^j is realized, then his second period incentive constraint may or may not hold, depending on whether $\Delta(\mu_0^j | \mu_1^j)$ is positive or negative. Thus his continuation utility is given by $V(y^j, 0)$. In contrast to the no commitment case, his continuation utility is not given by $V^+(y^j, 0) = \max\{ V(y^j, 0), \bar{u}\}$. Therefore, by deviating in period one, the agent gets second period expected utility equal to $\mathbf{E}(V(0))$.Thus the first period incentive constraint is given by

$$\lambda \sum_k (p_{1G}^k - p_{0G}^k) u\left(w_k^1\right) + (1-\lambda) \sum_k (p_{1B}^k - p_{0B}^k) u\left(w_k^1\right) + \mathbf{E}(V(0)) \geq c(1) - c(0). \quad (11)$$

$$V(y^k, 0) = \begin{cases} \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{1G}^j - p_{1B}^j) u\left(w_{kj}^2\right) & \text{if } \Delta(\mu_0^k | \mu_1^k) \geq 0 \\ \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{0G}^j - p_{0B}^j) u(w_{kj}) & \text{if } \Delta(\mu_0^k | \mu_1^k) < 0. \end{cases}$$

The optimal contract minimizes expected wage payments over the two periods subject to the first period IC, n signal contingent (on-path) second period ICs, and the overall IR constraint.

We now show that following any first period signal y_j, the profile of wages that follow that signal in period one and in period two, must satisfy the following condition

$$\frac{1}{u'(w_j^1)} = \sum_k \left[\mu_1^j p_{1G}^k + (1-\mu_1^j) p_{1B}^k\right] \frac{1}{u'\left(w_{jk}^2\right)}.$$

This condition is essentially the Lambert-Rogerson condition on the inverses of the marginal utilities. To prove that this condition must hold in the present context, consider a profile of wages and undertake the following experiment where the utility $u(w_j^1)$ is increased by ε, and the utility $u(w_{jk}^2)$ is increased by $-\varepsilon$, uniformly for every $k \in \{1, 2, .., n\}$. This does not affect the second period incentive constraint following signal y_j, (9). Furthermore, since the total utility, over the two periods, following signal y_j is unchanged, it also does not affect the overall individual rationality constraint. Finally, since the change in total utility following y_j is zero, independent of the probability distribution over second period signals, it also does not affect the first period incentive constraint. Since this change does not induce a violation of any of the constraints, it must be unprofitable at the optimum, and the standard argument shows that the martingale condition on the inverses of marginal utilities must be satisfied. Notice that this also implies that the full commitment contract is renegotiation proof. In the case where there are two signals, the martingale condition, the incentive constraints and the single IR constraint fully determine the contract wages – see Squintani (2008), for an analysis of the two signal case with commitment. More generally, the optimal contract is the solution to the general programing problems set out above. However, it is more interesting to compare outcomes in full commitment case with the case where only one period commitments, but where renegotiation is possible, since this allows optimal risk sharing.

Proposition 6 *The optimal consumptions under renegotiation coincides with the optimal contract with full commitment if and only if* $\ell_0^k \geq \ell_1^k \ \forall k$, *that is the agent is more optimistic on deviating after any signal.*

Proof. The only if part of the proposition is straightforward. If the incentive constraint binds in the full commitment contract, then it is violated in the case with renegotiation since the continuation value of the agent from deviating to $e = 0$ is $\mathbf{E}(\hat{V}^{+}(0))$, which is strictly greater than $\mathbf{E}(\hat{V}(0))$. To prove if, let the consumptions following renegotiation be identical with those in the full commitment contract. After every signal y^j, we need to find a wage w_j such that

$$u(\hat{w}_j^1) + \mu_1^j \sum_k p_{1G}^k u\left(w_{jk}^2\right) + (1 - \mu_1^j) \sum_k p_{1B}^k u\left(w_{jk}^2\right) - c(1) = u(w_j^1) + \bar{u},$$

Since u is strictly increasing and continuous and there are no limited liability constraints this can always be done. We now that the agent's optimal strategy is put high effort at $t = 1$, and to stay on the job after all signal realizations, and choose $e = 1$ also in period 2, independent of his effort choice at $t = 1$. If the agent deviates to $e = 0$ at $t = 1$, then he becomes more optimistic after all signals, and thus his expected continuation value is greater if he stays than if he goes. Given that he does not quit, his overall payoff is given by the long term contract, which by satisfies incentive constraints in each period. Thus the long term contract can be implemented by a short term contract followed by renegotiation. ■

This proposition clarifies the precise role of commitment. First, it permits intertemporal risk sharing, as in Lambert-Rogerson, but this can also be done if the principal is able to renegotiate at the end of the first period. The key difference is that it relaxes the first period IC in the case where the agent becomes more pessimistic after some realizations of the signal, since the agent cannot now walk away, an effect that does not arise in repeated moral hazard models without learning.

3.4 Random effort

We have assumed so far that the principal wants to induce high effort for sure at $t = 1$.Is there any advantage to the principal in inducing random effort, i.e. in the agent choosing high effort with a probability $\pi \in (0, 1)$? We have assumed that $R(1, \lambda)$ is sufficiently large relative to $R(0, \lambda)$, so that the revenue cost of inducing low effort is linear and decreasing in π. So inducing random effort can only help if it reduces expected wage payments for the principal. [3] We consider first the case where there are only single period

[3] Random effort at $t = 2$ is costly and cannot help in any way, since the IC and IR constraints are the same as for inducing high effort with probability one.

commitments.

Suppose that $\pi \in (0,1)$. This implies that at $t = 2$, there is asymmetric information, since the agent knows his chosen effort, while the principal does not. In particular, if signal y^k is realized, the principal believes that the agent has chosen $e = 1$ with probability

$$\theta^k = \Pr(e = 1|y^k) = \frac{\pi[\lambda p_{1G}^k + (1-\lambda)p_{1B}^k]}{\pi[\lambda p_{1G}^k + (1-\lambda)p_{1B}^k] + (1-\pi)\pi[\lambda p_{0G}^k + (1-\lambda)p_{0B}^k]}.$$

Therefore, the principal's second order belief assigns probability θ^k to the agent having first order belief μ_1^k and probability $1-\theta^k$ to the agent having first order belief μ_0^k. The principal therefore faces a classical mechanism design problem where the agent knows his "type" while the principal knows the probability distribution over these types, where a type is to be interpreted as the agent's belief about his own ability. Consider the mechanism design problem where agent has two possible beliefs, μ_1^k and with probabilities θ^k and $(1-\theta^k)$ respectively. The principal has limited screening possibilities. He can offer a contract which is acceptable only to the more optimistic type, i.e. the type with belief equal to $\max\{\mu_1^k, \mu_0^k\}$, without being required to pay a rent to this type, in which case the more pessimistic type will refuse the contract. This will be optimal if the probability assigned by the principal to this type (i.e. θ^k or $1-\theta^k$ as the case may be) is sufficiently low. Alternatively, he can offer a contract which is acceptable to the pessimistic type, i.e. the type with belief equal to $\min\{\mu_1^k, \mu_0^k\}$. In this case, he must pay an informational rent to the optimistic type, which has a similar form as $V(y^k, 0)$ defined earlier. That is, if $\mu_0^k > \mu_1^k$, the informational rent equals to the type with belief μ_0^k equals $V(y^k, 0, \mu_1^k)$, where we write this a function of μ_1^k to emphasize its dependence on the second period wages after making a false report. which depend. That is

$$\hat{V}(y^k, 0, \mu_1^k) = \begin{cases} \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{1G}^j - p_{1B}^j) u\left(\hat{w}_{kj}^2(\mu_1^k)\right) \text{ if } \Delta(\mu_0^k|\mu_1^k) \geq 0 \\ \left[\mu_0^k - \mu_1^k\right] \sum_j (p_{0G}^j - p_{0B}^j) u(\hat{w}_{kj}(\mu_1^k)) \text{ if } \Delta(\mu_0^k|\mu_1^k) < 0. \end{cases}$$

If $\mu_0^k < \mu_1^k$, the informational rent equals to the type with belief μ_1^k equals

$$V(y^k, 1, \mu_0^k) = \begin{cases} \left[\mu_1^k - \mu_0^k\right] \sum_j (p_{1G}^j - p_{1B}^j) u\left(w_{kj}^2(\mu_0^k)\right) \text{ if } \Delta(\mu_0^k|\mu_1^k) \leq 0 \\ \left[\mu_1^k - \mu_0^k\right] \sum_j (p_{0G}^j - p_{0B}^j) u(w_{kj}(\mu^k)) \text{ if } \Delta(\mu_0^k|\mu_1^k) > 0. \end{cases}$$

Consider first the case where $\mu_0^k \geq \mu_1^k$ for every signal y^k. In this case, one can show that the principal cannot gain by inducing random effort at $t = 1$. If θ_1^k is sufficiently close to one, then at $t = 2$, the principal will always want to ensure the participation of the belief type μ_1^k after every signal y^k. Thus he must pay an informational rent to type μ_0^k which equals $V(y^k, 0, \mu_1^k)$ after every signal y^k. Thus the increase in continuation value of the agent from choosing $e = 0$ is exactly equal to $\mathbf{E}(V^+(0))$, just as in the case where $e = 1$ is induced with probability one. Now since $e = 1$ must be optimal at $t = 1$, this implies that the incentive constraint corresponding to this is exactly the same as before. In other words, inducing random effort does not reduce the cost of provision of high effort, and only reduces revenue, since we have assumed that it is optimal to induce high effort. We have therefore established that if $\mu_0^k \geq \mu_1^k$ for every signal y^k, then it is not optimal to induce low effort with some small probability. On the other hand, if the principal induces $e = 0$ with sufficiently large probability, then it will be the case that after some y^k, θ^k may be sufficiently small. This may make it optimal to exclude the belief type μ_1^k, implying that the principal does not have to pay a rent to type μ_0^k after this signal. Thus the continuation value of the agent from choosing $e = 0$ is reduced, since he gets zero rather than $V(y^k, 0)$ after this specific signal y^k. However, the revenue cost of inducing low effort will be large, since π must be sufficiently low so as to ensure that exclusion of type μ_1^k is ex post optimal for the principal. Thus, if inducing high effort at $t = 1$ is sufficiently profitable, this will not be optimal.

Consider next the case where $\mu_0^k < \mu_1^k$ for some signal y^k. Suppose that θ^k is small enough that exclusion of the μ_0^k is not optimal after any signal y^k. In this case, the belief type μ_1^k gets an informational rent at $t = 2$ after signals y^k such that $\mu_0^k < \mu_1^k$. Therefore the agent's expected continuation utility from choosing $e = 1$ in the first period is given by

$$\mathbf{E}(V^+(1)) = \sum_k V^+(y^k, 1, \mu_0^k)(\lambda p_{1G}^k + (1 - \lambda)p_{1B}^k),$$

where

$$V^+(y^k, 1, \mu_0^k) = \max\{V(y^k, 1, \mu_0^k), 0\}.$$

This relaxes the incentive constraint for choosing $e = 1$ at $t = 1$, which is now given by

$$\lambda \sum_{k} (p_{1G}^{k} - p_{0G}^{k}) u\left(w_{k}^{1}\right) + (1-\lambda) \sum_{k} (p_{1B}^{k} - p_{0B}^{k}) u\left(w_{k}^{1}\right) \geq c(1) - c(0) + \mathbf{E}(V^{+}(0)) - \mathbf{E}(V^{+}(1)). \tag{12}$$

Note that the first period revenue cost of randomization is linear and decreasing in π, the probability of high effort. In the second period, π does not enter directly into the expressions for $\mathbf{E}(V^{+}(1))$ or $\mathbf{E}(V^{0}(1))$, since these depend only on the agent's beliefs (μ_0^k and μ_1^k) and not upon the principal's second order beliefs, which depend upon π. However, the principal's second order beliefs must assign sufficiently high probability to μ_0^k when it is lower than μ_1^k, or otherwise the principal will find it optimal to exclude belief type μ_0^k. Thus π must be sufficiently low such that after every signal y^k such $\mu_0^k < \mu_1^k$, the principal finds it optimal not to exclude type μ_0^k.

More generally, it may be the case that the principal induces sufficient randomization so that type μ_0^k is not excluded after some but not all signals y^k such that $\mu_0^k < \mu_1^k$. In this case, the expected rent of the agent must be modified appropriately. That is, for every signal y^k such that $\mu_0^k < \mu_1^k$, there is an associated maximum probability π^k with which $e = 1$ must be chosen so that a rent can be paid to the high effort type after this signal. Thus the expected rent of the agent from choosing high effort is given by

$$\mathbf{E}(V^{+}(1,\pi)) = \sum_{k:\pi \leq \pi^k} V^{+}(y^k, 1, \mu_0^k)(\lambda p_{1G}^k + (1-\lambda) p_{1B}^k).$$

Thus the first period incentive constraint is as in (12), with $\mathbf{E}(V^{+}(1,\pi))$ replacing $\mathbf{E}(V^{+}(1))$, so that tighter as a function of π. Revenue is a strictly increasing in π since we have assumed that $R(1,\lambda) > R(0,\lambda)$. Since there are finitely many signals, the optimal contract can be computed by comparing revenues and wage costs corresponding to the finitely many values π^k. We summarize our results in the following proposition.

Proposition 7 *With one period commitment, random effort is never optimal if $\ell_0^k \geq \ell_1^k$ $\forall k$, so that the agent is more optimistic on deviating after any signal. If $\ell_0^k < \ell_1^k$ for some k, random effort may help by relaxing the agent's incentive constraint, and may be part of the optimal contract.*

4 Many periods

We now consider the case where there are finitely many periods, T. We call this game $\Gamma^T(\lambda)$, to emphasize its dependence on the prior. The public history at date t, h^t is an element of $(Y)^{t-1}$. The private history at date t, $\tilde{h}^t$ is an element of $(Y \times \{0,1\})^{t-1}$. Let $h^1 = \tilde{h}^1$ be a singleton set.

Consider an a pure strategy equilibrium of the T period game, where the effort sequence on the equilibrium path is deterministic. Now both principal and agent update along this path using the equilibrium effort sequence. Let μ^t denote the realized belief in period t – this is random, since it depends upon the realization of output signals. Conditional on any type realization, e.g. $\alpha = G$, for any $\varepsilon > 0, \Pr(1 - \mu^t < \varepsilon | \alpha = G) \to 1$ as $t \to \infty$. Thus uncertainty vanishes, and with it, the scope for manipulating beliefs vanishes as well. While this is true, we show that as the period of interaction increases, it intensifies the incentive problem in the initial periods.

We focus on equilibria where the principal seeks to induce $e = 1$ in every period. Thus the principal's belief at $h^t = (y^1, y^2, .., y^{t-1})$ is given by

$$\mu(h^t) = \frac{\lambda \prod_{\tau=1}^{t-1} p_{1G}(y^\tau)}{\lambda \prod_{\tau=1}^{t-1} p_{1G}(y^\tau) + (1-\lambda) \prod_{\tau=1}^{t-1} p_{1B}(y^\tau)}.$$

The agent's belief at any $\tilde{h}^t = ((y^1, e^1), (y^2, e^2), .., (y^{t-1}, e^{t-1})$ is given by

$$\mu(\tilde{h}^t) = \frac{\lambda \prod_{\tau=1}^{t-1} [e^t p_{1G}(y^\tau) + (1-e^t) p_{0G}(y^\tau)]}{\lambda \prod_{\tau=1}^{t-1} [e^t p_{1G}(y^\tau) + (1-e^t) p_{0G}(y^\tau)] + (1-\lambda) \prod_{\tau=1}^{t-1} [e^t p_{1B}(y^\tau) + (1-e^t) p_{0B}(y^\tau)]}.$$

Now let us an arbitrary period t in the T period game with initial prior $\lambda, \Gamma^T(\lambda)$, and the $T+1$ period game with the same initial prior, $\Gamma^{T+1}(\lambda)$,where $t \leq T$. The set of possible t period histories is identical across these games. Furthermore, if the equilibrium effort sequence $(e^t)_{\tau=1}^{t-1}$ is the same, then $\mu(h^t)$ is the same across these games for the same realized public history.

Proposition 8 *In any period $t \leq T$ and after any public history h^t, the t-period incentive constraint is strictly more severe in $\Gamma^{T+1}(\lambda)$ than in $\Gamma^T(\lambda)$.*

The proof is by induction. We have already shown that the statement holds for $t = T$, for any history. Now suppose that the statement is true for any $\tau \in \{t+1, .., T\}$. Fix an equilibrium strategy of the T period game, and suppose that the agent deviates and chooses $e = 0$. Let $\hat{s}^T$ denote an optimal continuation strategy for the agent in the continuation game, given that he has deviated at date t. Suppose now that the agent deviates at h^t in Γ^{T+1} and chooses $e = 0$. Define his continuation strategy $\hat{s}^{T+1}$ as follows: it agrees with $\hat{s}^T$ at all h^τ such that $\tau \in \{t+1, .., T\}$, and in period $T+1$ it plays optimally. We now show that at every history where the agent makes a deviation gain using $\hat{s}^T$ in Γ^T, he makes a strictly larger deviation gain by using $\hat{s}^{T+1}$ in Γ^{T+1}.

At any period $\tau \in \{t+1, .., T\}$ and at any history h^τ, let $w_k^T(h^\tau)$ denote τ period wages in Γ^T and let $w_k^{T+1}(h^\tau)$ denote τ period wages in Γ^{T+1}. These wages coincide with the solution to the static contracting problem, but with different effort costs in the incentive constraint, where the effort cost is strictly greater in $w_k^{T+1}(h^\tau)$ as compared to $w_k^T(h^\tau)$, by an amount b. This implies that

$$\sum_j \left[\mu(h^\tau)\left(p_{1G}^j - p_{0G}^j\right) + (1-\mu(h^\tau))\left(p_{1B}^j - p_{0B}^j\right)\right]\left[u\left(w_j^{T+1}(h^\tau)\right) - u\left(w_j^T(h^\tau)\right)\right] = b > 0. \tag{13}$$

The rent after private history $\tilde{h}^t$ and public history h^t can be written as

$$V(\tilde{h}^t) = \left[\mu(\tilde{h}^t) - \mu(h^t)\right]\sum_j (p_{\tilde{e}G}^j - p_{\tilde{e}B}^j) u\left(w_j^T(h^t)\right),$$

where $\tilde{e} \in \{0,1\}$ is the optimal effort choice at belief $\mu(\tilde{h}^t)$. Thus if $\mu(\tilde{h}^t) - \mu(h^t) > 0$,the difference in rent at this history in Γ^{T+1} and Γ^T equals

$$\left[\mu(\tilde{h}^t) - \mu(h^t)\right]\sum_j (p_{\tilde{e}G}^j - p_{\tilde{e}B}^j)\left[u(w_k^{T+1}(h^\tau)) - u\left((w_k^T(h^\tau)\right)\right].$$

We now show that the above expression is strictly positive. Let $\mathbf{\Delta u}$ denote the vector $\left[u\left(w_j^{T+1}(h^\tau)\right) - u\left(w_j^T(h^\tau)\right)\right]_{j=1}^n$, and let $\mathbf{\Delta p}_\mu$ denote the vector

$[[\mu(h^\tau)\left(p^j_{1G}-p^j_{0G}\right)+(1-\mu(h^\tau))\left(p^j_{1B}-p^j_{0B}\right)]]^n_{j=1}$. Thus the inner product $\mathbf{\Delta u}.\mathbf{\Delta p}_\mu = b > 0$. Let $\mathbf{\Delta p}_{\tilde{e}}$ denote the vector $(p^j_{\tilde{e}G}-p^j_{\tilde{e}B})^n_{j=1}$. Since $\mathbf{\Delta p}_{\tilde{e}}$ is the difference between two probability distributions, its components sum to zero, i.e. $\mathbf{1}.\mathbf{\Delta p}_{\tilde{e}} = 0$, where $\mathbf{1}$ denotes a vector where every component is one. Write $\mathbf{\Delta u} = \mathbf{\Delta \tilde{u}} + c\mathbf{1}$, where $\mathbf{\Delta \tilde{u}}$ and the scalar c are chosen so that every component of $\mathbf{\Delta \tilde{u}}$ has the same sign as the corresponding component $\mathbf{\Delta p}_\mu$.Assumption A1 implies that every component of $\mathbf{\Delta p}_{\tilde{e}}$ has the same sign as the corresponding component of $\mathbf{\Delta p}_\mu$, and so $\mathbf{\Delta u}.\mathbf{\Delta p}_\mu = b > 0$ implies $\mathbf{\Delta u}.\mathbf{\Delta p}_{\tilde{e}} = \mathbf{\Delta \tilde{u}}.\mathbf{\Delta p}_{\tilde{e}} > 0$.

We have therefore shown that given any optimal deviation strategy at h^τ in the game Γ^T, there exists a deviation strategy that gives strictly higher payoffs in every period $\tau \leq T$ that the first strategy yields positive rents. Furthermore, the latter strategy also yields rents in period $T+1$, thereby proving the proposition.

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