

Informed seller in a Hotelling market*

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Abstract

We consider the problem of a monopolist who is selling a good and is privately informed about some of its attributes. We focus on the case where goods with different attributes are horizontally differentiated: in other words, they appeal to different segments of the market. Can the monopolist profit from concealing her private information? Is it optimal for her to reveal all good's attributes upfront?

We show that in many circumstances, the monopolist maximizes her profit by not disclosing any information, which is in contrast to insights from auction theory and the informed-principal literature. We characterize the optimal selling mechanism for the informed monopolist. The optimal selling mechanism depends on the shape of the transportation cost function and on the base consumption value of agents. Still, if the base value is sufficiently high, then it is optimal not to disclose any information, if the base value is sufficiently low, then it is optimal to disclose the location. For intermediate base values, one can implement the optimal mechanism as the two-item menu: buy the good at the fixed price without disclosure, or buy the information about the location with the option of purchasing the good afterwards at a predetermined exercise price.

1 Introduction

It is a common characteristic of several business negotiations that the interaction between sellers and buyers originally begins in a particular condition of asymmetric information: the buyers are uncertain about their valuation for the good on sale, the sellers own private information that would help the buyers to resolve their uncertainty. For example, a seller may be better informed about attributes of the good that are not immediately visible to the buyer: the ingredients used to cook a dish, the internal components of a technological device, the details of a financial investment plan. In these situations each seller may try to use her information strategically: revealing or hiding what she knows in order to maximize her profits.

When is profitable for the seller to disclose her private information? How much to reveal? To whom?

We consider the problem of a profit maximizing seller who owns private information in an horizontally differentiated market. Such a market is represented via a standard Hotelling

*Preliminary and incomplete.

(1929) model. Each buyer's and seller's private information consists in locations on the Hotelling line. The seller's type which can be thought of as a location of the good corresponds to a particular characteristic of the good that is relevant to the buyer but is the private information of the seller. The buyer's type describes his preferences over the characteristic so that his utility from consuming the good is a base consumption value minus a cost depending on the distance between the buyer and the good.

We derive the optimal (revenue maximizing) mechanism in a variety of settings, first restricting the information disclosure options for the seller to full revelation or no revelation and then solving for the optimal mechanism without restrictions. We show that under restricted disclosure option, it is always better to disclose the information if the base consumption value is low, and it is always better not to disclose the information if the base consumption value is sufficiently high, no matter what is the cost function considered. The optimal unrestricted mechanism may provide partial disclosure so that different buyers receive different information and thus it entails price discrimination across buyers in terms of their valuation for information. We propose mechanisms to implement the optimal outcome. In one of them, the seller offers a menu: buy the good with no information attached or buy the information about the good and have an option to buy the good at a predetermined exercise price after learning the seller's private information.

Our analysis inherits the complexity of the Informed Principal problem (Myerson (1983), Maskin & Tirole (1990), Maskin & Tirole (1992)). After the seller learns her type, she designs the mechanism to sell the good. Any choice she makes affects the buyer's beliefs over her types, and, thus, the buyer's willingness to pay for the good. However, the seller cannot manipulate arbitrarily the buyer's beliefs. Any signal the seller sends to the buyer has to pass through a credibility test. Whenever the seller wants to induce some probability distribution over her types, these probabilities should be consistent with what is indeed in the interest of each of the seller's types from the perspective (i.e. given the information set) of the buyer. In this sense, the actual seller (i.e. the true type of the seller) needs to consider what the other seller's type would do, if she wants to credibly reveal or conceal her identity.

We consider an environment in which the good can be located only at the two extremes of the line and the buyers have ex-ante beliefs that the good is at each extreme with equal probabilities. The symmetry that we are imposing by construction simplifies our analysis and allows us to bring to light the interaction between the seller's types in a clearer way. Indeed, given symmetry, the perspective of one type is the mirror-version of the other's. This implies that the setting is one of pure horizontal differentiation: there exists no "better" or "high value" type for the seller who finds advantageous to always reveal her identity.¹

In our environment, each type of the seller would like to reveal her location to the buyers located close by, and be pooled with the other type in the beliefs of the buyers located further away. This strategy would maximize each buyer's expected valuation for the good, but it is infeasible. The strategy requires that the two types of the seller pool together in the beliefs of some buyers. However, the seller's types disagree in terms of which set of buyers to pool together for. Therefore, the incentives of the seller's types are misaligned in terms of how to

¹The informed principal literature (see Yilankaya (1999)) considers several environments where the presence of high and low types of the principal hinders any strategic use of private information by the principal and supports a complete unraveling of information (i.e. full disclosure) as unique equilibrium.

use the private information to manipulate the buyer’s expected valuation for the good.

We show that, with linear or concave costs, the opposite is true considering the buyer’s valuation for information. Under certain conditions, both seller’s types recognize the buyer’s types that are located closer to the extremes as the ones who are willing to pay the most for their information. Accordingly, in the optimal mechanism both types of the seller decide to pool together and offer the good with no information disclosed — a product we may refer to as an *opaque good*²— and an option-like contract that allows the buyer to learn information and then, if he wants, to buy the good.

The conditions that support such result are about the buyer’s types utility functions. The disutility they suffer from consuming the good they like the least should be sufficiently large. We characterize these conditions by considering different cost functions in the Hotelling model.

By selling the opaque good and the actual information separately the true type of the seller finds the way to sell something to everyone in the market. The buyers that are almost indifferent between the two types of the seller choose to buy the opaque good. The buyers who have strong biases in their preferences buy the option. Out of this latter group, the ones that learn “positive” information (i.e. the actual good is the one that they like the most) buy the good at the exercise price of the option. The ones who learn “negative” information do not exercise the option.

By buying the option and learning the true type of the seller, a buyer avoids to consume the least preferred good. The more variable is a buyer’s valuation for the good (depending on the seller’s type), the higher is the same buyer’s valuation for information. This implies that, for a given type of the seller, the buyers who are willing to pay the most for her information are also the ones who value the least her actual good.

When the cost function is convex, the optimal mechanism is different. There are: (i) a first set of buyers located close to the endpoints of the Hotelling line who choose to pay for the good under full information; (ii) a second set of buyers located close to the segment middle-point who buy the opaque good; (iii) and a third set of buyers, located in an intermediate region, who buy lotteries with probability q to get the delivered the good if the good is of a specific type. In that way, they can choose a contract that with probability $1 - q$ will not deliver them the kind of good they like the least.

We show that, given the optimal mechanism, the distribution of informational rents among the buyers differ depending on the shape of the cost function considered. If the cost function is concave, the buyers with the highest valuation for the actual good have the highest rents; if the cost function is convex, then the highest information rents are gained by the buyers who are indifferent between the types of the seller. Furthermore, the shape of the cost function affects which constraints determine the prices of the contracts offered by the seller. When the costs are convex the price of the good, the prices of the lotteries and the price of the opaque good are pinned down by the individual rationality constraints of the types of buyers who are left with no surplus by the seller. When the costs are concave the price of the opaque good is determined by the individual rationality of the buyer located in the middle of the Hotelling line. Instead, the price of the option is derived from the incentive

²We borrow the opaque good term from the marketing literature. Opaque are goods that are offered on sale purportedly without disclosing relevant information about their attributes.

compatibility constraint of the buyer who is indifferent between the two contracts offered by the seller.

If the seller's types are asymmetric, then the characterization of the optimal mechanism is more complex. When the costs are linear or concave, for example, the seller's types disagree on which set of buyers to sell the option to. Still we show that there exists a set of mechanisms that guarantees each type a higher profit than what each of them would obtain revealing her identity. Like in the symmetric case, these mechanisms entail the simultaneous sale of an opaque good and an option. However, different mechanisms imply different distributions of this extra surplus among the types.

Our work contributes to different strands of literature. It is related to the Informed Principal literature (Myerson (1983), Maskin & Tirole (1990), Maskin & Tirole (1992), Yilankaya (1999), Skreta (2007), Tröger & Mylovanov (2008), Balestrieri (2008)) Indeed, in the jargon of Maskin & Tirole (1992), we consider an informed seller in a common value environment: the seller's type enters directly into the utility function of the buyers. Most of these works, however, consider environments with competition between the buyers as in auction models and vertical differentiation among the buyers' types. We contribute to this literature by considering an horizontally differentiated market. We offer an alternative approach to Myerson (1983) that allows to determine the optimal solution in a variety of new settings. The opportunity for the seller of pooling together her types is evaluated not only with respect of the resulting buyers' expected valuations for the good, but also considering the resulting buyers' willingness to pay for information.

Our analysis is also closely related to the literature about the optimal mechanism for a multi-product monopolist (McAfee & McMillan (1988), Thanassoulis (2004), Pavlov (2006), Balestrieri & João Leão (2008)). The uncertainty about the two types of the seller embedded in our environment happens to provide a natural link between our work and the ones that solve the profit maximization problem of a two-good monopolist. Indeed, our Hotelling environment with different transportation cost function specifications is similar to the one that appears in Balestrieri & João Leão (2008). Both in Pavlov (2006) and in Balestrieri & João Leão (2008) the optimal mechanism is characterized in terms of a set of lotteries. The buyers are price discriminated on the base of their degree of indifference between the two goods. The ones that are more indifferent prefer to buy a lottery with equal probability of winning each good. In the optimal mechanism, the goods are offered beside the lotteries for a higher price. Such extra value may be interpreted as the value of an informed (as opposed of random) purchase. The buyers are offered different bundles of information and the good. In our environment, instead, information and the good are unbundled. The problem of each type of the seller is how to extract revenue from the buyers who prefer the other type and the solution is to sell them information alone.

More broadly, our work contributes to a vast literature that considers mechanism design problems in environments in which the seller controls and manipulates the buyers's access to information in order to maximize her profits. This literature covers a space across two fields: industrial organization and auction theory. Like in our case, the problem that these studies tackle is determining if a profit maximizing seller should facilitate or not the buyer's acquisition of information. In general, each buyer's valuation is modeled as a function of the buyer's type and some extra factor. This additional component may be the private

information of the seller or an exogenous stochastic variable. In the first case, the problem is often the disclosure of quality related information (vertical differentiation) in the context of pre-specified mechanisms. In the second case, when a stochastic component is the source of the buyers' uncertainty, the seller controls the accuracy with which the buyers learn the value of the realized shock. In static models, the seller can add noise to the shock (without privately observing its realization); in dynamic models, where the shock realizes in the future, the seller can add different time-related options to her offers (advance selling options, refunds options).

Lewis & Sappington (1994) is a seminal contribution in the industrial organization side of this literature. They consider the trade-off faced by a seller who can control the accuracy with which buyers learn their valuation of the good on sale. More precise private information for the buyers brings new price discrimination opportunities to the seller, but it also leaves higher informational rents to the buyers. They characterize different settings in which extreme disclosure policies (i.e. maximum precision, noise) are optimal.

Such extreme results are also obtained by Johnson & Myatt (2006). In their work they show how the monopolist's profits are a U-shaped function of the dispersion of the buyer's valuation. Given that, the seller considers two strategies. She may reveal information in order to identify the high valuation buyers and charge them a high price (niche-market strategy). Otherwise she may hide information and charge low price to a large number of buyers (mass-market strategy). In other words, the information disclosure policy becomes a tool to transform (rotate) the buyers' demand and maximize profits through niche- or mass-market strategies. In a similar fashion, Anderson & Renault (2006) show that costless advertising may not be always profitable for the seller in an environment where advertisement has a twofold role: improve the precision of the buyer's expected valuation for the good and decrease the search costs incurred by the buyers. In our setting we can also identify a similar to these papers effect on the demand: by not revealing the information the seller lowers the buyer's expected willingness to pay and differently so for different types while gaining a new market.

In the auction theory literature, Milgrom & Weber (1982) raised the question of whether the seller should reveal her information and answered it positively for a general affiliated values setting. Bergemann & Pesendorfer (2007) and Esó & Szentes (2007) derive the optimal auction in environments in which the seller controls the precision with which the buyers learn about their valuation for the good³ and the buyers types are vertically differentiated. In both these works the degree of uncertainty left to the buyers is an endogenous variable, and in Esó & Szentes (2007) the optimal mechanism design entails the sale of information by the seller.⁴

³Shi (2007) characterizes an optimal auction with information acquisition. The cost of the information is an exogenous function of the signal precision and the revenues from selling information are not accrued by the seller. Several works in the literature about auctions with costly entry fee can be interpreted in terms of costly information acquisition for the bidders. As noticed in Esó & Szentes (2007), such works usually assume ex-ante homogeneity across the bidders.

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On top of that, each buyer’s expected valuation is a monotonically increasing function of the buyer’s private information. The provision of information by the auctioneer affects the degree of competition between bidders.⁵ This additional factor is crucial in determining the optimal disclosure policy. For example, Ganuza & Penalva (2010) consider the incentives of an auctioneer to provide private information to the bidders comparing different definitions of signal’s precision. Competition and precision appear to be complementary factors to maximize the auctioneer’s profits. The crucial distinction of our study from the auction models of the seller’s control of information channels and of the information acquisition is that in our setting the seller is the one that possesses information and has different incentives to share it depending on what she knows. In contrast to Milgrom & Weber (1982) we show that quite generally the seller would not want to disclose her information or do so only partially. Moreover, we show that instead of the conventional disclosure channels – reveal her information to everyone or affect the information of every type of a specific buyer in the same way, the seller with the private information chooses to disclose her information selectively, only to some types of the buyer and for a fee.

Price discrimination across buyers in terms of their valuation for information (instead of ex-post valuation for the good) is analyzed in models of mechanisms with refunds or advance-purchase discounts.⁶ In these models, like in ours, buyers pay higher prices for more information. However, differently from our work, there is an exogenous dynamic process according to which the buyers learn their true valuation for the good over time.

In the case of refunds, a buyer decides to buy a good on the base of his expectations over the good’s valuation. If the buyer actually buys the good, then he learns his valuation and decides if returning it in exchange for a refund. Courty & Li (2000) characterize the optimal selling mechanisms with refunds in settings where each buyer’s type is associated with a conditional probability distribution over valuations and the ranking of the types corresponds to a ranking of the conditional distributions in terms of first order stochastic dominance or mean-preserving spread.⁷

In the case of advance-purchase discounts, each buyer’s uncertainty about his valuation for the good is resolved over time, and such learning is not conditioned on the purchase of the good. This strand of literature refers mostly to capacity constrained sellers who face uncertain demand (Gale & Holmes (1992), Gale & Holmes (1993); Dana (1998), Dana (1999b), Dana (1999a), Dana (2001)). An exception is Nocke, Peitz & Rosar (2011), who determine necessary and sufficient conditions for an advance selling mechanism to be optimal.

Our environment differs from the ones in the refund or the advance-purchase literature, because, in our case, the uncertainty of the buyer is due to some information that belongs to the seller. Given that, even the selection of the mechanism becomes a vehicle of information transmission for the buyers (the informed principal problem). Moreover, the feasible information sets of the buyer are not two (no information before buying, full information after buying) but are a continuum and they are endogenously determined in equilibrium as a part

⁵Hoffmann & Inderst (2009) extends the work of Esó & Szentes (2007) to a setting in which the provision of information is costly for the seller, and buyers do not compete. The buyer’s type are vertically differentiated.

⁶Cremer & Khalil (1992), Cremer & Khalil (1994), Cremer, Khalil & Rochet (1998) study costly information acquisition by uncertain buyers. Such costs are not revenues for the seller of the good.

⁷Zhang (2008) considers an optimal auction problem with refunds.

of the design of the optimal mechanism.

In most of the aforementioned models buyers are vertically differentiated with respect to their types or, alternatively, ex-ante identical. There are few works that consider the disclosure of information about horizontally differentiable attributes of a good. Board (2009) and Li (2010) consider auction models with competition between the bidders and their results are driven by this factor that is absent in our environment. Koessler & Renault (2011) model each buyer's valuation through a general matching function of the seller's and the buyer's type. Sun (2011), Celik (2010) and Li (2010) use Hotelling models where the types are locations along the Hotelling line and the costs of transportations are linear. In all these works, differently from our analysis, any disclosure of information by the seller happens before the buyers make any purchasing decision and there is no price on information.

The rest of the paper is organized as follows. The model is set up in Section 2. In Section 3 we describe the optimal mechanism for the benchmark setting of complete information. In Section 4 we solve for the optimal mechanism under restricted disclosure policy, allowing the seller only full revelation or no revelation. We also consider specific settings of linear, convex, and concave cost functions. In Section 5 we set up the Informed Principal problem allowing for arbitrary disclosure policies and characterize the agent's incentive constraints. Then, in Section 6 we solve a simplified problem, where the principal's choice is limited to committing to reveal or not reveal her private information and then setting the price. Finally, we solve for a general optimal selling scheme and provide examples of the optimal mechanism for the specific cost functions. Section 7 concludes.

2 The model

There are two players, the principal (or the seller) and the agent (the buyer). The principal sells a good or service which the agent wants to purchase. There are two sources of incomplete information. First, the exact characteristics of the good are known to the principal but not fully known to the agent. Second, the agent's consumption utility (conditional on specifics of the good) is not fully known to the principal. In addition, the nature of the principal's information is such that no possible realization of such information (the principal's type) is a priori better than some other realization for all possible types of the agent. That is, the types of the principal are horizontally differentiated (e.g., by taste) rather than vertically (e.g., by quality). Such an environment can be conveniently modeled by a Hotelling-like model. For convenience, we would refer to the principal as she and to the agent as he.

The information of both the principal and the agent can be represented as a location on a line, with s denoting the location of the principal (the seller), and x – the one of the agent. Both the principal and the agent are risk neutral. The utility of the agent located at x from purchasing the good from principal s at price p is

$$U_A(x, s) = V - c(|x - s|) - p,$$

where V is the base value of the transaction, p is the price, and $c(\cdot)$ is the cost function specifying the loss of consumption value to the agent from the difference in the ideal and the actual characteristics of the good, $c(0) = 0$, and c is strictly increasing. The cost of

producing the good to the principal is without loss of any generality is taken to be 0, and so the principal’s utility from the transaction equals p . The outside participation values for both the principal and the agent are assumed to be 0.

For the main model we are going to assume that the principal’s type can take one of the two values $s \in \{0, 1\}$ with equal probabilities, while the agent’s type can take a continuum of values $x \in [0, 1]$ and is drawn from a uniform distribution. Accordingly, one can interpret this setup as the principal facing a continuum of agents of a unit mass, uniformly located over the segment. Thus, each of the players knows her own type, while types are independently drawn and the distributions are commonly known.

The interaction between the principal and the agent proceeds as follows. The principal offers a mechanism. The agent decides whether to participate in it. If he participates, both players play by the rules of the mechanism to generate an outcome (q, p) consisting of the quantity to be transacted (possibly random) and the transfer from the agent to the principal. If the agent refuses the mechanism, no transaction will take place. We do not impose any restrictions on the set of mechanisms that can be offered but the requirement of the outcome function specifying the transaction details. The equilibrium notion we use throughout the paper is Bayesian-Nash equilibrium.

We are going to assume that the nature of the principal’s information is such that it is not verifiable to the agent, so that each type of the principal can pretend to be any other type. In particular, no type of the principal has actions available only to her that can be used to credibly convey her information to the agent without regard to incentive constraints, and the agent cannot learn the value of the good to him *ex interim*. For instance, the principal cannot provide documentation or offer the following “warranty” mechanism: “I claim that I am of type s , I offer to sell my good at price p . Once you receive the good, you can return it and receive the full refund plus bonus if you find that I have lied.” Clearly, if the agent were able to assess the exact value of the good to him, the principal of $s' \neq s$ would not be able to offer such a mechanism as she would be caught when pretending to be s . Any credible information communication must be incentive compatible. Assuming that the information is not verifiable ensures that the same set of mechanisms is available to each type of the principal.

3 Optimal selling mechanism under complete information

In this section we derive the optimal selling scheme from the perspective of the principal under the assumption that her information is known to the agent. This is the classic bilateral trade setting with one-sided information problem, and the optimal mechanism is a posted price. For the sake of completeness and to introduce the notation and the logical steps of the approach we derive the optimal mechanism explicitly. Our problem is a special case of the optimal auction problem (Myerson (1981)). Our exposition follows the treatment of the optimal auction in Krishna (2002).

Without loss of any generality we can assume that the principal is located at $s = 0$. By the Revelation Principle, for any mechanism and an equilibrium of it offered by the principal

there exist an outcome-equivalent direct mechanism in which all types of the agents report their valuations truthfully. Thus, in order to find the maximal possible expected revenue to the principal and the corresponding allocation and payment rules, it suffices to limit the search over the direct mechanisms.

Any direct mechanism can be characterized by a pair of functions (q, p) , where $q(z)$ is the probability of the sale (the allocation function) and $p(z)$ is the expected payment of the agent reporting z . Let $U(z|x)$ denote the expected utility the agent of type x obtains in the mechanism when he reports z . For the truthtelling to be an equilibrium, the pair (q, p) has to satisfy the following incentive compatibility (IC) and individual rationality (IR) constraints:

IC constraints: $\forall x, z \in [0, 1]$,

$$U(x) \triangleq U(x|x) = q(x)(V - c(x)) - p(x) \geq q(z)(V - c(x)) - p(z) = U(z|x). \quad (1)$$

IR constraints: $\forall x \in [0, 1]$,

$$U(x) = q(x)(V - c(x)) - p(x) \geq 0. \quad (2)$$

For any pair x, z , the combination of IC constraints $U(x) \geq U(z|x)$ and $U(z) \geq U(x|z)$ gives

$$[U(x) - U(x|z)] - [U(z|x) - U(z)] \geq 0 \implies (q(x) - q(z))(c(z) - c(x)) \geq 0. \quad (3)$$

Therefore, the first implication of incentive compatibility is that $q(x)$ must be non-increasing since $c(x)$ is strictly increasing.

Since

$$\begin{aligned} q(x)(c(x) - c(z)) &= U(x|z) - U(x) \leq U(z) - U(x) \\ &\leq U(z) - U(z|x) = q(z)(c(x) - c(z)), \end{aligned}$$

we obtain that the derivative of U exists almost everywhere and that U can be expressed as an integral of its derivative:

$$U'(x) = -q(x)c'(x), \quad (4)$$

$$U(x) = U(0) - \int_0^x q(t)c'(t)dt. \quad (5)$$

Accordingly, from equations (1) and (5), the payment $p(x)$ of consumer located at x is expressed as follows (this is essentially the Revenue Equivalence Theorem, as in Myerson (1981))

$$p(x) = q(x)(V - c(x)) - U(0) + \int_0^x q(t)c'(t)dt. \quad (6)$$

The expected revenue to the principal from any incentive compatible mechanism with implied probabilities of sale $q(x)$ is

$$ER = \int_0^1 p(x)dx = -U(0) + \int_0^1 q(x)(V - c(x))dx + \int_0^1 \int_0^x q(t)c'(t)dt dx. \quad (7)$$

By changing the order of integration

$$\int_0^1 \int_0^x q(t)c'(t)dt dx = \int_0^1 \left(\int_t^1 q(t)c'(t)dx \right) dt = \int_0^1 q(t)c'(t)(1-t)dt, \quad (8)$$

$$ER = -U(0) + \int_0^1 q(x) [V - c(x) + c'(x)(1-x)] dx. \quad (9)$$

Thus, the problem of choosing the optimal selling mechanism reduces to the problem of choosing $q(x)$ that maximizes (9) subject to IR constraints (2). From equation (5) and $q(x) \geq 0$, if IR holds for $x = 1$, then it holds for all x . From equation (5),

$$U(0) = U(1) + \int_0^1 q(t)c'(t)dt, \quad (10)$$

and so equation (9) becomes

$$ER = -U(1) + \int_0^1 q(x) [V - c(x) - c'(x)x] dx. \quad (11)$$

Now, ignoring necessity of $q(x)$ to be non-increasing for a moment, ER is maximized by setting $U(1) = 0$ (as low as possible) and setting $q(x) = 1$ for all x with $\Psi(x) = V - c(x) - c'(x)x \geq 0$, and $q(x) = 0$ for all other x . The virtual valuation function is actually a marginal revenue function, indeed $\Psi(x) = [x(V - c(x))]'$. Certainly, the function $c(x)$ can be such that the virtual valuation function $\Psi(x)$ is not monotone, can cross 0 several times, and so $q(x)$ defined above may not be non-increasing. But in this case (which corresponds to the case where SOC condition for maximization of $x(V - c(x))$ does not hold globally), the familiar ironing technique should be used (or the appropriate global maximum should be chosen), see Myerson (1981).

Note also that the expression (9) for the expected revenue cannot be simply optimized as $U(0)$ also depends on the marginal type of the buyer to whom the good is sold. If buyer x is the marginal type whose IR constraint binds, the price of the good is $P = V - c(x)$. By changing the marginal type, there is the marginal effect on price equal to $-c'(x)$. This is why the function under the integral differs from the virtual value $\Psi(x)$ by exactly $c'(x)$ to account for this extra marginal effect. When solving for the optimal mechanism under incomplete information such extra effects are going to appear and will have to be carefully considered.

3.1 More general settings

One way to generalize the above model is to allow for an arbitrary distribution of the agent's types. We are going to assume that $x \sim F[0, 1]$ with symmetric around 1/2 density f , that is $f(x) = f(1-x)$ for all $x \in [0, 1]$, and that distribution of x is independent of the distribution of the principal's type.

Clearly, the incentive compatibility and individual rationality constraints remain the same, and so the implications of the incentive compatibility are not affected. What is

affected is the computation of the expected revenue, equation (7),

$$ER = \int_0^1 p(x)f(x)dx = -U(0) + \int_0^1 q(x)(V - c(x))f(x)dx + \int_0^1 \int_0^x q(t)c'(t)f(x)dt dx.$$

Respectively,

$$\begin{aligned} \int_0^1 \int_0^x q(t)c'(t)f(x)dt dx &= \int_0^1 \left(\int_t^1 q(t)c'(t)f(x)dx \right) dt = \int_0^1 q(t)c'(t)(1 - F(t))dt \\ ER &= -U(0) + \int_0^1 q(x)\Phi(x)f(x)dx, \end{aligned} \quad (13)$$

where

$$\Phi(x) = V - c(x) + c'(x) \frac{1 - F(x)}{f(x)}.$$

Given that the lowest-value type of the agent is $x = 1$ and using (10), we can rewrite (13) as

$$ER = -U(1) + \int_0^1 q(x)\Phi(x)f(x)dx - \int_0^1 q(x)c'(x)dx = -U(1) + \int_0^1 q(x)\Psi(x)f(x)dx, \quad (14)$$

where

$$\Psi(x) = V - c(x) - c'(x) \frac{F(x)}{f(x)}. \quad (15)$$

This is the virtual valuation function appropriately derived for the considered model. If it is not monotone, ironing (and so deriving a monotone quasi-virtual valuation function) may be needed.

The second possible generalization is to allow for the possibility of having random base consumption values, $V \sim H[0, \bar{V}]$ with density h , and independent of the other distributions. Despite the now two-dimensional agent types, (V, x) , the analysis remains the same, as only $V - c(x)$ matters in determining the consumption value to the agent. Indeed, for any pair (V, x) a consumption-equivalent type (\bar{V}, x') can be associated, requiring $\bar{V} - c(x') = V - c(x)$. Clearly, by extending the range of x beyond 1, and by extending $c(x)$ for the values $x > 1$, such an equivalence can be well defined. But this is only so for the case of complete information, as with incomplete information, consumption value equivalence also depends on the perceived probabilities of buying from different types of the principal. (We will deal with this later, once we analyzed the main model for the incomplete information case.)

4 To reveal or not to reveal

Suppose now that the choice of the selling scheme to the principal is limited by either revealing her information and then running the optimal selling scheme of Section 3 or not revealing her info and then running the same selling scheme for all the types of the principal. We are considering this restricted problem in order to gain track on the effects of informational disclosure under various cost structures. Besides, in many settings, the seller may be limited to no or full disclosure policies.

Given the assumption of unverifiability of the principal's information, a specific type of the principal can credibly reveal her identity to the agent if and only if such information disclosure is supported by proper incentives. In other words, the principal does not (cannot) reveal her information by simply communicating her type to the agent. Instead, the principal can offer a mechanism that is available to both of her types, which both of her types find optimal to offer, and in which the information is disclosed or not. Mechanisms and information disclosure policies are strictly intertwined, so we can say that the principal chooses over disclosure policies (i.e. to reveal or not to reveal), even though her choice is in terms of mechanisms.

Let us derive the best selling scheme conditional on the information not being revealed and then compare whether it is better to reveal the information or not to reveal. By the Revelation principle, for any mechanism and its equilibrium that is offered by the seller there exists a direct mechanism in which the agent finds it optimal to report his type truthfully. Such a direct mechanism can be represented by a pair of functions (q, p) , where $q(z)$ and $p(z)$ denote, respectively, the probability of getting the good and the price paid given the report $z \in [0, 1]$.

The implication of the IC constraints, inequality (20), becomes: $\forall x, z \in [0, 1]$,

$$(q(x) - q(z)) ([c(z) - c(x)] + [c(1 - z) - c(1 - x)]) \geq 0. \quad (16)$$

For the general cost functions, the simplest way to solve for the optimal selling scheme is to reorder the agent's types according to their expected distance costs. For each type x of the agent we can assign type $y(x) = \frac{1}{2}c(x) + \frac{1}{2}c(1 - x)$. Letting $y_0 = \min_{x \in [0, 1]} \frac{1}{2}c(x) + \frac{1}{2}c(1 - x)$ and $y_1 = \max_{x \in [0, 1]} \frac{1}{2}c(x) + \frac{1}{2}c(1 - x)$, we have that the new types of the agent belong to the segment $[y_0, y_1]$, the utility from purchasing the good to the agent of type y net of the price is $V - y$, and the distribution of the types y , F_y is given by $F_y(z) = \Pr(y < z) = \Pr(\frac{1}{2}c(x) + \frac{1}{2}c(1 - x) < z)$. In turn, inequality (16) becomes

$$\forall y, z \in [y_0, y_1], \quad (q(y) - q(z)) (z - y) \geq 0.$$

Thus, in any incentive compatible scheme, the lower is the expected distance costs of the agent the higher must be the probability of him receiving the good.

By following the same steps as in Sections 3 and 3.1, we obtain

$$ER = -U(y_1) + \int_{y_0}^{y_1} q(z) \left[V - z - \frac{F_y(z)}{f_y(z)} \right] f_y(z) dz. \quad (17)$$

As long as the virtual value $\Psi^{nr}(z) = V - z - \frac{F_y(z)}{f_y(z)}$ is monotone, the optimal selling mechanism is obtained by setting $q(z) = 1$ whenever $\Psi^{nr}(z) \geq 0$ (note that $\Psi^{nr}(y_0) > 0$), which means selling the good at the price $P = V - y^*$, where y^* solves $\Psi^{nr}(y^*) = 0$ or $y^* = y_1$ if $\Psi^{nr}(y_1) \geq 0$. If $\Psi^{nr}(z)$ is not monotone, the ironing procedure needs to be applied to compute the monotone quasi virtual valuation function $\hat{\Psi}^{nr}(z)$, and then the optimal mechanism is derived in a similar way. In any case, the optimal selling scheme is the posted price.

Ultimately, one has to compare the optimal mechanism under no disclosure policy with the one under full disclosure. Whichever generates the most revenue will be selected by each type of the principal, as each type prefers the same disclosure policy.

Notice that this result is in part due to the perfect symmetry between the principal's types (i.e. each one is located at one extreme of the segment) as their incentives to disclose information are perfectly aligned. If the types were located asymmetrically or were vertically differentiated, the optimal disclosure policies could have been different for each type: one might have preferred to reveal her info, while the other to conceal. The strategic interaction between the types of the principal is the essence of the Informed Principal problem, that we set up and analyze in the next section.

In order to get a better feeling of how the optimal solution looks under no revelation of information and to compare it with the full revelation of the information option, let us consider three subcases of the cost function: linear, convex, and concave.

4.1 Linear costs

For the case of linear costs, $c(x) = cx$, the resulting optimal scheme is trivial. Indeed, $\frac{1}{2}c(x) + \frac{1}{2}c(1-x) = \frac{c}{2}$, that is all the agents have the same expected utility from the good. Thus, as long as $V - \frac{c}{2} > 0$, it is optimal for the principal to set $P = V - \frac{c}{2}$ and serve the whole market. Thus, the profit to each type of the principal is $\pi = V - \frac{c}{2}$. If the principal reveals her private information, then the optimal cut-off type computed for the principal's type $s = 0$ is determined from the equation $V - c(x) - c'(x)x = 0$, that is $x^* = \frac{V}{2c}$ or $x^* = 1$ if $V > 2c$. The price and the profit are $P = V - cx^* = \frac{V}{2}$ and $\pi = \frac{V^2}{4c}$ if $x^* < 1$ or $P = V - c$ and $\pi = V - c$ if $V > 2c$.

By comparing $\frac{V^2}{4c}$ and $V - \frac{c}{2}$, we obtain that for a given c , if $V > 2c - \sqrt{4c^2 - 2}$, then it is better not to reveal the information about the principal's type. The basic trade-off in choosing whether to reveal the information or not is whether to serve only the "local" market—the types of the agents that are "nearby" and charge them a higher price or expand and serve the whole market but at a possibly lower price. If V is large and the whole market is served even when the information is revealed, then it is strictly better not to reveal the information, as then the higher price can be charged. For a specific value of $c = 1$, the cutoff value is $\hat{V} = 2 - \sqrt{2} \approx 0.6$ in which case around 30% of the market is served when the type of the principal is revealed. By not revealing the information, each type of the principal will be able to serve the whole market, and this explains why the cut-off value of V for this to happen is relatively small.

4.2 Convex costs

For the case of convex costs, i.e. strictly increasing $c'(x)$, we have $y(x) = \frac{1}{2}c(x) + \frac{1}{2}c(1-x)$ is decreasing on $x \in [0, \frac{1}{2}]$ as $y'(x) = \frac{1}{2}(c'(x) - c'(1-x)) < 0$. Thus, $y_0 = c(\frac{1}{2})$ and $y_1 = \frac{1}{2}c(1)$. It is clear that density $f_y(z)$ is bounded from away from 0. Indeed, $F_y(z) = 2\Pr(z > y(x) \geq y_0 | x \leq \frac{1}{2})$, and as $|y'(x)|$ is bounded from above $f_y(z)$ is bounded from below, $f_y(z) \geq \frac{2}{\max|y'(x)|}$. Thus, $\frac{F_y(z)}{f_y(z)}$ and $z + \frac{F_y(z)}{f_y(z)}$ are bounded.

Consider the virtual value $\Psi^{nr}(z) = V - z - \frac{F_y(z)}{f_y(z)}$ from equation (17). If $V > y_0$, then $\Psi^{nr}(y_0) > 0$, and so a positive revenue can be earned if the principal reveals no information. In the optimal scheme, the principal will set up a price $P^* > V - y_0$, in which case agent's

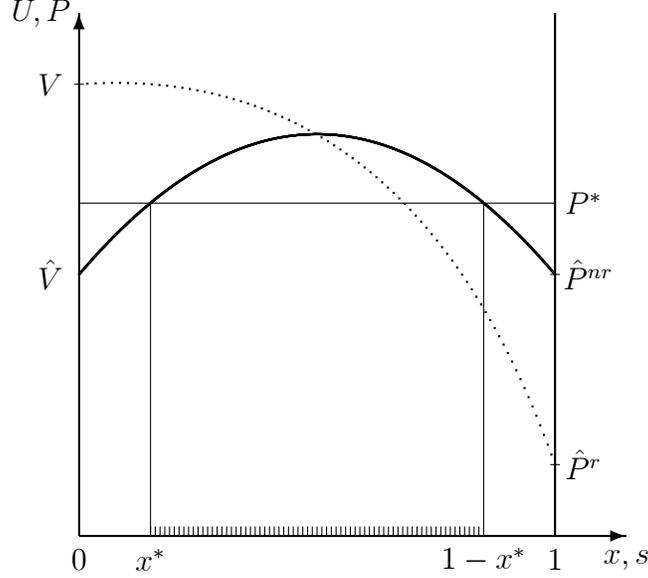


Figure 1: No information disclosure policy under convex costs.

types in the middle of the segment (with $y(x)$ close to y_0) will purchase the good, while those who are at the edges may be left out. Note that if V is sufficiently high, then $\Psi^{nr}(z) > 0$ for all $z \in [y_0, y_1]$, in which case in the optimal scheme all the agents buy the good, and the optimal price $P^{nr} = V - y_1 = V - \frac{1}{2}c(1)$.

Compared to the case when all the information is revealed, it is clear that for $V \leq y_0$ and by continuity for $V < y_0 + \delta$ for some $\delta > 0$, it is better to reveal all the information. However, if V is sufficiently high, for instance when $\Psi^{nr}(z) > 0$ for all $z \in [y_0, y_1]$ and $\Psi(x) > 0$ for all x (from equation (15)), the principal sells to all the agents at the price of $P^{nr} = V - \frac{1}{2}c(1)$ when no information is revealed and at the price $P^r = V - c(1)$ if all the information is revealed. Clearly, it is better not to reveal any information.

The expected valuations of the agent, and possible optimal schemes are shown on Figure 1. Here the solid curve is the expected value of the agent under no revelation disclosure policy. The dotted curve is the expected value of the agent if the type of the seller were $s = 0$ and known. If the base value V is in the intermediate range, the typical optimal solution will be to set price P^* , $\hat{V} = V - \frac{1}{2}c(1) < P < V - c(\frac{1}{2})$, in which case the agents to the left of x^* and to the right of $1 - x^*$ will buy the good. Prices P^{nr} and P^r show what the optimal prices would be under no revelation and under revelation disclosure policies if V is sufficiently high.

4.3 Concave costs

For the case of concave costs, i.e. strictly decreasing $c'(x)$, we have $y(x) = \frac{1}{2}c(x) + \frac{1}{2}c(1-x)$ is increasing on $x \in [0, \frac{1}{2}]$ as $y'(x) = \frac{1}{2}(c'(x) - c'(1-x)) < 0$. Thus, $y_0 = \frac{1}{2}c(1)$ and $y_1 = c(\frac{1}{2})$. The only qualitative difference with the case of convex costs is that now it is agent's types at the edges that have the lowest expected costs, and if the optimal price is in between y_0 and y_1 those at the edges buy the good, while those in the middle are not. Similarly, if $V < y_0 + \delta$

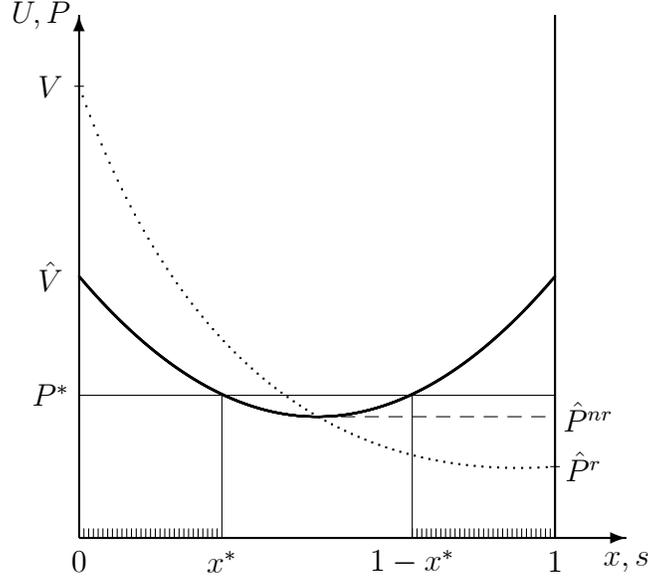


Figure 2: No information disclosure policy under convex costs.

for some $\delta > 0$, it is better to reveal all the information, while if V is sufficiently high, the price at which all the agent's types are served is higher when no information is revealed.

We can state a more general result considering only two possibilities for the principal: to reveal the information about her location or not to reveal it.

Lemma 1 *If the base consumption value V is sufficiently low, then it is better for the principal to reveal her information. If the base consumption value is sufficiently high, then it is better not to reveal anything.*

Proof. Clearly, $y_0 > 0$, and if $V \leq y_0$ then by not revealing any information no revenue can be collected. Revealing is better and will remain better by continuity for V slightly above y_0 . If V is sufficiently high then both $\Psi^{nr}(y(x))$ and $\Psi(x)$ are strictly positive for all x , thus every type of the agent will be served in each informational treatment. Clearly, $y_1 = \max_{x \in [0,1]} \frac{1}{2}c(x) + \frac{1}{2}c(1-x) < c(1)$ as $c(x)$ is increasing, and so $\pi^{nr} = P^{nr} = V - y_1 > V - c(1) = P = \pi$. ■

If V is small, then when no information is revealed the expected value of the good for each type of the agent is very low. Thus, it is better to reveal all information and extract higher revenue from the nearby types. When V is intermediate, when she does not reveal her location the principal gains the access to a larger market though selling at a low price. When V gets larger the “new” market effect starts to dominate higher price effect. When V is large then the whole market is being served, but on top of it, the principal is able to charge a higher price when no information is revealed.

The expected valuations of the agent, and possible optimal schemes are shown on Figure 2. The difference with the convex costs is that for the intermediate values of V , in the optimal mechanism under no informational disclosure only the agent's types in the middle, from x^* to $1 - x^*$ buy the good.

5 The Informed Principal Problem

5.1 Inscrutability Principle

The characterization of the optimal mechanism under incomplete information and without restrictions on disclosure policy is a complex exercise, as we have multiple types of the principal and we assume that the principal selects the mechanism after learning her true type. Indeed, each type of the principal wants to maximize her own revenue. By offering a specific mechanism the principal may try to influence beliefs of the agent about her type in the way that is more profitable for her. On the other hand, given the mechanism offered, the agent may reason about which type of the principal have offered the mechanism, and adjust his behavior accordingly. The principal can still commit to the rules of the mechanism she is offering but cannot commit or force the agent to believe that she would have offered a specific mechanism if she were the other type.

How to deal with the mechanism selection issue and alignment of simultaneous objectives of all of the principal's types is the heart of the Informed Principal problem.

By the Inscrutability Principle (see Myerson (1983)) we can always represent the menu of the mechanisms offered by the principal (depending on her type) as a single inscrutable mechanism, in which agents infer nothing about the type of the principal when the mechanism (and its equilibrium) is offered. The rationale behind the Inscrutability Principle is based on the observation that any information transmitted by the principal through the selection of a specific mechanism can be conveyed through the application of specific rules inside a more general mechanism. By the Revelation Principle, for any such inscrutable mechanism and its equilibrium, there exists a direct inscrutable mechanism with truth-telling as an equilibrium.

Accordingly, we can limit our search of the optimal incentive scheme for all types of the principal to the set of inscrutable direct mechanisms. An inscrutable direct mechanism is a function $\mu : (s, x) \rightarrow (q, p)$ that maps a report s from the principal and a report x from the agent into a tuple composed by the (possibly random) traded quantity q , and the transfer p paid by the agent to the principal. Direct mechanism μ is *incentive compatible (IC)* if each type of each player is willing to report her or his type truthfully given that the other player reports his or her type truthfully and *individually rational (IR)* if each type of each player is willing to participate in it. In our setup an inscrutable direct mechanism can be represented by a collection of functions $(Q_0, P_0; Q_1, P_1)$, where for all $s \in \{0, 1\}$, $Q_s(x)$ is the probability of sale and $P_s(x)$ is the expected payment of the agent reporting x when the reported type of the principal is s . From the perspective of the agent, who due to the inscrutability of the mechanism cannot distinguish principal's types, such a direct mechanism can be represented as a triple of functions (q_0, q_1, p) , which are, respectively, the probabilities of receiving the good if the principal is of type 0 and of type 1, and the expected payment, all functions of the agent's report. Thus, $q_s(z) = Q_s(z)$, for all $s \in \{0, 1\}$, and $p(z) = \frac{1}{2}P_0(z) + \frac{1}{2}P_1(z)$.

IC and IR constraints for the agent. Letting $U(z|x)$ stand for the expected utility of the agent of type x reporting z and assuming all other players' types report their types truthfully,

we have: $\forall x, z \in [0, 1]$,

$$\begin{aligned}
U(x) &\triangleq U(x|x) = \frac{1}{2}q_0(x)(V - c(x)) + \frac{1}{2}q_1(x)(V - c(1 - x)) - p(x) \\
&\geq U(z|x) = \frac{1}{2}q_0(z)(V - c(x)) + \frac{1}{2}q_1(z)(V - c(1 - x)) - p(z); \\
U(x) &\geq 0.
\end{aligned} \tag{18}$$

Similarly to (3), we obtain

$$(q_0(x) - q_0(z))(c(z) - c(x)) + (q_1(x) - q_1(z))(c(1 - z) - c(1 - x)) \geq 0. \tag{20}$$

Unlike the complete information case, one cannot establish monotonicity of $q(x)$, since for $z > x$, $c(z) > c(x)$ and $c(1 - z) < c(1 - x)$. However, if $q_0(x)$ is constant at around some x , then $q_1(x)$ is increasing; and if $q_1(x)$ is constant at around x , then $q_0(x)$ is increasing. Intuitively, in any incentive compatible mechanism there is a pressure to sell the good from principal $s = 0$ (or $s = 1$) more often to agents closer to 0 (or 1).

Similarly⁸ to (4) and (5) we have

$$U'(x) = -\frac{1}{2}q_0(x)c'(x) + \frac{1}{2}q_1(x)c'(1 - x), \tag{21}$$

$$U(x) = U(0) - \frac{1}{2} \int_0^x q_0(t)c'(t)dt + \frac{1}{2} \int_0^x q_1(t)c'(1 - t)dt. \tag{22}$$

Accordingly, the expected payment from the agent of type x is

$$\begin{aligned}
p(x) &= \frac{1}{2}q_0(x)(V - c(x)) + \frac{1}{2}q_1(x)(V - c(1 - x)) \\
&\quad - U(0) + \frac{1}{2} \int_0^x q_0(t)c'(t)dt - \frac{1}{2} \int_0^x q_1(t)c'(1 - t)dt.
\end{aligned} \tag{23}$$

5.2 Individual rationality of the principal

In the previous section we considered the constraints of the agent, here we look to the constraints of the principal. The principal's constraints are of different nature as the principal designs the mechanism. It is convenient to analyze the strategic situation between principal and agent as happening on two levels: a *game* and a *meta-game*.

The *game* represents the interaction between the players inside a given mechanism. A direct mechanism μ is incentive compatible for the principal of type s if she is better off by reporting s than by reporting some alternative s' , given all the agents report their type truthfully. Thus, a direct revelation mechanism is incentive compatible for the principal if it implies a game in which truth-telling is the best strategy for the principal, given the agent's strategy.

⁸The implementability conditions are satisfied in this setting: $\sum_{s=0,1} \frac{\partial}{\partial x} \left(\frac{\partial U / \partial q_s}{\partial U / \partial p} \right) \frac{\partial q}{\partial x} \geq 0$.

The fact that the principal owns private information brings a specific connection between the principal’s incentive compatibility and the agent’s beliefs over the principal types. If a mechanism μ' is incentive compatible given a principal’s true type s but not compatible for any other true type s' , then, after observing μ' , the agent can update their beliefs expecting the principal’s type to be s . In other words, the offered mechanism must be incentive compatible for all the types of the principal in the support of the agent’s posterior beliefs over the principal’s types, but not so for the other types of the principal.

As discussed in the previous section, without loss of generality, we can limit our search for the optimal mechanism to the set of inscrutable mechanisms. This implies that any feasible mechanism should not provide the agent with any information that would allow him to update his prior over the principal’s types. To guarantee that the selected mechanism is indeed inscrutable, the principal’s incentive compatibility constraint need to hold for any type s . More formally, letting $ER_s(\mu; s')$ to denote expected revenue of the principal of type s submitting a report s' in a direct inscrutable mechanism μ that is IC and IR for the agent, incentive compatibility condition for the principal takes the form

$$\forall s, s' \neq s, \quad ER_s(\mu) \triangleq ER_s(\mu; s) \geq ER_s(\mu; s').$$

The *meta-game* formalizes the principal’s choice over different mechanisms. The principal’s ability to choose over different mechanisms entails a new specification for her individual rationality constraint.

The conventional game-theoretic way to deal with IR constraint of the principal is to specify agent’s beliefs about the principal’s type and an action to be played for any alternative (μ') out-of-equilibrium move by the principal as it is done, for instance, in the informed principal mechanism selection game studied in Maskin-Tirole or in dynamic signaling games. Applying such approach to our setting, the main objective would be to make sure that no type of the principal wants to deviate from μ . Accordingly, as μ' is an out-of-equilibrium choice, the agent’s beliefs and his response given the beliefs can be selected in the most adversarial way to the principal. Following this approach, any equilibrium becomes possible, as long as it is possible to find appropriate agent’s off-equilibrium beliefs to support it. On top of it, this is not a fully satisfactory way to set IR constraints in our opinion, as it is at odds with classical principal-agent paradigm endowing the principal with ultimate commitment and bargaining powers to select and commit to the mechanism and suggest an equilibrium to be played. Why would the principal lose these powers off-equilibrium?

We set the IR constraint of the principal considering two different utility levels and we require that any solution to the Informed Principal problem should give each type of the principal no less than the highest of the two. Notice that, as for the incentive compatibility constraints, the individual rationality constraints have to hold simultaneously for all the types of the principal. This is *also* because of their role in the agent’s beliefs: a mechanism cannot be expected to be selected by all the principal’s types (and ergo be inscrutable) if it is not individually rational for some of them. The first utility level is the one associated with the outside option, as in standard principal-agent models. We assume that it is exogenous and we allow it to be type-specific. The second utility level is the maximal payoff each type of the principal can guarantee to herself. How to determine this value is an issue per se. It depends on finer details of the setting considered, such as whether the principal’s information

is verifiable. If the principal's info is verifiable, at the least the principal should be guaranteed her complete information optimal expected revenue. Even if she cannot verifiably reveal her type as we assume in this paper it may happen that she may offer a mechanism in which she can guarantee herself the same revenue.⁹ Depending on the beliefs of the agent, a type of the principal may also guarantee herself the expected revenue associated with a mechanism that entails partial (or no) information disclosure. Indeed, as long as the agent recognizes the convenience for multiple types of the principal to select the same mechanism and conceal their identities, then each of these types should be regarded as capable of assuring herself the expected revenue associated with that mechanism. In our setting with two equally likely types, this is equivalent to say that if there is a mechanism that is beneficial to both types and requires each type not to reveal her identity, then any solution should provide each type at least the expected revenue associated with such a mechanism. To summarize, we assume that, if there are two mechanisms μ and μ' , both IC and IR for the agent and IC for the principal, and μ guarantees higher expected revenues than μ' (i.e. μ dominates μ') to all principal's types,¹⁰ then μ' is not individually rational for the principal.

Note that by its nature, the IR constraints are endogenous. The maximum expected revenue that each type of the principal can guarantee herself in the meta-game depends on the set of mechanisms that can be credibly selected by all the types of the principal. What can be credibly selected by the different types of the principal is in turn determined by what is each type's equilibrium strategy.

As a justification for this formalization of the principal's individual rationality constraint, consider the following argument based on plausibility and consistency of agent's beliefs. Suppose the principal offers mechanism μ' in place of the expected (in equilibrium) mechanism μ . Let $\Pr_{\mu'}(s)$ be the posterior probability the agent assigns to the principal to be of type s given μ' is offered. Suppose both types of the principal gain by offering μ' under this belief, that is μ is strictly Pareto-dominated by μ' in terms of expected revenue to each type. Then we should expect the agent to believe that all the principal's types would want to deviate from μ to μ' and so $\Pr_{\mu'}(\cdot)$ must be his prior belief about the principal's type. Therefore, μ is not individually rational for the principal, because there is not a single type s for whom it makes sense to choose μ instead of μ' . If only one principal's type gains by offering μ' , say type s , we should expect the agent's posterior to be concentrated on the type that gains, $\Pr_{\mu'}(\cdot) = 1$. In all other cases, whatever is the posterior no type of the principal gains. Thus, if μ' does not dominate μ , at most one type of the principal can gain. If principal s is the one gaining, then $\Pr_{\mu'}(s) = 1$ and $ER_s(\mu) \leq ER_s^*$, where ER_s^* is the expected revenue associated with the common knowledge optimal mechanism.

Letting \bar{U}_s stand for the outside option utility of the principal of type $s = 0, 1$, the individual rationality of the principal for the direct incentive compatible mechanism μ requires:

⁹For the auction setting with the seller who has differential private preferences over selling to specific buyers, Balestrieri (2008) presents an indirect dynamic mechanism in which the seller can obtain the first-best expected revenue as if her preferences were commonly known no matter what prior beliefs the buyers have and without disclosing her preferences in the process.

¹⁰With more than two types of the principal, additional constraints will have to be added to account for a possibility for a subset (coalition) of the principal's types to offer a mechanism that benefits them and only them.

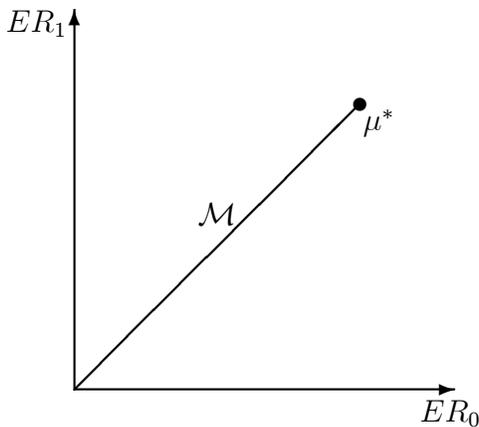


Figure 3: The set of feasible mechanisms.

$$\begin{aligned} \text{IR(i)} & \quad \forall s : ER_s(\mu) \geq \bar{U}_s, \\ \text{IR(ii)} & \quad \nexists \mu' \in \mathcal{M}, \forall s : ER_s(\mu') > ER_s(\mu), \end{aligned}$$

where \mathcal{M} is the set of direct inscrutable mechanisms that are ex interim IC and IR for the agent and ex interim IC for the principal.

Note that the set \mathcal{M} is non-empty as it includes μ^r and μ^{nr} , respectively, the optimal mechanisms under full information disclosure and under no disclosure policies. In our symmetric setting, the incentive compatibility for the principal implies that unless one type of the principal chooses her outside option both types obtain the same payoff. In what follows we are going to assume that the revenue for the outside option for each type s does not exceed $ER_s(\mu^r)$, and so the constraint IR(i) does not bind for all types of the principal as $\mu^r \in \mathcal{M}$.

We can represent the set of mechanisms M in a graph with the expected revenue of each principal's type s on each axis, see Figure 3. Notice that the incentive compatibility of the principal implies that any feasible mechanism μ assigns equal shares of revenues to the two types of the principal. In other words, any feasible mechanism has to lie on the bisector. Constraint IR(ii) defines the Pareto frontier of the set M . In the symmetric case, IR(ii) identifies the most extreme point on the bisector (the furthest from the origin), that is the solution of the principal maximization problem.

Even though, after looking to all constraints, we know that only one point in the set M is a feasible solution to the principal's maximization problem, given the endogenous nature of IR(ii) we still need to determine it. To do so we need to consider simultaneously each principal's type maximization problem subject to IR(ii).

5.3 The Informed Principal problem

Once defined the set of feasible inscrutable mechanisms in terms of incentive compatibility and individual rationality of both the agent and the principal, we are left with the problem

of how to select the optimal mechanism for a given type of the principal.

Mechanism $\mu^* \in \mathcal{M}$ is the solution to the Informed Principal problem if it solves the following constrained maximization

$$ER_s(\mu) \longrightarrow \max_{\mu \in \mathcal{M}}, \quad (24)$$

$$\text{s.t. } \nexists \mu' \in \mathcal{M}, \forall s : ER_s(\mu') > ER_s(\mu). \quad (25)$$

The solution to the informed principal problem is equivalent to the solution of the following maximization problem

$$ER(\mu) = \sum_{s \in S} \Pr(s) ER_s(\mu) \longrightarrow \max_{\mu \in \mathcal{M}}, \quad (26)$$

where $\Pr(s)$ is the prior probability of type s , and S is the set of the principal's types. In our setting, $S = \{0, 1\}$ and $\Pr(0) = \Pr(1) = \frac{1}{2}$. As such, the solution is also the mechanism maximizing the ex ante expected revenue subject to all ex interim IC and IR constraints.

This is the solution because in our setting any mechanism μ' that is ex interim IC and IR but does not maximize the ex ante expected revenue among all the feasible mechanisms in \mathcal{M} will be strictly dominated by some other mechanism in \mathcal{M} and so it will violate the principal's IR(ii).

Proposition 2 $\arg \max \{ER_s(\mu) : \mu \in \mathcal{M}; \text{s.t. IR(ii)}\} = \arg \max \{ER(\mu) : \mu \in \mathcal{M}\}$

Proof. Suppose $\mu' = (Q'_0, P'_0; Q'_1, P'_1)$ and so, from the agent's perspective, it is represented by a triple of functions (q'_0, q'_1, p') . Suppose also that mechanism $\mu \in \mathcal{M}$ such that $\mu = (Q_0, P_0; Q_1, P_1)$ is maximizing the ex-interim constrained ex ante revenue (26) and is represented by (q_0, q_1, p) . Notice that since $\mu \in \mathcal{M}$, (q_0, q_1, p) is IC and IR for the agent. Since the ex-interim constrained ex ante expected revenue is equal to the expected payment from the agent, $ER(\mu) = \int_0^1 p(x) dx > \int_0^1 p'(x) dx = ER(\mu')$. While the total revenue under μ is higher we need to guarantee that each type of the principal earns more to have μ' dominated. Construct mechanism $\mu'' = (Q''_0, P''_0; Q''_1, P''_1)$ as follows: given (q_0, q_1, p) , set $Q''_0 = q_0$, $Q''_1 = q_1$, and find P''_0, P''_1 , such that $p(z) = \frac{1}{2}P''_0(z) + \frac{1}{2}P''_1(z)$, while $ER_s(\mu'') = \int_0^1 P''_s(x) dx > \int_0^1 P'_s(x) dx = ER_s(\mu')$ for $s = 0, 1$. For instance, set $P''_s(z) = 2 \frac{ER_s(\mu')}{ER(\mu')} p(z)$ (and so $P''_0(z) + P''_1(z) = 2p(z)$). The extra profits $ER(\mu) - ER(\mu')$ are divided between different types of the principal in proportion to their revenues in mechanism μ' . Then, each type of the principal instead of offering μ' can offer μ'' and gain strictly more in expectation. Notice that, from the point of view of the agent, $\mu'' = \mu = (q_0, q_1, p)$. Ergo μ'' is IR and IC for the agents and IR and IC for the principal, i.e. $\mu'' \in \mathcal{M}$. ■

In addition, due to the symmetry of our setup, we would search for the solution in which each type of the principal earns the same revenue, that is, the principal's types split the ex ante revenue equally. This does not necessarily mean that the optimal mechanism needs to be symmetric, though.

Theorem 3 *The solution to problem (26) exists.*

Proof. The proof is rather straightforward. Set \mathcal{M} is convex, closed, and non-empty. Accordingly, the set of feasible payoffs available to different types of the principal is bounded from above and is a compact. Thus, there exists a maximum. ■

Remark 4 *The logic behind the definition of the solution to IP problem would apply for any support of the principal's types, whether a finite set or continuum. If there is any "money left on the table" to the agent, then there exists a mechanism extracting it, in which extra revenues can be divided among all the types of the principal so as to make all of them strictly better off. This can be done because the agent is risk-neutral, and his payment can be represented as a lottery on the principal's type, which allows to distribute the surplus among different types of the principal.*

Remark 5 *The possibility to distribute the surplus among different types of the principal allows for monetary transfers between them. In turn, this, in principle, allows for the whole family of solutions to the IP problem (if one does not impose the symmetry in payoffs). In all of them the total surplus over the principal's types is the same, but different types may earn different payoffs within the bounds given by IR constraints.*

6 Optimal mechanism

Notice that $ER(\mu) = \sum_{s \in S} \Pr(s) ER_s(\mu) = \frac{1}{2} ER_{s=0}(\mu) + \frac{1}{2} ER_{s=1}(\mu) = \int_0^1 p(x) dx$.

The expected revenue collected from the agent in the incentive compatible mechanism $q_0(x)$, $q_1(x)$, $p(x)$ is

$$\begin{aligned} ER &= \int_0^1 p(x) dx = -U(0) + \frac{1}{2} \int_0^1 q_0(x) (V - c(x)) dx + \frac{1}{2} \int_0^1 q_1(x) (V - c(1-x)) dx \\ &\quad + \frac{1}{2} \int_0^1 \int_0^x q_0(t) c'(t) dt dx - \frac{1}{2} \int_0^1 \int_0^x q_1(t) c'(1-t) dt dx. \end{aligned} \quad (27)$$

By changing the order of integration as in equations (7-9), we can express

$$\begin{aligned} ER &= -U(0) + \frac{1}{2} \int_0^1 q_0(x) [V - c(x) + c'(x)(1-x)] dx + \\ &\quad + \frac{1}{2} \int_0^1 q_1(x) [V - c(1-x) - c'(1-x)(1-x)] dx \end{aligned} \quad (28)$$

Finding the optimal mechanism boils down to maximizing (28) subject to the agent's IR and IC constraints (20), and $q_0(x) \in [0, 1]$, $q_1(x) \in [0, 1]$ for all $x \in [0, 1]$. Similarly to the complete information case, if we let x^* be a type which has the lowest utility from the mechanism, using (22), we can express

$$U(0) = U(x^*) + \frac{1}{2} \int_0^{x^*} q_0(t) c'(t) dt - \frac{1}{2} \int_0^{x^*} q_1(t) c'(1-t) dt.$$

Therefore,

$$ER = -U(x^*) + \frac{1}{2} \int_0^{x^*} [q_0(x)A(x) + q_1(x)C(x)] dx + \frac{1}{2} \int_{x^*}^1 [q_0(x)B(x) + q_1(x)D(x)] dx, \quad (29)$$

where

$$\begin{aligned} A(x) &= V - c(x) - c'(x)x, \\ B(x) &= V - c(x) + c'(x)(1-x) = A(x) + c'(x), \\ C(x) &= V - c(1-x) + c'(1-x)x, \\ D(x) &= V - c(1-x) - c'(1-x)(1-x) = C(x) - c'(1-x). \end{aligned}$$

These four components can be interpreted as follows. Function $A(x)$ is the complete information virtual valuation or marginal revenue from selling the good located at $s = 0$ to the agent of type x assuming all the agent's types closer to $s = 0$ also purchase the good. Indeed, for price $P = V - c(x)$ and revenue $R = (V - c(x))x$, the marginal revenue is $MR = V - c(x) - c'(x)x$. Similarly, given $s = 1$, $D(x)$ is the marginal revenue under assumption that all agent's types from x to 1 buy the good. Note that $D(1-x) = A(x)$.

Function $C(x)$ can be interpreted as the complete information lost marginal revenue from not selling the good, given $s = 1$, to types closer to $s = 0$. Indeed, if the price is $P = V - c(1-x)$, then the types in the interval $[0, x]$ do not buy the good, and the lost revenue is $LR = (V - c(1-x))x$. Similarly $B(x)$ can be interpreted as the complete information lost marginal revenue from not selling the good, given $s = 0$, to types closer to $s = 1$.

Alternatively, we can interpret the functions $C(x)$ and $B(x)$ in the light of different information disclosure policies. Function $\frac{1}{2}(A(x) + C(x))$ represents the marginal revenue from selling the good to agent x without revealing any information about the principal's type s , under assumption that all agents from 0 to x buy it as well. Similarly, function $\frac{1}{2}(D(x) + B(x))$ represents the marginal revenue from selling the good to agent x without revealing any information under assumption that all agents from x to 1 buy it as well.

The maximization of the expected revenue is non-trivial in at least two respects. One has to determine the value of the types for whom IR binds, x^* , and respect incentive constraints when assigning values for $q_0(x)$ and $q_1(x)$ based on functions A, B, C, D .

To solve the general problem of revenue maximization one can maximize (29) for any given $x^* \in [0, 1]$ under IC constraints (20) and IR constraints and then maximize over $x^* \in [0, 1]$. Solutions of (29) for different x^* will have many similarities as neither functions $A(x), B(x), C(x)$, and $D(x)$ nor constraints (20) depend on x^* . As we shall see, this will greatly simplify the problem in specific settings, as it will often be clear that the specific x^* cannot be part of the optimal mechanism. Note also that there can be multiple x^* for which IR binds. If this happens, the maximization over possible x^* becomes non-trivial. Once we perturb one specific x^* , we may need to perturb some of the others as well so as to keep IR satisfied. As we shall see that would be exactly the case when we consider the case of the convex costs. On the other hand, we can show that in the search of the optimal mechanism and the correct x^* it suffices to check optimality of x^* locally.

Lemma 6 *Let $\mu(x^*) \in M$ a mechanism that maximizes (26) subject to having x^* as the type for which IR constraint binds. If $ER(\mu(x^*))$ has a local maximum at x^* , then $\mu(x^*)$ is the global maximum and so the solution to the Informed Principal problem.*

Proof. Suppose not, that is the global maximum is different and gives strictly higher revenue. Let X^* be the set of types for which IR binds in the local maximum corresponding to x^* , and so $x^* \in X^*$, and let Z^* be the set of types for which IR binds in the global maximum. It must be the case that $Z^* \cap X^* = \emptyset$ as otherwise x^* is not a local maximum. Consider mechanism $\mu = (1 - \varepsilon)\mu(x^*) + \varepsilon\mu(z^*)$ for any $z^* \in Z^*$ and some $\varepsilon \in (0, 1)$. Clearly, $\mu \in M$ and $ER(\mu) = (1 - \varepsilon)ER(\mu(x^*)) + \varepsilon ER(\mu(z^*)) > ER(\mu(x^*))$. It may happen that the IR constraint in μ is not binding for all agent's types, in which case one can further improve its revenue. As ε goes to 0, by continuity, the set of types with the lowest utility from mechanism μ must have a limiting point in the set X^* , which means that the solution with x^* is not a local maximum. ■

To illustrate our approach, let us consider a simple optimization problem for the case of complete information. Equation (29) takes the following form

$$ER = -U(x^*) + \int_0^{x^*} q(x)A(x)dx + \int_{x^*}^1 q(x)B(x)dx. \quad (30)$$

Assume that we are in the regular case, that is $A(x)$ is strictly monotone (decreasing). Let \hat{x}_A solve $A(x) = 0$ or be equal to 1 if $A(1) > 0$. Consider a given x^* . As optimization over choice of $U(x^*)$ is separate from one over choice of $q(x)$, we set $U(x^*) = 0$, extracting all the surplus from type x^* , so his IR constraint binds. Suppose first $x^* \geq \hat{x}_A$. Then for all $x \leq \hat{x}_A$ we would like to set $q(x) = 1$, and for $x \in (\hat{x}_A, x^*]$, set $q(x) = 0$. By doing so, we do not violate IR constraint for all $x \leq x^*$, as $U(x) \geq U(x^*)$ (see (5)). For $x > x^*$, even if for some of them $B(x) > 0$, we cannot set $q(x) > 0$ as then IC constraint dictates we set $q(z) \geq q(x)$ for all $z \in (x^*, x)$ and we would violate the IR constraint for type x . Therefore, $q(x) = 0$ for all $x > x^*$. Similarly, if $x^* < \hat{x}_A$, we would like to set $q(x) = 1$ for all $x \leq x^*$ as $A(x) > 0$, but $q(x) = 0$ for $x > x^*$ as otherwise IR will be violated.

What is the optimal x^* . From (30) we see that the marginal effect from adjusting x^* depends on whether it is lower or higher than \hat{x}_A . Note that we cannot simply differentiate as $q(x)$ may change at x^* . If $x^* \geq \hat{x}_A$, $q(x) = 0$ before and after x^* , so the effect is 0. If $x^* < \hat{x}_A$, $q(x) = 1$ before x^* and 0, after, so the effect is $A(x)$. Since $A(x) > 0$, it is optimal to increase x^* . Therefore the first-best x^* can take any value from \hat{x}_A to 1.

Before we proceed with computing the optimal mechanism for specific shapes of cost functions, we can formulate two simple yet important general results.

If the base value V is sufficiently low, then it is optimal to fully disclose the information. If the base value V is sufficiently high, then it is optimal not to disclose any information. Formally,

Proposition 7 *There exist bounds $V^r > 0$ and $V^{nr} > 0$, such that if $V < V^r$, then the solution to the informed principal problem is mechanism μ^r , and if $V > V^{nr}$, then the*

solution is mechanism μ^{nr} . Mechanisms μ^r and μ^{nr} are defined as follows

$$\mu^r = \begin{cases} \begin{cases} Q_0(x) = 1, \text{ for } x \in [0, \hat{x}], \\ Q_0(x) = 0, \text{ for } x \in (\hat{x}, 1], \end{cases} & P_0(x) = V - c(\hat{x}); \\ \begin{cases} Q_1(x) = 0, \text{ for } x \in [0, 1 - \hat{x}], \\ Q_1(x) = 1, \text{ for } x \in [1 - \hat{x}, 1], \end{cases} & P_1(x) = V - c(\hat{x}). \end{cases}$$

$$\mu^{nr} = \begin{cases} Q_0(x) = 1, \text{ for } x \in [0, 1], \\ Q_1(x) = 1, \text{ for } x \in [0, 1], \end{cases} \quad P_0(x) = P_1(x) = V - y^*.$$

Here \hat{x} is the threshold type in the optimal mechanism for the principal of type $s = 0$ under complete information, and y^* is defined as in Section 4.

Proof. To be added. ■

Now we proceed with deriving optimal mechanisms for the special functional forms of the cost function: linear, convex and concave. To ease the notation, we would use $\vec{q}(x) = (q_0(x), q_1(x))$.

6.1 Linear costs

The linear costs case presents an ideal setting on which to demonstrate in detail how the optimal mechanism is derived and show that the optimal mechanism is non-trivial.

So, suppose the cost function is $c(x) = cx$. We have $A(x) = V - 2cx$, $B(x) = V + c - 2cx$, $C(x) = V + 2cx - c$, and $D(x) = V + 2cx - 2c$. Note that: $A(x)$ is decreasing and $A(x) = 0$ at $\hat{x}_A = \frac{V}{2c}$, while $C(x)$ is increasing and $C(x) = 0$ at $\hat{x}_C = \frac{c-V}{2c}$.

Assume for a moment that $x^* = \frac{1}{2}$ (a good guess from the previous analysis and having in mind that the optimal mechanism is likely to be symmetric). If $V \geq c$, then $\hat{x}_A \geq \frac{1}{2}$ and $\hat{x}_C \leq 0$, so that $A(x) \geq 0$, $C(x) \geq 0$ for all $x \in [0, \frac{1}{2}]$, and it is optimal to set $q_0(x) = q_1(x) = 1$ for all $x \in [0, \frac{1}{2}]$. Similarly, looking at $x > x^*$, by symmetry (as $D(1-x) = A(x)$ and $C(1-x) = B(x)$), it is also optimal to set $q_0(x) = q_1(x) = 1$ for all $x \in [\frac{1}{2}, 1]$. That is, no information is revealed and all types of the agent buy the good from each type of the principal. It is obvious that this solution satisfies all IC and IR constraints. The expected utility for all types of the agents in this mechanism is 0, so IR binds for all of them. Clearly, we cannot gain by choosing a different x^* given $q_0(x) = q_1(x) = 1$ for all x . What if the maximization starting from a different x^* gave us a different result? For $x^* \in [1 - \hat{x}_A, \hat{x}_A]$ the argument is exactly the same as before, so no improvement is possible. Consider $x^* > \hat{x}_A$. By looking at $A(x)$ and $C(x)$ we would like to set $q_0(x) = 0$ for $\hat{x}_A < x \leq x^*$. Once we do that, the IC constraint dictates that $q_0(x) = 0$ for all $x > x^*$. At the same time, by looking for $x > x^*$, since $B(x)$ is positive we would like to set $q_0(x) = 1$ for all $x > x^*$. Clearly, both cannot coexist, the combined effect from setting $q_0(x) = 1$ instead of $q_0(x) = 0$ is

$$A(x) + B(x) = 2V + c - 2cx \geq 2V + c - 2c = 2V - c > 0.$$

Therefore it is optimal to set $q_0(x) = q_1(x) = 1$ for all x . Similar argument applies if we consider $x^* < 1 - \hat{x}_A$.

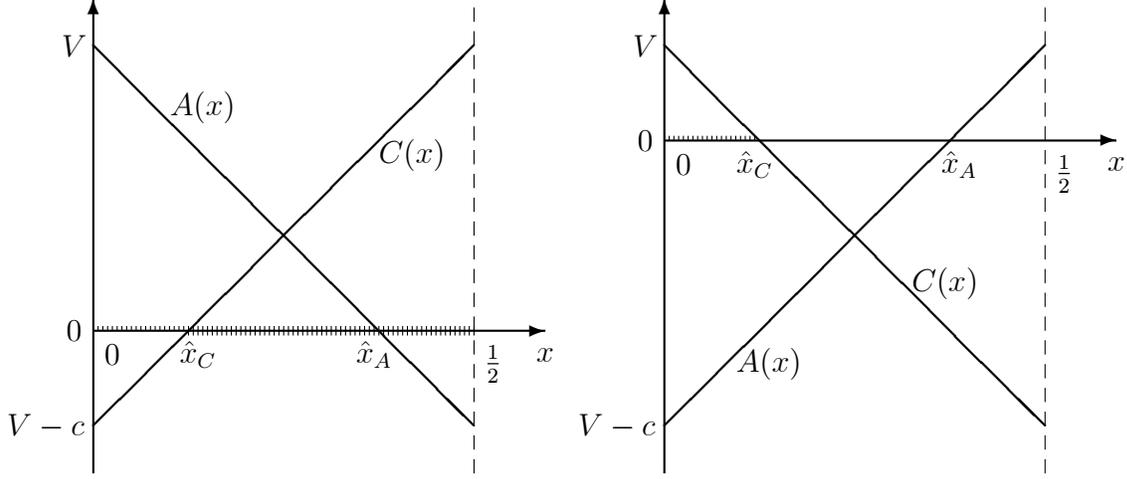


Figure 4: Linear costs optimal mechanism.

If $V < c$, two cases are possible: $\hat{x}_C \leq \hat{x}_A$ or $\hat{x}_C > \hat{x}_A$, see the left and the right side of Figure 4, respectively. For a moment, assume a symmetric solution. Symmetry together with IC constraints implies that for all $x < \frac{1}{2}$, $q_0(x) \geq q_1(x)$, and for $x > \frac{1}{2}$, $q_0(x) \leq q_1(x)$. That is, the probability of receiving the good from the nearest type of the principal should be higher.

Consider the case $\hat{x}_C \leq \hat{x}_A$ first. If we ignore IC constraints, we would like to set $\vec{q}(x) = (1, 0)$ for $x \leq \hat{x}_C$, $\vec{q}(x) = (1, 1)$ for $x \in (\hat{x}_C, \hat{x}_A]$, and $\vec{q}(x) = (0, 1)$ for $x \in (\hat{x}_A, \frac{1}{2}]$. But the latter violates $q_0(x) \geq q_1(x)$. Therefore, the extra constraint comes into play, namely $q_0(x) = q_1(x)$ for $x \in (\hat{x}_A, \frac{1}{2}]$. As $A(x) + C(x) > 0$, it is optimal to set $\vec{q}(x) = (1, 1)$ for $x \in (\hat{x}_A, \frac{1}{2}]$. Therefore, the optimal solution for $x \in [0, \frac{1}{2}]$ assuming $x^* = \frac{1}{2}$ and symmetry (and symmetrically for the other half) is

$$q_0(x) = 1, \text{ for } x \in \left[0, \frac{1}{2}\right]; \quad q_1(x) = \begin{cases} 0, & \text{for } x \in \left[0, \frac{c-V}{2c}\right] \\ 1, & \text{for } x \in \left[\frac{c-V}{2c}, \frac{1}{2}\right] \end{cases}; \quad (31)$$

$$p(x) = \begin{cases} \frac{3V}{4} - \frac{c}{4}, & \text{for } x \in \left[0, \frac{c-V}{2c}\right] \\ V - \frac{c}{2}, & \text{for } x \in \left[\frac{c-V}{2c}, \frac{1}{2}\right] \end{cases}.$$

Thus, the principal of type 0 sells to all agents to whom she is the nearest, while the principal of type 1 sells to all but the furthers from her types. Accordingly, the agent's types in the middle effectively by the *opaque* good, as they purchase it from both types of the principal, at a price that extracts their all surplus. The agents at extremes of the segment purchase only when the good is from preferred type of the seller and are left with the surplus. Effectively, they have an option to buy after they learn who is the principal. The marginal type \hat{x}_C is indifferent between purchasing the option or the opaque good. His surplus from the option is $\frac{1}{2}(V - c(\hat{x}_C))$, and this determines the price of the option.

Next, consider the case $\hat{x}_C > \hat{x}_A$. In this case, we would like to set $\vec{q}(x) = (1, 0)$ for $x \leq \hat{x}_A$, $\vec{q}(x) = (0, 0)$ for $x \in (\hat{x}_A, \hat{x}_C]$, and $\vec{q}(x) = (0, 1)$ for $x \in (\hat{x}_C, \frac{1}{2}]$. Again, the

latter violates $q_0(x) \geq q_1(x)$. Since $A(x) + C(x) < 0$, it is optimal to set $\vec{q}(x) = (1, 0)$ for $x \in (\hat{x}_A, \frac{1}{2}]$. Therefore, the optimal solution in this case is

$$\begin{aligned} q_0(x) &= \begin{cases} 1, & \text{for } x \in [0, \frac{V}{2c}] \\ 0, & \text{for } x \in (\frac{V}{2c}, \frac{1}{2}] \end{cases} ; & q_1(x) &= 0, \text{ for } x \in x \in \left[0, \frac{1}{2}\right], \\ p(x) &= \begin{cases} \frac{V}{4}, & \text{for } x \in [0, \frac{V}{2c}] \\ 0, & \text{for } x \in [\frac{V}{2c}, \frac{1}{2}] \end{cases} . \end{aligned}$$

This coincides with the solution in complete information case, and so, in this case, each type of the principal would like to disclose her information fully and sell only to her local market.

What remains to be shown is that the found solutions for these two cases are indeed optimal once we do not impose symmetry and allow for any x^* . [TO BE SHOWN, is not particularly hard but a bit tedious]

6.2 Linear costs: Example.

Let us compute a numerical example for the optimal selling mechanism for the case of linear costs, $c(x) = x$. If $V \leq \frac{1}{2}$, the optimal mechanism is to reveal the type of the principal, $p = \frac{V}{2}$, and the profits $\pi = \frac{V}{2} \times \frac{V}{2} = \frac{V^2}{4}$. In the case $V > \frac{1}{2}$, $x_1^* = \frac{1}{2} - \frac{V}{2}$, $p(\frac{1}{2}) = V - \frac{1}{2}$; $p_I = \frac{1-V}{4}$; and $p_G = 2V - 1$. Altogether, the profits are

$$\begin{aligned} \pi &= \frac{1-V}{2} p_G + 2 \frac{1-V}{2} p_I + V \left(V - \frac{1}{2} \right) \\ &= \frac{1-V}{2} (2V - 1) + 2 \frac{1-V}{2} \frac{1-V}{4} + V^2 - \frac{V}{2} = \frac{V^2}{4} + \frac{V}{2} - \frac{1}{4}. \end{aligned}$$

For instance, if $V = 0.8$, we have $x_1^* = 0.1$, $p_I = 0.05$, $p_G = 0.6$, the price of the good without information revelation is 0.3. Altogether the profits are 0.31, which is higher than 0.16 coming from full revelation, and 0.30 from no information revelation. If $V = 0.6$, then $x_1^* = 0.2$, $p_I = 0.1$, $p_G = 0.2$, no revelation price is 0.1. Total profits are 0.14 compared to 0.09 from full revelation and 0.1 from no revelation.

6.3 Concave and convex costs

[to be added]

7 Conclusion

We consider a seller who holds private information about some good's characteristics with respect of which the market is horizontally differentiated. We show that the seller can strategically use her private information to increase her profits. Depending on the shape of the buyers' utility function, the seller's optimal mechanism may entail disclosing, hiding, or selling information.

If the utility from consuming the good is high enough for everyone in the market independently from its specific characteristics, then the seller prefers not to reveal his private

information. We show that in many settings the seller maximizes her profits offering simultaneously an opaque good and an option. Purchasing the option, the buyer learns the seller's private information and acquires the right to buy the good at a predetermined exercise price. Selling information allows the seller to cover a wider market, appealing to customers whose willingness to pay for the good is highly sensitive to the content of the seller's private information. Indeed, some customers may be willing to pay for information alone in order to avoid to buy the good if they discover characteristics they dislike.

In different markets we observe practices that resemble the optimal mechanism that we characterized. In the market of financial services, for example, customers are offered both investment products and consulting services by the same provider. Customers may buy consulting services to better learn the characteristics of different investment products and make a final purchase from a more informed standpoint. In the market of education, customers can buy a limited number of introductory classes or enroll directly in a full-length course. In the case of food and beverages, retailers sell both small and big packages of several products. Through the offer of introductory classes and small packages goods, the sellers allow the customers to get a taste of the good before buying it (e.g. wine-tasting). In the travel market, tickets and vacation packages are offered with the option of buying an additional insurance product. Buying the insurance, the customer retains the right to cancel the travel and receive a refund. More recently, technology companies have started to offer "buy-back programs": paying a fee, customers purchase the right to return their product over time and get a pro-rated refund.

One of the common denominators of these business practices is that they all entail the opportunity for the customers of learning their valuation for the good before buying it (or buying only a sample of it, or retaining the right of asking for a refund). Indeed, several characteristics of the good may not be known to the potential buyers. However, customers' valuations may be differently sensitive to this extra information. Some customers may not be affected and prefer to buy directly the final good, other customers may choose to defer such purchase until after they have acquired more information. In that way, if they discover of not liking the good, they can limit their disutility.

We offer a rationale for these kinds of mechanisms to appear based on the analysis of the Informed Principal Problem. The choice by the seller of the mechanism may reveal her private information. We consider mechanisms in which the degree of information disclosure to different customers may be different. In horizontally differentiated markets, the optimal mechanism chosen by an informed seller may entail price-discrimination across customers based on their valuation for the seller's private information about the good's characteristics.

A Numerical Examples

We present here some numerical examples to show how the sale of the opaque good and the option maximizes the profit of the seller. We compare three different disclosure strategies: full information disclosure, no information revelation, and selective information revelation at a price (i.e. the option). We assume $V = 0.8$ and we consider three different cost functions: $c(x) = x$, $c(x) = x^2$, and $c(x) = \sqrt{x}$.

A.1 Full information disclosure

When the cost function is linear, each type of the principal maximizes the same profit function $\pi = (0.8 - x)x$. The maximum profit $\pi_L = 0.16$ is obtained at price $p = 0.4$.

When the cost function is concave, the profit $\pi = (0.8 - \sqrt{x})x$ is maximized at $\pi_{CV} \approx 0.076$ at price $p \approx 0.27$.

When the cost function is convex, the profit $\pi = (0.8 - x^2)x$ is maximized at $\pi_{CX} \approx 0.28$ at price $p \approx 0.53$.

A.2 No information disclosure

When the cost function is linear, the principal maximizes her profits selling the good to all the market at price $p = 0.3$. The resulting profit is $\pi_L = 0.3$.

When the cost function is concave, $p = \pi_{CV} \approx 0.093$. The price is determined by the individual rationality constraint of the buyer located in the middle of the Hotelling line.

When the cost function is convex, $p = \pi_{CX} = 0.3$. The price is determined by the individual rationality constraint of the buyers located at the extremes of the Hotelling line.

A.3 Option

The principal sells the good without providing info about her type at price l (opaque good); and she sells an option. The option costs k to the buyer and entitles him to buy the good at exercise price p after learning the information about the principal's location. Notice that, in principle, arbitrage opportunities arise if the prices l , k , and p are not carefully chosen or if the environment is not enriched with some additional enforceable rules to prevent these arbitrage opportunities to be exploited. We are going to require $p < l$, so that the agent will not find convenient to learn information at price k , not exercise the option, and go back to the market and buy the opaque good.

Linear costs. We derive first the price of the opaque good from the individual rationality constraints of the buyer's types.

$$\frac{1}{2}(0.8 - x) + \frac{1}{2}(0.8 - (1 - x)) = 0.3 = l$$

To find prices k and p of the option, we consider the incentive compatibility constraint of a threshold type x^* that is indifferent between buying the opaque good and the option.

$$\frac{1}{2} \max\{0.8 - x^* - p, 0\} + \frac{1}{2} \max\{(0.8 - (1 - x^*) - p), 0\} - k = 0$$

We determine $x^* \leq \frac{1}{2}$ from the condition $\frac{1}{2}(0.8 - x - p) - k = 0$. Ergo $x^* = 0.8 - p - 2k$. Given k, p , the expected profits to the informed seller are

$$\pi(k, p) = (1 - 2x)l + x^*(k + p) + x^*k.$$

Solving for optimal k and p , we obtain that there are multiple possible solutions expressed in the following relation

$$k + \frac{1}{2}p = 0.35. \quad (32)$$

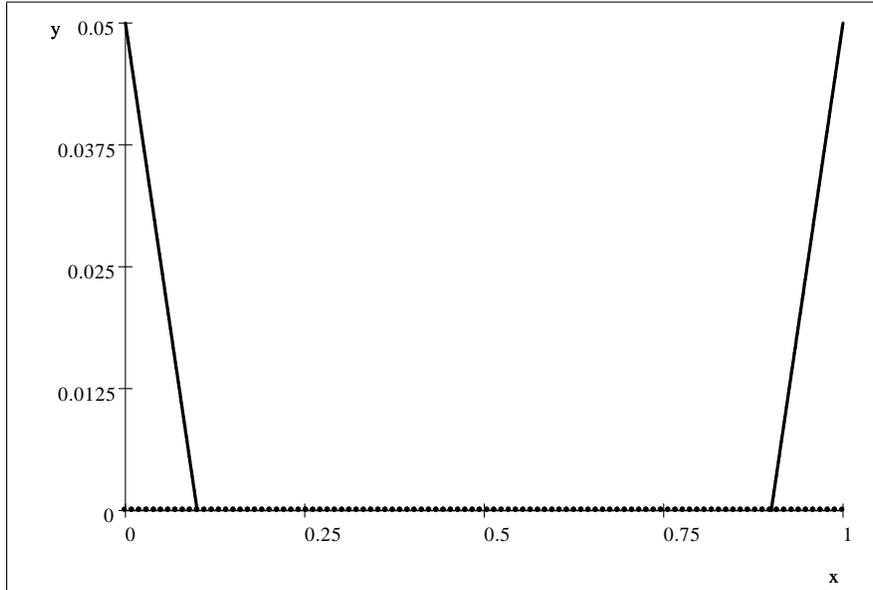
and the threshold type is

$$x^* = 0.8 - p - 2 \left(0.35 - \frac{1}{2}p \right) = 0.1$$

Adding the no arbitrage constraint $p < l$, we have that the set of prices is, in addition to (32) must satisfy $p \leq 0.3$. For example, a candidate solution is $l = p = 0.3$, $k = 0.2$.

For any optimal combination of k and p , the total profit is $\pi_L^* = 0.31$, and this is higher than with full or no disclosure.

In the following graph we represent the agent's expected surplus distribution across his types from buying the option (solid line) and the opaque good (dots). The types that are located closer to the extremes of the Hotelling line buy the options and have positive expected surplus, the types located closer to the mid-point of the segment buy the opaque good and have zero expected surplus. All the agents who buy the opaque good are left with no surplus.



Concave costs. The cost structure is such that the buyer's type located in the middle of the Hotelling line is the one with the lowest utility for any information. It is not hard to show that it is optimal to sell (something) to the whole market. Accordingly, the price of the opaque good is determined from the IR constraint of $x = \frac{1}{2}$.

$$\frac{1}{2} \left(0.8 - \sqrt{\frac{1}{2}} \right) + \frac{1}{2} \left(0.8 - \sqrt{\left(1 - \frac{1}{2} \right)} \right) = l,$$

$$l = 0.8 - \frac{1}{2}\sqrt{2} \approx 0.093.$$

From the incentive compatibility of the threshold type x^* who is indifferent between buying the opaque good and the option and the FOC of the profit maximization problem of the informed seller, we derive the relation between the price and the exercise price of the option.

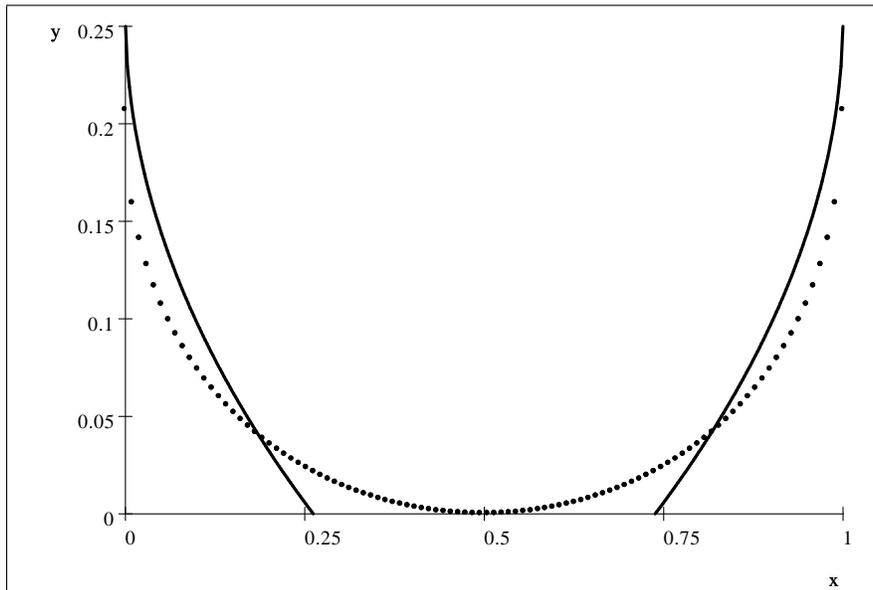
$$\begin{aligned} & \frac{1}{2} \max \left\{ 0.8 - \sqrt{x^*} - p, 0 \right\} + \frac{1}{2} \max \left\{ (0.8 - \sqrt{1 - x^*} - p), 0 \right\} - k \\ &= \frac{1}{2} \left(0.8 - \sqrt{x^*} \right) + \frac{1}{2} \left(0.8 - \sqrt{(1 - x^*)} \right) - l. \end{aligned}$$

Given k and p , we have $x^* = - \left(2k + p + \sqrt{2} - \frac{4}{5} \right)^2 + 1$.

Again, there is a whole family of possible optimal k and p , $k + \frac{1}{2}p \approx 0.14$, and $\pi_{CV}^* \approx 0.11$. Because of the no arbitrage condition, we also require that $p < l = 0.093$. The profit π_{CV}^* is higher than with full or no disclosure, $x^* = 0.18528$.

In the following graph, we represent the expected surplus distribution across the agent's types. The dotted line represents the distribution of expected surplus across the types who buy the opaque good, the solid line represents the distribution of expected surplus across the types who buy the option. Notice that types that are closer to the endpoints of the Hotelling line have higher expected surplus from buying the option instead of the opaque good. The opposite is true for the types that are closer to the middle of the segment.

Compared to the linear transportation costs case, only one type does not gain any surplus. In the concave case, there are types who buy the opaque good and have a positive expected surplus. Moreover, the individual rationality constraint of the type who is indifferent between the option and the opaque good does not bind.



A.3.1 Convex costs

In the case of convex costs the seller offers a different set of type contingent options. The agent can choose to learn for free the type of the good and pay p to buy it, if he like it. For

a price $p(q, s)$, the agent can buy a lottery (q, s) such that, if the good is s , it gets delivered with probability q , if the good is $t \neq s$, it gets delivered for sure.

Given the symmetry between good $s = 0$ and $s = 1$, we drop s from the price function and we index it only by q . Notice that, if $q = 1$, then the option is equivalent to the opaque good.

Notice that the most convenient move by the agent would be to learn the information for free and then, if he likes the good, buy the good at the cheapest price. For example, if $s = 0$, the cheapest way to buy it would be through the lottery $(0.11803, s = 1)$ that costs $p(0.11803) = 0.39443$. In this section we assume that the agent who learns the good's type is not allowed to buy it in any other way than paying its price p .

Through this set of contracts the seller extracts all the expected surplus from a set of agents located in $x \in [\hat{x}, \tilde{x}]$. For these agents' both IR and IC bind.

They buy option with $q = \frac{x}{1-x}$ at a price

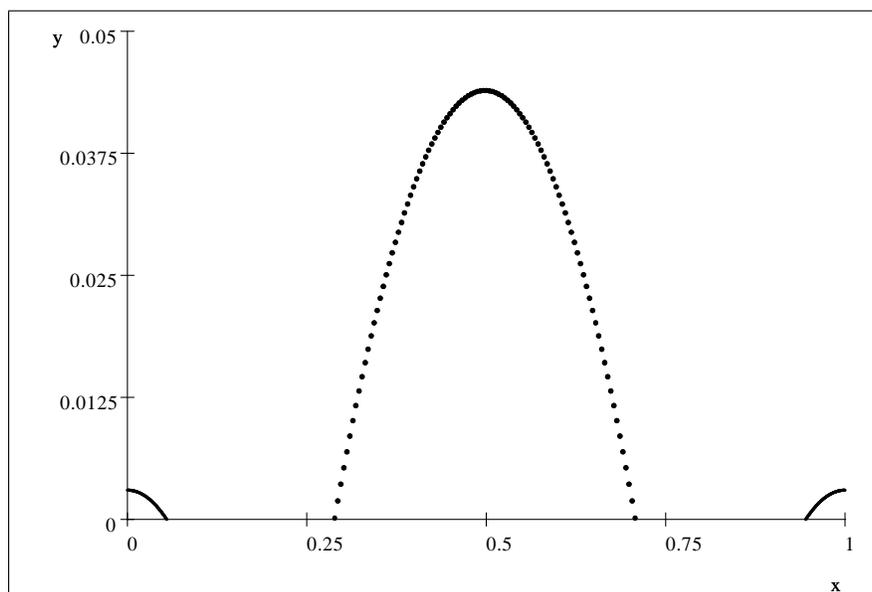
$$p(q) = \frac{1}{2} \left(0.8 - \left(\frac{q}{q+1} \right)^2 \right) + \frac{1}{2} q \left(0.8 - \left(1 - \frac{q}{q+1} \right)^2 \right).$$

The agents located in $x \in [0, \hat{x}] \wedge [1 - \hat{x}, 1]$ where $\hat{x} \approx 0.052$, buy the good with full information at price $p(0) \approx 0.797$.

The agent located in $x \in [\tilde{x}, 1 - \tilde{x}]$, where $\tilde{x} \approx 0.29$, buys the opaque good at price $p(1) \approx 0.506$.

The overall profit is $\pi_{CX}^* \approx 0.445$, that is higher than with full or no disclosure.

The graph represents the distribution of the expected surplus across the agent's types. The solid line represents the surplus of the types who buy the good after learning its type. The dotted line represents the expected surplus of the types who buy the opaque good. The types who buy the lotteries (q, s) are left with no expected surplus. Notice how, differently from any other scenario, in the convex case the types who are more indifferent between the two types of good are the ones who have the highest expected surplus.



References

- Anderson, S. P. & Renault, R. (2006), ‘Advertising content’, *American Economic Review* **96**(1), 93–113.
- Balestrieri, F. (2008), ‘A modified english auction for an informed buyer’, MIT, mimeo.
- Balestrieri, F. & João Leão (2008), ‘The market for surprises: Selling substitute goods through lotteries’, MIT, mimeo.
- Bergemann, D. & Pesendorfer, M. (2007), ‘Information structures in optimal auctions’, *Journal of Economic Theory* **137**(1), 580–609.
- Board, S. (2009), ‘Revealing information in auctions: the allocation effect’, *Economic Theory* **38**(1), 125–135.
- Celik, L. (2010), ‘Information unraveling revisited: Disclosure of horizontal attributes’, CERGE-EI, mimeo.
- Courty, P. & Li, H. (2000), ‘Sequential screening’, *Review of Economic Studies* **67**(4), 697–717.
- Cremer, J. & Khalil, F. (1992), ‘Gathering information before signing a contract’, *American Economic Review* **82**(3), 566–78.
- Cremer, J. & Khalil, F. (1994), ‘Gathering information before the contract is offered: The case with two states of nature’, *European Economic Review* **38**(3-4), 675–682.
- Cremer, J., Khalil, F. & Rochet, J.-C. (1998), ‘Strategic information gathering before a contract is offered’, *Journal of Economic Theory* **81**(1), 163–200.
- Dana, J. D. (1998), ‘Advance-purchase discounts and price discrimination in competitive markets’, *Journal of Political Economy* **106**(2), 395–422.
- Dana, J. D. (1999a), ‘Equilibrium price dispersion under demand uncertainty: The roles of costly capacity and market structure’, *RAND Journal of Economics* **30**(4), 632–660.
- Dana, J. D. (1999b), ‘Using yield management to shift demand when the peak time is unknown’, *RAND Journal of Economics* **30**(3), 456–474.
- Dana, J. D. (2001), ‘Monopoly price dispersion under demand uncertainty’, *International Economic Review* **42**(3), 649–70.
- Esó, P. & Szentes, B. (2007), ‘Optimal information disclosure in auctions and the handicap auction’, *Review of Economic Studies* **74**(3), 705–731.
- Gale, I. L. & Holmes, T. J. (1992), ‘The efficiency of advance-purchase discounts in the presence of aggregate demand uncertainty’, *International Journal of Industrial Organization* **10**(3), 413–437.

- Gale, I. L. & Holmes, T. J. (1993), ‘Advance-purchase discounts and monopoly allocation of capacity’, *American Economic Review* **83**(1), 135–46.
- Ganuja, J. J. & Penalva, J. (2010), ‘Signal orderings based on dispersion and the supply of private information in auctions’, *Econometrica* **78**(3), 1007–1030.
- Hoffmann, F. & Inderst, R. (2009), ‘Price discrimination and the provision of information’, University of Frankfurt, mimeo.
- Hotelling, H. (1929), ‘Stability in competition’, *Economic Journal* **39**(153), 41–57.
- Johnson, J. P. & Myatt, D. P. (2006), ‘On the simple economics of advertising, marketing, and product design’, *American Economic Review* **96**(3), 756–784.
- Koessler, F. & Renault, R. (2011), ‘When does a firm disclose product information?’, Paris School of Economics, mimeo.
- Krishna, V. (2002), *Auction Theory*, Academic Press, San Diego, California.
- Lewis, T. R. & Sappington, D. (1994), ‘Supplying information to facilitate price discrimination’, *International Economic Review* **35**(2), 309–27.
- Li, D. Z. (2010), ‘Cheap-talk information disclosure in auctions’, University of Oxford, mimeo.
- Maskin, E. & Tirole, J. (1990), ‘The principal-agent relationship with an informed principal: The case of private values’, *Econometrica* **58**(2), 379–409.
- Maskin, E. & Tirole, J. (1992), ‘The principal-agent relationship with an informed principal, ii: Common values’, *Econometrica* **60**(1), 1–42.
- McAfee, R. P. & McMillan, J. (1988), ‘Multidimensional incentive compatibility and mechanism design’, *Journal of Economic Theory* **46**(2), 335–354.
- Milgrom, P. R. & Weber, R. J. (1982), ‘A theory of auctions and competitive bidding’, *Econometrica* **50**(5), 1089–1122.
- Myerson, R. B. (1981), ‘Optimal auction design’, *Mathematics of Operations Research* **6**(1), 58–73.
- Myerson, R. B. (1983), ‘Mechanism design by an informed principal’, *Econometrica* **51**(6), 1767–97.
- Nocke, V., Peitz, M. & Rosar, F. (2011), ‘Advance-purchase discounts as a price discrimination device’, *Journal of Economic Theory* **146**(1), 141–162.
- Pavlov, G. (2006), Optimal mechanism for selling substitutes, Boston University - Department of Economics - Working Papers Series WP2006-014, Boston University - Department of Economics.

- Shi, X. (2007), Optimal auctions with information acquisition, Working Papers tecipa-302, University of Toronto, Department of Economics.
- Skreta, V. (2007), On the informed seller problem: Optimal information disclosure, Levine's Bibliography 84364400000000222, UCLA Department of Economics.
- Sun, M. J. (2011), 'Disclosing multiple product attributes', *Journal of Economics and Management Strategy* **20**(1).
- Thanassoulis, J. (2004), 'Haggling over substitutes', *Journal of Economic Theory* **117**(2), 217–245.
- Tröger, T. & Mylovanov, T. (2008), Optimal auction design and irrelevance of private information, Bonn Econ Discussion Papers bgse21_2008, University of Bonn, Germany.
- Yilankaya, O. (1999), 'A note on the seller's optimal mechanism in bilateral trade with two-sided incomplete information', *Journal of Economic Theory* **87**(1), 267–271.
- Zhang, J. (2008), 'Auctions with refund policies as optimal selling mechanisms', Queen's University, mimeo.